Dalmia Hr. Sec. School. Dalmiapuram.
Public Examination – May – 2022
–: XII – Business maths key :–

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PART–I

1. (a) $\begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$

2. (d) consistent and has a unique solution.
(ஒருங்கமைவு உடையது மற்றும் ஒரே ஒரு தீர்வு பெற்றுள்ளது.)

3. (c) $1/2$

4. (b) $MC - MR = 0$

5. (a) $-\frac{1}{\eta_d}$

6. (d) $9/2$

7. (b) 2 and 6

8. (b) $x = vy$

9. (a) $f(x+h) - f(x)$

10. (c) $f(a) - f(a-h)$

11. (b) Zero (பூஜ்ஜியம்)

12. (a) $1/15$

13. (c) $2.5e^{-1}$

14. (c) $1/81$

15. (d) finite subset
(வரம்புறு உட்கணம்)

16. (d) random sample
(சமவாய்ப்பு கூறு)

17. (a) $\frac{\sigma}{\sqrt{n}}$

18. (a) Either positive or Negative.
(மிகை அல்லது குறை)

19. (b) Fisher Index number
(∴ பிஷர் குறியீட்டு எண்)

20. (d) all of the above
(மேற்கூறிய அனைத்தும்.)

PART – II

21. $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{pmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ $R_3 \to R_3 - R_2$

$\therefore \rho(A) = 2$

22. $f'(x) = 8x^3 - 2x$

$\int f'(x)\,dx = \int (8x^3 - 2x)\,dx$

$f(x) = 8\frac{x^4}{4} - 2\frac{x^2}{2} + C$

$f(x) = 2x^4 - x^2 + C$

Given: $f(2) = 8 \Rightarrow 8 = 2(2)^4 - 2^2 + C$

$\Rightarrow C = -20$

$\therefore f(x) = 2x^4 - x^2 - 20$

23. $y = mx \to (1)$ $\{m\}$

$\frac{dy}{dx} = m$

$\therefore (1) \Rightarrow y = \left(\frac{dy}{dx}\right)x$

24. LHS:

$[E^{-1}(\Delta)]x^3 = \Delta[E^{-1}(x^3)]$

$= \Delta(x-1)^3$

$= (x-1+1)^3 - (x-1)^3$

$= x^3 - (x^3 - 3x^2 + 3x - 1)$

$= 3x^2 - 3x + 1$

$=$ RHS

Hence proved.

25. $S = \{(1,1), (1,2) \ldots (6,6)\}$

$n(S) = 36$

Let X be the sum of the upturned faces.

$\therefore X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.$

$\therefore P(X=2) = \frac{1}{36}$, $P(X=3) = \frac{2}{36}$

$P(X=4) = \frac{3}{36}$, $P(X=5) = \frac{4}{36}$,

$P(X=6) = \frac{5}{36}$, $P(X=7) = \frac{6}{36}$

$P(X=8) = \frac{5}{36}$, $P(X=9) = \frac{4}{36}$

$P(X=10) = \frac{3}{36}$, $P(X=11) = \frac{2}{36}$

$P(X=12) = \frac{1}{36}$

$\therefore$ The p.m.f is

X	2	3	4	5	6
P(x)	1/36	2/36	3/36	4/36	5/36

X	7	8	9	10	11	12
P(x)	6/36	5/36	4/36	3/36	2/36	1/36

(26) $E(X) = \sum x\,p(x)$

$= (0\times0.2) + (1\times0.1) + (2\times0.4) + (3\times0.3)$

$E(X) = 1.8$

(27) Mean $= np = 20 \rightarrow (1)$

$SD = \sqrt{npq} = 4$

$npq = 16 \rightarrow (2)$

$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{16}{20}$

$q = \frac{4}{5}$

$\therefore p = 1 - \frac{4}{5} = \frac{1}{5}$

From (1), $np = 20$

$n = \frac{20}{p} = \frac{20}{1/5}$

$n = 100$

(28) $n = 50$, $S = 6.3$, $\sigma = 6$

$S.E = \sqrt{\frac{\sigma^2}{2n}} = \sqrt{\frac{6^2}{2(50)}} = 0.6$

(29) (i) It is an important tool for the formulating decision and management policies.

(ii) It helps in studying the trends and tendencies

(iii) It determines the inflation and deflation in an economy.

