M. M. Hr. Sec. School, Thirunagar, Madurai-6.

Public Examination - 2022.

MATHEMATICS. - Answer key.

TYPE-A

	option	Answer
1	b.	1
2	c.	F(x) is real valued decreasing function.
3	d.	$\frac{1}{(x+1)^2}\,dx$.
4	d.	$\frac{1}{10100}$
5	d.	$\pi/6$.
6	a.	19
7	b.	$-9/2$
8	a.	x^2+y^2
9	c.	9
10	a.	0
11	d.	-3
12	d.	$x^2+y^2+6x+8y+16=0$.
13	d.	$y = ce^{-\int p\,dx}$
14	c.	$2/27$.
15	a.	(1, 0)
16	a.	$\pi/2$
17	a.	multiplication.
18	a.	If A is a sq. matrix of order n, λ is a scalar, $adj(\lambda A) = \lambda^n adj A$.
19	c.	cot x
20	b.	24.

TYPE-B.

Q.No.	option	Answer.
1.	d.	$\frac{1}{10100}$
2.	d.	$\frac{1}{(x+1)^2}\,dx$.
3	a.	If A is a sq. matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj A$.
4	b.	24
5	c.	$2/27$
6.	d.	$y = ce^{-\int p\,dx}$.
7	a.	x^2+y^2
8	a	19
9	c.	cot x.
10.	a.	multiplication.
11.	c.	F(x) is real valued decreasing function
12.	c.	9
13	a.	0
14	b.	$-9/2$
15	d.	$x^2+y^2+6x+8y+16=0$
16.	b.	1
17	a.	$\pi/2$
18	a	(1, 0)
19.	d.	$\pi/6$.
20.	d.	-3.

PART-II

21. $z = x+iy$, $\bar{z} = x-iy$

$z+\bar{z} = 2x$ $\quad z-\bar{z} = 2iy$

$\frac{z+\bar{z}}{2} = Re(z)$ $\quad \frac{z-\bar{z}}{2i} = Im(z)$

22. $2-\sqrt{3}$ is a root, another root is $2+\sqrt{3}$

$S.R = 4$, $\quad P.R = 4-3 = 1$.

$x^2 - 4x + 1 = 0$.

23. $\tan^{-1}(\sqrt{3}) = y$, $\tan y = \sqrt{3}$, $\quad y = \pi/3 \in (-\pi/2, \pi/2)$

24. Slope of $y = x$ is 1. $\quad y = x^3 - 3x^2 + x - 2$

$y' = 3x^2 - 6x + 1 = 1$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, (0, -2)$

$x = 2, (2, -4)$

25. $f(x) = x^2 + 3x$,

$df = (2x+3)\,dx = (4+3)(0.1) = 0.7$

26. $y = Ae^x + Be^{-x}$

$\frac{dy}{dx} = Ae^x - Be^{-x}$.

$\frac{d^2y}{dx^2} = Ae^x + Be^{-x} = y$

$\frac{d^2y}{dx^2} - y = 0$.

27. $\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{\sqrt{1-x^2}}\,dx$, $\quad \sin^{-1}y = \sin^{-1}x + c$.

28. $\sum_{i=1}^{6} f(x_i) = 1 \Rightarrow 30k = 1$, $\quad k = \frac{1}{30}$.

29. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$ $\quad n(S) = 8$.

Let its R.V. x denote its no. of tails.

$x : 0, 1, 2, 3$.

$x : x$	0	1	2	3	Total
No. of inverse images.	1	3	3	1	8

30. $p = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$

$= \frac{|7|}{\sqrt{9+36+4}} = \frac{7}{7} = 1$

PART - III.

31. $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ (applying elementary transformation)

$\rho(A) = 3$.

32. $\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$AX = B$.

$|A| = 4 \neq 0$, $\text{adj } A = \begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}$

$A^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}$

$X = A^{-1}B$.

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $x = 1$, $y = 4$

34. 1 is a root.

1	2	−9	10	−3
	0	2	−7	3
	2	−7	3	|0

$2x^2 - 7x + 3 = 0$

$(x - \frac{1}{2})(x - 3) = 0$

∴ The roots are $1, \frac{1}{2}, 3$.

