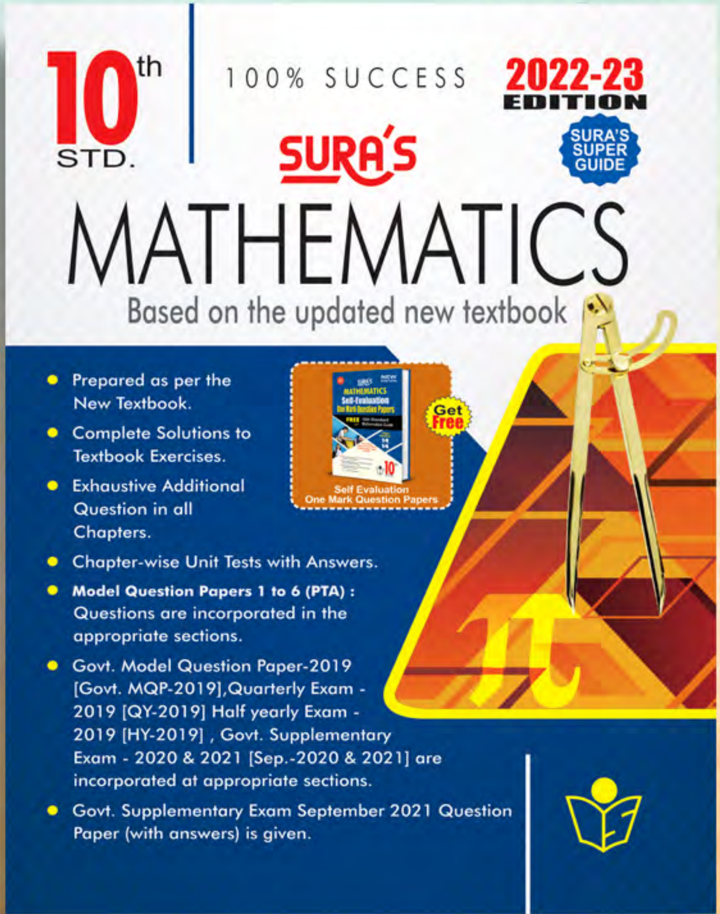
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PREFACE

The woods are lovely, dark and deep.
But I have promises to keep, and
miles to go before I sleep - Robert Frost

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In complete cognizance of the dedicated role of Teachers, I completely believe that our students will learn the subject effectively with this guide and prove their excellence in Board Examinations.

I once again sincerely thank the Teachers, Parents and Students for supporting and valuing our efforts.

God Bless all.

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Chapter 1

RELATIONS AND FUNCTIONS

FORMULAE TO REMEMBER

- **Vertical line test :**

  A curve drawn in a graph represents a functions, if every vertical line intersects the curve in at most one point.

- **Horizontal line test :**

  A function represented in a graph is one - one, if every horizontal line intersect the curve in at most one point.

- Linear functions has applications in Cryptography as well as in several branches of Science and Technology.

EXERCISE 1.1

1. Find A× B, A×A and B ×A
(i) A = {2,−2,3} and B ={1,−4} (ii) A= B ={p,q} (iii) A= {m,n} ; B = ϕ [PTA - 1]

Sol. (i) A = {2, –2, 3}, B = {1, –4}

A × B = {(2, 1), (2, –4), (–2, 1), (–2, –4), (3, 1), (3, –4)}

A × A = {(2, 2), (2, –2), (2, 3), (–2, 2), (–2, –2), (–2, 3), (3, 2), (3, –2), (3, 3)}

B × A = {(1, 2), (1, –2), (1, 3), (–4, 2), (–4, –2), (–4, 3)}

(ii) A = B = {(p,q)}

A × B = {(p, p), (p, q), (q, p), (q, q)}

A × A = {(p, p), (p, q), (q, p), (q, q)}

B × A = {(p, p), (p, q), (q, p), (q, q)}

(iii) A = {m,n} , B = ϕ

A × B = { }

A × A = {(m,m), (m,n), (n, m), (n, n)}

B × A = { }

2. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A× B and B × A.

Sol. A = {1, 2, 3}, B = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

3. If B × A={(−2, 3),(−2, 4),(0, 3),(0, 4),(3, 3), (3, 4)} find A and B. [Qy - 2019]

Sol. Given B × A ={(–2, 3), (–2, 4), (0, 3), (0, 4), (3, 3), (3, 4)}

Here B = { –2, 0, 3}

[All the first elements of the order pair]

and A = {3, 4}

[All the second elements of the order pair]

4. If A ={5, 6}, B = {4, 5, 6} , C ={5, 6, 7}, Show that A×A = (B × B)∩(C ×C).

Sol. A = {5, 6}, B = {4, 5, 6},C = {5, 6, 7}

A × A = {(5, 5), (5, 6), (6, 5), (6, 6)} ...(1)

B × B = {(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} ...(2)

C × C = {(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)} ...(3)

(B × B) ∩ (C × C) = {(5, 5), (5, 6), (6, 5), (6, 6)} ...(4)

(1) = (4)

A × A = (B × B) ∩ (C × C). It is proved.

5. Given A ={1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if (A∩C) × (B∩D) = (A×B)∩(C × D) is true? [Qy - 2019]

Sol. LHS = {(A ∩ C) × (B ∩ D)

A ∩ C = {3}

B ∩ D = {3, 5}

(A ∩ C) × (B ∩ D) = {(3, 3) , (3, 5)} ...(1)

RHS = (A × B) ∩ (C × D)

A × B = {(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)}

C × D ={(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)}

(A × B) ∩ (C × D) = {(3, 3), (3, 5)} ...(2)

∴ (1) = (2) ∴ It is true.

6. Let A={$x \in \mathbb{W}$ |$x < 2$}, B = {$x \in \mathbb{N}$ |$1 < x \leq 4$} and C = {3, 5} . Verify that
(i) A× (B∪C) =(A × B)∪(A × C) [PTA - 2]
(ii) A×(B∩C) = (A × B)∩(A × C) [PTA - 5; Sep - 2021]
(iii) (A∪B) × C = (A×C)∪(B × C)

Sol. **(i) A × (B ∪ C) = (A × B) ∪ (A × C)**

A = {$x \in \mathbb{W}$ |$x < 2$}= {0, 1}

[Whole numbers less than 2]

B = {$x \in \mathbb{N}$ |$1 < x \leq 4$}={2, 3, 4}

C = {3, 5}

[Natural numbers from 2 to 4]

LHS = A × (B ∪ C)

B ∪ C = {2, 3, 4} ∪ {3, 5}
= {2, 3, 4, 5}

A × (B ∪ C) = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4),(1, 5)} ...(1)

RHS = (A × B) ∪(A × C)

(A × B) = {(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)}

(A × C) = {(0, 3), (0, 5), (1, 3), (1,5)}

(A × B) ∪ (A × C) = {(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4),(1, 5)} ...(2)

(1) = (2), LHS = RHS Hence it is proved.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS = $A \times (B \cap C)$

$(B \cap C)$ = {3}

$A \times (B \cap C)$ = {(0, 3), (1, 3)} ...(1)

RHS = $(A \times B) \cap (A \times C)$

$(A \times B)$ = {(0, 2), (0, 3),(0, 4), (1, 2), (1, 3), (1, 4)}

$(A \times C)$ = {(0, 3), (0, 5), (1, 3), (1,5)}

$(A \times B) \cap (A \times C)$ = {(0, 3), (1, 3)} ...(2)

(1) = (2) ⇒ LHS = RHS.

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

LHS = $(A \cup B) \times C$

$A \cup B$ = {0, 1, 2, 3, 4}

$(A \cup B) \times C$ = {(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)} ...(1)

RHS = $(A \times C) \cup (B \times C)$

$(A \times C)$ = {(0, 3), (0, 5), (1, 3), (1, 5)}

$(B \times C)$ = {(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)}

$(A \times C) \cup (B \times C)$ = {(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)}...(2)

(1) = (2)

∴ LHS = RHS. Hence it is verified.

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ [Sep. - 2020]

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ [PTA - 1]

A = {1, 2, 3, 4, 5, 6, 7}

B = {2, 3, 5, 7}

C = {2}

[∵ 2 is the only even prime number]

Sol. (i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS = $(A \cap B) \times C$

$A \cap B$ = {2, 3, 5, 7}

$(A \cap B) \times C$ = {(2, 2), (3, 2), (5, 2), (7, 2)} ...(1)

RHS = $(A \times C) \cap (B \times C)$

$(A \times C)$ = {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)}

$(B \times C)$ = {(2, 2), (3, 2), (5, 2), (7, 2)}

$(A \times C) \cap (B \times C)$ = {(2, 2), (3, 2), (5, 2), (7, 2)} ...(2)

(1) = (2)

∴ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

LHS = $A \times (B - C)$

$(B - C)$ = {3, 5, 7}

$A \times (B - C)$ = {(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)} ...(1)

RHS = $(A \times B) - (A \times C)$

$(A \times B)$ = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7,7)}

$(A \times C)$ = {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)}

$(A \times B) - (A \times C)$ = {(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)} ...(2)

(1) = (2) ⇒ LHS = RHS. Hence it is verified.