(30) $y = x^3 + 3x$

$y(1) = 1^3 + 3(1) = 4$

$y(2) = 2^3 + 3(2) = 14$

$y(3) = 3^3 + 3(3) = 36$

$y(4) = 4^3 + 3(4) = 64 + 12 = 76$

$y(5) = 5^3 + 3(5) = 125 + 15 = 140$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	4				
		10			
2	14		12		
		22		6	
3	36		18		0
		40		6	
4	76		24		
		64			
5	140				

PART-III

(31) Transition Probability matrix

$$T = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} \end{array}$$

At equilibrium, $(A\ B)T = (A\ B)$

$(A\ B)\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} = (A\ B)$

$0.9A + 0.3B = A$

$0.9A + 0.3(1-A) = A$

$0.9A + 0.3 - 0.3A = A$

$0.6A + 0.3 = A$

$0.4A = 0.3$

$A = \frac{0.3}{0.4} = \frac{3}{4}$ $\therefore A = 75\%$ $\therefore B = 25\%$

32)

$$\int_{-1}^{1} \frac{2x+3}{x^2+3x+7}\,dx$$

$$= \left[\log(x^2+3x+7)\right]_{-1}^{1}$$

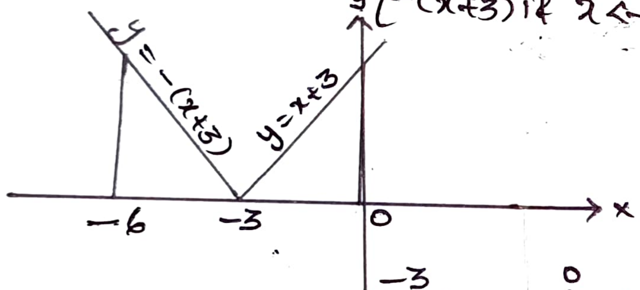
$$= \log(1+3+7) - \log(1-3+7)$$

$$= \log 11 - \log 5$$

$$= \log \frac{11}{5}$$

33)

$$y = |x+3| = \begin{cases} x+3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$$



$$\text{Required Area} = \int_{-6}^{-3} y\,dx + \int_{-3}^{0} y\,dx$$

$$= \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$$

$$= -\left[\frac{(x+3)^2}{2}\right]_{-6}^{-3} + \left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$$

$$= -\left[0 - \frac{9}{2}\right] + \left[\frac{9}{2} - 0\right]$$

$= 9$ sq. units.

34)

$$(1-x)\,dy - (1+y)\,dx = 0$$

$$(1-x)\,dy = (1+y)\,dx$$

$$\int \frac{1}{1+y}\,dy = \int \frac{1}{1-x}\,dx$$

$$\log(1+y) = \frac{\log(1-x)}{-1} + \log C$$

$$\log(1+y) + \log(1-x) = \log C$$

$$\log[(1+y)(1-x)] = \log C$$

$$(1+y)(1-x) = C$$

35)

$$\Delta^4(y_0) = 0$$

$$(E-1)^4 y_0 = 0$$

$$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$$

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$81 - 4y_3 + 6(9) - 4(3) + 1 = 0$$

$$81 + 54 - 11 = 4y_3$$

$$4y_3 = 124$$

$$y_3 = 31$$

36)

(i) $E(a) = a$, $a \to$ constant

(ii) $E(aX) = aE(X)$

(iii) $E(aX+b) = aE(X) + b$

(iv) If $X > 0$ then $E(X) > 0$

(37) The required conditions are

(i) n, the number of trials is indefinitely large

(ie) $n \to \infty$

(ii) p, the constant probability of success in each trial is very small.

(ie), $p \to 0$

(iii) $np = \lambda$ is finite.

Thus $p = \frac{\lambda}{n}$ and $q = 1 - \frac{\lambda}{n}$

(38)

Commodities	Price 2012 (P_0)	Price 2015 (P_1)	Weight (V)	$P = \frac{P_1}{P_0} \times 100$	PV
Rice	250	280	10	112	1120
Wheat	70	85	5	121.43	607.15
Corn	150	170	6	113.33	679.98
oil	25	35	4	140	560
Dhal	85	90	3	105.88	317.64
		Total	28		3284.77

$$C.L.I = \frac{\Sigma PV}{\Sigma V} = \frac{3284.77}{28}$$

$$= 117.31$$

C.L.I is increased for Years 2012 by 17.31% compared to 2015

(39)

Strategy	States of nature E_1	States of nature E_2	Maximum	Minimum
S_1	40	60	60	40
S_2	10	-20	10	-20
S_3	-40	150	150	-40

(ii) Min (60, 10, 150) = 10

The best alternative = S_2

(i) Max (40, -20, -40) = 40

The best alternative = S_1

(40)