35. $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r} = -2\hat{i} + \hat{k}$

$\vec{r} \times \vec{F} = -\hat{i} - 2\hat{k}$

$|\vec{\tau}| = \sqrt{5}$

$\text{DCs} = \left(\frac{-1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}\right)$

36. $\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$, ∞/∞

By L'Hospital Rule,

$\lim_{x \to \infty} \frac{4x}{2x - 5} = \frac{4}{2 - 0} = 2$

37. Cross Section

Area $A = \pi r^2$

$dA = 2\pi r\, dr$

$= 2\pi(2)(0.1)$

$= (0.4)\pi \text{ mm}^2$

38. $\int_0^{\pi/3} \frac{\sec x \tan x}{1 + \sec^2 x}\, dx$

$\sec x = u$

$\sec x \tan x\, dx = du$

$x = 0,\ u = 1$

$x = \pi/3,\ u = 2$

$\int_0^2 \frac{1}{1 + u^2}\, du$

$= \tan^{-1}(2) - \tan^{-1}(1)$

$= \tan^{-1}(2) - 1$.

39. $a * b = a + b + ab - 7 \in R$, $\forall a, b \in R$

∴ $*$ is a binary operation on R

$3 * \left(\frac{-7}{15}\right) = 3 - \frac{7}{15} - 3\left(\frac{7}{15}\right) - 7$

$= -7/15 - 7/5 - 4$

$= -88/15$

40. $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$(x + 4)(x + 1) + (y + 2)(y + 1) = 0$

$x^2 + 5x + 4 + y^2 + 3y + 2 = 0$

$x^2 + y^2 + 5x + 3y + 6 = 0$.

PART – IV

41. a) $\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{vmatrix} = 0.$

As $\Delta = 0$, $x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$ } undefined

Cramer's Rule is not applicable.

(or)

b) $f(x) = 4x^6 - 6x^4$

$f'(x) = 24x^5 - 24x^3$

$f''(x) = 120x^4 - 72x^2$

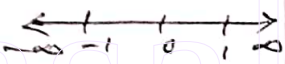
$= 24x^2(5x^2 - 3)$

$f'(x) = 0 \Rightarrow x = -1, 0, 1.$ (critical nos)

$f''(-1) = 48$

$f''(0) = 0 \rightarrow$ no information.

$f''(1) = 4.$



At $x = -1$, local Min $= -2$

$x = 0$, local Max $= 0$

$x = 1$, local Min $= -2$

42. $|z+i| = |z-1|$

$\Rightarrow x^2 + (y+1)^2 = (x-1)^2 + y^2$

$x + y = 0$, which is the locus of $z = x + iy$.

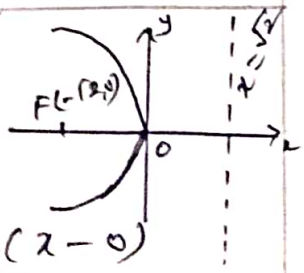
(or)

$$I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx = \int_0^a \frac{f(a-x)}{f(a-x)+f(x)}\,dx.$$

$$2I = \int_0^a dx = [x]_0^a = a$$

$$I = a/2$$

43. open leftward axis x-axis's. vertex $(0,0)$



$(y-0)^2 = -4\sqrt{2}(x-0)$

$y^2 = -4\sqrt{2}x.$

(or)

b) $\alpha = \cot^{-1}(1) = \pi/4$

$\beta = \sin^{-1}(-\sqrt{3}/2) = -\pi/3$

$\gamma = \sec^{-1}(-\sqrt{2})$

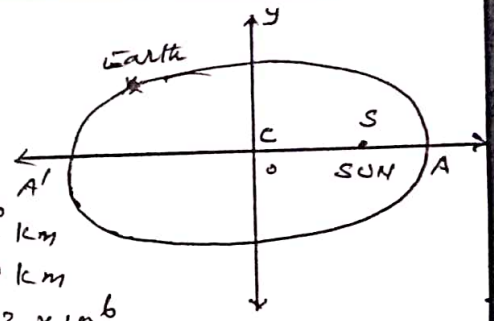
$\sec\gamma = -\sqrt{2}$, $\cos\gamma = -1/\sqrt{2}$

$\cos\gamma = \cos(\pi - \pi/4)$

$\gamma = 3\pi/4$

$\therefore \pi/4 - \pi/3 - 3\pi/4 = -5\pi/6.$

44) a)



$AS = 94.5 \times 10^6$ km

$SA' = 152 \times 10^6$ km

$a + c = 152 \times 10^6$

$a - c = 94.5 \times 10^6$

$2c = 57.5 \times 10^6 = 575 \times 10^5$ km.

Distance from the sun to the other focus $= 575 \times 10^5$ km.

(or)

b) $\hat{a} = \overrightarrow{OA}$, $\hat{b} = \overrightarrow{OB}$, unit vectors making angles A, B with x-axis respectively.

