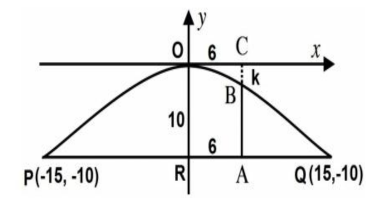
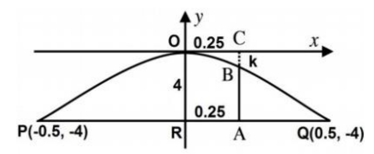
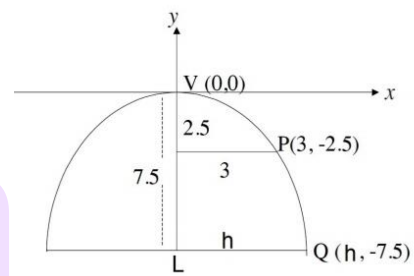
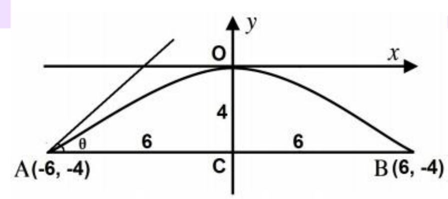
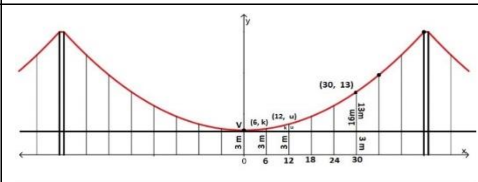
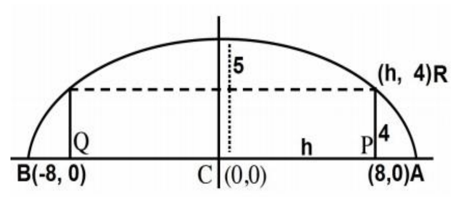
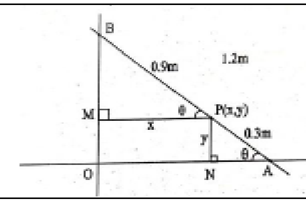
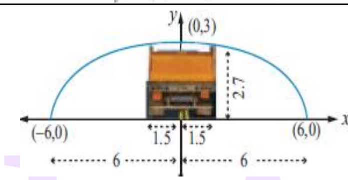
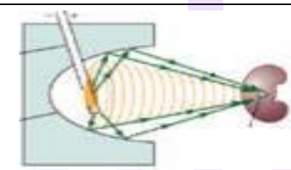
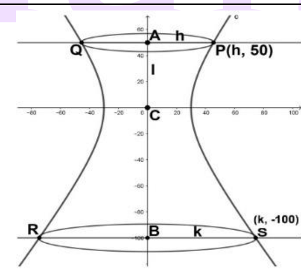
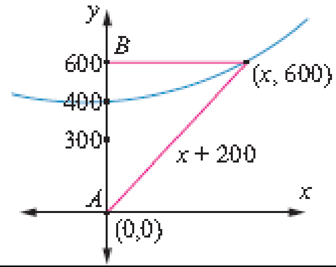
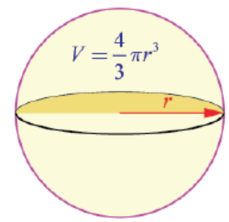
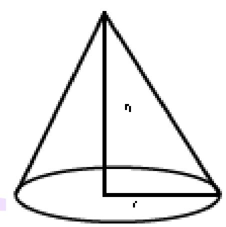
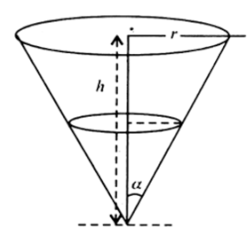
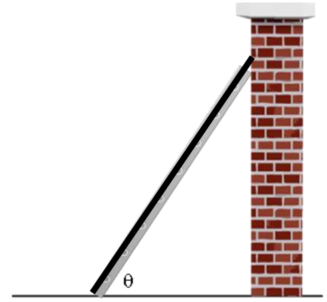
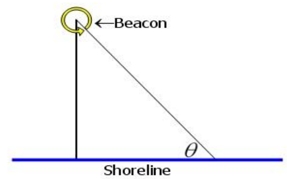
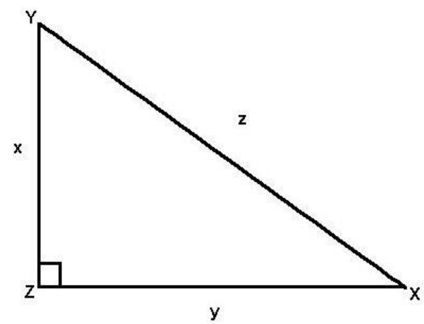
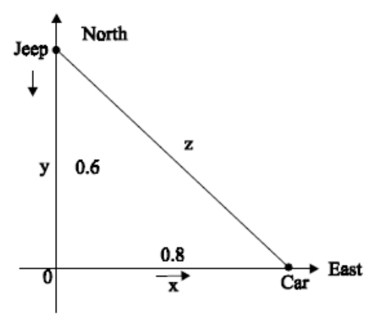
12 MATHEMATICS