$X = +6, -3$

$P(X=6) = 1/2$

$P(X=-3) = 1/2$

$X=x$	+6	-3
$P(X=x)$	1/2	1/2

$$E(X) = \Sigma x\, p(x).$$
$$= 6 \times \frac{1}{2} - 3 \times \frac{1}{2}$$
$$E(X) = \frac{6}{2} - \frac{3}{2} = \frac{3}{2} = 1.5$$
$$E(X^2) = \Sigma x^2 p(x)$$
$$= 36 \times \frac{1}{2} + 9 \times \frac{1}{2}$$
$$= \frac{36}{2} + \frac{9}{2}$$
$$E(X^2) = \frac{45}{2}$$
$$V(X) = E(X^2) - [E(X)]^2$$
$$= \frac{45}{2} - \left(\frac{3}{2}\right)^2$$
$$= 45/2 - 9/4 = \frac{90}{4} = 22.5$$

Part-IV

41. (a)

Let the amount of investment in each bond be ₹x, ₹y and ₹z respectively.

Given: $x+y+z=5000$

$6x+7y+8z=35800$

$6x+7y-8z=7000$

$$\begin{pmatrix}1&1&1\\6&7&8\\6&7&-8\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5000\\35800\\7000\end{pmatrix}$$

$$(A,B)=\begin{pmatrix}1&1&1&5000\\6&7&8&35800\\6&7&-8&7000\end{pmatrix}$$

$$\sim\begin{pmatrix}1&1&1&5000\\0&1&2&5800\\0&1&-14&-23000\end{pmatrix}\begin{array}{l}R_2\to R_2-6R_1\\R_3\to R_3-6R_1\end{array}$$

$$\sim\begin{pmatrix}1&1&1&5000\\0&1&2&5800\\0&0&-16&-28800\end{pmatrix}R_3\to R_3-R_2$$

$\rho(A)=\rho(A,B)=n=3$

$$\begin{pmatrix}1&1&1\\0&1&2\\0&0&-16\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5000\\5800\\-28800\end{pmatrix}$$

$x+y+z=5000 \to (1)$

$y+2z=5800 \to (2)$

$-16z=-28800 \to (3)$

After solving (1), (2) and (3)

$x=1000$, $y=2200$ and

$z=1800$

41. (b)

(i) $\sum p(x)=1$

$K+2K+2K+3K+K^2+2K^2+7K^2+K=1$

$10K^2+9K-1=0$

$(10K-1)(K+1)=0$

$10K-1=0$

$10K=1$

$\boxed{K=\frac{1}{10}}$; $K=-1$ not possible.

(ii) $P(X<6)=\sum_{x=0}^{5}p(x)$

$=K^2+8K$

$=\left(\frac{1}{10}\right)^2+8\left(\frac{1}{10}\right)=\frac{81}{100}$

$P(X\geq 6)=\sum_{x=6}^{7}p(x)=9K^2+K$

$=\frac{9}{100}+\frac{1}{10}=\frac{19}{100}$

$P(0<x<5)=\sum_{x=1}^{4}p(x)=8K=\frac{8}{10}$

(iii) $P(X\leq x)=F(x)$

$F(0)=0$; $F(1)=\frac{1}{10}$

$F(2)=\frac{3}{10}$; $F(3)=\frac{5}{10}=\frac{1}{2}$

$F(4)=\frac{8}{10}>\frac{1}{2}$

$\therefore$ min value of $x=4$.

42 (a)

$$\int x^3 e^{3x}\,dx = uv - u'v_1 + u''v_2 - u'''v_3$$

Let $u = x^3$ | $dv = e^{3x}dx$

$u' = 3x^2$ | $v = \frac{e^{3x}}{3}$

$u'' = 6x$ | $v_1 = \frac{e^{3x}}{9}$

$u''' = 6$ | $v_2 = \frac{e^{3x}}{27}$

$v_3 = \frac{e^{3x}}{81}$

$$\therefore \int x^3 e^{3x}\,dx = x^3\frac{e^{3x}}{3} - 3x^2\frac{e^{3x}}{9} + 6x\frac{e^{3x}}{27} - 6\frac{e^{3x}}{81} + C$$

$$= e^{3x}\left[\frac{x^3}{3} - \frac{x^2}{3} + \frac{2x}{9} - \frac{2}{27}\right] + c$$

42 (b)

