EXAM NO:

STD: XI	MATHEMATICS	MARKS: 90
	GOVT. PUBLIC EXAM (MAY 2022)	TIME: 3.00 hr

PART – A

Choose the correct answer **20 x 1 = 20**

		TYPE A			TYPE B
1.	(d)	discontinuous	**1.**	(c)	$\frac{2}{3}$
2.	(d)	A + B is symmetric	**2.**	(d)	$1-2^{-n}$
3.	(b)	e^4	**3.**	(a)	$2x^2e^{\frac{x}{2}}-8xe^{\frac{x}{2}}+16e^{\frac{x}{2}}+c$
4.	(d)	18	**4.**	(c)	N
5.	(c)	10	**5.**	(b)	e^4
6.	(c)	N	**6.**	(d)	∞
7.	(c)	does not exist	**7.**	(d)	discontinuous
8.	(d)	∞	**8.**	(d)	5^5
9.	(c)	$\frac{2x^3}{3}-x^2+x+c$	**9.**	(b)	$-2\hat{i}-\hat{j}+9\hat{k}$
10.	(a)	0.56	**10.**	(a)	$\sec\theta$
11.	(a)	$\sec\theta$	**11.**	(d)	8
12.	(b)	$-2\hat{i}-\hat{j}+9\hat{k}$	**12.**	(c)	8
13.	(a)	$2x^2e^{\frac{x}{2}}-8xe^{\frac{x}{2}}+16e^{\frac{x}{2}}+c$	**13.**	(c)	does not exist
14.	(d)	5^5	**14.**	(d)	$\frac{1}{2},-2$
15.	(c)	$\frac{2}{3}$	**15.**	(a)	0.56
16.	(d)	$\frac{1}{2},-2$	**16.**	(c)	10
17.	(d)	$\vec{b}-\vec{a}$	**17.**	(d)	A + B is symmetric
18.	(d)	8	**18.**	(d)	$\vec{b}-\vec{a}$
19.	(c)	8	**19.**	(d)	18
20.	(d)	$1-2^{-n}$	**20.**	(c)	$\frac{2x^3}{3}-x^2+x+c$

PART - B

Answer any SEVEN questions

Question number 30 is compulsory **7 x 2 = 14**

21. Solve $|2x-17|=3$ for x.

Solution:

$$|2x-17|=3 \qquad \because |x|=r \implies x=\pm r$$

$$2x-17=\pm 3$$

$2x-17=-3$	$2x-17=3$
$2x=-3+17$	$2x=3+17$
$2x=14$	$2x=20$
$x=7$	$x=10$

22. Express $\sin 50° + \sin 20°$ as a product.

Solution:

$$\text{We know that } \sin C+\sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

Take $C=50°$ and $D=20°$. We have

$$\sin 50°+\sin 20° = 2\sin\frac{50°+20°}{2}\cos\frac{50°-20°}{2}$$

$$\sin 50°+\sin 20° = 2\sin 35°\cos 15°$$

23. Find the value of $\cos 135°$.

Solution:

$$\cos 135° = \cos(90°+45°) = -\sin(45°) = -\frac{1}{\sqrt{2}}$$

$$\text{(or) } \cos 135° = \cos(180°-45°) = -\cos(45°) = -\frac{1}{\sqrt{2}}$$

24. Find the total number of outcomes when 5 coins are tossed once.

Solution:

When a coin is tossed, the outcomes are in two ways which are {Head, Tail}.

By the rule of product rule,

The number of outcomes when 5 coins are tossed is $2\times2\times2\times2\times2=2^5=32$.