EXERCISE 1.2

1. Let A = {1,2,3,7} and B = {3,0,–1,7}, which of the following are relation from A to B ?

(i) $\mathbb{R}_1$ = {(2,1), (7,1)}

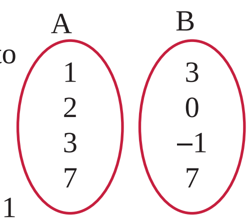
(ii) $\mathbb{R}_2$ = {(–1,1)}

(iii) $\mathbb{R}_3$ = {(2,–1), (7,7), (1,3)}

(iv) $\mathbb{R}_4$ = {(7,–1), (0,3), (3,3), (0,7)}

Sol. Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

(i) R_1 = {(2, 1), (7, 1)}



2 and 7 cannot be related to 1 since 1 ∉ B

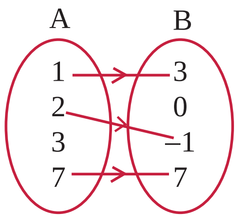
∴ R_1 is not a relation.

(ii) R_2 = {(–1, 1)}

–1 cannot be related to 1 since –1 ∉ A and 1 ∉ B

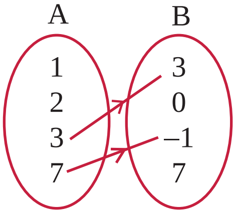
∴ R_2 is not a relation.

(iii) R_3 = {(2, –1), (7, 7), (1, 3)}



R_3 is a relation since 2 is related to –1, 7 is related to 7 and 1 is related to 3.

(iv) R_4= {(7, –1), (0, 3), (3, 3), (0, 7)}



7 is related to –1
3 is related to 3
Since 0 ∉ A, 0 cannot be related to 3 and 7.
∴ R_4 is not a relation.

2. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "is square of a number" on A. Write ℝ as a subset of A × A. Also, find the domain and range of ℝ. **[Sep - 2021]**

Sol. Given A ={1, 2, 3, 4, . . . 45}
∴ A × A = {(1, 1) (1, 2) (1,3) ... (1, 45)
(2, 1) (2, 2) ... (2, 45) (45, 1) (45, 2)
(45, 3) ... (45, 45)} ... (1)
R is defined as "is square of "
∴ R = {(1,1) (2,4) (3,9) (4,16) (5, 25) (6,36)}... (2)
[∵ 1 is the square of 1, 2 is the square of 4 and so on]
From (1) and (2), R is the subset of A × A
∴R ⊂ A × A
Domain of R = {1, 2, 3, 4, 5, 6}
[All the first elements of the order pair in (2)]
Range of R = {1, 4, 9, 16, 25, 36}
[All the second elements of the order pair in (2)]

3. A Relation ℝ is given by the set {(x, y) /y = x + 3, x ∈{0, 1, 2, 3, 4, 5}}. Determine its domain and range. **[PTA - 5]**

Sol. Given ℝ = {(x, y) /$y = x + 3$} and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ [∵ $y = x + 3$]
When $x = 1$, $y = 1 + 3 = 4$
When $x = 2$, $y = 2 + 3 = 5$
When $x = 3$, $y = 3 + 3 = 6$
When $x = 4$, $y = 4 + 3 = 7$
When $x = 5$, $y = 5 + 3 = 8$

∴ ℝ ={(0, 3),(1, 4),(2, 5),(3, 6),(4, 7), (5, 8)}
∴Domain of ℝ = {0, 1, 2, 3, 4, 5}
[All the first element in ℝ]
Range of ℝ = {3, 4, 5, 6, 7, 8}
[All the second element in ℝ]

4. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

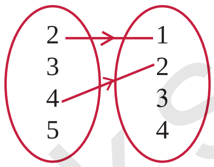
(i) {(x, y)|x = 2y, x ∈ {2, 3, 4, 5}, y ∈ {1, 2, 3, 4}}

(ii) {(x, y)|y = x + 3, x, y are natural numbers < 10}

Sol. (i) R = {(x, y)| $x = 2y$, $x \in$ {2, 3, 4, 5} and $y \in$ {1, 2, 3, 4}}

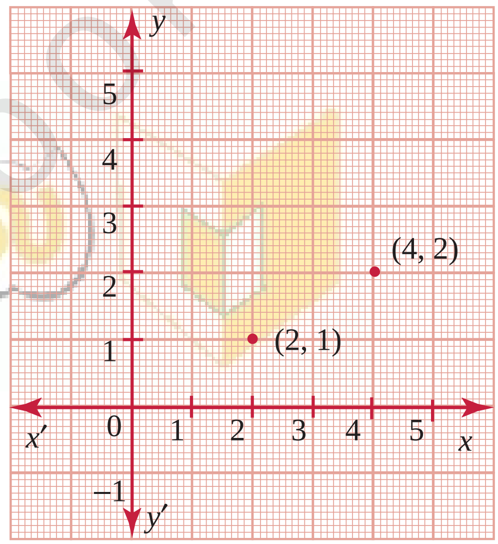
When $x = 2$, $y = \frac{x}{2} = \frac{2}{2} = 1$
[∵ $x = 2y \Rightarrow y = \frac{x}{2}$]
When $x = 3$, $y = \frac{3}{2}$
When $x = 4$, $y = \frac{4}{2} = 2$
When $x = 5$, $y = \frac{5}{2}$

(a) an arrow diagram



3 cannot be related to $\frac{3}{2}$ and 5 cannot be related to $\frac{5}{2}$.

(b) a graph



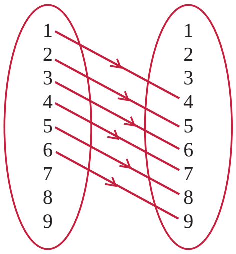
(c) Roster form : R = {(2, 1), (4, 2)}

(ii) R = {(x, y)|$y = x + 3$,
x and y are natural numbers <10}
x = {1, 2, 3, 4, 5, 6, 7, 8, 9
y = {1, 2, 3, 4, 5, 6, 7, 8, 9}
[∵ x and y are natural numbers less than 10]
Given $y = x + 3$
When $x = 1$, $y = 1 + 3 = 4$
When $x = 2$, $y = 2 + 3 = 5$
When $x = 3$, $y = 3 + 3 = 6$
When $x = 4$, $y = 4 + 3 = 7$
When $x = 5$, $y = 5 + 3 = 8$
When $x = 6$, $y = 6 + 3 = 9$

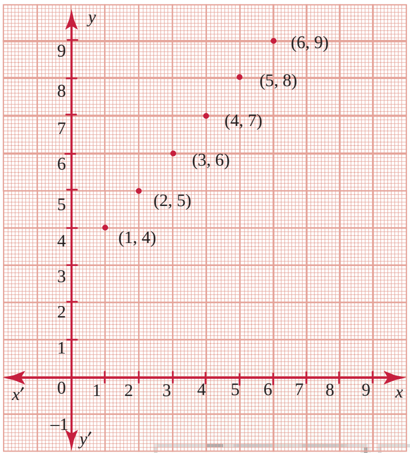
$$\left.\begin{array}{l}\text{When } x = 7, \quad y = 7 + 3 = 10\\ \text{When } x = 8, \quad y = 8 + 3 = 11\\ \text{When } x = 9, \quad y = 9 + 3 = 12\end{array}\right\}[10, 11, 12 \notin y]$$

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

(a) an arrow diagram



(b) a graph



(c) Roster form :

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation $\mathbb{R}$ is defined by $x\mathbb{R}y$, where x is the salary given to person y, express the relation $\mathbb{R}$ through an ordered pair and an arrow diagram.

Sol. A – Assistants $\rightarrow$ A_1, A_2, A_3, A_4, A_5

C – Clerks $\rightarrow$ C_1, C_2, C_3, C_4

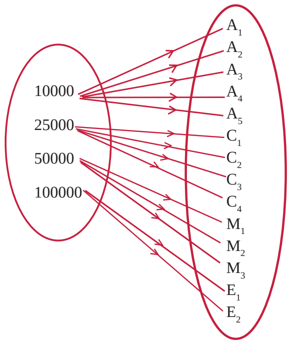
M – Managers $\rightarrow$ M_1, M_2, M_3

E – Executive officer $\rightarrow$ E_1, E_2

xRy is defined as x is the salary for assistants is ₹10,000, clerks is ₹25,000, Manger is ₹50,000 and for the executing officer ₹1,00,000.

(a) $\therefore$R = {(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5), (25,000, C_1), (25,000, C_2), (25,000,C_3), (25,000, C_4) (50,000, M_1), (50,000, M_2), (50,000, M_3), (1,00,000, E_1), (1,00,000, E_2)}

(b)



EXERCISE 1.3

1. Let $f = \{(x, y)|x, y \in \mathbb{N}$ and $y = 2x\}$ be a relation on $\mathbb{N}$. Find the domain, co-domain and range. Is this relation a function?