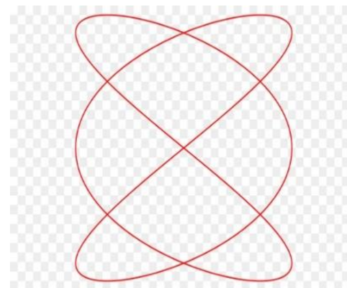
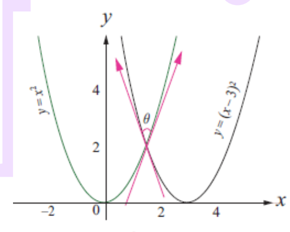
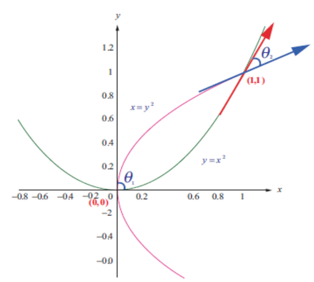
(5 Mark question and answer for slow learners Only)

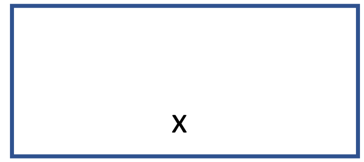
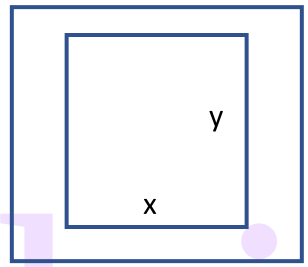
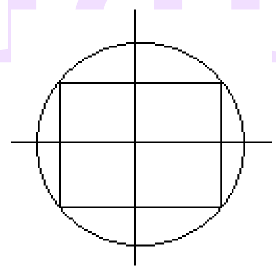
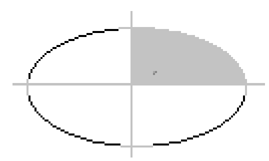
1) A bridge has a parabolic arch that is 10m high at the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre on either sides.		**Equation of Parabola** $x^2 = -4ay$ a $= \frac{225}{40}$ **height = 8.4 m**
2) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0 5 . m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 075.m from the point of origin.		**Equation of Parabola** $x^2 = -4ay$ a $= \frac{0.25}{16}$ **height = 3 m**
3) Assume that water issuing from the end of a horizontal pipe, 7 5 . m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2 5 . m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?		**Equation of Parabola** $x^2 = -4ay$ a $= \frac{9}{10}$ **height =** $3\sqrt{3}$ **m**
4) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.		**Equation of Parabola** $x^2 = -4ay$ a $= \frac{9}{4}$ **Angle =** $\tan^{-1}\left(\frac{4}{3}\right)$
5) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.		**Equation of Parabola** $x^2 = 4ay$ 4a $= \frac{900}{13}$ **length = 5.08 m**

