LONDON KRISHNAMOORTHI MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

+1 MATHS Q-BANK

(VOLUME - 2)

Expected Questions Only

LONDON KRISHNAMOORTHI MATRIC HIGHER SECONDARY SCHOOL, ORATHANADU.

CHAPTER 7 MATRICES AND DETERMINANTS

2 Marks

SET 1

1. Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by (i) $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$, (ii) $a_{ij} = \frac{|3i-4j|}{4}$ with $m = 3, n = 4$. **(EX 7.1 - 1)**

2. If A is a 3×4 matrix and B is a matrix such that both $A^T B$ and BA^T are defined, what is the order of the matrix B? **(EX 7.1 - 16)**

3. Find the value of x, $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric. **(EX 7.1. 20)**

4. Let A and B be two symmetric matrices. Prove that $AB = BA$ if and only if AB is a symmetric matrix. **(EX 7.1. 22)**

5. If A and B are symmetric matrices of same order, prove that (i) $AB + BA$ is a symmetric matrix. (ii) $AB - BA$ is a skew-symmetric matrix. **(EX 7.1. 23)**

6. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, find the value of $|3AB|$. **(EX 7.2 - 17)**

SET 2

7. Compute $|A|$ using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$. **(EG 7.17)**

8. If the area of the triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 square units, find the values of k . **(EG 7.32)**

9. Find the area of the triangle whose vertices are $(-2, -3)$, $(3, 2)$, and $(-1, -8)$. **(EG 7.33)**

10. Identify the singular and non-singular matrices: $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$, **(EX 7.4 - 3)**

11. Determine the values of a and b so that the following matrices are singular:
(i) $A = \begin{bmatrix} 7 & 3 \\ -2 & a \end{bmatrix}$ (ii) $\begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$. **(EX 7.4. 4)**

12. If A is a square matrix and $|A| = 2$, find the value of $|AA^T|$. **(EX 7.2 - 16)**

3 Marks

SET 1

1. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A^T = \begin{bmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$. **(EX 7.1 - 18)**

2. If $\lambda = -2$, determine the value of $\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2 + 1 \\ -1 & 6\lambda - 1 & 0 \end{vmatrix}$. **(EX 7.2 - 18)**

3. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ is a matrix such that $AA^T = 9I$, find the values of x and

4. For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric.

5. If $\begin{bmatrix} 0 & p & 3 \\ 2 & q^2 & -1 \\ r & 1 & 0 \end{bmatrix}$ is skew-symmetric, find the values of p, q, and r. **(EX 7.1 - 20)**

6. Find the value of x if $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$. **(EG 7.21)**

SET 2

7. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$. **(EG 7.22)**

8. If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$. **(EX 7.2 - 14)**

9. Determine the roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$. **(EX 7.2 – 19)**

10. Verify that $|AB| = |A||B|$ if $A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$, $B = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$. **(EG 7.27)**

11. If $\cos 2\theta = 0$, determine $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$. **(EX 7.4 - 5)**

5 Marks

SET 1

1. If $A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & -1 \\ 3 & -1 & 4 \\ -1 & 2 & 1 \end{bmatrix}$ verify

   $(i)\ (AB)^T = B^T A^T$ $\quad (ii)\ (A+B)^T = A^T + B^T$

   $(iii)\ (A-B)^T = A^T - B^T$ $\quad (iv)\ (3A)^T = 3A^T$ **(EG 7.12)**

2. Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices. **(EG 7.13)**

3. Verify the property $A(B+C) = AB + AC$, when the matrices A, B, and C are given by $A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$. **(EX 7.1 - 13)**

4. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$. **(EX 7.2 - 3)**

5. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. **(EX 7.2 - 4)**

6. Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$. **(EG 7.23)**

7. Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$. **(EG 7.24)**

SET 2

8. Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$. **(EG 7.25)**

9. In a triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A\,(1+\sin A) & \sin B\,(1+\sin B) & \sin C\,(1+\sin C) \end{vmatrix} = 0$, prove that ΔABC is an isosceles triangle. **(EG 7.26)**

