1 MARK

Q.NO		
1	d.	Discontinuous
2	d.	A+B is symmetric
3	b	e^4
4	d	18
5	c	10
6	c	N
7	c	Does not exist
8	d	∞
9	c	$\frac{2x^3}{3} - x^2 + x + C$
10	a	0.56

Q.NO		
11	a	$\sec\theta$
12	b	$-2\hat{i} - \hat{j} + 9\hat{k}$
13	a	$2x^2e^{x/2} - 8xe^{x/2} + 16e^{x/2} + C$
14	d	5^5
15	c	$2/3$
16	d	$1/2, -2$
17	d	$\vec{b} - \vec{a}$
18	d	8
19	c	8
20	d	$1-2^{-n}$.

2 MARK

21. Solu

$|2x-17| = 3$

$2x-17 = \pm 3$

$2x-17 = 3$

$2x = 3+17$

$2x = 20$

$\boxed{x=10}$

$2x-17 = -3$

$2x = -3+17$

$2x = 14$

$\boxed{x=7}$

22. Solu.

$\sin 50^\circ + \sin 20^\circ$

$\boxed{\sin C + \sin D = 2\sin \frac{C+D}{2} \cos \frac{C-D}{2}}$

$2\sin \frac{50-20}{2} \cos \frac{50-20}{2}$

$2\sin \frac{70}{2} \cos \frac{30}{2}$

$2\sin 35^\circ \cos 15^\circ$.

23. Solu

$\cos 135^\circ$

$= \cos(180^\circ - 45^\circ)$

$= \cos 45^\circ$

$= 1/\sqrt{2}$.

24. Solu

5 coins are tossed once

$= 2^5$

$= 32$.

25. Solu.

$a_1 = 1$

$a_2 = 2$

$a_3 = 3$

$a_4 = a_{4-1} + a_{4-2} + a_{4-3}$

$= a_3 + a_2 + a_1$

$= 3 + 2 + 1$

$a_4 = 6$.

$1, 2, 3, 6, \ldots\ldots$

26. Solu

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$

$(1,1)$ & $(-2,3)$

$\frac{x-1}{-2-1} = \frac{y-1}{3-1}$

$\frac{x-1}{-3} = \frac{y-1}{2}$

$2x-2 = -3y+3$

$2x+3y-5=0$

27. Solu

$$= 0 - \sin\alpha \begin{vmatrix} \sin\alpha & \sin\beta \\ \cos\alpha & 0 \end{vmatrix} + \cos\alpha \begin{vmatrix} \sin\alpha & 0 \\ \cos\alpha & -\sin\beta \end{vmatrix}$$

$$= 0 - \sin\alpha(0 - \sin\beta\cos\alpha) + \cos\alpha(-\sin\alpha\sin\beta - 0)$$

$$= \sin\alpha\sin\beta\cos\alpha - \sin\alpha\sin\beta\cos\alpha$$

$$= 0.$$

28. Solu

$$\vec{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$$

$$|\vec{a}| = \sqrt{5^2 + 3^2 + 4^2}$$

$$= \sqrt{25 + 9 + 16}$$

$$|\vec{a}| = \sqrt{50}.$$

$$\frac{\hat{n}}{|\hat{n}|}$$

$$= \frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{50}}.$$

29. Solu

$$\frac{dy}{dx} = \frac{d}{dx}(x)^3 + \frac{d}{dx}(5x^2) + \frac{d}{dx}(3x) + \frac{d}{dx}(7)$$

$$\frac{dy}{dx} = 3x^2 + 5x + 3.$$

30. Solu

$$I = \int (x-11)^7$$

$$= \frac{(x-11)^8}{8}.$$

3 MARK.

31. Solu

$$n = \{2, 3, 4, 5\}$$

$$x = 4n + 1$$

$$4(2) + 1 = 9$$

$$4(3) + 1 = 13$$

$$4(4) + 1 = 17$$

$$4(5) + 1 = 21$$

$$A = \{9, 13, 17, 21\}$$

$$n(A) = 4$$

$$P(A) = 2^{n(A)}$$

$$= 2^4$$

$$= 2 \times 2 \times 2 \times 2$$

$$= 16.$$

32. Solu

$$\frac{x}{(x+3)(x-4)} = \frac{A}{(x+3)} + \frac{B}{(x-4)}$$

$$x = A(x-4) + B(x+3)$$

$x = 4$: $A = 7B$, $B = 4/7$

$x = -3$: $-3 = -7A$, $A = 3/7$

$$\frac{x}{(x+3)(x-4)} = \frac{3}{7(x+3)} + \frac{4}{7(x-4)}.$$

33. Solu.

Total letter = 13

A = 1 C = 2 E = 1 S = 2

I = 3 B = 1 L = 1 T = 1 Y = 1.

$$\frac{13!}{2!2!3!}$$

= 259,459,200.