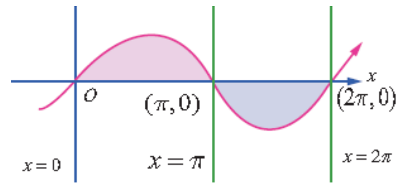
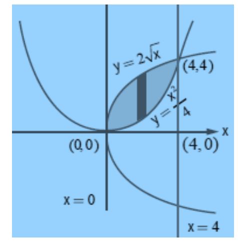
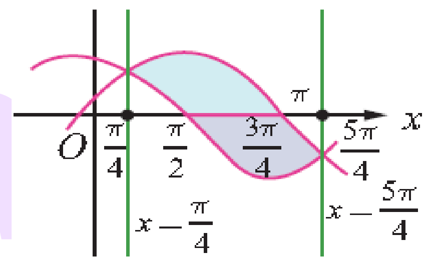
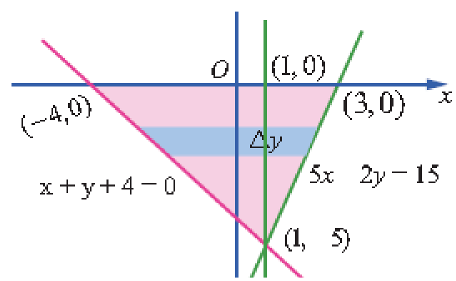
6) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m and the height at the edges of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. how wide must the opening be ?		**Equation of Ellipse** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $b^2 = 25$, $a = 13.33$ **Wide = 26.7 m**
7) A rod of length 1 2 . m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0 3 . m from the end in contact with x -axis is an ellipse. Find the eccentricity.		**Equation of Ellipse** $\frac{x^2}{0.9^2}+\frac{y^2}{0.3^2}=1$ **eccentricity** $e = \frac{2\sqrt{2}}{3}$
8) A semielliptical archway over a one-way road has a height of 3m and a width of 12m. The truck has a width of 3m and a height of 27.m. Will the truck clear the opening of the archway?		**Equation of Ellipse** $\frac{x^2}{6^2}+\frac{y^2}{3^2}=1$ **height = 2.9 m**
9) If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?		$a^2 = 484$ $b^2 = 64$ $ae = \sqrt{a^2-b^2}$ $ae = \sqrt{420} = 20.5$ cm
10) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$ The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.		**Equation of hyperbola** $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$ **top diameter = 90.82 m** **base diameter = 148.98 m**

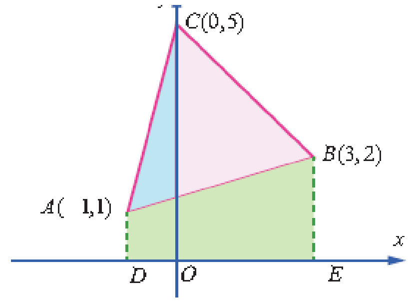
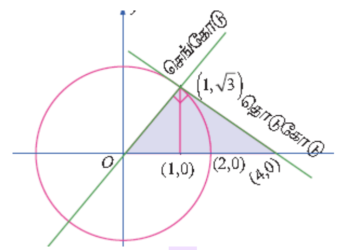
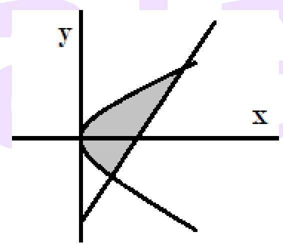
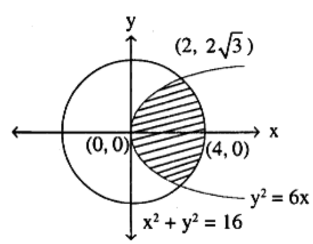
11) Two coast guard stations are located 600 km apart at points A(,) 00and B(,) 0600. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B . Determine the equation of hyperbola that passes through the location of the ship.		$a^2 = 10000$ $b^2 = 80000$ **Equation of hyperbola** $\dfrac{(y-300)^2}{10000} - \dfrac{x^2}{80000} = 1$
12) If we blow air into a balloon of spherical shape at a rate of 1000 3 cm per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.		$\dfrac{dV}{dt} = 1000 \quad \& \quad r = 7$ **change in radius** $\dfrac{dr}{dt} = \dfrac{250}{49\pi}$ Change in surface area $\dfrac{dS}{dt} = \dfrac{2000}{7}$
13) Salt is poured from a conveyer belt at a rate of 30cubic meter per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10meter high?		$\dfrac{dV}{dt} = 30 \quad \& \quad h = 10$ **radius** $r = \dfrac{h}{2}$ **hight increases** $\dfrac{dh}{dt} = \dfrac{6}{5\pi}$ meter/min
14) A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 meters deep?		$\dfrac{dV}{dt} = 10 \quad \& \quad h = 8$ radius $r = \dfrac{5h}{12}$ **depth of the water increases** $\dfrac{dh}{dt} = \dfrac{9}{10\pi}$