10. Show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x-a)^2(x+2a)$. **(EX 7.3 - 1)**

11. Show that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$. **(EX 7.3 - 2)**

12. Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$. **(EX 7.3 - 4)**

13. Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$. **(EX 7.3 - 6)**

14. Find the value of the product: $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$. **(EX 7.4 - 6)**

CHAPTER 8 VECTOR ALGEBRA – I

SET 1

2 Marks

1. Represent graphically the displacement of (i) 30 km 60° west of north (ii) 60 km 50° south of east. **(EG 8.1)**

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2. If $\vec{a}$ and $\vec{b}$ are vectors represented by two adjacent sides of a regular hexagon, then find the vectors represented by other sides. **(EG 8.2)**
3. Prove that the relation R defined on the set V of all vectors by '$\vec{a}R\vec{b}$ if $\vec{a} = \vec{b}$' is an equivalence relation on V. **(EX 8.1 - 2)**
4. Find a unit vector along the direction of the vector $5\vec{i} - 3\vec{j} + 4\vec{k}$. **(EG 8.4)**
5. Find a direction ratio and direction cosines of the following vectors. $3\vec{i} - 4\vec{k}$ **(EG 8.5)**
6. Can a vector have direction angles $30°$,$45°$, $60°$? **(EG 8. 6)**
7. Find the direction cosines of $\overrightarrow{AB}$ where A is $(2, 3, 1)$ and B is $(3, -1, 2)$. $(EG\ 8.6)$
8. Find a point whose position vector has magnitude 5 and parallel to the vector $4\vec{i} - 3\vec{j} + 10\vec{k}$ **(EG 8.8)**
9. If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$ are the direction cosines of some vector, then find a. **(EX 8.2 - 5)**
10. Find the value of λ for which the vectors $\vec{a} = 3\vec{i} + 2\vec{j} + 9\vec{k}\hat{}$ and $\vec{b} = \vec{i} + \lambda\vec{j} + 3\vec{k}$ are parallel. **(EX 8.2 - 8)**
11. Show that the following vectors are coplanar $\vec{i} - 2\vec{j} + 3\vec{k}$, $-2\vec{i} + 3\vec{j} - 4\vec{k}$ and $-\vec{j} + 2\vec{k}$ **(EX 8.2 - 9)**
12. Find $\vec{a}.\vec{b}$ when (i) $\vec{a} = \vec{i} - \vec{j} + 5\vec{k}$ and $\vec{b} = 3\vec{i} - 2\vec{k}$ **(EG 8.11)**
13. The position vectors $\vec{a}, \vec{b}, \vec{c}$ of three points satisfy the relation $2\vec{a} - 7\vec{b} + 5\vec{c} = \vec{0}$ Are these points collinear? **(EX 8.2 - 14)**

SET 2

14. Find the value or values of m for which $m\ (\vec{i} + \vec{j} + \vec{k})$ is a unit vector. **(EX 8.2 - 16)**
15. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$prove that and $\vec{a}$ and $\vec{b}$ are perpendicular. **(EG 8.14)**
16. For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}.\vec{i})\vec{i} + (\vec{r}.\vec{j})\vec{j} + (\vec{r}.\vec{k})\vec{k}$. **(EG 8.15)**
17. Find the angle between the vectors $5\vec{i} + 3\vec{j} + 4\vec{k}$ and $6\vec{i} - 8\vec{j} - \vec{k}$. **(EG 8.16)**
18. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 10, |\vec{b}| = 15$ and $\vec{a}.\vec{b} = 75\sqrt{2}$ Find the angle between $\vec{a}$ and $\vec{b}$. **(EX 8.3 - 31**
19. Show that the points $(2, -1, 3)$, $(4, 3, 1)$ and $(3, 1, 2)$ are collinear. **(EX 8.3 - 9)**
20. Find the projection of the vector $\vec{i} + 3\vec{j} + 7\vec{k}$ on the vector $2\vec{i} + 6\vec{j} + 3\vec{k}$. **(EX 8.3 - 12)**
21. Find λ when the projection of $\vec{a} = \lambda\vec{i} + \vec{j} + 4\vec{k}$ on $\vec{b} = 2\vec{i} + 6\vec{j} + 3\vec{k}$ is 4 units. **(EX 8.3 - 13)**
22. Find $|\vec{a} \times \vec{b}|$ where $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = \vec{i} + \vec{j} + \vec{k}$. **(EG 8.20)**
23. Find the area of the parallelogram whose adjacent sides are $\vec{a} = 3\vec{i} + \vec{j} + 4\vec{k}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$. **(EG 8.24)**
24. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$. **(EG 8.25**