34. Solu

$(x+2)^{-2/3}$

$(x+a)^{-n} = 1 - nx + \frac{n(n-1)}{2!}x^2 \ldots\ldots$

$= 1 - (-2/3)(x) + (-2/3)(-2/3-1)x^2 \ldots$

$= 1 + \frac{2x}{3} + \frac{(-2/3)(-5/3)x^2}{2!} \ldots$

$= 1 + 2x/3 + 10/18\, x^2 \ldots$

35. Solu.

(1,2)

$5x + 12y - 3 = 0$

$d = \left| \frac{ax+by+c}{\sqrt{a^2+b^2}} \right|$

$= \left| \frac{5x+12y-3}{\sqrt{25+144}} \right|$

$= \left| \frac{5(1)+12(2)-3}{13} \right|$

$= \left| \frac{5+24-3}{13} \right|$

$= \left| \frac{26}{13} \right|$

= 2 units.

36. Solu.

$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

$x - y = 0$

$x = y$

$\begin{vmatrix} 1 & 1 & 1 \\ y & y & z \\ y^2 & y^2 & z^2 \end{vmatrix}$ $(C_1 \equiv C_3)$

$= 0$

$(x-y)$ is also a factor.

$|||^{ly}$ $(y-z)(z-x)$ is also a factor.

$m = 3-3$

$= 0$

So another factor must be k.

$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = k(x-y)(y-z)(z-x)$

Equating the coefficient

$k = 1$

$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

Hence proved.

37. Solu.

$\vec{a} = 5\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b} = 6\hat{i} - 8\hat{j} - \hat{k}$

Product of the vector

$= (5\hat{i} + 3\hat{j} + 4\hat{k})(6\hat{i} - 8\hat{j} - \hat{k})$

$= 30 - 24 - 4 = 2$

$|\vec{a}| = \sqrt{5^2 + 3^2 + 4^2}$

$= \sqrt{25 + 9 + 16}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$|\vec{b}| = \sqrt{6^2 + (-8)^2 + (-1)^2}$

$= \sqrt{36 + 64 + 1}$

$= \sqrt{101}$

$$\cos\theta = \frac{a \cdot b}{|\vec{a}||\vec{b}|}$$

$$= \frac{2}{5\sqrt{2} \times \sqrt{101}}$$

$$= \frac{\sqrt{2} \times \sqrt{2}}{5\sqrt{2} \times \sqrt{101}}$$

$$= \frac{\sqrt{2}}{5\sqrt{101}}$$

$$\theta = \cos^{-1}\frac{\sqrt{2}}{5\sqrt{101}}$$

38. Solu

$$\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = -x$$

$$\frac{dy}{dx} = -x/y.$$

39. Solu

$\int f'(x) = \int f(x)$

$\int f'(x) = \int 4x - 5\,dx$

$f(x) = \frac{4x^2}{2} - 5x + C$

$f(x) = 2x^2 - 5x + C$

$f(2) = 1$

$\Rightarrow 2(2)^2 - 5(2) + C = 1$

$8 - 10 + C = 1$

$-2 + C = 1$

$C = 3.$

$f(x) = 2x^2 - 5x + 3.$

40. Solu.

Let A be the event of getting even number.

Let B be the event of getting number 6.

$S = \{1,2,3,4,5,6\}$

$n(S) = 6.$

i, Getting even number.

$A = \{2,4,6\}.$

$n(A) = 3$

$P(A) = 3/6$

ii. Getting number 6.

$B = \{6\}$

$A \cap B = \{6\}$

$P(A \cap B) = 1/6.$

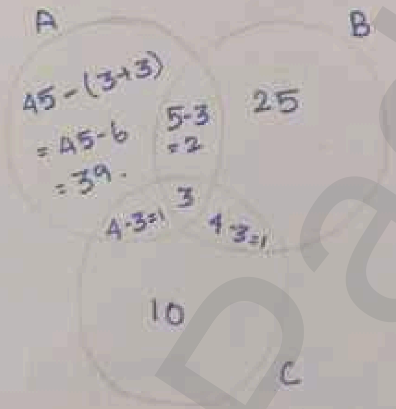
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/6}{3/6}$$

$$= 1/3.$$

5 MARK.

41 a.



$$= \frac{39}{100} \times 5000$$

$= 1950.$

$\therefore$ 1950 people knows the language A.