Sol. Given $f = \{(x, y) |x, y \in \mathbb{N}$ and $y = 2x\}$

When $x = 1$, $y = 2(1) = 2$

When $x = 2$, $y = 2(2) = 4$

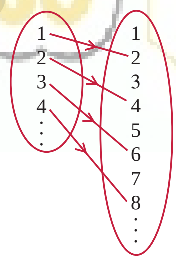
When $x = 3$, $y = 2(3) = 6$

When $x = 4$, $y = 2(4) = 8$ and so on.

R = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10),...}

Domain of R = {1, 2, 3, 4,...},

Range of R = {2, 4, 6, 8,...}



Since all the elements of domain are related to some elements of co-domain, this relation f is a function.

2. Let X = {3, 4, 6, 8}. Determine whether the relation $\mathbb{R} = \{(x, f(x)) |x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbb{N}$?

Sol. x = {3, 4, 6, 8}

R = $((x, f(x))|x \in X, f(x) = x^2 + 1\}$

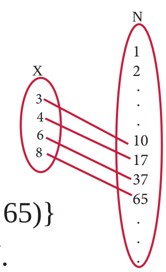
$f(x) = x^2 + 1$

$f(3) = 3^2 + 1 = 10$

$f(4) = 4^2 + 1 = 17$

$f(6) = 6^2 + 1 = 37$

$f(8) = 8^2 + 1 = 65$



R = {(3, 10), (4, 17), (6, 37), (8, 65)}

Yes, R is a function from X to $\mathbb{N}$.

Since all the elements of X are related to some elements of $\mathbb{N}$.

3. Given the function $f : x \to x^2 - 5x + 6$, evaluate
(i) $f(-1)$ (ii) $f(2a)$
(iii) $f(2)$ (iv) $f(x-1)$

Give the function $f : x \to x^2 - 5x + 6$.

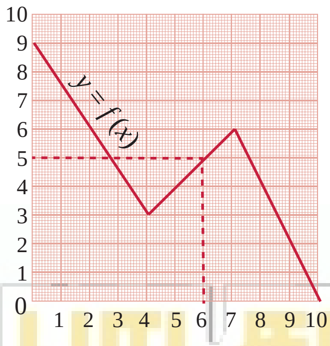
(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

(iv) $f(x-1) = (x-1)^2 - 5(x-1) + 6$
$= x^2 - 2x + 1 - 5x + 5 + 6$
$= x^2 - 7x + 12$

4. A graph representing the function $f(x)$ is given in figure it is clear that $f(9) = 2$.



(i) Find the following values of the function
(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$
(ii) For what value of x is $f(x) = 1$?
(iii) Describe the following (i) Domain (ii) Range.
(iv) What is the image of 6 under f?

Sol. (i) From the graph

(a) $f(0) = 9$ (c) $f(2) = 6$

(b) $f(7) = 6$ (d) $f(10) = 0$

(ii) At $x = 9.5$, $f(x) = 1$

(iii) Domain $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$= \{x \mid 0 \le x \le 10, x \in \mathbb{R}\}$

Range $= \{x \mid 0 \le x \le 9, x \in \mathbb{R}\}$
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iv) The image of 6 under f is 5. Since when you draw a line at $x = 6$, it meets the graph at 5.

5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\dfrac{f(x+2) - f(2)}{x}$.

Sol. Given $f(x) = 2x + 5$, $x \neq 0$. $\dfrac{f(x+2) - f(2)}{x}$

$f(x) = 2x + 5$

$\Rightarrow f(x+2) = 2(x+2) + 5$
$= 2x + 4 + 5 = 2x + 9$

$\Rightarrow f(2) = 2(2) + 5 = 4 + 5 = 9$

$\therefore \dfrac{f(x+2) - f(2)}{x} = \dfrac{2x + 9 - 9}{x} = \dfrac{2x}{x} = 2$

6. A function f is defined by $f(x) = 2x - 3$
(i) find $\dfrac{f(0) + f(1)}{2}$
(ii) find x such that $f(x) = 0$.
(iii) find x such that $f(x) = x$.
(iv) find x such that $f(x) = f(1 - x)$.

Sol. Given $f(x) = 2x - 3$

(i) $\dfrac{f(0) + f(1)}{2}$

$f(0) = 2(0) - 3 = -3$

$f(1) = 2(1) - 3 = -1$

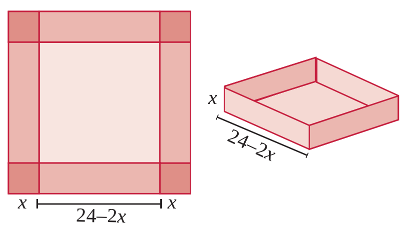
$\therefore \dfrac{f(0) + f(1)}{2} = \dfrac{-3-1}{2} = \dfrac{-4}{2} = -2$

(ii) $f(x) = 0 \Rightarrow 2x - 3 = 0$
$2x = 3$
$x = \dfrac{3}{2}$

(iii) $f(x) = x \Rightarrow 2x - 3 = x \Rightarrow 2x - x = 3$
$x = 3$

(iv) $f(x) = f(1 - x)$
$2x - 3 = 2(1 - x) - 3$
$2x - 3 = 2 - 2x - 3$
$2x + 2x = 2 - \cancel{3} + \cancel{3}$
$4x = 2$
$x = \dfrac{\cancel{2}}{\cancel{4}\,2}$
$x = \dfrac{1}{2}$

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x.



Sol. Volume of the box = Volume of the cuboid
$= l \times b \times h$ cu. units

Here $l = 24 - 2x$
$b = 24 - 2x$
$h = x$

$\therefore V = (24 - 2x)(24 - 2x) \times x$
$= (576 - 48x - 48x + 4x^2)x$
$V = 4x^3 - 96x^2 + 576x$

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Sol. Given $f(x) = 3 - 2x$

Also, it is given that $f(x^2) = [f(x)]^2$

$$f(x^2) = 3 - 2x^2 \text{[Replacing } x \text{ by } x^2] \quad \dots (1)$$

$$[f(x)]^2 = (3 - 2x)^2 = 9 - 12x + 4x^2 \quad \dots (2)$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2]$$

From (1) and (2),

$\Rightarrow \quad 9 - 12x + 4x^2 = 3 - 2x^2$

$\Rightarrow \quad 9 - 12x + 4x^2 - 3 - 2x^2 = 0$

$\Rightarrow \quad 6x^2 - 12x + 6 = 0$

1, -2, -1, -1

Dividing by 6, we get $x^2 - 2x + 1 = 0$

On factorizing we get, $(x - 1)(x - 1) = 0$

$\Rightarrow \quad x = 1$

9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

Sol. $\text{Speed} = \dfrac{\text{distance covered}}{\text{time taken}}$

$\Rightarrow \quad \text{distance} = \text{Speed} \times \text{time}$

$\Rightarrow \quad d = 500 \times t \; [\because \text{time} = t \text{ hrs}]$

$\Rightarrow \quad d = 500\,t$

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as $y = ax + b$, where a, b are constants. [PTA - 4]

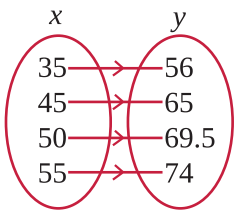
Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.
(ii) Find a and b.
(iii) Find the height of a person whose forehand length is 40 cm.
(iv) Find the length of forehand of a person if the height is 53.3 inches.

Sol. Given relation is $y = ax + b$ ...(1)

(i) The given ordered pairs are

R = { (35, 56) (45, 65) (50, 69.5) (55, 74)}



Since all the elements of x are related to some elements of y, the given relation is a function.

(ii) Consider any two ordered pairs (35, 56) and (45, 65)

Substitute $\overset{x}{35}, \overset{y}{56}$ i.e. (35, 56) in $y = ax + b$ we get,

$$56 = a(35) + b \quad \dots (1)$$

Similarly substitute (45, 65) in $y = ax + b$, we get

$$65 = a(45) + b \quad \dots(2)$$

$(2) \rightarrow \quad 65 = 45a + \cancel{b} \quad \dots.(2)$

$(1) \rightarrow \quad \underline{56 = 35a + \cancel{b}} \quad \dots(3)$

Substituting, $9 = 10a$

$\Rightarrow \quad a = \dfrac{9}{10} = 0.9$

Substituting $a = 0.9$ in (1) we get

$56 = 35(0.9) + b$

$\Rightarrow \quad 56 = 31.5 + b$

$\Rightarrow \quad b = 56 - 31.5 = 24.5$

Since $y = ax + b$

We get $y = 0.9x + 24.5$

(iii) When the length of the forehand $x = 40$ cm,

$y = 0.9(40) + 24.5$

$\Rightarrow \quad y = 36 + 24.5 = 60.5$ inches

$\therefore$ The required height of the person is 60.5 inches.