25. Write the first 4 terms of the sequences whose n^{th} term is given as

$$a_n=\begin{cases} n & \text{if } n \text{ is } 1,2 \text{ or } 3 \\ a_{n-1}+a_{n-2}+a_{n-3} & \text{if } n>3 \end{cases}$$

Solution:

$$a_1=1\ ;\ a_2=2\ ;\ a_3=3$$

$$a_4=a_3+a_2+a_1=3+2+1=6$$

The first 4 terms of the sequences are 1, 2, 3 and 6

26. Find the equation of the lines passing through the point $(1,1)$ and $(-2,\ 3)$.

Solution:

$$\text{Here } (x_1,y_1)=(1,1) \text{ and } (x_2,y_2)=(-2,3)$$

$$\text{Equation of the line is } \frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \implies \frac{y-1}{3-1}=\frac{x-1}{-2-1}$$

$$-3y+3=2x-2 \implies 2x+3y-5=0$$

27. Find $|A|$ if $A = \begin{bmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$.

Solution:

$$|A| = \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$$

$$= 0 - \sin\alpha\,(0 - \cos\alpha\sin\beta) + \cos\alpha\,(-\sin\alpha\sin\beta - 0) = 0$$

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28. Find a unit vector along the direction of the vector $5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

Solution:

$$\text{Here } \vec{a} = 5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$

$$\text{unit vector} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{|5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}|} = \frac{5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{5^2 + 3^2 + 4^2}} = \frac{5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{50}}$$

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29. Differentiate $y = x^3 + 5x^2 + 3x + 7$ with respect to x.

Solution:

$$y = x^3 + 5x^2 + 3x + 7$$

$$\frac{dy}{dx} = 3x^2 + 10x + 3$$

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30. Integrate $(x - 11)^7$ with respect to x.

Solution:

$$\int (x - 11)^7\,dx = \frac{(x - 11)^8}{8} + c$$

PART – C

Answer any SEVEN questions

Question number 40 is compulsory **7 x 3 = 21**

31. Find the number of subsets of A if $A = \{x : x = 4n + 1,\ 2 \le n \le 5, n \in N\}$.

Solution:

$$A = \{x : x = 4n + 1, 2 \le n \le 5, n \in N\}$$

$$A = \{9, 13, 17, 21\}$$

$$n(A) = 4$$

$$\text{Number of subsets} = 2^{n(A)} = 2^4 = 16$$

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32. Resolve into partial fractions : $\frac{x}{(x+3)(x-4)}$.

Solution:

$$\text{Let } \frac{x}{(x+3)(x-4)} = \frac{A}{(x+3)} + \frac{B}{(x-4)} \rightarrow ①$$

$$x = A(x-4) + B(x+3)$$

Put $x = -3$

$$-3 = A(-3-4) + 0$$

$$-3 = A(-7) \Rightarrow A = \frac{3}{7}$$

Put $x = 4$

$$4 = 0 + B(4+3)$$

$$4 = B(7) \Rightarrow B = \frac{4}{7}$$

Sub A, B values in ①

$$\frac{x}{(x+3)(x-4)} = \frac{\frac{3}{7}}{(x+3)} + \frac{\frac{4}{7}}{(x-4)}$$

$$\therefore \frac{x}{(x+3)(x-4)} = \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$$

33. Find the distinct permutations of the letters of the word ACCESSIBILITY.

Solution:

ACCESSIBILITY

Total number of letters $= 13$

Number of C's $= 2$

Number of S's $= 2$

Number of I's $= 3$

$$\text{Required number of arrangements} = \frac{13!}{2!\,2!\,3!} = 259459200$$

34. Expand $(x+2)^{-\frac{2}{3}}$ in powers of x.

Solution:

$$(x+2)^{-\frac{2}{3}} = (2+x)^{-\frac{2}{3}} = 2^{-\frac{2}{3}}\left(1+\frac{x}{2}\right)^{-\frac{2}{3}} = 2^{-\frac{2}{3}}(1+y)^{-\frac{2}{3}} \quad \text{Let } y = \frac{x}{2}$$

$$= 2^{-\frac{2}{3}}\left[1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \cdots \ldots \ldots\right] \text{if } |y| < 1$$

$$= 2^{-\frac{2}{3}}\left[1 + \left(-\frac{2}{3}\right)\left(\frac{x}{2}\right) + \frac{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}-1\right)}{2!}\left(\frac{x}{2}\right)^2 + \frac{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}-1\right)\left(-\frac{2}{3}-2\right)}{3!}\left(\frac{x}{2}\right)^3 + \cdots \ldots \ldots\right]$$

$$= 2^{-\frac{2}{3}}\left[1 - \frac{x}{3} + \frac{5}{9}\left(\frac{x^2}{4}\right) - \frac{2}{3}\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)\left(\frac{1}{6}\right)\left(\frac{x^3}{8}\right) + \cdots \ldots \ldots\right]$$

$$(x+2)^{-\frac{2}{3}} = 2^{-\frac{2}{3}}\left[1 - \frac{x}{3} + \frac{5}{36}x^2 - \frac{5}{81} - x^3 + \cdots \cdots\right] \text{if } |x| < 2$$