25. Show that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$. **(EX 8.4 - 2)**

3 Marks

SET 1

1. If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$. **(EX 8.1 - 4)**
2. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side. **(EX 8.1 - 5)**
3. Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram. **(EX 8.1 - 6)**
4. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$. **(EX 8.1 - 9)**
5. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$. **(EX 8.1 - 10)**
6. Show that the points whose position vectors are $2\vec{i} + 3\vec{j} - 5\vec{k}$, $3\vec{i} + \vec{j} - 2\vec{k}$ and $6\vec{i} - 5\vec{j} + 7\vec{k}$ are collinear. **(EG 8.7)**
7. Show that the vectors $5\vec{i} + 6\vec{j} + 7\vec{k}$, $7\vec{i} - 8\vec{j} + 9\vec{k}$ and $3\vec{i} + 20\vec{j} + 5\vec{k}$ are coplanar. **(EG 8.10)**
8. A triangle is formed by joining the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. Find the direction cosines of the medians. **(EX 8.2 - 4)**
9. Show that the vectors $2\vec{i} - \vec{j} + \vec{k}$, $3\vec{i} - 4\vec{j} - 4\vec{k}$ and $\vec{i} - 3\vec{j} - 5\vec{k}$ form a right angled triangle. **(EX 8.2 - 7)**
10. Show that the points whose position vectors $4\vec{i} + 5\vec{j} + \vec{k}$, $-\vec{j} - \vec{k}$, $3\vec{i} + 9\vec{j} + 4\vec{k}$ and $-4\vec{i} + 4\vec{j} + 4\vec{k}$ are coplanar. **(EX 8.2 - 10)**

SET 2

11. The position vectors of the points P, Q, R, S are $\vec{i} + \vec{j} + \vec{k}$, $2\vec{i} + 5\vec{j}$, $3\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{i} - 6\vec{j} - \vec{k}$ respectively. Prove that the line PQ and RS are parallel. **(EX 8.2 - 15)**
12. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + 2\vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$. **(EX 8.3 - 5)** ‘
13. Show that the vectors $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}, \vec{b} = 6\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{c} = 3\vec{i} - 6\vec{j} + 2\vec{k}$ are mutually orthogonal. **(EX 8.3 - 6)**
14. If $|\vec{a}| = 5, |\vec{b}| = 6, |\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$. **(EX 8.3 - 8)**
15. If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that
$(i)\ sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$ $\quad (ii)\ cos\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} + \vec{b}\right|$ $\quad$ **(EG 8.6)**

16. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$. **(EX 8.3 - 11)**
17. If $\vec{a} = -3\vec{\imath} + 4\vec{\jmath} - 7\vec{k}$ and $\vec{b} = 6\vec{\imath} + 2\vec{\jmath} - 3\vec{k}$,verify (i) $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other. (ii) $\vec{b}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other. **(EG 8.21)**
18. Find the vectors of magnitude 6 which are perpendicular to both vectors $\vec{a} = 4\vec{\imath} - \vec{\jmath} + 3\vec{k}$ and $\vec{b} = -2\vec{\imath} + \vec{\jmath} - 2\vec{k}$. **(EG 8.22)**
19. For any vector $\vec{a}$ prove that $|\vec{a} \times \vec{\imath}|^2 + |\vec{a} \times \vec{\jmath}|^2 + |\vec{a} \times \vec{k}|^2 = 2|\vec{a}|^2$. **(EX 8.4 - 8)**
20. Find the angle between the vectors $2\vec{\imath} + \vec{\jmath} - \vec{k}$ and $\vec{\imath} + 2\vec{\jmath} + \vec{k}$ by using cross product. **(EX 8.4 - 10)**