15) A ladder 17 meter long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s when the base of the ladder is 8 meters from the wall. (i) How fast is the top of the ladder moving down the wall ? (ii) At what rate, the area of the triangle formed by the ladder ,wall and the floor is changing.		$\frac{dx}{dt}=5$ & $x=8$ $y=15$, $\frac{dy}{dt}=\frac{-8}{3}$ Area changes $\frac{dA}{dt}=\frac{1}{2}\left[x\frac{dy}{dt}+y\frac{dx}{dt}\right]=26.83$
16) A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?		$\frac{d\theta}{dt}=\frac{2\pi}{10}=\frac{\pi}{5}$ & $\theta=45^\circ$ $\tan\theta=\frac{x}{5}\Rightarrow x=5\tan\theta$ **Beam changes at** $\frac{dx}{dt}=2\pi\ km/\sec$
17) A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 km to the north of P and traveling at 80 km/hr, while car B is 15 km to the east of P and traveling at 100km/hr. How fast is the distance between the two cars changing?		$x=10,\ \frac{dx}{dt}=80$ **km/hr** $y=15,\ \frac{dy}{dt}=100$ **km/hr** $z^2=x^2+y^2\Longrightarrow z=5\sqrt{13}$ **distance changes** $\frac{dz}{dt}=127.6$ **km/hr**
18) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20km/hr. If the jeep is moving at 60km/hr at the instant of measurement, what is the speed of the car?		$x=0.6,\quad y=0.8$ $\frac{dz}{dt}=20$ **km/hr** $z^2=x^2+y^2\Longrightarrow z=1$ **speed of the car** $\frac{dx}{dt}=70$ **km/hr**

19) Find the equation of the tangent and normal to the Lissajous curve given by $\boldsymbol{x = 2cos3t}$ **and** $\mathbf{y = 3sin2t}$ **, tϵR**		$m = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{6\cos 2t}{-6\sin 3t} = \frac{-\cos 2t}{\sin 3t}$ **Tangent** $y - y_1 = m(x - x_1)$ $y - 3\sin 2t = \frac{-\cos 2t}{\sin 3t}(x - 2\cos 3t)$ Normal $y - y_1 = \frac{-1}{m}(x - x_1)$ $y - 3\sin 2t = \frac{\sin 3t}{\cos 2t}(x - 2\cos 3t)$
20)) Find the acute angle between $\boldsymbol{y = x^2}$ **and** $\boldsymbol{y = (x-3)^2}$		**intersecting point** $(\mathbf{x, y}) = \left(\frac{\mathbf{3}}{\mathbf{2}}, \frac{\mathbf{9}}{\mathbf{4}}\right)$ **slopes** $m_1 = 3$; $m_2 = -3$ $\theta = \tan^{-1}\left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $= \tan^{-1}\left(\frac{3}{4}\right)$
21) Find the acute angle between the curves $\mathbf{y = x^2}$ **and** $\mathbf{x = y^2}$ at their points of intersection (0,0), (1,1)		$y = x^2$ $\quad$ $x = y^2$ $\frac{dy}{dx} = 2x$ $\quad$ $\frac{dy}{dx} = \frac{1}{2y}$ **at (0,0) ;** $\theta = \frac{\pi}{2}$ **at (1,1) ;** $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

22) A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.		$A = xy = x(20-x) = 20x - x^2$ $\frac{dA}{dx} = 20 - 2x$ & $\frac{d^2A}{dx^2} = -2 < 0$ $\frac{dA}{dx} = 0 \Rightarrow x = 10$ & $y = 10$ Max area $A = 100\ m^2$
23) A rectangular page is to contain 24 sq.cm of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the is 1cm. What should be the dimensions of the page so that the area of the paper used is minimum.		$A = (x+2)(y+3)$ $A = 3x + \frac{48}{x} + 30$ $\frac{dA}{dx} = 3 - \frac{48}{x^2}$ & $\frac{d^2A}{dx^2} = \frac{96}{x^3} > 0$ $\frac{dA}{dx} = 0 \Rightarrow x = 4$ & $y = 6$ $\therefore\ x + 2 = 6$ & $y + 3 = 9$
24) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.		$x = 20\cos\theta$ & $y = 20\sin\theta$ $A = (2x)(2y)$ $A = 200\sin 2\theta$ $\frac{dA}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{4}$ $\therefore\ L = 2x = 10\sqrt{2}$ & $B = 2y = 10\sqrt{2}$
25) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$		**area** $A = \int_a^b y\ dx$ $= 4\int_0^a y\ dx$ $= \pi ab$