41 b. Solu

$$\int \frac{2x+4}{x^2+4x+6}\,dx.$$

$$2x+4 = A\frac{d}{dx}(x^2+4x+6)+B$$

$2x+4 = A(2x+4)+B$

$2x+4 = 2Ax+4A+B$

$2Ax = 2x$ $\qquad$ $4A+B=4$

$A=1$ $\qquad$ $4+B=4$

$B=0.$

$2x+4 = (2x+4)+0$

$$I = \int \frac{2x+4}{x^2+4x+6}$$

$$= \int \frac{2x}{x^2+4x+6} + 4\int \frac{dx}{x^2+4x+6}.$$

$$= 2|x^2+4x+6| + 4\int \frac{dx}{(x+2)^2+4-6}$$

$$= 2|x^2+4x+6| + 4\int \frac{dx}{(x+2)^2-(\sqrt{2})^2}$$

$$\boxed{x^2-a^2 = \log|x| + |\sqrt{x^2-a^2}| + c}$$

$$= 2|x^2+4x+6| + 4\log|x+2+\sqrt{(x+2)^2-(\sqrt{2})^2}| + C.$$

42 a.

LHS

$$= \frac{\cot\theta \quad \cos\theta \quad \cos\theta}{\cos\theta \quad \tan\theta \quad \operatorname{cosec}\theta}$$

$$= \cot\theta \quad \cos\theta \left[\frac{\cos\theta}{\sin\theta} \times \sin\theta\right]$$

$$= \cot\theta \quad \cos\theta \quad \cos\theta$$

$$= \cos^2\theta \quad \cot\theta$$

LHS = RHS

Hence proved.

42 b. Solu.

$$\int x \cos dx.$$

$u = x$

$du = dx$

$$\int dv = \int \cos x \; dx$$

$v = \sin x$

$$\int u\,dv = uv - \int v\,du.$$

$$I = x \sin x - \int \sin x \; dx$$

$$= x \sin x + \cos x + C.$$

43 a. Solu.

$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

now put

$n = 1.$

LHS

$1^3 + 2^3 + \ldots + n^3$

$n^3 = 1^3$

$= 1.$

RHS

$$\left(\frac{n(n+1)}{2}\right)^2$$

$$= \left(\frac{1(2)}{2}\right)^2$$

$= 1^2$

$= 1.$

Put $n = k$.

$$1^3 + 2^3 + 3^3 + \ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$$

assume it is true.

Put $n = (k+1)$.

$$1^3 + 2^3 + 3^3 + \ldots + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$$

To prove

LHS

$1^3+2^3+3^3+\dots+K^3+(K+1)^3$

$= \left(\frac{K(K+1)}{2}\right)^2 + (K+1)^3$

$= \frac{K^2(K+1)^2}{4} + (K+1)^3$

$= \frac{K^2(K+1)^2 + 4(K+1)^3}{4}$

$= \frac{(K+1)^2(K^2+4K+4)}{4}$

$= \frac{(K+1)^2(K+2)(K+2)}{4}$

$= \left(\frac{(K+1)(K+2)}{2}\right)^2$

LHS = RHS

Hence proved.

b. soln.

$\frac{dy}{dx}$

$\frac{dx}{dt} = a(0-\cos t)$

$= -a\cos t$

$\frac{dy}{dt} = a(0-(-\sin t))$

$= a\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= a\sin t \times \frac{1}{-a\cos t}$

$= -\tan t.$

44. a. soln

$12x^2+2kxy+2y^2+11x-5y+2=0 \to (1)$

$ax^2+2hxy+by^2+2gx+2fy+c=0 \to (2)$

Compare (1) & (2).

$a=12 \quad 2h=2k \quad b=2$

$h=k$

$2g=11 \quad 2f=-5 \quad c=2.$

$g=11/2 \quad f=-5/2$

Condition

$abc+2fgh-af^2-bg^2-ch^2=0$

$12(2)(2)+2(-5/2)(11/2)k-12(-5/2)^2$
$-2(11/2)^2-2(k)^2=0$

$48-\frac{55}{2}k-75-\frac{121}{2}-2k=0$

$96-55k-150-121-4k=0$

$4k^2+55k+175=0$

$(k+5)(4k+35)=0$

$k=-5$

$k=-35/4.$

44.
b.

$$\lim_{x\to 0} \frac{\tan 2x}{\sin 5x}$$

$$\lim_{x\to 0} \frac{\frac{\sin 2x}{\cos 2x}}{\sin 5x}$$

$$\lim_{x\to 0} \frac{\frac{\sin 2x}{\cos 2x} \times \frac{2x}{2x}}{\frac{\sin 5x}{5x} \times 5x}$$

$$\lim_{x\to 0} \frac{\frac{\sin 2x}{2x} \times \frac{2x}{\cos 2x}}{\frac{\sin 5x}{5x} \times 5x}$$

$$= \frac{1 \times \frac{2x}{\cos 2x}}{1 \times 5x}$$

$$= \frac{2x}{\cos 2x \times 5x}$$

$$= \frac{2x}{\cos 2(0) \times 5x}$$

$$= \frac{2x}{5x}$$

$$= 2/5.$$

45.
a)

RHS.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times (-1) \begin{vmatrix} -a & -b & -c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$R_3 \leftrightarrow R_2$

$$\begin{vmatrix} -a^2+bc+cb & -ab+ba+c^2 & -ac+b^2+ca \\ -ab+c^2+ab & -b^2+ca+ac & -cb+cb+a^2 \\ -ac+ac+b^2 & -cb+a^2+bc & -c^2+ab+ba \end{vmatrix}$$

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$

Hence proved.