35. Find the distance from a point $(1, 2)$ to the line $5x + 12y - 3 = 0$.

Solution:

The distance between the point (x_1, y_1) and the line $ax + by + c = 0$ is

$$D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$\therefore$ The distance between the point $(1, 2)$ and the line $5x + 12y - 3 = 0$ is

$$D = \left|\frac{5(1) + 12(2) - 3}{\sqrt{5^2 + 12^2}}\right| = 2$$

36. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.

Solution:

$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{vmatrix} \quad \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$$

$$= (y-x)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$$

$$= (y-x)(z-x)[(z+x)-(y+x)]$$

$$= (y-x)(z-x)(z-y)$$

$$= (x-y)(y-z)(z-x) = \text{RHS}$$

37. Find the angle between the vectors $5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $6\hat{\imath} - 8\hat{\jmath} - \hat{k}$.

Solution:

Let $\vec{a} = 5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 6\hat{\imath} - 8\hat{\jmath} - \hat{k}$

Let θ be the agle between them

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{30-24-4}{\sqrt{50}\sqrt{101}} = \frac{\sqrt{2}}{5\sqrt{101}} \Rightarrow \theta = \cos^{-1}\left[\frac{\sqrt{2}}{5\sqrt{101}}\right]$$

38. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 1$.

Solution:

$$x^2 + y^2 = 1$$

We differentiate both sides of the equation,

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

39. If $f'(x) = 4x - 5$ and $f(2) = 1$, find $f(x)$.

Solution:

$$f'(x) = 4x - 5$$

$$\int f'(x)\,dx = \int (4x-5)\,dx$$

$$f(x) = 4\frac{x^2}{2} - 5x + c = 2x^2 - 5x + c$$

$$f(2) = 1$$

$$2(2^2) - 5(2) + c = 1$$

$$c = 3$$

$$\therefore f(x) = 2x^2 - 5x + 3$$

40. A die is rolled. If it shows an even number, then find the probability of getting 6.

Solution:

Sample space S = {1,2,3,4,5,6}

Let A be the event of die shows an even number

Let B be the event of getting 6

Then, $A = \{2,4,6\}, B = \{6\}, \text{and } A \cap B = \{6\}$

$$\therefore P(A) = \frac{3}{6} \text{ and } P(A \cap B) = \frac{1}{6}$$

$$P(\text{getting } 5/ \text{ die shows an odd number}) = P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{3}{6}}$$

$$P(B/A) = \frac{1}{3}$$

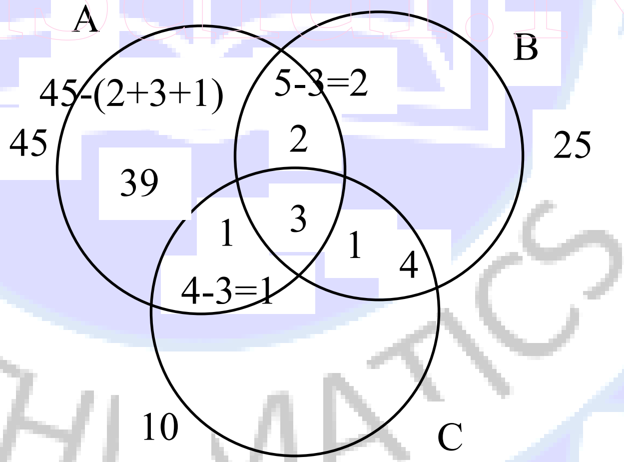
PART – D

Answer ALL questions **7 x 5 = 35**

41. (a) In a survey of 5000 persons in a town, it was found that 45% of the persons know Language A, 25% know Language B, 10% know Language C, 5% know Languages A and B, 4% know Languages B and C, and 4% know Languages A and C. If 3% of the persons know all the three Languages, find the number of persons who knows only Language A.