(iv) When the length of the forehand $y = 53.3$ inches,

$$53.3 = 0.9x + 24.5$$

$$[\because y = 0.9x + 24.5]$$

$\Rightarrow \quad 53.3 - 24.5 = 0.9x$

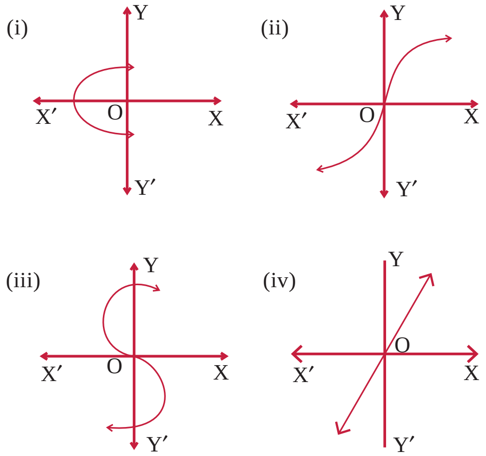
$\Rightarrow \quad 28.8 = 0.9x$

$\Rightarrow \quad x = \dfrac{28.8 \times 10}{0.9 \times 10}$

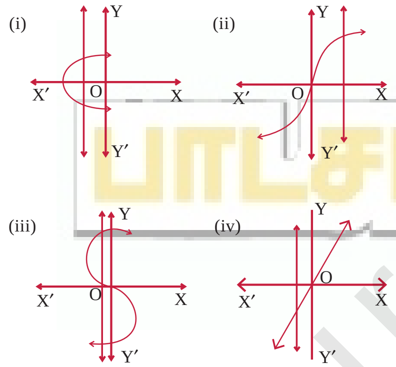
$\Rightarrow \quad x = \dfrac{288}{9} = 32$ cm

EXERCISE 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



Sol.



(i) It is not a function. The graph meets the vertical line at more than one points.

(ii) It is a function as the curve meets the vertical line at only one point.

(iii) It is not a function as it meets the vertical line at more than one points.

(iv) It is a function as it meets the vertical line at only one point.

2. Let $f: A \to B$ be a function defined by $f(x) = \dfrac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$. Represent f by [Govt. MQP - 2019]

(i) set of ordered pairs;
(ii) a table;
(iii) an arrow diagram;
(iv) a graph

Sol. $f: A \to B$

$A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$

$f(x) = \dfrac{x}{2} - 1$, $\quad f(2) = \dfrac{2}{2} - 1 = 0$

$f(4) = \dfrac{4}{2} - 1 = 1$, $\quad f(6) = \dfrac{6}{2} - 1 = 2$

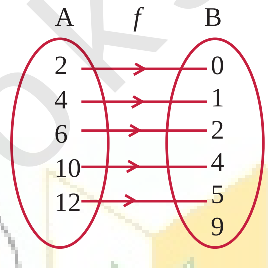
$f(10) = \dfrac{10}{2} - 1 = 4$, $\quad f(12) = \dfrac{12}{2} - 1 = 5$

(i) Set of ordered pairs
$= \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

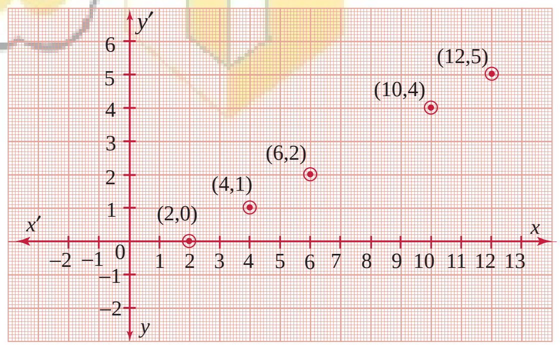
(ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) an arrow diagram;



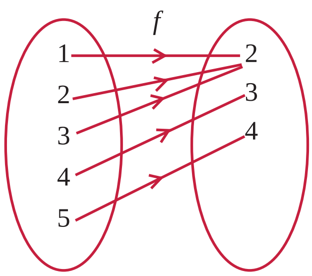
(iv) a graph



3. Represent the function $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$ through
(i) an arrow diagram
(ii) a table form
(iii) a graph

Sol. $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$

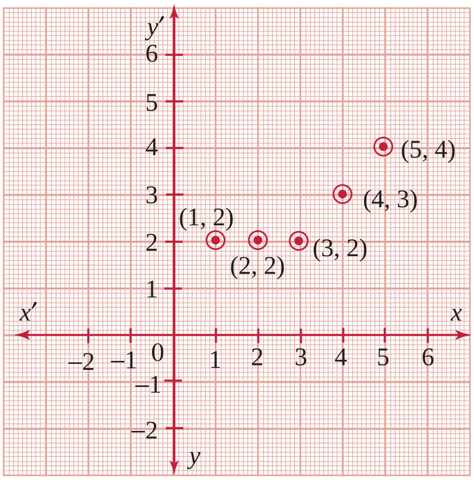
(i) An arrow diagram.



(ii) a table form

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) A graph representation.



4. **Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$ is one - one but not onto.**

Sol. $f : \mathbb{N} \to \mathbb{N}$

$f(x) = 2x - 1$

$\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$

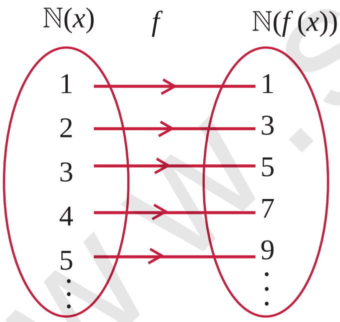
$f(1) = 2(1) - 1 = 1$

$f(2) = 2(2) - 1 = 3$

$f(3) = 2(3) - 1 = 5$

$f(4) = 2(4) - 1 = 7$

$f(5) = 2(5) - 1 = 9$



In the figure, for different elements in x, there are different images in $f(x)$.

Hence $f : \mathbb{N} \to \mathbb{N}$ is a one-one function.

A function $f : \mathbb{N} \to \mathbb{N}$ is said to be onto function if the range of f is equal to the co-domain of f.

Range = {1, 3, 5, 7, 9, ...}

Co-domain = {1, 2,3,..}

But here the range is not equal to co-domain. Therefore it is one-one but not onto function.

5. **Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one - one function.**

Sol. $f : \mathbb{N} \to \mathbb{N}$

$f(m) = m^2 + m + 3$

$\mathbb{N} = \{1, 2, 3, 4, 5, . . .\}, m \in \mathbb{N}$

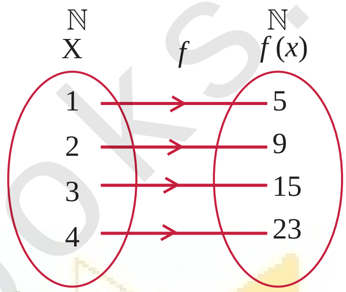
$f(m) = m^2 + m + 3$

$f(1) = 1^2 + 1 + 3 = 5$

$f(2) = 2^2 + 2 + 3 = 9$

$f(3) = 3^2 + 3 + 3 = 15$

$f(4) = 4^2 + 4 + 3 = 23$



In the figure, for different elements in the (X) domain, there are different images in $f(x)$. Hence $f : \mathbb{N} \to \mathbb{N}$ is a one to one but not onto function as the range of f is not equal to co-domain.

Co-domain = $\mathbb{N}$

Range = {5, 9, 15, 23}

Hence it is proved.

6. **Let A ={1, 2, 3, 4} and B = $\mathbb{N}$. Let f : A→ B be defined by $f(x) = x^3$ then,** **[Hy - 2019]**

(i) find the range of f

(ii) identify the type of function

Sol. A = {1, 2, 3, 4}

B = $\mathbb{N}$

$f : A \to B, f(x) = x^3$

(i) $f(1) = 1^3 = 1$

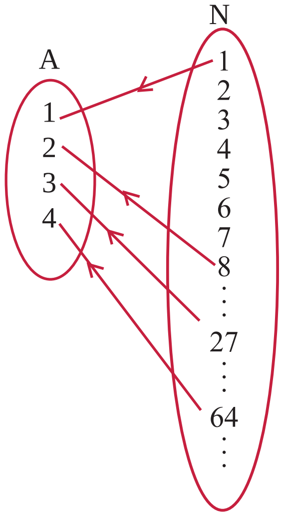
$f(2) = 2^3 = 8$

$f(3) = 3^3 = 27$

$f(4) = 4^3 = 64$

(ii) The range of f = {1, 8, 27, 64,....}

(iii)



Here co-domain = N = {1, 2, 3 ...}
Range = {1, 8, 27, 64}
Different elements have different images and co-domain ≠ Range.
∴ The given function is one - one into function.

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

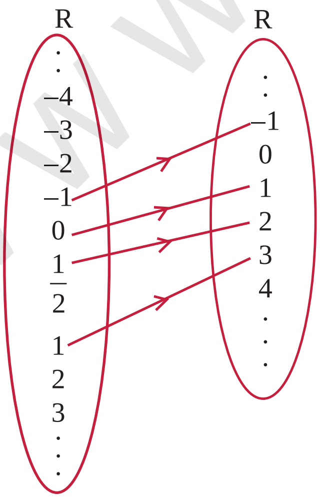
(i) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 1$

(ii) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x^2$

Sol. Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2x + 1$

(i) When $x = 1$,

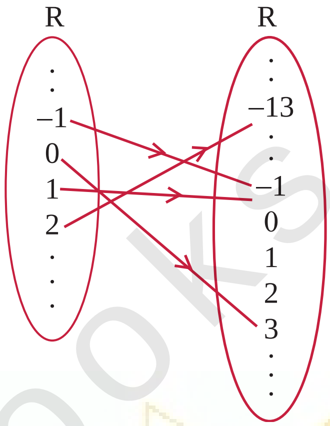
$$\begin{aligned} f(1) &= 2(1) + 1 = 3 \\ f(2) &= 2(2) + 1 = 5 \\ f(0) &= 2(0) + 1 = 1 \\ f(-1) &= 2(-1) + 1 = -2 + 1 = -1 \\ f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \text{ and so on} \end{aligned}$$



Here, different element in domain have different images in B and Co-domain = Range = R.
∴ f is a bijective function.