<table>
<tr>
<td>26) Find the area of the region bounded by x – axis, the sine curve $y = \sin x$, the lines $x = 0$ and $\boldsymbol{x = 2\pi}$</td>
<td>
</td>
<td>area $A = \int_a^b y\ dx$
$= \int_0^{\pi} \sin x\ dx - \int_{\pi}^{2\pi} \sin x\ dx$
$= 4$</td>
</tr>
<tr>
<td>27) Find the area of the region bounded between the parabolas $\boldsymbol{y^2 = 4x}$ and $\boldsymbol{x^2 = 4y}$</td>
<td>
</td>
<td>intersecting points = (0,0), (4,4)
Area between two curve
$A = \int_a^b [\,y_U - y_L\,]\,dx$
$= \frac{16}{3}$</td>
</tr>
<tr>
<td>28) Find the area of the region bounded by y=cosx, y=sinx, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$</td>
<td>
</td>
<td>Area between two curves
$A = \int_a^b [\,y_U - y_L\,]\,dx$
$= \int_{\pi/4}^{5\pi/4} [\sin x - \cos x\,]\,dx$
$= 2\sqrt{2}$</td>
</tr>
<tr>
<td>29) Find by integration the area of the region bounded by the lines $5x - 2y = 15, x + y + 4 = 0$ and the x-axis.</td>
<td>
</td>
<td>Lines intersect at = (1,-5)
Lines meet the x-axis at
= (3,0), (-4,0)
Area
$A = \left|\int_{-4}^{1} y\ dx\right| + \left|\int_{1}^{3} y\ dx\right|$
$= \frac{35}{2}$</td>
</tr>
</table>

30) Using integration find the area of the region bounded by triangle ABC, whose vertices A,B and C are (-1,1), (3,2) and (0,5) respectively.		**Equation of straight lines** $y = 4x + 5$ $y = -x + 5$ $y = \frac{1}{4}(x + 5)$ **Area** $A = \int_{-1}^{0} (4x + 5)\ dx + \int_{0}^{3} (-x + 5)\ dx - \frac{1}{4} \int_{0}^{\pi} (x + 5)\ dx$ $= \frac{15}{2}$
31) Using integration, find the area of the region which is bounded by x-axis, the tangent and normal to the circle $x^2 + y^2 = 4$ **drawn at** $(1, \sqrt{3})$		**Tangent equation** $x + y\sqrt{3} = 4$ **Normal equation** $y = \sqrt{3}x$ **Area** $A = \int_{0}^{1} y\ dx + \int_{1}^{4} y\ dx = 2\sqrt{3}$
32) Find the area of the region bounded by the parabola $y^2 = x$ **and the line** $y = x - 2$		**limit on y-axis y= -1,2** **Area between two curves** $A = \int_{c}^{d} [x_R - x_L]\, dy$ $= \int_{-1}^{2} [y + 2 - y^2]\, dy = \frac{9}{2}$
33) Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$		**Intersecting points** $= (2, 2\sqrt{3}), (2, -2\sqrt{3})$ **Area between two curves** $A = \int_{c}^{d} [x_R - x_L]\, dy$ $= \int_{-2\sqrt{3}}^{2\sqrt{3}} \left[\sqrt{16 - y^2} - \frac{y^2}{6} \right] dy$ $= \frac{4}{3}(4\pi + \sqrt{3})$

34) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0\ ;\Rightarrow C=A_0$ $t=5\ ;\Rightarrow e^{5k}=3$ $t=10\ ;\Rightarrow A=9A_0$
35) Find the population of a city at any time *t*, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0\Rightarrow C=3,00,000$ $t=40\Rightarrow k=\frac{1}{40}\log\left(\frac{4}{3}\right)$ $A=3,00,000\left(\frac{4}{3}\right)^{t/40}$
36) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?	$\frac{dA}{dt}=kA$ $A=Ce^{0.05t}$	$t=0\ ;\Rightarrow C=10,000$ $t=1.5;\Rightarrow A=10,000\,e^{0.075}$
37) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0\ ;\Rightarrow C=100$ $t=100\ ;\ e^{100k}=\frac{9}{10}$ $t=1000;\Rightarrow A=\frac{9^{10}}{10^{8}}\ \%$
38) A radioactive isotope has an initial mass 200mg, which two years later is 150mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half- life means the time taken for the radioactivity of a specified isotope to fall to half its original value)	$\frac{dA}{dt}=kA$ $A=Ce^{kt}$	$t=0\Rightarrow\ C=200\ \ \&\ t=2\Rightarrow k=\frac{-1}{2}\log\left(\frac{4}{3}\right)$ $A(t)=200e^{\frac{-t}{2}\log\left(\frac{4}{3}\right)}\ \ \&\ \ t=\frac{2\log\left(\frac{1}{2}\right)}{\log\left(\frac{4}{3}\right)}$

