PUBLIC EXAM - 2022

MATHEMATICS ANSWER KEY

R. VIJAYARAGAVAN M.Sc., M.Ed., D.T.Ed., M.Sc (Psy)., M.Sc (Yoga).
PG ASST IN MATHEMATICS - PATTUKKOTTAI.

	TYPE - A		TYPE - B
1	b) 1	1	d) $1/10100$
2	c) f(x) is Real valued dec function	2	d) $1/(x+1)^3\,dx$
3	d) $1/(x+1)^3\,dx$	3	a) If A is a sq...
4	d) $1/10100$	4	b) 24
5	d) $\pi/6$	5	c) $2/27 - \int p\,dx$
6	a) 19	6	d) $y = ce$
7	b) $-9/7$	7	a) x^2+y^2
8	a) x^2+y^2	8	a) 19
9	c) 9	9	c) $\cot x$
10	a) 0	10	a) multiplication
11	d) -3	11	c) f(x) is real...
12	d) $x^2+y^2+6x+8y+16=0$	12	c) 9
13	d) $y = ce^{-\int p\,dx}$	13	a) 0
14	c) $2/27$	14	b) $-9/7$
15	a) (1,0)	15	d) $x^2+y^2+6x+8y+16=0$
16	a) $\pi/2$	16	b) 1
17	a) multiplication	17	a) $\pi/2$
18	a) If A is a square matrix of order n, λ is scalar adj $(\lambda A) = \lambda^{n-1}$ adj A	18	a) (1,0)
19	c) $\cot x$	19	d) $\pi/6$
20	b) 24	20	d) -3

PART - II

21. Let $z = x + iy$

$\bar{z} = x - iy$

$z + \bar{z} = 2x$ $\qquad z - \bar{z} = 2iy$

$\frac{z+\bar{z}}{2} = Re(z)$ $\qquad \frac{z-\bar{z}}{2i} = Im(z)$

22 Given $2 - \sqrt{3}$ is a root

another root is $2 + \sqrt{3}$

Sum of roots = 4

Product of roots = 1

eqn: $x^2 - 4x + 1 = 0$

23 Let $y = \tan^{-1}(\sqrt{3})$

$\tan y = \tan \pi/3$

$y = \pi/3 \in (-\pi/2, \pi/2)$

24. slope of $y = x$ is 1.

$y = x^3 - 3x^2 + x - 2$

$y' = 3x^2 - 6x + 1$ $\qquad x = 0, (0, -2)$

$3x^2 - 6x = 0$ $\qquad x = 2, (2, -4)$

$x^2 - 2x = 0$

$x(x-2) = 0$

25 $f(x) = x^2 + 3x$

$f'(x) = 2x + 3$

$dy = (2x+3)\,dx = (4+3)(0.1) = 0.7$

26 $y = Ae^{x} + Be^{-x}$

$\frac{dy}{dx} = Ae^{x} - Be^{-x}$

$\frac{d^2y}{dx^2} = Ae^{x} + Be^{-x} = y$

$$\boxed{\frac{d^2y}{dx^2} - y = 0}$$

27 $\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{\sqrt{1-x^2}}\,dx.$

$\sin^{-1} y = \sin^{-1} x + C$

28 $\sum_{i=1}^{6} f(x_i) = 1 \rightarrow 30k = 1 \Rightarrow \boxed{k = 1/30}$

29. Sample space $= \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

$X = 0, 1, 2, 3$

x	0	1	2	3	Total
No of inverse images	1	3	3	1	8

30. $$P = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|7|}{\sqrt{9+36+4}} = \frac{7}{7} = 1$$

PART - III

31. $$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\rho(A) = 3}$

32. $$\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$AX = B$$

$$|A| = 4 \neq 0 \qquad \text{adj } A = \begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}$$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$\boxed{x = 1}$

$\boxed{y = 4}$

33 Let $z_1 = 1+i$

$z_2 = 10-8i$

$z_3 = 11+6i$

$|z_1 - z_2| = |-9+9i| = 9\sqrt{2}$

$|z_1 - z_3| = |-10-5i| = \sqrt{5}\sqrt{5} = 5\sqrt{5}$

$5\sqrt{5} > 9\sqrt{2}$

$|z_1 - z_2| > |z_1 - z_3|$

$11+6i$ is closest to $1+i$.