Solution:

We draw the Venn diagram using percentage



From the diagram

The percentage of persons who knows only Language A is 39.

The required number of persons $= 39\% \times 5000$

$$= \frac{39}{100} \times 5000 = 1950$$

OR

(b) Evaluate : $\int \frac{2x+4}{x^2+4x+6} dx$.

Solution:

$$\text{Let } I = \int \frac{2x+4}{x^2+4x+6} dx$$

$$\text{Put } x^2+4x+6=u, \text{ then } (2x+4)dx=du$$

$$\text{Thus, } I=\int\frac{du}{u}=\log|u|+c=\log|x^2+4x+6|+c$$

$$\therefore \int\frac{2x+4}{x^2+4x+6}dx=\log|x^2+4x+6|+c$$

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42. (a) Prove that $\frac{\cot(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\tan(-\theta)\operatorname{cosec}(360°+\theta)}=\cos^2\theta\cot\theta.$

Solution:

$$\text{L.H.S}=\frac{\cot(180°+\theta)\sin(90°-\theta)\cos(-\theta)}{\sin(270°+\theta)\tan(-\theta)\operatorname{cosec}(360°+\theta)}$$

$$=\frac{(\cot\theta)(\cos\theta)(\cos\theta)}{(-\cos\theta)(-\tan\theta)(\operatorname{cosec}\theta)}$$

$$=\frac{\cot\theta\cos^2\theta}{(\cos\theta)\left(\frac{\sin\theta}{\tan\theta}\right)\left(\frac{1}{\sin\theta}\right)}$$

$$=\cos^2\theta\cot\theta=\text{R.H.S}$$

OR

(b) Evaluate : $\int x\cos x\ dx.$

Solution:

$$\text{Let } I=\int x\cos x\,dx$$

$$u=x\ ;\ dv=\cos x\,dx$$

$$du=dx\ ;\ v=\sin x$$

$$\int u\,dv=uv-\int v\,du$$

$$\int x\cos x\,dx=x\sin x-\int\sin x\,dx$$

$$\int x\cos x\,dx=x\sin x+\cos x+c$$

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43. (a) By the principle of mathematical induction, prove that, for $n\geq 1$

$1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2.$

Solution:

$$P(n):=1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

$$P(1):=1^3=\left(\frac{(1)(1+1)}{2}\right)^2$$

$$1=1$$

$$\therefore P(1) \text{ is true}$$

Let us assume the statement is true for $n=k$

$$\text{i.e., } P(k):=1^3+2^3+3^3+\cdots..+k^3=\left(\frac{k(k+1)}{2}\right)^2$$

$$P(k+1)=1^3+2^3+3^3+\cdots...+k^3+(k+1)^3$$

$$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$
$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$
$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$
$$= \frac{(k+1)^2[k^2 + 4(k+1)]}{4}$$
$$= \frac{(k+1)^2[k^2 + 4k + 4]}{4}$$
$$= \frac{(k+1)^2(k+2)^2}{4}$$

P(k + 1) is true

∴ P(k + 1) is true whenever P(k) is true

Hence, By the principle of mathematical induction, prove that, for $n \geq 1$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

OR

(b) Find $\frac{dy}{dx}$ if $x = a(t - \sin t),\ y = a(1 - \cos t)$.

Solution:

$$x = a(t - \sin t)\ ;\ y = a(1 - \cos t)$$
$$\frac{dx}{dt} = a(1 - \cos t)\ ;\ \frac{dy}{dt} = a \sin t$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a \sin t}{a(1 - \cos t)} = \frac{\sin t}{(1 - \cos t)}$$

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44. (a) For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight lines.