5 Marks

<u>SET 1</u>

1. **(Section Formula - Internal Division)**
2. The medians of a triangle are concurrent. **(TH 8.3)**
3. If $ABCD$ is a quadrilateral and E and F are the mid-points of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$. **(EX 8.1 - 12)**
4. Prove that the points whose position vectors $2\vec{\imath} + 4\vec{\jmath} + 3\vec{k}$, $4\vec{\imath} + \vec{\jmath} + 9\vec{k}$ and $10\vec{\imath} - \vec{\jmath} + 6\vec{k}$ form a right angled triangle. **(EG 8.9)**
5. The position vectors of the vertices of a triangle are $\vec{\imath} + 2\vec{\jmath} + 3\vec{k}$, $3\vec{\imath} - 4\vec{\jmath} + 5\vec{k}$ and $-2\vec{\imath} + 3\vec{\jmath} - 7\vec{k}$ Find the perimeter of the triangle. **(EX 8.2 - 12)**
6. Show that the points $A\ (1,1,1), B(1,2,3)$ and $C(2,-1,1)$ are vertices of an isosceles triangle. **(EX 8.2 - 17)**
7. Three vectors $\vec{a}, \vec{b}$and $\vec{c}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a}.\vec{b} + 3\vec{b}.\vec{c} + 3\vec{c}.\vec{a}$. **(EX 8.3 - 14)**
8. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a triangle ABC, then prove that the area of triangle ABC is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. Also deduce the condition for collinearity of the points A, B and C. **(EX 8.4 - 7)**
9. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$. **(EX 8.4 - 9)**

CHAPTER 9 LIMITS AND CONTINUITY

<u>SET 1</u>

2 Marks

1. Calculate $\lim_{x \to 0}|x|$. **(EG 9.1)**

2. If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? **(EX 9.1 - 20)**

3. Evaluate: $\lim_{x\to 3}\frac{x^2-9}{x-3}$ if it exists by finding $f(3^-)$ and $f(3^+)$. **(EX 9.1 - 22)**

4. Calculate $\lim_{x\to 3}(x^2 - 2x + 6)$. **(EG 9.7)**

5. Evaluate $\lim_{x\to 0}\frac{\sqrt{1-x}-1}{x^2}$. **(EX 9.2 - 13)**

6. Evaluate $\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$. **(EX 9.2 - 14)**

SET 2

7. Find $\lim_{t\to 1}\frac{\sqrt{t}-1}{t-1}$. **(EG 9.17)**

8. Find $\lim_{x\to 0}\frac{(2+x)^5-2^5}{x}$ **(EG 9.18)**

9. Find the positive integer n so that $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = 27$. **(EG 9.19)**

10. Calculate $\lim_{x\to\infty}\frac{1-x^3}{3x+2}$. **(EG 9.24)**

11. Evaluate $\lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}$. **(EX 9.3 - 4)**

12. Prove that $\lim_{x\to 0}\sin x = 0$. **(EG 9.30)**

SET 1

3 Marks

1. Check if $\lim_{x\to -5} f(x)$ exists or not, where $f(x) = \begin{cases} \frac{|x+5|}{x+5}, & for\ x \neq -5 \\ 0, & for\ x = -5 \end{cases}$. **(EG 9.5)**

2. Evaluate $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$, m and n are integers. **(EX 9.2 - 2)**

3. Evaluate $\lim_{\sqrt{x}\to 3}\frac{x^2-81}{\sqrt{x}-3}$. **(EX 9.2 - 3)**

4. Evaluate $\lim_{x\to 1}\frac{\sqrt{x}-x^2}{1-\sqrt{x}}$. **(EX 9.2 - 7)**

5. Evaluate $\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$. **(EX 9.2 - 8)**

6. Evaluate $\lim_{x\to 1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$. **(EX 9.2 - 10)**

7. Show that $\lim_{n\to\infty}\frac{1+2+3+\dots\dots+n}{3n^2+7n+2} = \frac{1}{6}$, **(EX 9.3 - 8)**

SET 2

8. Show that $\lim_{n\to\infty}\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots\dots+\frac{1}{n(n+1)} = 1$. **(EX 9.3 - 8)**