(ii) Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3 - 4x^2$

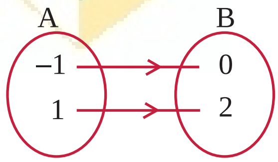
$$\begin{aligned} f(1) &= 3 - 4(1^2) = 3 - 4\,(1) \\ &= 3 - 4 = -1 \\ f(2) &= 3 - 4(2^2) = 3 - 4\,(4) \\ &= 3 - 16 = -13 \\ f(0) &= 3 - 4(0)^2 = 3 - 0 = 3 \\ f(-1) &= 3 - 4(-1)^2 = 3 - 4(1) \\ &= 3 - 4 = -1 \end{aligned}$$



Here, different element in domain do not have different images in B. Since 1 and –1 are related to –1.
∴ f is not one - one.
Hence, f is not a bijective function.

8. Let A = {–1, 1}and B ={0, 2}. If the function f :A → B defined by $f(x) = ax + b$ is an onto function? Find a and b.

Sol. Given A = {–1, 1}, B = {0, 2} and $f : A \to B$ is defined by $f(x) = ax + b$ is an onto function.



$$\begin{aligned} f(-1) &= 0 \\ \Rightarrow \quad a(-1) + b &= 0 \quad [\because \text{Sub } x = -1, y = 0 \text{ in } y = ax + b] \\ \Rightarrow \quad -a + b &= 0 \quad \ldots (1) \\ \text{Also } f(1) &= 2 \\ \Rightarrow \quad a(1) + b &= 2 \quad [\because \text{Sub } x = 1, y = 2 \text{ in } y = ax + b] \\ \Rightarrow \quad \cancel{a} + b &= 2 \quad \ldots (2) \\ (1) \Rightarrow \quad -\cancel{a} + b &= 0 \\ \hline \text{Adding,} \quad 2b &= 2 \\ \Rightarrow \quad b &= \frac{2}{2} = 1 \end{aligned}$$

Substituting $b = 1$ in (2) we get

$$\begin{aligned} a + 1 &= 2 \quad \Rightarrow a = 2 - 1 = 1 \\ \therefore a &= 1, b = 1 \end{aligned}$$

Unit Test

Time : 45 Minutes **Marks : 25**

Section - A $5 \times 1 = 5$

1. n R = $\{(x, x^2) \mid x$ is a prime number less than 13$\}$ is
   (A) $\{2, 3, 5, 7\}$ (B) $\{2, 3, 5, 7, 11\}$
   (C) $\{4,9,25,49,121\}$ (D) $\{1,4, 9, 25, 49, 121\}$
2. A=$\{a, b, p\}$, B = $\{2, 3\}$, C = $\{p, q, r, s\}$ then $n[(A\cup C) \times B]$ is
   (A) 8 (B) 20 (C) 12 (D) 16
3. Let A = $\{1,2,3,4\}$ and B = $\{4,8,9,10\}$. A function f:A→ B given by $f = \{(1, 4),(2, 8),(3, 9),(4, 10)\}$ is a
   (A) Many-one function
   (B) Identity function
   (C) One-to-one function
   (D) Into function
4. If $g = \{(1, 1),(2, 3),(3, 5),(4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
   (A) $(-1, 2)$ (B) $(2, -1)$
   (C) $(-1, -2)$ (D) $(1,2)$
5. The range of the relation R = $\{(x, x^2) \mid x$ is a prime number less than 13$\}$ is
   (A) $\{2,3,5,7\}$ (B) $\{2,3,5,7,11\}$
   (C) $\{4,9,25,49,121\}$ (D) $\{1,4,9,25,49,121\}$

Section - B $5 \times 2 = 10$

1. Let A=$\{1, 2, 3, 4,...,45\}$ and R be the relation defined as "is square of" on A. Write $\mathbb{R}$ as a subset of A × A. Also, find the domain and range of $\mathbb{R}$.
2. Let $f = \{(x, y) \mid x, y \in \mathbb{N}$ and $y = 2x\}$ be a relation on $\mathbb{N}$. Find the domain, co-domain and range. Is this relation a function?
3. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.
4. Let A,B,C $\subseteq \mathbb{N}$ and a function f: A→ B be defined by $f(x) = 2x + 1$ and g : B → C be defined by $g(x) = x^2$. Find the range of fog and gof.
5. Let A = $\{-1, 1\}$and B =$\{0, 2\}$. If the function f :A → B defined by $f(x) = ax + b$ is an onto function? Find a and b.

Section - C $2 \times 5 = 10$

1. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$ is one - one but not onto.
2. Represent the function f =$\{(1, 2),(2, 2),(3, 2),(4,3), (5,4)\}$ through
   (i) an arrow diagram
   (ii) a table form
   (iii) a graph

Answers

SECTION - A

1. (C) $\{4,9,25,49,121\}$
2. (C) 12
3. (C) One-to-one function
4. (B) $(2, -1)$
5. (C) $\{4,9,25,49,121\}$

Section - B

1. Refer Sura's Guide Exercise 1.2; Q.No.2
2. Refer Sura's Guide Exercise 1.3; Q. No.1
3. Refer Sura's Guide Exercise 1.3; Q. No.8
4. Refer Sura's Guide Exercise 1.5; Q. No.5
5. Refer Sura's Guide Exercise 1.4; Q. No.8

Section - C

1. Refer Sura's Guide Exercise 1.4; Q.No.4
2. Refer Sura's Guide Exercise 1.4; Q.No.3

Chapter 2

NUMBERS AND SEQUENCES

FORMULAE TO REMEMBER

- Euclid's Division Lemma Let a and b ($a > b$) be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r$, $0 \le r < b$.
- If a, b are two positive integers with $a > b$ then G.C.D of (a, b) = G.C.D of $(a - b, b)$.
- Number of terms in A.P., $n = \frac{l - a}{d} + 1$
- If the sum of three consecutive terms of an A.P is given, then they can be taken as $a - d$, a and $a + d$.
- If a prime number p divides ab then either p divides a or p divides b, that is p divides atleast one of them.
- Two integers a and b are congruence modulo n if they differ by an integer multiple of n. That is $b - a = kn$ for some integer k. This can also be written as $a \equiv b \pmod{n}$.
- A real valued sequence is a function defined on the set of natural numbers and taking real values.
- Let a and d be real numbers. Then the numbers of the form a, $a + d$, $a + 2d$, $a + 3d$, $a + 4d$,... is said to form Arithmetic progression.

EXERCISE 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2.

Sol. The positive integers when divided by 3 leaves remainder 2.

By Euclid's division lemma

$a = bq + r, 0 \le r < b.$

Here $a = 3q + 2$, where $0 \le q < 3$.

When $q = 0$, $a = 3(0) + 2 = 2$

When $q = 1$, $a = 3(1) + 2 = 5$

When $q = 2$, $a = 3(2) + 2 = 8$

When $q = 3$, $a = 3(3) + 2 = 11$

The required positive numbers are 2, 5, 8, 11,...

2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

Sol. By Euclid's division algorithm,

$a = bq + r, \ 0 \le r < b.$

Here $532 = 21q + r$ ... (1)

$\Rightarrow$ $532 = 21(25) + 7$ ... (2)

$\therefore q = 25$, and $r = 7$

[Comparing (1) and (2)]

$$\therefore \begin{array}{r|l} & 25 \\ \hline 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & 7 \end{array}$$

$\therefore$ Number of completed rows = 25 and the leftover flower pots = 7.

3. Prove that the product of two consecutive positive integers is divisible by 2.

Sol. Let $n - 1$ and n be two consecutive positive integers. Then their product is $(n - 1)n$.

$(n - 1)(n) = n^2 - n.$

We know that any positive integer is of the form $2q$ or $2q + 1$ for some integer q. So, following cases arise.

Case I. When $n = 2q$. In this case, we have

$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q - 1)$

$\Rightarrow n^2 - n = 2r$, where $r = q(2q - 1)$

$\Rightarrow n^2 - n$ is divisible by 2.

Case II : When $n = 2q + 1$. In this case, we have

$n^2 - n = (2q + 1)^2 - (2q + 1)$

$= (2q + 1)(2q + 1 - 1) = 2q(2q + 1)$

$\Rightarrow n^2 - n = 2r$, where $r = q(2q + 1)$.

$\Rightarrow n^2 - n$ is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

Hence it is proved.

4. When the positive integers a, b and c are divided by 13, the respective remainders are 9, 7 and 10. Show that $a + b + c$ is divisible by 13.

Sol. When a is divided by 13, the remainder is 9.