34. 1 is a root

1	2	-9	10	-3
	0	2	-7	3
	2	-7	3	0

$2x^2 - 7x + 3 = 0$

$(x - 1/2)(x - 3) = 0$

Roots are $1, 1/2, 3$

35 $\vec{L} = \vec{r} \times \vec{F}$

$= -\hat{i} - 2\hat{k}$

$|\vec{L}| = \sqrt{5}$

Direction Cosines $(-1/\sqrt{5}, 0, -2/\sqrt{5})$

36 $\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$, $\frac{\infty}{\infty}$

By L'Hopital's rule

$\lim_{x \to \infty} \frac{4x}{2x - 5} = \frac{4}{2 - 0} = 2$

37. Cross Section Area $= \pi r^2$

$dA = 2\pi r\, dr$

$= 2\pi (2)(0.1)$

$= 0.4\pi \text{ mm}^2$

38. $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\, dx$

Let $\sec x = u$

$\sec x \tan x\, dx = du$

$x = 0 \quad u = 0$

$x = \pi/3 \quad u = 2$

$\int_0^2 \frac{1}{1+u^2}\, du$

$= \tan^{-1}(2) - \tan^{-1}(1)$

$= \tan^{-1}(2) - 1$

39. $a * b = a + b + ab - 7 \in R, \forall a, b \in R$

$*$ is binary operation on R

$3 * (-7/15) = 3 - 7/15 - 3(7/15) - 7$

$= -88/15$

40. $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$(x+4)(x+1) + (y+2)(y+1) = 0$

$\boxed{x^2 + y^2 + 5x + 3y + 6 = 0}$

PART - IV

A1 (a)

$$\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{vmatrix} = 0$$

As $\Delta = 0$

$x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$ } undefined

Cramer's Rule cannot be applicable.

A1 (b)

$f(x) = 4x^6 - 6x^4$

$f'(x) = 24x^5 - 24x^3$

$f''(x) = 120x^4 - 72x^2$

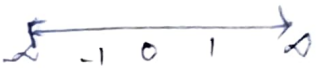
$= 24(5x^2 - 3)$

$f'(x) = 0 \Rightarrow x = -1, 0, 1$

$f''(-1) = 48$

$f''(0) = 0$

$f''(1) = 48$



At $x = -1$ local minimum $= -2$

$x = 0$ local max $= 0$

$x = 1$ local min $= -2$

A2 (a) $|z+i| = |z-1|$

$\Rightarrow x^2 + (y+1)^2 = (x-1)^2 + y^2$

$x + y = 0$

which is the locus of $z = x + iy$

A2 (b) $I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx$

$= \int_0^a \frac{f(a-x)}{f(a-x) + f(x)} dx$

$2I = \int_0^a dx \Rightarrow [x]_0^a = a$

$\boxed{I = a/2}$

A3 (a)

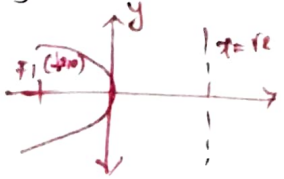
Open leftward

axis - x axis

vertex (0,0)

eqn: $(y-0)^2 = -4\sqrt{2}(x-0)$

$y^2 = -4\sqrt{2}x$



A3 (b)

$\alpha = \cot^{-1}(1) = \pi/4$

$\beta = \sin^{-1}(-\sqrt{3}/2) = -\pi/3$

$\gamma = \sec^{-1}(-\sqrt{2})$

$\sec\gamma = -\sqrt{2} \quad \cos\gamma = -1/\sqrt{2}$

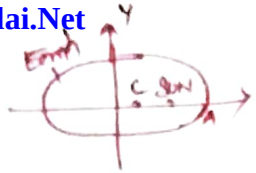
$\cos\gamma = \cos(\pi - \pi/4)$

$\gamma = 3\pi/4$

$\pi/4 - \pi/3 - 3\pi/4 = -5\pi/6$

A4

(a)



$AS = 94.5 \times 10^6$ km

$SA' = 152 \times 10^6$ km

$a + c = 152 \times 10^6$

$a - c = 94.5 \times 10^6$

$2c = 57.5 \times 10^6$

$= 575 \times 10^5$ km

Distance from the Sun to the other focus $= 575 \times 10^5$ km

A4

(b) $\vec{a} = \overrightarrow{OA}$ $\vec{b} = \overrightarrow{OB}$

Unit vectors.