9. Evaluate $\lim_{x\to 0}(1+\sin x)^{2cosec\, x}$. **(EG 9.32)**

10. Do the limits of functions exist as $x \to 0$? State reasons for your answer. $\frac{\sin|x|}{x}$ **(EG 9.35)**

11. Evaluate: $\lim_{x\to 0}\frac{3^x-1}{\sqrt{1+x}-1}$. **(EG 9.36)**
12. Evaluate $\lim_{x\to 0}\frac{\sin\alpha x}{\sin\beta x}$. **(EX 9.4 - 7)**
13. Evaluate $\lim_{x\to 0}\frac{3^x-1}{\sqrt{x+1}-1}$. **(EX 9.4 - 16)**
14. If f and g are continuous functions with $f(3)=5$ and $\lim_{x\to 3}[2f(x)-g(x)]=4$, find $g(3)$. **(EX 9.5 - 8)**

SET 1

5 Marks

1. Show that $\lim_{x\to 0^+} x\left[\left\lfloor\frac{1}{x}\right\rfloor+\left\lfloor\frac{2}{x}\right\rfloor+\ldots\ldots+\left\lfloor\frac{15}{x}\right\rfloor\right]=120$. **(EG 9.31)**
2. Show that function $f(x)=\begin{cases}\frac{x^3-1}{x-1}, & if\ x\neq 1\\ 3, & if\ x=1\end{cases}$, is continuous on $(-\infty,\infty)$. **(EX 9.5 - 5)**
3. For what value of α is this function $f(x)=\begin{cases}\frac{x^4-1}{x-1}, & if\ x\neq 1\\ \alpha, & if\ x=1\end{cases}$, continuous at $x=1$? **(EX 9.5 - 6)**
4. A function f is defined as follows: $f(x)=\begin{cases}0, & for\ x<0\\ x, & for\ 0\le x<1\\ -x^2+4x-2, & for\ 1\le x<3\\ 4-x, & for\ x\ge 3\end{cases}$. Is the function continuous? **(EX 9.5 - 10)**

CHAPTER 10 – DIFFERENTIAL CALCULUS - DIFFERENTIABILITY AND METHODS OF DIFFERENTIATION

2 Marks

SET 1

1. Find the slope of the tangent line to the graph of $f(x)=7x+5$ at any point $(x_0,f(x_0))$. **(EG 10.1)**
2. Find the derivatives of the functions using first principle. $f(x)=-x^2+2$ **(EX 10.1 - 1)**
3. Differentiate the following with respect to x: $(i)\ y=x^3+5x^2+3x+7$ $(ii) y=e^x+\sin x+2$ **(EG 10.7)**
4. Differentiate $g(t)=4\sec t+\tan t$ **(EX 10.2 - 6)**
5. Differentiate $y=\frac{\sin x}{1+\cos x}$ **(EX 10.2 - 9)**
6. Differentiate $y=\frac{\sin x}{x^2}$ **(EX 10.2 - 12)**

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SET 2

7. Find $F'(x) if\ F(x) = \sqrt{x^2+1}$. **(EG 10.8)**
8. Differentiate: $y = e^{\sin x}$. **(EG 10.14)**
9. Differentiate 2^x. **(EG 10.15)**
10. Differentiate $y = xe^{-x^2}$ **(EX 10.3 - 15)**
11. Differentiate $y = e^{x\cos x}$ **(EX 10.3 - 27)**
12. Find $\frac{dy}{dx}$ if $x = at^2, y = 2at, t \neq 0$. **(EG 10.26)**
13. Find y''' if $y = \frac{1}{x}$. **(Eg. 10.32)**
14. Differentiate: $x = a\cos^3 t\,, y = a\sin^3 t$ **(EX. 10.4 - 13)**

SET 1

3 Marks

1. If Determine whether the function is differentiable at the indicated values. $(iii) f(x) = |x| + |x-1|\ at\ x = 0, 1 \quad (iv) f(x) = sin|x|\ at\ x = 0$. **(EX 10.1 - 3)**
2. Show that the functions are not differentiable at the indicated value of x. $f(x) = \begin{cases} -x+2, x \leq 2 \\ 2x-4, x > 2 \end{cases}, x = 2$ **(EX 10.1 - 4)**
3. Draw the function $f'(x)$ if $f(x) = 2x^2 - 5x + 3$ **(EX 10.2 - 20)**
4. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y'. **(EG 10.16)**
5. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ **(EX 10.3 - 28)**
6. Find $\frac{dy}{dx}$ if $x^4 + x^2y^3 - y^5 = 2x + 1$. **(EG 10.19)**
7. Find $\frac{dy}{dx}$ if $\sin y = y\cos 2x$. **(EG 10.20)**
8. Differentiate: $= x^{\sqrt{x}}$. **(EG 10.23)**