Solution:

$$12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$$

Here $a = 12\ ;\ 2h = 2k\ ;\ b = 2\ ;\ 2g = 11\ ;\ 2f = -5\ ;\ c = 2$

$$h = k\ ;\ g = \frac{11}{2}\ ;\ f = -\frac{5}{2}$$

condition for pair straight line is $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

$$\begin{vmatrix} 12 & k & \frac{11}{2} \\ k & 2 & -\frac{5}{2} \\ \frac{11}{2} & -\frac{5}{2} & 2 \end{vmatrix} = 0$$

$$12\left(4-\frac{25}{4}\right)-k\left(2k+\frac{55}{4}\right)+\frac{11}{2}\left(-\frac{5k}{2}-11\right)=0$$
$$12(16-25)-k(8k+55)+11(-5k-22)=0$$
$$-108-8k^2-55k-55k-242=0$$
$$-8k^2-110k-350=0$$
$$4k^2+55k+175=0$$
$$\left(k+\frac{20}{4}\right)\left(k+\frac{35}{4}\right)=0$$
$$(k+5)\left(k+\frac{35}{4}\right)=0$$
$$k=-5\ ;\ k=-\frac{35}{4}$$

OR

(b) Evaluate: $\lim_{x\to 0}\frac{\tan 2x}{\sin 5x}$.

Solution:

$$\lim_{x\to 0}\frac{\tan 2x}{\sin 5x}=\lim_{x\to 0}\frac{2x\frac{\tan 2x}{2x}}{5x\frac{\sin 5x}{5x}}=\frac{2}{5}\frac{\lim\limits_{\substack{x\to 0\\2x\to 0}}\frac{\tan 2x}{2x}}{\lim\limits_{\substack{x\to 0\\5x\to 0}}\frac{\sin 5x}{5x}}=\frac{2\,(1)}{5\,(1)}=\frac{2}{5}$$

45. (a) Show that $\begin{vmatrix}2bc-a^2 & c^2 & b^2\\ c^2 & 2ca-b^2 & a^2\\ b^2 & a^2 & 2ab-c^2\end{vmatrix}=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}^2$.

Solution:

$$\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}^2=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}\times\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}$$
$$=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}\times(-1)\begin{vmatrix}a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}=\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}\times\begin{vmatrix}-a & -b & -c\\ c & a & b\\ b & c & a\end{vmatrix}$$
$$=\begin{vmatrix}-a^2+bc+cb & -ab+ab+c^2 & -ac+b^2+ac\\ -ab+c^2+ab & -b^2+ac+ac & -bc+bc+a^2\\ -ac+ac+b^2 & -bc+a^2+bc & -c^2+ab+ab\end{vmatrix}$$
$$=\begin{vmatrix}2bc-a^2 & c^2 & b^2\\ c^2 & 2ca-b^2 & a^2\\ b^2 & a^2 & 2ab-c^2\end{vmatrix}$$

OR

(b) Prove $\log\frac{75}{16}-\ 2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$.

Solution:

$$\text{LHS}=\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}$$
$$=\log 75-\log 16-2\log 5+2\log 9+\log 32-\log 243$$
$$=\log 3+\log 25-\log 16-\log 25+\log 81+\log 16+\log 2-\log 81-\log 3$$
$$=\log 2=\text{RHS}$$

46. (a) Prove that the points whose position vectors $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$ and $10\hat{i}-\hat{j}+6\hat{k}$ form a right angled triangle.

Solution:

Let A, B, C be the given points and O be the point of reference or origin.

$$\text{Then } \overrightarrow{OA}=2\hat{i}+4\hat{j}+3\hat{k}, \overrightarrow{OB}=4\hat{i}+\hat{j}+9\hat{k} \text{ and } \overrightarrow{OC}=10\hat{i}-\hat{j}+6\hat{k}$$