By Euclid's lemma, $a = bq + r, 0 \le r < b$

$\Rightarrow a = 13q + 9$ ... (1)

Similarly when the positive integers b and c are divided by 13, the remainders are 7 and 10.

$\therefore b = 13q + 7$ ... (2)

and $c = 13q + 10$ ... (3)

Adding (1), (2) and (3) we get,

$a + b + c = 13q + 9 + 13q + 7 + 13q + 10$

$= 39q + 26 = 13(3q + 2)$

Which is divisible by 13.

$\therefore a + b + c$ is divisible by 13.

5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.

Sol. Let x be any integer.

The square of x is x^2.

Case (i) : Let x be an even integer.

$\Rightarrow x = 2q$ [$\because x$ is even]

Where q is some integer

$\Rightarrow x^2 = (2q)^2 = 4q^2$

$\Rightarrow x^2 = 4(q^2)$

$\Rightarrow x^2$ is divisible by 4.

$\therefore$ When x is an even integer, x^2 is divisible by 4.

$\Rightarrow x^2$ leaves the remainder 0 when divided by 4.

Case (ii) : Let x be an odd integer.

$\therefore x = 2k + 1$ for some integer k.

$\therefore x^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

$= 4k(k + 1) + 1$

$= 4q + 1$ where $q = k(k + 1)$

$\Rightarrow x^2 = 4q + 1$ leaves the remainder 1 when divided by 4.

From Case (i) and Case (ii), the square of any integer leaves the remainder either 0 or 1 when divided by 4.

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of

(i) 340 and 412 (ii) 867 and 255

(iii) 10224 and 9648 (iv) 84, 90 and 120

Sol. (i) To find the HCF of 340 and 412. Using Euclid's division algorithm.

We get $412 = 340 \times 1 + 72$

The remainder $72 \neq 0$

Again applying Euclid's division algorithm

$340 = 72 \times 4 + 52$

The remainder 52 $\neq$ 0.
Again applying Euclid's division algorithm

$$72 = 52 \times 1 + 20$$

The remainder 20 $\neq$ 0.
Again applying Euclid's division algorithm,

$$52 = 20 \times 2 + 12$$

The remainder 12 $\neq$ 0.
Again applying Euclid's division algorithm.

$$20 = 12 \times 1 + 8$$

The remainder 8 $\neq$ 0.
Again applying Euclid's division algorithm

$$12 = 8 \times 1 + 4$$

The remainder 4 $\neq$ 0.
Again applying Euclid's division algorithm

$$8 = 4 \times 2 + 0$$

The remainder is zero.
Therefore HCF of 340 and 412 is 4.

(ii) To find the HCF of 867 and 255, using Euclid's division algorithm.

$$867 = 255 \times 3 + 102$$

The remainder 102 $\neq$ 0.
Again using Euclid's division algorithm

$$255 = 102 \times 2 + 51$$

The remainder 51 $\neq$ 0.
Again using Euclid's division algorithm

$$102 = 51 \times 2 + 0$$

The remainder is zero.
Therefore the HCF of 867 and 255 is 51.

(iii) To find HCF 10224 and 9648. Using Euclid's division algorithm.

$$10224 = 9648 \times 1 + 576$$

The remainder 576 $\neq$ 0.
Again using Euclid's division algorithm

$$9648 = 576 \times 16 + 432$$

Remainder 432 $\neq$ 0.
Again applying Euclid's division algorithm

$$576 = 432 \times 1 + 144$$

Remainder 144 $\neq$ 0.
Again using Euclid's division algorithm

$$432 = 144 \times 3 + 0$$

The remainder is zero.
There HCF of 10224 and 9648 is 144.

(iv) To find HCF of 84, 90 and 120.
Using Euclid's division algorithm

$$90 = 84 \times 1 + 6$$

The remainder 6 $\neq$ 0.
Again using Euclid's division algorithm

$$84 = 6 \times 14 + 0$$

The remainder is zero.
$\therefore$ The HCF of 84 and 90 is 6. To find the HCF of 6 and 120 using Euclid's division algorithm.

$$120 = 6 \times 20 + 0$$

The remainder is zero.
Therefore HCF of 120 and 6 is 6
$\therefore$ HCF of 84, 90 and 120 is 6.

7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Sol. The required number is the HCF of the numbers.

$$1230 - 12 = 1218,$$
$$1926 - 12 = 1914$$

First we find the HCF of 1218 & 1914 by Euclid's division algorithm.

$$1914 = 1218 \times 1 + 696$$

The remainder 696 $\neq$ 0.
Again using Euclid's algorithm

$$1218 = 696 \times 1 + 522$$

The remainder 522 $\neq$ 0.
Again using Euclid's algorithm.

$$696 = 522 \times 1 + 174$$

The remainder 174 $\neq$ 0.
Again by Euclid's algorithm

$$522 = 174 \times 3 + 0$$

The remainder is zero.
$\therefore$ The HCF of 1218 and 1914 is 174.
$\therefore$ The required number is 174.

8. If *d* is the Highest Common Factor of 32 and 60, find *x* and *y* satisfying $d = 32x + 60y$.

Sol. Applying Euclid's division lemma to 32 and 60, we get

$$60 = 32 \times 1 + 28 \quad \text{...(i)}$$

The remainder is 28 $\neq$ 0.
Again applying division lemma

$$32 = 28 \times 1 + 4 \quad \text{...(ii)}$$

The remainder 4 $\neq$ 0.
Again applying division lemma

$$28 = 4 \times 7 + 0 \quad \text{...(iii)}$$

The remainder zero.
$\therefore$ HCF of 32 and 60 is 4.
From (ii), we get

$$32 = 28 \times 1 + 4$$
$$\Rightarrow \quad 4 = 32 - 28 \times 1$$
$$\Rightarrow \quad 4 = 32 - (60 - 32 \times 1) \times 1 \quad [\because 28 = (60 - 32) \times 1]$$
$$\Rightarrow \quad 4 = 32 - 60 + 32$$
$$\Rightarrow \quad 4 = 32 \times \underline{2} + (-1) \times \underline{60}$$
$$\therefore \quad x = 2 \text{ and } y = -1$$

9. A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

Sol. Let the positive integer be x.

$$x = 88 \times y + 61$$
$$61 = 11 \times 5 + 6$$
$$(\because 88 \text{ is multiple of } 11)$$

$\therefore$ 6 is the remainder. (When the number is divided by 88 giving the remainder 61 and when divided by 11 giving the remainder 6).

10. Prove that two consecutive positive integers are always coprime.

Sol. Let the two consecutive integers be n and $n+1$.

Suppose HCF $(n, n+1) = p$

$\Rightarrow p$ divides n ... (1)

and p divides $(n+1)$ ... (2)

$\Rightarrow p$ divides $(n + 1 - n)$ [From (2) – (1)]

$\Rightarrow p$ divides 1

There is no number which divides 1 except 1

$\Rightarrow p = 1$ $\therefore$ HCF $(n, n+1) = 1$.

$\Rightarrow n$ and $(n+1)$ are Coprime.

EXERCISE 2.2

1. For what values of natural number n, 4^n can end with the digit 6?

Sol. $4^n = (2 \times 2)^n = 2^n \times 2^n$

2 is a factor of 4^n.

So, 4^n is always even and end with 4 and 6.

When n is an even number say 2, 4, 6, 8 then 4^n can end with the digit 6.

Example:

$4^2 = 16$	$4^3 = 64$
$4^4 = 256$	$4^5 = 1{,}024$
$4^6 = 4{,}096$	$4^7 = 16{,}384$
$4^8 = 65{,}536$	$4^9 = 262{,}144$

2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?

Sol. $2^n \times 5^m$

2^n is always even for all values of n.

5^m is always odd and ends with 5 for all values of m.

But $2^n \times 5^m$ is always even and ends in 0.

[$\because$ even number X odd number = even number]

$\therefore$ $2^n \times 5^m$ cannot end with the digit 5 for any values of m. No value of m will satisfy $2^n \times 5^m$ ends in 5.

3. Find the HCF of 252525 and 363636.

Sol. To find the HCF of 252525 and 363636

Using Euclid's Division algorithm

$$363636 = 252525 \times 1 + 111111$$

The remainder $111111 \neq 0$.

$\therefore$ Again by division algorithm

$$252525 = 111111 \times 2 + 30303$$

The remainder $30303 \neq 0$.

$\therefore$ Again by division algorithm.

$$111111 = 30303 \times 3 + 20202$$

The remainder $20202 \neq 0$.

$\therefore$ Again by division algorithm

$$30303 = 20202 \times 1 + 10101$$

The remainder $10101 \neq 0$.

$\therefore$ Again using division algorithm

$$20202 = 10101 \times 2 + 0$$

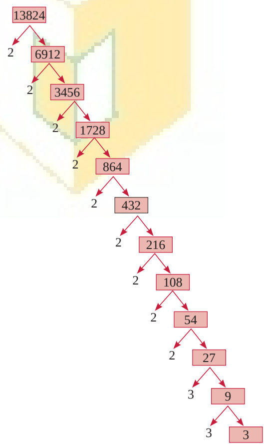
The remainder is 0.