$p = \frac{18}{100} = 0.18$; $q = 1 - 0.18 = 0.82$

and $n = 4$

(i) $P(X=x) = nC_x p^x q^{n-x}$

$P(X=1) = 4C_1 (0.18)^1 (0.82)^3$

$P(X=1) = 0.39698$

(ii) $P(X=0) = 4C_0 (0.18)^0 (0.82)^4$

$P(X=0) = 0.45212$

(iii) $P(X \le 2) = P(0) + P(1) + P(2)$

$= 0.45212 + 0.39698 + 4C_2 (0.18)^2 (0.82)^2$

$= 0.8491 + 0.13071$

$= 0.97981$

43 (a)

$$\eta_d = \frac{p + 2p^2}{100 - p - p^2}$$

$$-\frac{p}{x}\frac{dx}{dp} = \frac{p(2p+1)}{100 - p - p^2}$$

$$-\frac{dx}{x} = \frac{-(2p+1)}{p^2 + p - 100}dp$$

$$\int \frac{dx}{x} = \int \frac{2p+1}{p^2 + p - 100}dp$$

$$\log x = \log[p^2 + p - 100] + \log k$$

$$\therefore x = k[p^2 + p - 100]$$

when $x = 70$, $p = 5$

$\therefore 70 = k(25 + 5 - 100)$

$\Rightarrow k = -1$

Hence $x = 100 - p - p^2$

$R = px$

$\therefore$ Revenue $= p(100 - p - p^2)$

43 (b)

Let μ is the mean length of the components in the population.

Confidence interval is

$$\left(\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

Here, $\sigma = 1.6$ $Z_{\alpha/2} = 1.96$

$\bar{x} = 90$ and $n = 64$

$\therefore$ confidence interval is

$$\left[90 - \left(1.96 \times \frac{1.6}{8}\right) < \mu < 90 + \left[1.96 \times \frac{1.6}{8}\right]\right]$$

$$\Rightarrow \left[90 - (1.96 \times 0.2) < \mu < (90 + 1.96 \times 0.2)\right]$$

$$\Rightarrow (89.61 < \mu < 90.39)$$

(ie) $(89.61, 90.39)$

$\therefore$ 95% confidence interval ensures acceptance of the component by the consumer.

44 (a)

$$(y^3 - 2yx^2)\,dx + (2xy^2 - x^3)\,dy = 0$$

$$\frac{dy}{dx} = -\left(\frac{y^3 - 2yx^2}{2xy^2 - x^3}\right)$$

Put $y = vx$ then

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = -\left(\frac{v^3x^3 - 2vx\,x^2}{2x\,v^2x^2 - x^3}\right)$$

$$v + x\frac{dv}{dx} = \frac{-x^3(v^3 - 2v)}{x^3(2v^2 - 1)}$$

$$x\frac{dv}{dx} = -\frac{v^3 - 2v}{2v^2 - 1} - v$$

$$x\frac{dv}{dx} = \frac{-3v^3 + 3v}{2v^2 - 1}$$

$$\int\frac{(2v^2 - 1)\,dv}{v^3 - v} = -3\int\frac{dx}{x}$$

$$\int\frac{2v^2 - 1}{v(v+1)(v-1)}\,dv = -3\int\frac{dx}{x}$$

$$\int\left(\frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1}\right)dv = -3\int\frac{dx}{x}$$

$$\int\left(\frac{1}{v} + \frac{1/2}{v+1} + \frac{1/2}{v-1}\right)dv = -3\int\frac{dx}{x}$$

$$\log v + \frac{1}{2}\log(v+1) + \frac{1}{2}\log(v-1) = -3\log x + \log C$$

$$\log v + \frac{1}{2}\log|v^2 - 1| + 3\log x = \log C$$

Multiply by 2,

$$2\log v + \log(v^2-1) + 6\log x = 2\log c$$

$$\log[v^2(v^2-1)x^6] = \log c^2$$

$$v^2(v^2-1)x^6 = c^2$$

$$\frac{y^2}{x^2}\left[\frac{y^2}{x^2}-1\right]x^6 = c^2$$

$$\frac{y^2}{x^2}\left[\frac{y^2-x^2}{x^2}\right]x^6 = c^2$$

$$x^2y^2(y^2-x^2) = c^2 \rightarrow (1)$$

The curve passes through $(1,2)$ (ie) put $x=1$, $y=2$ in (1)

$$(1)^2(2)^2[2^2-1^2] = c^2$$

$$c^2 = 12$$

$\therefore$ (1) $\Rightarrow$ $x^2y^2(y^2-x^2) = 12$

(or)

$$xy\sqrt{y^2-x^2} = 2\sqrt{3}$$

45 (a)