SET 2

9. Find $f'(x)$ if $f(x) = \cos^{-1}(4x^3 - 3x)$. **(EG 10.25)**
10. Find $\frac{dy}{dx}$ if $x = a(t - sin\ t), y = a(1 - cos\ t)$. **(EG 10.27)**
11. Find the derivative of $\tan^{-1}(1+x^2)$ with respect to $x^2 + x + 1$. **(EG 10.29)**
12. Find the second order derivative if x and y are given by $x = a\cos t\,, y = a\sin t$. **(EG 10.35)**
13. Find $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 4$. **(EG 10.36)**
14. $x = a(\cos t + t\sin t),\ \ y = a(\sin t - t\cos t)$ **(EX 10.4 - 14)**
15. If $y = \sin^{-1} x$ then find y''. (**EX 10.4 − 23**)

5 Marks

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1. If $u = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ and $v = \tan^{-1} x$, find $\frac{du}{dv}$. **(EX 10.4 - 21)**
2. If $y = e^{\tan^{-1} x}$ show that $(1+x^2)y'' + (2x-1)y' = 0$. **(EX 10.4 - 24)**
3. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$. **(EX 10.4 - 25)**
4. If $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$ then P.T at, $\theta = \frac{\pi}{2}, y'' = \frac{1}{a}$. **(EX 10.4 - 26)**
5. If $\sin y = x\sin(a+y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}, a \neq n\pi$. **(EX 10.4 - 27)**
6. If $y = (\cos^{-1} x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$. **(EX 10.4 - 28)**

CHAPTER 11 – INTEGRAL CALCULUS

2 Marks

SET 1

1. Evaluate the with respect to x: $\int(4x+5)^6 dx$ **(EG 11.4)**
2. Integrate the with respect to x: $\frac{1}{\sqrt{1-25x^2}}$ **(EG 11.7)**
3. Integrate the following with respect to x: $5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}}$ **(EG 11.8)**
4. If $f'(x) = 3x^2 - 4x + 5$ and $f(1) = 3$,then find $f(x)$. **(EG 11.10)**
5. If $f''(x) = 12x - 6$ and $f(1) = 30, f'(1) = 5$ find $f(x)$.**(EX 11.4 - 3)**

SET 2

6. Evaluate:(i) $\int a^x e^x dx$ (ii) $\int e^{x\log 2} e^x dx$. **(EG 11.26)**
7. *Evaluate* $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ **(EX 11.5 - 2)**
8. Evaluate $e^{x\log a} e^x$ **(EX 11.5 – 13)**
9. Integrate the with respect to x: $\int \frac{2x+4}{x^2+4x+6} dx$
10. *Evaluate* $\frac{\sin^{-1} x}{\sqrt{1-x^2}}$ **(EX 11.6 - 9)**
11. Evaluate $\frac{\sqrt{x}}{1+\sqrt{x}}$ **(EX 11.6 - 10)**

3 Marks

SET 1

1. A tree is growing so that, after t − years its height is increasing at a rate of $\frac{18}{\sqrt{t}}$ cm per year. Assume that when $t = 0$, the height is 5 cm. (i) Find the height of the tree after 4 years. (ii) After how many years will the height be149 cm? **(EG 11.13)**
2. A wound is healing in such a way that $t$days since Sunday the area of the wound has been decreasing at a rate of $\frac{-3}{(t+2)^2}$ cm^2per day. If on Monday the area of the

wound was $2cm^2$ (i)What was the area of the wound on Sunday? (ii) What is the anticipated area of the wound on Thursday if it continues to heal at the same rate?**(EX 11.4 - 5)**