Therefore HCF. of 252525 and 363636 is 10101.

4. If $13824 = 2^a \times 3^b$ then find a and b.

Sol. If $13824 = 2^a \times 3^b$

Using the prime factorisation tree



$$13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= 2^9 \times 3^3 = 2^a \times 3^b$$

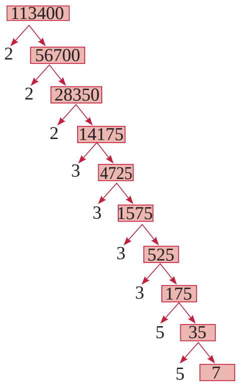
$\therefore$ $a = 9$, $b = 3$.

5. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4.

Sol. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

p_1, p_2, p_3, p_4 are primes in ascending order, x_1, x_2, x_3, x_4 are integers.

Using prime factorisation tree.



$113400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$

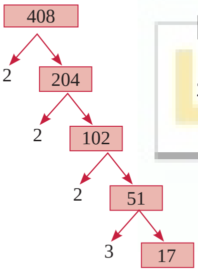
$= 2^3 \times 3^4 \times 5^2 \times 7^1$

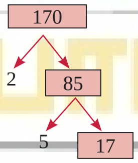
$= p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$

$\therefore\ p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1.$

6. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Sol. 408 and 170.





$408 = 2^3 \times 3^1 \times 17^1$

$170 = 2^1 \times 5^1 \times 17^1$

[By fundamental arithmetic theorem, every natural no except, can be factorized as a product of primes and it is unique]

Common Prime Factors	Least Exponents
2	1
17	1

$\therefore$ HCF $= 2^1 \times 17^1 = 34.$

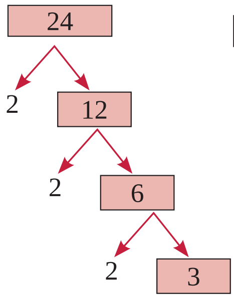
To find LCM, we list all prime factors of 408 and 170, and their greatest exponents as follows.

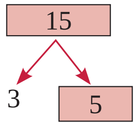
Prime factors of 408 and 170	Greatest Exponents
2	3
3	1
5	1
17	1

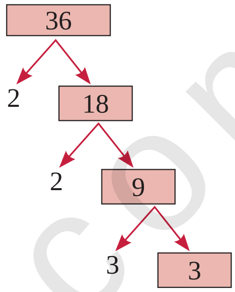
$\therefore$ LCM $= 2^3 \times 3^1 \times 5^1 \times 17^1 = 2040.$

7. Find the greatest number consisting of 6 digits which is exactly divisible by 24,15,36?

Sol. To find LCM of 24, 15, 36







$24 = 2^3 \times 3$

$15 = 3 \times 5$

$36 = 2^2 \times 3^2$

Prime factors of 24, 15, 36	Greatest Exponents
2	3
3	2
5	1

$\therefore$ LCM $= 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

If a number has to be exactly divisible by 24, 15, and 36, then it has to be divisible by 360. Greatest 6 digit number is 999999.

Common multiplies of 24, 15, 36 with 6 digits are 103680, 116640, 115520, ...933120, 999720 with six digits.

$\therefore$ The greatest number consisting 6 digits which is exactly divisible by 24, 15, 36 is 999720.

8. What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?

Sol.

$35 = 5 \times 7$

$56 = 2 \times 2 \times 2 \times 7$

$91 = 7 \times 13$

LCM of 35, 56, 91 $= 5 \times 7 \times 2 \times 2 \times 2 \times 13$

$= 3640$

$\therefore$ Required number $= 3647$ which leaves remainder 7 in each case.

9. Find the least number that is divisible by the first ten natural numbers.

Sol. The least number that is divisible by the first ten natural numbers is 2520.

Hint:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10
The least multiple of 2 & 4 is 8
The least multiple of 3 is 9
The least multiple of 7 is 7
The least multiple of 5 is 5
LCM of $8 \times 9 \times 7 \times 5 = 40 \times 63 = 2520.$

Unit Test

Time : 45 Minutes **Marks : 25**

SECTION - A (5 × 1 = 5)

1. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are
   (A) 0, 1, 8 (B) 1, 4, 8
   (C) 0, 1, 3 (D) 1, 3, 5
2. The sum of the exponents of the prime factors in the prime factorization of 1729 is
   (A) 1 (B) 2 (C) 3 (D) 4
3. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P
   (A) 4551 (B) 10091
   (C) 7881 (D) 13531
4. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?
   (A) 6 (B) 7 (C) 8 (D) 9
5. The value of $(1^3 + 2^3 + 3^3 + ... + 15^3) - (1 + 2 + 3 + ... + 15)$ is
   (A) 14400 (B) 14200
   (C) 14280 (D) 14520

SECTION - B (5 × 2 = 10)

1. Find all positive integers which when divided by 3 leaves remainder 2.
2. For what values of natural number n, $4n$ can end with the digit 6?
3. What is the time 100 hours after 7 a.m.?
4. Find the 19th term of an A.P. –11, –15, –19,...
5. The sum of the squares of the first n natural numbers is 285, while the sum of their cubes is 2025. Find the value of n.

SECTION - C (2 × 5 = 10)

1. Sivamani is attending an interview for a job and the company gave two offers to him.
   Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.
   Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.
   What is his salary in the 4^{th} year with respect to the offers A and B?
2. Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹2 to mail one letter, find the amount spent on postage when 8th set of letters is mailed.

Answers

SECTION - A

1. (A) 0, 1, 8
2. (A) 1
3. (C) 7881
4. (C) 8
5. (C) 14280

SECTION - B

1. Refer Sura's Guide Exercise 2.1, Q.No.1
2. Refer Sura's Guide Exercise 2.1, Q.No.1
3. Refer Sura's Guide Exercise 2.3, Q.No.5
4. Refer Sura's Guide Exercise 2.5 Q.No.4
5. Refer Sura's Guide Exercise 2.9 Q.No.

SECTION - C

1. Refer Sura's Guide Exercise No.2.7, Q.No.11
2. Refer Sura's Guide Exercise 2.8 Q.No.8

❁❁❁❁❁

Chapter 3

ALGEBRA

FORMULAE TO REMEMBER

- $f(x) \times g(x) = \text{LCM}\ [f(x), g(x)] \times \text{GCD}\ [f(x), g(x)]$
- Roots of the quadratic equation, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- General Form of an equation : x^2 – (sum of the roots)x + product of the roots = 0

EXERCISE 3.1

1. Solve the following system of linear equations in three variables

(i) $x + y + z = 5$; $2x - y + z = 9$; $x - 2y + 3z = 16$ **[PTA - 5; Hy - 2019; Sep - 2021]**

(ii) $\frac{1}{x} - \frac{2}{y} + 4 = 0$; $\frac{1}{y} - \frac{1}{z} + 1 = 0$; $\frac{2}{z} + \frac{3}{x} = 14$

(iii) $x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$

Sol. Let

(i)

$$x + y + z = 5 \quad ...(1)$$
$$2x - y + z = 9 \quad ...(2)$$
$$x - 2y + 3z = 16 \quad ...(3)$$

$(1) + (2) \Rightarrow$

$$x + \cancel{y} + z = 5$$
$$2x - \cancel{y} + z = 5$$

Adding, $3x + 2z = 14$ ...(4)

$(2) \times 2 \Rightarrow 4x - \cancel{2y} + 2z = 18$

$(3) \Rightarrow x - \cancel{2y} + 3z = 16$
$\quad (-)\ (+)\ (-)\ (-)$

Subtracting $3x - z = 2$ ...(5)

$(4) - (5) \Rightarrow 3x + 2z = 14$

$\Rightarrow 3x - z = 2$
$\quad (-)\ (+)\ (-)$

Subtracting, $3z = 12$

$z = 4$

Substitute $z = 4$ in (4)

$$3x + 2(4) = 14$$
$$3x + 8 = 14$$
$$3x = 6$$
$$x = 2$$

Substitute $x = 2$, $z = 4$ in (1)

$2 + y + 4 = 5 \Rightarrow y = -1$

$\boxed{x = 2,\ y = -1,\ z = 4}$

(ii)

$$\frac{1}{x} - \frac{2}{y} + 4 = 0 \quad ...(1)$$
$$\frac{1}{y} - \frac{1}{z} + 1 = 0 \quad ...(2)$$
$$\frac{2}{z} + \frac{3}{x} = 14 \quad ...(3)$$

Put $\frac{1}{x} = a$

$\frac{1}{y} = b$

$\frac{1}{z} = c$ in (1), (2) & (3)

$a - 2b + 4 = 0 \Rightarrow a - 2b = -4$ ...(1)

$b - c + 1 = 0 \Rightarrow b - c = -1$ ...(2)

$2c + 3a = 14 \Rightarrow 2c + 3a = 14$ ...(3)

$(1) \Rightarrow a - \cancel{2b} = -4$

$(2) \times 2 \Rightarrow -2c + \cancel{2b} = -2$

$a - 2c = -6$ ...(4)