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(4\hat{i}+\hat{j}+9\hat{k})-(2\hat{i}+4\hat{j}+3\hat{k})=2\hat{i}-3\hat{j}+6\hat{k}$$

$$AB=|\overrightarrow{AB}|=\sqrt{2^2+(-3)^2+6^2}=\sqrt{4+9+36}=7$$

$$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(10\hat{i}-\hat{j}+6\hat{k})-(4\hat{i}+\hat{j}+9\hat{k})=6\hat{i}-2\hat{j}-3\hat{k}$$

$$BC=|\overrightarrow{BC}|=\sqrt{6^2+(-2)^2+(-3)^2}=\sqrt{36+4+9}=7$$

$$\overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}=(2\hat{i}+4\hat{j}+3\hat{k})-(10\hat{i}-\hat{j}+6\hat{k})=-8\hat{i}+5\hat{j}-3\hat{k}$$

$$CA=|\overrightarrow{CA}|=\sqrt{(-8)^2+5^2+(-3)^2}=\sqrt{64+25+9}=\sqrt{98}$$

$$BC^2=49, CA^2=98, AB^2=49$$

$$CA^2=BC^2+AB^2$$

Therefore, the given points form a right angled triangle.

OR

(b) Find the values of (i) cos 15° and (ii) tan 165°.

Solution:

$$\text{(i) } \cos 15^\circ=\cos(45^\circ-30^\circ)=\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ$$

$$=\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\frac{1}{2}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\text{(ii) } \tan 165^\circ=\tan(120^\circ+45^\circ)=\frac{\tan 120^\circ+\tan 45^\circ}{1-\tan 120^\circ\tan 45^\circ}$$

$$\text{But, } \tan 120^\circ=\tan(90^\circ+30^\circ)=-\cot 30^\circ=-\sqrt{3} \text{ and } \tan 45^\circ=1$$

$$\text{Thus, } \tan 165^\circ=\frac{1-\sqrt{3}}{1+\sqrt{3}}$$

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47. (a) There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

Solution:

Let A_1 be the event of selecting urn – I

Let A_2 be the event of selecting urn – II

Let B be the event of selecting a black ball

$$P(A_1)=\frac{1}{2} \quad ; \quad P(A_2)=\frac{1}{2}$$

$$P(B/A_1)=\frac{6}{10} \quad ; \quad P(B/A_2)=\frac{2}{4}=\frac{1}{2}$$

$$\text{(i) } P(B)=P(A_1).P(B/A_1)+P(A_2).P(B/A_2)$$

$$P(B)=\frac{1}{2}\times\frac{6}{10}+\frac{1}{2}\times\frac{1}{2}=\frac{3}{10}+\frac{1}{4}=\frac{12+10}{40}=\frac{22}{40}=\frac{11}{20}$$

$$\text{(ii) } P(A_1/B)=\frac{P(A_1).P(B/A_1)}{P(A_1).P(B/A_1)+P(A_2).P(B/A_2)}$$

$$= \frac{P(A_1).P(B/A_1)}{P(B)} = \frac{\frac{1}{2} \times \frac{6}{10}}{\frac{11}{20}} = \frac{6}{11}$$

$$P(A_1/B) = \frac{6}{11}$$

OR

(b) Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form.

Solution:

The required form $x\cos\alpha + y\sin\alpha = p$

Given form $-\sqrt{3}x - y = 4$ ($\because$ p is always positive)

Since both represent the same equation

$\therefore$ The coefficients are proportional. We get,

$$\frac{\cos\alpha}{-\sqrt{3}} = \frac{\sin\alpha}{-1} = \frac{p}{4}$$

$$\frac{\cos\alpha}{-\sqrt{3}} = \frac{\sin\alpha}{-1} = \frac{p}{4} = \frac{\sqrt{\cos^2\alpha + \sin^2\alpha}}{\sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}} = \frac{1}{2}$$

$$\left(\begin{array}{c}\text{In componendo and dividendo,}\\ \text{whenever a term is square off,}\\ \text{then it should be square root off it.}\end{array}\right)$$

$$\cos\alpha = \frac{-\sqrt{3}}{2}, \sin\alpha = \frac{-1}{2} \text{ and } p = \frac{4}{2}$$

$$\alpha = 210^\circ = \frac{7\pi}{6} \text{ and } p = 2$$

Normal form of the equation is

$$x\cos\frac{7\pi}{6} + y\sin\frac{7\pi}{6} = 2$$

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