45.
b. LHS

$$\log\left(\frac{75}{16}\right) - 2\log(5/9) + \log\left(\frac{32}{243}\right)$$

$$= \log\left(\frac{75}{16}\right) - \log\left(\frac{25}{81}\right) + \log\left(\frac{32}{243}\right)$$

$$= \log\left(\frac{75}{16} \times \frac{81}{25}\right) + \log\left(\frac{32}{243}\right)$$

$$= \log\left(\frac{75}{16} \times \frac{81}{25} \times \frac{32}{243}\right)$$

$$= \log 2.$$

Hence proved.

46

b. $\cos(15°)$

$\cos(45-30)$

$\cos 45° \cos 30° + \sin 45° \sin 30°$

$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\tan 165°$

$\tan(180-15°)$

$= \tan 15°$

$= \tan(45-30)$

$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$= \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

a. $\vec{OA} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{OB} = 2\hat{i} + \hat{j} + 9\hat{k}$

$\vec{OC} = 10\hat{i} + \hat{j} + 6\hat{k}$.

$\vec{AB} = \vec{OB} - \vec{OA}$

$= 4\hat{i} + \hat{j} + 9\hat{k} - 2\hat{i} - 4\hat{j} - 3\hat{k}$

$= 2\hat{i} - 3\hat{j} + 6\hat{k}$

$|\vec{AB}| = \sqrt{2^2+3^2+6^2}$

$= \sqrt{49}$

$= 7$.

$\vec{BC} = \vec{OC} - \vec{OB}$

$= 10\hat{i} - \hat{j} + 6\hat{k} - 4\hat{i} - \hat{j} - 9\hat{k}$

$= 6\hat{i} - 2\hat{j} - 3\hat{k}$

$|\vec{BC}| = \sqrt{6^2+2^2+3^2}$

$= \sqrt{49}$

$= 7$.

$\vec{AC} = \vec{OC} - \vec{OA}$

$= 10\hat{i} - \hat{j} + 6\hat{k} - 2\hat{i} - 4\hat{j} - 3\hat{k}$.

$= 8\hat{i} - 5\hat{j} + 3\hat{k}$

$|\vec{AC}| = \sqrt{8^2 + 25 + 3^2}$

$= \sqrt{98}$.

To prove

$|\vec{AC}|^2 = |\vec{AB}|^2 + |\vec{BC}|^2$

$(\sqrt{98})^2 = 7^2 + 7^2$

$98 = 98$

Hence proved

47.
a.

$P(A_1) = 1/2$ $\quad$ $P(B/A_1) = 6/10$

$P(A_2) = 1/2$ $\quad$ $P(B/A_2) = 2/4$

$P(A_1)P(B/A_1) + P(A_2)P(B/A_2)$

$= (1/2)(6/10) + (1/2)(2/4)$

$= \frac{12}{40} + \frac{10}{40}$

$= 11/20$

Bayes theorem

$$= \frac{P(A_1)P(B/A_1)}{P(A_1)(P(B/A_1)+P(A_2)(P(B/A_2))}$$

$$= \frac{(1/2)(6/10)}{11/20}$$

$$= \frac{6/20}{11/20}$$

$= 6/11$

b. $\sqrt{3}x - y + 4 = 0$

$-\sqrt{3}x + y = 4 \rightarrow ①$

$Ax + By + C = 0$

$A = -\sqrt{3}$; $B = 1$

$\sqrt{A^2+B^2} = \sqrt{3+1}$

$= 2$

① ÷ by ②

$\frac{-\sqrt{3}}{2}x + \frac{1}{2}y = 2 \rightarrow ②$

$x\cos\alpha + y\sin\alpha = p \rightarrow ③$

$\Rightarrow ② \times ③$

$\cos\alpha = -\sqrt{3}/2$

$\alpha = \cos^{-1}(-\sqrt{3}/2) = 150°$

$\alpha = 5\pi/6$

$\sin\alpha = 1/2$

$\alpha = \sin^{-1}(1/2)$

$\alpha = 150$

$P = 2$.

$x\cos\frac{5\pi}{6} + y\sin\frac{\pi}{6} = 2$.