39) Water at temperature $\mathbf{100^\circ C}$ cools in 10 minutes to $\mathbf{80^\circ C}$ in a room temperature of $\mathbf{25^\circ C}$ .Find **(i)** The temperature of water after 20 minutes **(ii)** The time when the temperature is $\mathbf{40^\circ C}$	$\frac{dT}{dt}=k\left(T-25\right)$ $T=25+Ce^{kt}$	$t=0\ \Rightarrow C=75$ $t=20\min\ \Rightarrow T=65.33^\circ C$ $T=40^\circ C\ \Rightarrow t=51.89\min$
40) A pot of boiling water at $100^\circ C$ is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to $80^\circ C$, and another 5 minutes later it has dropped to $65^\circ C$. Determine the temperature of the kitchen.	$\frac{dT}{dt}=k\left(T-S\right)$ $T=S+Ce^{kt}$	$t=0\ \Rightarrow C=100-S$ $t=5\ \Rightarrow e^{5k}=\frac{80-S}{100-S}$ **Kitchen temperature** $S=20^\circ$ C
41) In murder investigation, a corpse was found by detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be $70^\circ F$. Two hours later, the detective measured the body temperature again and found it to be $60^\circ F$. If the room temperature is $50^\circ F$ and assuming that the body temperature of the person before death was $98.6^\circ F$, at what time did the murder occur?	$\frac{dT}{dt}=k\left(T-50\right)$ $T=50+Ce^{kt}$	$t=0;\Rightarrow C=20$ $t=2\Rightarrow k=\frac{1}{2}\log\left(\frac{1}{2}\right)$ **The person died at** **5:30 pm**
42) A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (*Brine is a high-concentration solution of salt (usually sodium chloride) in water*) runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t .	$\frac{dx}{dt}=IN\ -\ OUT$ $\frac{dx}{dt}=50-0.01x$ $x=5000+Ce^{-0.01t}$	$t=0\ ;\ C=-4900$ **Amount of salt at t** $x=5000-4900e^{-0.01t}$

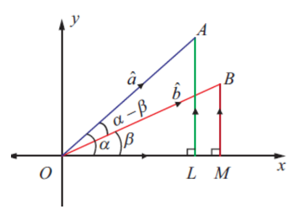
<table>
<tr><td>43) A tank initially contains 50 liters of pure water. Starting at time t = 0 a brine containing with 2 grams of dissolved salt per liter flows into the tank at the rate of 3 liters per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time t > 0.</td><td>$\frac{dx}{dt} = IN - OUT$
$\frac{dx}{dt} = 6 - \frac{3}{50}x$
$x = 100 + Ce^{\frac{-3t}{50}}$</td><td>$t = 0\ ;\ C = -100$
Amount of salt at t
$x = 100 - 100e^{-\frac{3t}{50}}$</td></tr>
<tr><td>44) Verify (i) Closure property (ii) commutative property (iii) Associative property (iv) Existence of Identity (v) Existence of Inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.</td><td>

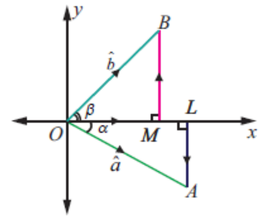
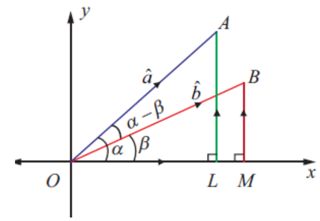
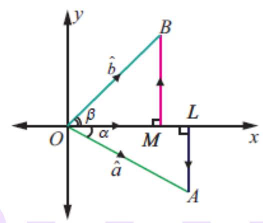
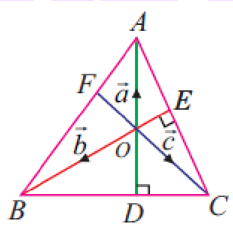
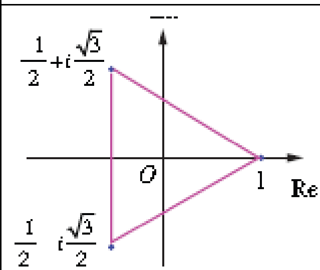
$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