3. Evaluate:$\int \frac{\sin x}{1+\sin x} dx$ **(EG 11.19)**
4. Evaluate:$\int \frac{(x-1)^2}{x^3+x} dx$ **(EG 11.21)**
5. Evaluate:$\int \frac{1-\cos x}{1+\cos x} dx$ **(EG 11.23)**
6. Evaluate:$\int (x-3)\sqrt{x+2}dx$. **(EG 11.27)**
7. Evaluate:$\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$. **(EG 11.28)**
8. Evaluate: (ii) $\int \frac{x+3}{(x+2)^2(x+1)} dx$ **(EG 11.29)**
9. $\frac{x+1}{(x+2)(x+3)}$**(EX 11.5 - 17)**
10. $\frac{x^3}{(x-1)(x-2)}$.**(EX 11.5 - 20)**

SET 2

11. Integrate the following with respect to x:(12) $\alpha\beta x^{\alpha-1}e^{-\beta x^{\alpha}}$ **(EX. 11.6 - 12)**
12. $\int e^{-x^2} x dx$ $(EG\ 11.30)$
13. $\int x(a-x)^8 dx$ **(EG 11.30)**
14. Evaluate:$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$ **(EG 11.34)**
15. Integrate (i) $\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$
16. Integrate $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right) dx$
17. Integrate $\int \frac{1}{x^2-2x+5} dx$
18. *Integrate* $\int \frac{1}{\sqrt{12+4x-x^2}} dx$
19. Evaluate the following x $\int \sqrt{x^2+x+1}dx$ **(EG 11.41)**
20. Integrate $\sqrt{81+(2x+1)^2}$**(EX 11.12 − 2)**

5 Marks

SET- 1

1. Evaluate : (i) $\int \frac{3x+7}{x^2-3x+2} dx$ (ii) $\int \frac{x+3}{(x+2)^2(x+1)} dx$ **(Eg. 11.29)**
2. Integrate the following functions with respect to x: $\frac{3x-9}{(x-1)(x+2)(x^2+1)}$. (EX. 11.5 –9)
3. Integrate the following with respect to x: $e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)$. **(EX. 11.9 - 5)**
4. Integrate the following with respect to x: $\frac{\log x}{(1+\log x)^2}$. **(EX. 11.9 - 6)**
5. Evaluate the following integral: $\int \frac{3x+5}{x^2+4x+7} dx$. **(Eg. 11.40(i))**
6. Evaluate the following integral: $\int \frac{x+1}{x^2-3x+1} dx$. **(Eg. 11.40(ii))**

SET -2

7. Evaluate the following integral: $\int \frac{2x+3}{\sqrt{x^2+x+1}} dx$. **(Eg. 11.40(iii))**
8. Evaluate the following integral: $\int \frac{5x-7}{\sqrt{3x-x^2-2}} dx$ **(Eg. 11.40(iv)))**
9. Integrate the following with respect to x: $\frac{5x-2}{2+2x+x^2}$ **(EX. 11.11 – 1(ii))**
10. Integrate the following with respect to x: $\frac{3x+1}{2x^2-2x+3}$. **(EX. 11.11 – 1(iii))**
11. Integrate the following with respect to x: $\frac{2x+1}{\sqrt{9+4x-x^2}}$. **(EX. 11.11 – 2(i))**
12. Integrate the following with respect to x:) $\frac{2x+3}{\sqrt{x^2+4x+1}}$ **(EX. 11.11 – 2(iii))**

CHAPTER 12 - PROBABILITY THEORY

2 Marks

1. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer chosen is a prime or multiple of 8? **(EX. 12.1 - 6)**
2. Given that $P(A) = 0.52, P(B) = 0.43$, and $P(A \cap B) = 0.24$, find $(i)\ P(A \cap \bar{B})$ $(ii)\ P(A \cup B)$ $(iii)\ P(\bar{A} \cap \bar{B})$ $(iv)\ P(\bar{A} \cup \bar{B})$ **(Eg. 12.14)**
3. If $P(A) = 0.6, P(B) = 0.5$, and $P(A \cap B) = 0.2$, Find $(i)\ P(A/B)$ $(ii)\ P(\bar{A}/B)$ $(iii)\ P(A/\bar{B})$. **(Eg. 12.16)**
4. If A and B are two independent events such that $P(A) = 0.4$ and $P(A \cup B) = 0.9$. Find $P(B)$. **(Eg. 12.20)**
5. If A and B are two events such that $P(A \cup B) = 0.7, P(A \cap B) = 0.2$, and $P(B) = 0.5$, then show that A and B are independent. **(EX. 12.3 - 2)**
6. Given $P(A) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(B)$ if (i) A and B are mutually exclusive **(EX. 12.3 – 10(i))**
7. Given $P(A) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(B)$ if A and B are independent events **(EX. 12.3 – 10(ii))**
8. Given $P(A) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(B)$ if $P(A/B) = 0.4$ $(iv)\ P(B/A) = 0.5$ **(EX. 12.3 – 10(iii))**