$\hat{a} = \cos A\,\hat{i} - \sin A\,\hat{j}$

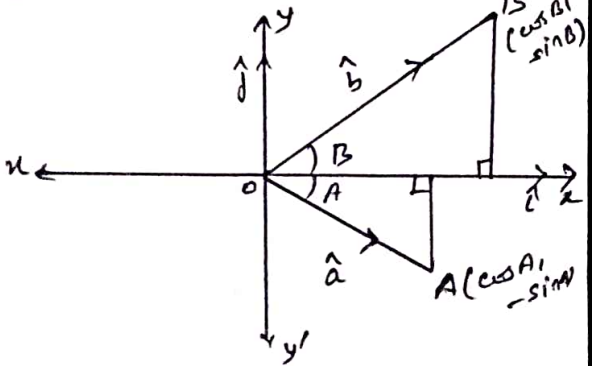
$\hat{b} = \cos B\,\hat{i} + \sin B\,\hat{j}$ — (1)

$\hat{a} \times \hat{b} = (\sin A\cos B + \cos A \sin B)\,\hat{k}$

By Defn $\hat{a} \times \hat{b} = |\hat{a}|\,|\hat{b}| \sin(A+B)\,\hat{k}$

$= \sin(A+B)\,\hat{k}$ — (2)

$\sin(A+B) = \sin A\cos B + \cos A \sin B.$



45) a) $\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{c}$

$= (1-s)(2\hat{i}+2\hat{j}+\hat{k}) + s(9\hat{i}+3\hat{j}+6\hat{k}) + t(2\hat{i}+6\hat{j}+6\hat{k})$

s, t are scalars

C.E $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0.$

$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$

$3x + 4y - 5z - 9 = 0.$

(or)

b) $y = x^2$ $\quad x = y^2$

$\frac{dy}{dx} = 2x = m_1$ $\quad 1 = 2y\, dy/dx$

$\frac{1}{2y} = \frac{dy}{dx} = m_2$

Pt. of Xn. (1, 1)

$m_1 = 2, \quad m_2 = 1/2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2 - 1/2}{1 + 2 \cdot 1/2}\right|$

$= 3/4$

$\theta = \tan^{-1}(3/4)$

46. a) i) $P(X < 3) = 1/2 = F(3)$

ii) $P(2 < X < 4)$

$= F(4) - F(2)$

$= 3/4 - 1/4$

$= 1/2$

iii) $P(3 \le X)$

$= P(X \ge 3) = 1 - P(X < 3)$

$= 1 - F(3)$

$= 1 - 1/2$

$= 1/2.$

(5)

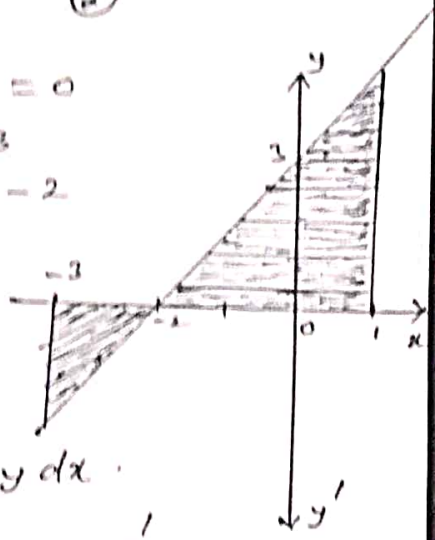
(or)

b) $3x - 2y + 6 = 0$

$x = 0, \quad y = 3$

$y = 0, \quad x = -2$

$y = \frac{3x+6}{2}.$



Required Area

$= \int_{-3}^{-2} (-y)\,dx + \int_{-2}^{1} y\,dx.$

$= -\int_{-3}^{-2} \left(\frac{3x+6}{2}\right) dx + \int_{-2}^{1} \left(\frac{3x+6}{2}\right) dx.$

$= -\frac{1}{2}\left[\left(\frac{12}{8} - 12\right) - \left(\frac{27}{2} - 18\right)\right] + \frac{1}{2}\left[\left(\frac{3}{2} + 6\right) - \left(\frac{12}{2} - 12\right)\right]$

$= \frac{15}{2}$

47) a) $(1+x^2)\frac{dy}{dx} = 1 + y^2$

$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + c.$

(or)

P	q	r	q→r	P→(q→r)	p∧q	(p∧q)→r
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T
① = ②				①		②

— V. Gurunathan (P.G. ASST)

— T. Kathiresan (B.T. ASST.)