</td><td>) Closure Property - true
ii) Commutative Property - true
iii) Associative Property-true
iv) Identity - exist $e = 0$
v) Inverse exist
inverse of 0 = 0
inverse of 1 = 4
inverse of 2 = 3
inverse of 3 = 2
inverse of 4 = 1</td></tr>
<tr><td>45) Verify (i) Closure property (ii) commutative property (iii) Associative property (iv) Existence of Identity (v) Existence of Inverse for the operation $\times_{11}$ on a subset A = { 1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, ,5 6, 7, 8, 9, 10}</td><td>

$\times_{11}$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

</td><td>i) Closure Property - true
ii) Commutative Property - true
iii) Associative Property-true
iv) Identity - exist e=1
v) Inverse exist
inverse of 1 = 1
inverse of 3 = 4
inverse of 4 = 3
inverse of 5 = 9
inverse of 9 = 5</td></tr>
</table>

<table>
<tr>
<td>46) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let * be the matrix multiplication. Determine whether M is closed under *. If so examine the (i) Commutative property (ii) Associative property (iii) Existence of Identity
(iv) Existence of inverse property for the operation * on M.</td>
<td>i) Closure Property - true
ii) Commutative Property - true
iii) Associative Property- true
iv)Identity - exist
$E = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \in M$</td>
<td>v) Inverse Property - Inverse exist
$= \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \in M$</td>
</tr>
<tr>
<td>47) Let A be Q\{1}. Define * on A by x*y = x + y - xy. Is * binary on A? If so, examine the the (i) Commutative property (ii) Associative property (iii) Existence of Identity (iv) Existence of inverse property for the operation * on A.</td>
<td>i) Closure Property - true
ii) Commutative Property
- true
iii) Associative Property
- true</td>
<td>iv) Identity - exist
e = 0
v) Inverse property - inverse exist
inverse of x is $= \frac{-x}{1-x} \in A$</td>
</tr>
<tr>
<td>48) A random variable X has the following probability mass function
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$f(x)$</td><td>k</td><td>$2k$</td><td>$6k$</td><td>$5k$</td><td>$6k$</td><td>$10k$</td></tr>
</table>
then find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$
(iii) $P(X \le 4)$ iv) $P(3 < X)$</td>
<td>$\sum p_i = 1$
$k = \frac{1}{30}$
i) $P(2 < X < 6) = \frac{17}{30}$</td>
<td>ii) $P(2 \le X < 5) = \frac{13}{30}$
iii) $P(X \le 4) = \frac{14}{30}$
iv) $P(3 < X) = \frac{21}{30}$</td>
</tr>
<tr>
<td>49) A random variable X has the following probability mass function.
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>$f(x)$</td><td>k^2</td><td>$2k^2$</td><td>$3k^2$</td><td>$2k$</td><td>$3k$</td></tr>
</table>
Find (i) value of k (ii) $P(2 \le X < 5)$
(iii) $P(3 > X)$</td>
<td>i) $\sum p_i = 1$
$k = \frac{1}{6}$</td>
<td>(ii) $P(2 \le X < 5) = \frac{17}{36}$
(iii) $P(3 > X) = \frac{5}{6}$</td>
</tr>
<tr>
<td>50) Prove by vector method that
$Cos(\alpha - \beta) = Cos\alpha Cos\beta + Sin\alpha Sin\beta$</td>
<td>
</td>
<td>$\hat{a} = cos\alpha\ \hat{\imath} + sin\alpha\ \hat{\jmath}$
$\hat{b} = cos\beta\ \hat{\imath} + sin\beta\ \hat{\jmath}$
$\hat{a} \cdot \hat{b} = \cos(\alpha - \beta)$
$\hat{a} \cdot \hat{b} = Cos\alpha Cos\beta + Sin\alpha Sin\beta$
$\mathrm{Cos}(\alpha - \beta) = \mathrm{Cos}\alpha \mathrm{Cos}\beta + \mathrm{Sin}\alpha \mathrm{Sin}\beta$</td>
</tr>
</table>