3 Marks

1. Five mangoes and 4 apples are in a box. If two fruits are chosen at random, find the probability that (i) one is a mango and the other is an apple (ii) both are of the same variety. **(EX. 12.1 - 3)**
2. If $\bar{A}$ is the complementary event of A, then $P(\bar{A}) = 1 - P(A)$ **(TH. 12.4)**
3. If A and B are two events associated with a random experiment for which $P(A) = 0.35, P(A\ or\ B) = 0.85$, and $P(A\ and\ B) = 0.15$. Find $(i)\ P(only\ B)$ $(ii)\ P(\bar{B})$ $(iii)\ P(only\ A)$ **(EX. 12.2 - 2)**
4. If $P(A) = 0.6, P(B) = 0.5$, and $P(A \cap B) = 0.2$, Find $(i)\ P(A/B)$ $(ii)\ P(\bar{A}/B)$ $(iii)\ P(A/\bar{B})$. **(Eg. 12.16)**

5. If A and B are two independent events such that $P(A \cup B) = 0.6, P(A) = 0.2$, find $P(B)$. **(EX. 12.3 - 3)**
6. If $P(A) = 0.5, P(B) = 0.8$ and $P(B/A) = 0.8$, find $P(A\ /\ B)$ and $P(A \cup B)$. **(EX. 12.3 - 4)**
7. If for two events A and $B, P(A) = \frac{3}{4}, P(B) = \frac{2}{5}$ and $A \cup B = S$ (sample space), find the conditional probability $P(A/B)$. **(EX. 12.3 - 5)**

5 Marks

SET -1

1. Suppose ten coins are tossed. Find the probability to get (i) exactly two heads (ii) at most two heads (iii) at least two heads **(Eg. 12.4)**
2. Three candidates X, Y and Z are going to play in a chess competition to win $FIDE$ (World Chess Federation) cup this year. X is thrice as likely to win as Y and Y is twice as likely as to win Z. Find the respective probability of X, Y and Z to win the cup. **(Eg. 12.7)**
3. If A and B are independent then (i) $\bar{A}$ and $\bar{B}$ are independent. (ii) A and $\bar{B}$ are independent. $(iii)\bar{A}$ and B are also independent. **(TH. 12.8)**
4. A coin is tossed twice. Events E and F are defined as follows E =Head on first toss, F = Head on second toss. Find $(i) P(E \cup F\ )(ii) P\left(\frac{E}{F}\right)$ (iii) $P(\bar{E}/F)$ (iv) Are the events E and F independent? **(Eg. 12.19)**
5. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane in the first, second, third, and fourth shot are respectively 0.2, 0.4, 0.2 and 0.1. Find the probability that the gun hits the plane. **(Eg. 12.21)**
6. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact? **(Eg. 12.22)**

SET -2

7. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$ $and \frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it? **(EX. 12.3 - 6)**
8. A construction company employs 2 executive engineers. Engineer−1 does the work for 60% of jobs of the company. Engineer−2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer−1 does the work is 0.03, whereas the probability of an error in the work of engineer−2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work? **(Eg. 12.27)**
9. The chances of X, Y and Z becoming managers of a certain company are 4: 2: 3. The probabilities that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.4 respectively. If the bonus scheme has been introduced, what is the probability that Z was appointed as the manager? **(Eg. 12.28)**

10. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N? **(Eg. 12.29)**

11. (3) A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production, (i) find the probability that the selected pipe is a defective one. (ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y ? **(EX. 12.4 - 3)**

12. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television. **(EX. 12.4 - 5)**