Angles A and B

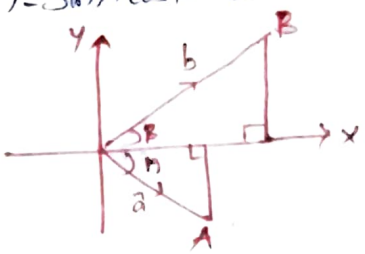
$\hat{a} = \cos A\,\hat{i} - \sin A\,\hat{j}$

$\hat{b} = \cos B\,\hat{i} + \sin B\,\hat{j}$

$\vec{a} \times \vec{b} = (\sin A \cos B + \cos A \sin B)\,\hat{k}$ — (1)

$= |\vec{a}|\,|\vec{b}| \sin(A+B)\,\hat{k}$ — (2)

$\sin(A+B) = \sin A \cos B + \cos A \sin B$



A5

(a)

$\vec{r} = (1-s)\vec{a} + s\vec{b} + t\vec{c}$

$= (1-s)(2\hat{i}+2\hat{j}+\hat{k}) + s(9\hat{i}+3\hat{j}+6\hat{k}) + t(2\hat{i}+6\hat{j}+6\hat{k})$

s, t are scalars

Cartesian Equation

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$3x + 4y - 5z - 9 = 0$

A5

(b) $y = x^2$

$\frac{dy}{dx} = 2x = m_1$

$x = y^2$

$1 = 2y\frac{dy}{dx}$

$\frac{1}{2y} = \frac{dy}{dx} = m_2$

Point (1, 1)

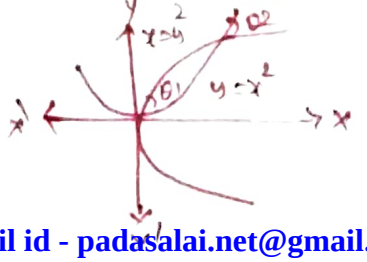
$m_1 = 2$ $m_2 = \frac{1}{2}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{2 - \frac{1}{2}}{1 + 2 \cdot \frac{1}{2}}\right|$

$= \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$



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(a)

(i) $P(X < 2) = 1/2 = F(2)$

(ii) $P(2 < X < 4) = F(4) - F(2)$

$= 3/4 - 1/4 = 1/2$

(iii) $P(3 \leq X) = 1 - P(X < 3)$

$= 1 - 1/2$

$= 1/2$

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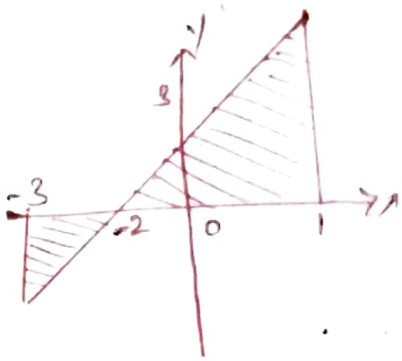
(b) $3x - 2y + 6 = 0$

$x = 0 : y = 3$

$y = 0 : x = -2$

$y = \frac{3x+6}{2}$



Required Area,

$= \int_{-3}^{-2} y\,dx + \int_{-2}^{1} y\,dx$

$= -\int_{-3}^{-2} \left(\frac{3x+6}{2}\right)dx + \int_{-2}^{1} \left(\frac{3x+6}{2}\right)dx$

$= -1/2 \left[12/2 - 12 \right] - \left(\frac{27}{2} - 18 \right)] +$

$1/2 [(3/2 + 6) - (12/2 - 12)$

$= 15/2$ units.

47 (a)

$(1+x^2)\,dy/dx = 1+y^2$

$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\tan^{-1} y = \tan^{-1} x + C$

(or)

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(b)

p	q	r	$q \to r$	$(p \to (q \to r))$	$(p \wedge q)$	$(p \wedge q) \to r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

① ②

From ① & ②

$P \to (q \to r) \equiv (p \wedge q) \to r$.