(i) $T = \begin{array}{c|cc} & A & B \\ \hline A & 0.65 & 0.35 \\ B & 0.45 & 0.55 \end{array}$

$$(A \quad B)\,T = (A \quad B)$$

$$(0.15 \quad 0.85)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$$

$$= \begin{array}{cc} A & B \\ (0.48 & 0.52) \end{array}$$

$\therefore$ $A = 48\%$ & $B = 52\%$

At equilibrium,

$$(A \quad B)\,T = (A \quad B)$$

$$(A \quad B)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} = (A \quad B)$$

$$0.65A + 0.45B = A$$

$$0.65A + 0.45(1-A) = A$$

$$0.45 = 0.80A$$

$$\therefore A = \frac{0.45}{0.80} = 0.5625$$

$$\therefore B = 1 - 0.5625$$

$$B = 0.4375$$

$\therefore$ $A = 56.25\%$

$B = 43.75\%$

45 (b)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$\boxed{1951}$	$\boxed{35}$			
		$\boxed{7}$		
1961	42		$\boxed{9}$	
		16		$\boxed{1}$
1971	58		10	
		26		
1981	84			

The Newton's forward formulae is

$$y = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0.$$

where $n = \frac{x - x_0}{h} = \frac{1955-1951}{10}$

$$n = \frac{4}{10} = 0.4$$

$$\therefore y = 35 + \frac{0.4}{1!}(7) + \frac{0.4(0.4-1)}{2!}(9) + \frac{0.4(0.4-1)(0.4-2)}{3!}(1)$$

$$\therefore y = 35 + 2.8 - 1.08 + 0.064$$

$y = 36.784$ lakhs.

46 (a)

By partial fraction,

$$\frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{1}{x+3} + \frac{2x}{x^2+1}$$

$$\therefore I = \int \frac{3x^2+6x+1}{(x+3)(x^2+1)} = \int \frac{dx}{x+3} + \int \frac{2x}{x^2+1}$$

$$\Rightarrow I = \log|x+3| + \log|x^2+1| + C$$

$$I = \log|(x+3)(x^2+1)| + C$$

$$I = \log|x^3+3x^2+x+3| + C$$

46 (b)

Total Demand = 90
Total Supply = 90

$\therefore$ It is balanced Transportation problem.

Final Allocation

	D1	D2	D3	D4	
O1	(16) 5	(3) 3	6	2	19/3/0
O2	4	(15) 7	(22) 9	1	37/22/0
O3	3	4	(9) 7	(25) 5	34/25/0
	16/0	18/15/0	31/9/0	25/0	

Transportation Shedule:

$O_1 \to D_1$;

$O_1 \to D_2$;

$O_2 \to D_2$;

$O_2 \to D_3$;

$O_3 \to D_3$;

$O_3 \to D_4$

Transportation cost =

$(16\times5) + (3\times3) + (15\times7) + (22\times9) + (9\times7) + (25\times5)$

$= 80 + 9 + 105 + 198 + 63 + 125$

$=$ Rs. 580

47 (a).

At equilibrium price,

$P_d = P_s$

$25 - 3x = 5 + 2x$

$x = 4 \quad \therefore x_0 = 4$

$P_d = 25 - 3(4) = 13$

$\therefore P_0 = 13$

$\therefore P_0 x_0 = 13 \times 4 = 52$

$CS = \int_0^{x_0} f(x)dx - P_0 x_0$

$= \int_0^4 (25 - 3x)dx - 52$

$= \left(25x - 3\frac{x^2}{2}\right)_0^4 - 52$

$= 25(4) - \frac{3(4)^2}{2} - 52$

$CS = 24$ units.

$PS = P_0 x_0 - \int_0^{x_0} g(x)dx$

$= 52 - \int_0^4 (5 + 2x)dx$

$= 52 - [5x + x^2]_0^4$

$= 52 - [5(4) + 16]$

$= 52 - 36$

$PS = 16$ units

47 (b).

Commodity	Price 2000 P_0	Price 2010 P_1	Quantity 2000 q_0	Quantity 2010 q_1	P_0q_0	P_0q_1	P_1q_0	P_1q_1
A	12	14	18	16	216	192	252	224
B	15	16	20	15	300	225	320	240
C	14	15	24	20	336	280	360	300
D	12	12	29	23	348	276	348	276
Total					1200	973	1280	1040

Laspeyre's price index $= \frac{\sum P_1q_0}{\sum P_0q_0} \times 100$

$P_{01}^L = \frac{1280}{1200} \times 100 = 106.667$

Paasche's price index $= \frac{\sum P_1q_1}{\sum P_0q_1} \times 100$

$P_{01}^P = \frac{1040}{973} \times 100 = 106.886$

Fisher's price index $= P_{01}^F$

$= \sqrt{\frac{\sum P_1q_0}{\sum P_0q_0} \times \frac{\sum P_1q_1}{\sum P_0q_1}} \times 100$

$= \sqrt{\frac{1280 \times 1040}{1200 \times 973}} \times 100$

$P_{01}^F = 106.776$