51) Prove by vector method that $Cos(\alpha+\beta) = Cos\alpha Cos\beta - Sin\alpha Sin\beta$		$\hat{a} = cos\alpha\,\hat{\imath} - sin\alpha\,\hat{\jmath}$ $\hat{b} = cos\beta\,\hat{\imath} + sin\beta\,\hat{\jmath}$ $\hat{a}\,.\,\hat{b} = \cos(\alpha+\beta)$ $\hat{a}\,.\,\hat{b} = Cos\alpha Cos\beta - Sin\alpha Sin\beta$ $\mathbf{Cos(\alpha+\beta) = Cos\alpha Cos\beta - Sin\alpha Sin\beta}$
52) Prove by vector method that $Sin(\alpha-\beta) = Sin\alpha\,Cos\beta - Cos\alpha\,Sin\beta$		$\hat{a} = cos\alpha\,\hat{\imath} + sin\alpha\,\hat{\jmath}$ $\hat{b} = cos\beta\,\hat{\imath} + sin\beta\,\hat{\jmath}$ $\hat{b} \times \hat{a} = \hat{k}\,\mathrm{Sin}(\alpha-\beta)$ $\hat{b} \times \hat{a} = \hat{k}(Sin\alpha Cos\beta - Cos\alpha Sin\beta)$ $\mathbf{Sin(\alpha-\beta) = Sin\alpha Cos\beta - Cos\alpha Sin\beta}$
53) Prove by vector method that $Sin(\alpha+\beta) = Sin\alpha\,Cos\beta + Cos\alpha\,Sin\beta$		$\hat{a} = cos\alpha\,\hat{\imath} - sin\alpha\,\hat{\jmath}$ $\hat{b} = cos\beta\,\hat{\imath} + sin\beta\,\hat{\jmath}$ $\hat{b} \times \hat{a} = \hat{k}\,\mathrm{Sin}(\alpha+\beta)$ $\hat{b} \times \hat{a} = \hat{k}(Sin\alpha Cos\beta + Cos\alpha Sin\beta)$ $\mathbf{Sin(\alpha+\beta) = Sin\alpha Cos\beta + Cos\alpha Sin\beta}$
54) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.		$\vec{a}\,.\,\vec{c} - \vec{a}\,.\,\vec{b} = 0$ $\vec{a}\,.\,\vec{b} - \vec{b}\,.\,\vec{c} = 0$ **Adding** $\vec{a}\,.\,\vec{c} - \vec{b}\,.\,\vec{c} = 0$ $\overrightarrow{CO}\,.\,\overrightarrow{AB} = 0$
55) Show that the point 1 , $\frac{-1}{2}+\frac{i\sqrt{3}}{2}$ and $\frac{-1}{2}-\frac{i\sqrt{3}}{2}$ are the vertices of an equilateral triangle.		$z_1 = 1$; $z_2 = \frac{-1}{2}+\frac{i\sqrt{3}}{2}$; $z_3 = \frac{-1}{2}-\frac{i\sqrt{3}}{2}$ $\lvert z_1 - z_2\rvert = \sqrt{3}$ $\lvert z_1 - z_2\rvert = \sqrt{3}$ $\lvert z_1 - z_2\rvert = \sqrt{3}$ **sides are equal . Hence equilateral traiangle.**

56) Show that $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.	$\frac{19+9i}{5-3i}=2+3i$ $\frac{8+i}{1+2i}=2-3i$	$z=(2+3i)^{15}-(2-3i)^{15}$ $\bar{z}=(2-3i)^{15}-(2+3i)^{15}$ $\bar{z}=-z$ purely imaginary
57) Show that $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ **is purely real.**	$\frac{19-7i}{9+i}=2-i$ $\frac{20-5i}{7-6i}=2+i$	$z=(2-i)^{12}+(2+i)^{12}$ $\bar{z}=(2+i)^{12}+(2-i)^{12}$ $\bar{z}=z$ purely real
58) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ **if one of the root is** $\frac{1}{3}$	$\frac{1}{3}$ | 6 -5 -38 -5 6 0 2 -1 -13 -6 3 | 6 -3 -39 -18 0 0 18 45 18 -2 | 6 15 6 0 0 -12 -6 $\frac{-1}{2}$ | 6 3 0 0 -3 6 0	**Solutions are =** $\frac{1}{3}$ **, 3 ,** $\frac{-1}{2}$**, -2**
59) Solve: $6x^4-35x^3+62x^2-35x+6=0$	| 6 -35 62 -35 6 2 | 0 12 -46 32 -6 | 6 -23 16 -3 0 $^1/_2$ | 0 3 -10 3 | 6 -20 6 0 3 | 0 18 -6 | 6 -2 0 $^1/_3$ | 0 2 | 6 0	**Solutions are = 2 ,** $\frac{1}{2}$**, 3,** $\frac{1}{3}$