Adding, $3a + 2c = 14$

$(4) + (3) \Rightarrow 4a = 8$

$a = 2$

Substitute $a = 2$ in (1), we get

$$2 - 2b = -4$$
$$-2b = -6$$
$$b = 3$$

Substitute $b = 3$ in (2), we get

$$3 - c = -1$$
$$-c = -4 \Rightarrow c = 4$$

$$a = \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$$
$$b = \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$$
$$c = \frac{1}{z} = 4 \Rightarrow z = \frac{1}{4}$$

$\boxed{x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{4}}$ Solution set $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

(iii) Given equations are

I II III IV

$$x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$$

Consider $x + 20 = \frac{3y}{2} + 10$ [From I & II]

$\Rightarrow x = \frac{3y}{2} + 10 - 20$

$\Rightarrow x = \frac{3y}{2} - 10$

Multiply by 2, we get, $2x = 3y - 20$

$\Rightarrow 2x - 3y = -20$

From [I and III]

Now, $x + 20 = 2z + 5$

$\Rightarrow x - 2z = 5 - 20$

$\Rightarrow x - 2z = -15$ ... (2)

From [I and IV]

Also $x + 20 = 110 - (y + z)$

Also $x + 20 = 110 - y - z$

$\Rightarrow x + y + z = 110 - 20$

$\Rightarrow x + y + z = 90$ ... (3)

$2 \times (3) \Rightarrow 2x - 2y + 2z = 180$

$(2) \Rightarrow x - 2z = -15$

Adding $3x + 2y = 165$ ... (4)

Consider equations (1) and (4)

$$\begin{aligned}(1)\times 3 \Rightarrow \quad 6x - 9y &= -60\\ (4)\times 2 \Rightarrow \quad \underset{(-)}{6x} + \underset{(-)}{4y} &= \underset{(-)}{330}\end{aligned}$$

Subtracting, $-13y = -390$

$\Rightarrow \quad y = \dfrac{-390}{-13} = 30$

$\Rightarrow \quad \boxed{y = 30}$

Substituting $y = 30$ in (1), we get

$\Rightarrow \quad 2x - 3(30) = -20$

$2x - 90 = -20$

$\Rightarrow \quad 2x = -20 + 90 = 70$

$\Rightarrow \quad x = \dfrac{70}{2} = 35$

$\boxed{\therefore x = 35}$

Substituting $x = 35$ in (2) we get,

$35 - 2z = -15$

$\Rightarrow \quad 35 + 15 = 2z$

$\Rightarrow \quad 50 = 2z$

$\Rightarrow \quad z = \dfrac{50}{2}$

$\boxed{\therefore z = 25}$

Solution set is {35, 30, 25}

Hence, the system has unique solution

2. Discuss the nature of solutions of the following system of equations

(i) $x + 2y - z = 6$; $-3x - 2y + 5z = -12$; $x - 2z = 3$

(ii) $2y + z = 3(-x + 1)$; $-x + 3y - z = -4$

$3x + 2y + z = -\dfrac{1}{2}$

(iii) $\dfrac{y+z}{4} = \dfrac{z+x}{3} = \dfrac{x+y}{2}$; $x + y + z = 27$

Sol. (i)

$x + 2y - z = 6$ ...(1)

$-3x - 2y + 5z = -12$ ...(2)

$x - 2z = 3$ ...(3)

Consider (1) and (2)

$x + \cancel{2y} - z = 6$ ...(1)

$-3x - \cancel{2y} + 5z = -12$ ...(2)

Adding, $-2x + 4z = -6$

$-x + 2z = -3$ [Divided by 2]

$x - \cancel{2z} = 3$

(3) $\underset{(-)}{x} - \underset{(+)}{\cancel{2z}} = \underset{(-)}{3}$

$0 = 0$

We see that the system has an infinite number of solutions.

(ii) Given equations are $2y + z = 3(-x + 1)$; $-x + 3y - z = -4$, $3x + 2y + z = -\dfrac{1}{2}$

Consider $2y + z = 3(-x + 1)$

$\Rightarrow \quad 2y + z = -3x + 3$

$\Rightarrow \quad 3x + 2y + z = 3$ ... (1)

Now, $-x + 3y - z = -4$ ... (2)

Also, $3x + 2y + z = -\dfrac{1}{2}$

Multiplying by 2 we get,

$6x + 4y + 2z = -1$ ... (3)

Consider (1) and (2)

$(1) \Rightarrow \quad 3x + 2y + z = 3$

$(2)\times 3 \Rightarrow \quad -3x + 9y - 3z = -12$

Adding $11y - 2z = -9$ ... (4)

Consider (2) and (3)

$(2)\times 6 \Rightarrow -6x + 18y - 6z = -24$

$(3) \Rightarrow \quad 6x + 4y + 2z = -1$

Adding $22y - 4z = -25$ ... (5)

Now $(4)\times 2 \Rightarrow \quad 22y - 4z = -18$

$(5) \Rightarrow \quad \underset{(-)}{22y} - \underset{(+)}{4z} = \underset{(+)}{-25}$

Subtracting $0 \neq -7$

Since we have got a false equation, the system is inconsistent and has no solution.

(iii) Given equation are

$$\underset{\text{I}}{\frac{y+z}{4}} = \underset{\text{II}}{\frac{z+x}{3}} = \underset{\text{III}}{\frac{x+y}{2}} \text{ and } x + y + z = 27$$

Let $x + y + z = 27$ .... (1)

From I and II, we get

$\dfrac{y+z}{4} = \dfrac{z+x}{3}$

Cross multiplying we get, $3(y + z) = 4(z + x)$

$\Rightarrow \quad 3y + 3z = 4z + 4x$

$\Rightarrow \quad 3y + 3z - 4z - 4x = 0$

$\Rightarrow \quad -4x + 3y - z = 0$ ... (2)

From II and II, we get,

$\dfrac{z+x}{3} = \dfrac{x+y}{2}$

Cross multiplying we get, $2(z + x) = 3(x + y)$

$\Rightarrow \quad 2z + 2x = 3x + 3y$

$\Rightarrow 2z + 2x - 3x - 3x - 3y = 0$

$\Rightarrow \quad -x - 3y + 2z = 0$ ... (3)

Chapter 4

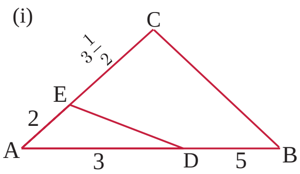
GEOMETRY

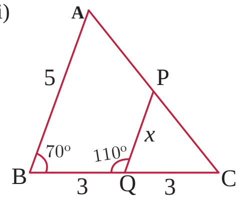
FORMULAE TO REMEMBER

- In a right angle triangle, the squares on the hypotenuse is equal to the sum of the squares on the other two sides.
- **AA criterion of similarity:**
  If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar, because the third angle in both triangles must be equal. Therefore, AA similarity criterion is same as the AAA similarity criterion.
- **SAS criterion of similarity:**
  If one angle of a triangle is equal to one angle of another triangle and if the sides including them are proportional then the two triangles are similar.

EXERCISE 4.1

1. Check whether the which triangles are similar and find the value of x.





Sol. Consider ΔADE and ΔABC

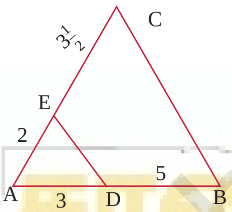
(i) Consider $\frac{AE}{AC} = \frac{AD}{AB}$ (for similar triangle)

$$\left[\because 3\frac{1}{2} + 2 = 5\frac{1}{2} = \frac{11}{2}\right]$$

But here, $\frac{2}{\frac{11}{2}} \neq \frac{3}{8}$

$2 \times \frac{2}{11} \neq \frac{3}{8}$

$\frac{4}{11} \neq \frac{3}{8}$



$\therefore$ They are not similar triangles

(ii) In ΔABC and ΔPQC,

$\angle PQC = 180 - 110° = 70°$

$\angle ABC = \angle PQC = 70°$

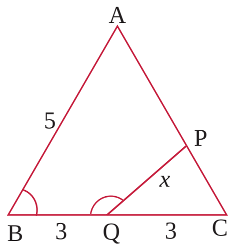
$\angle C = \angle C$ (common angle)

$\therefore \angle A = \angle QPC$ ($\therefore$ AAA criterion)

$\therefore \Delta ABC$ and ΔPQC are similar triangles

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QC}$

$\frac{5}{x} = \frac{3+3}{3}$



Cross multiplying $6x = 15$

$x = \frac{15}{6} = 2.5$

2. A girl looks the reflection of the top of the lamp post on the mirror which is 6.6 m away from the foot of the lamppost. The girl whose height is 1.25 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamppost are in a same line, find the height of the lamp post.

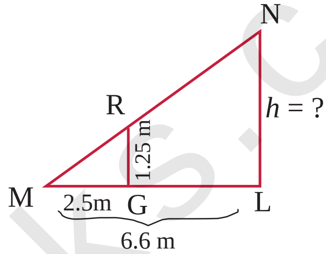
Sol. Let M be the position of the mirror, G be the position of the girl and L be the position of the lamp.

Let GR be the height of the girl

$\Rightarrow$ GR = 1.25 m and let NL be the height of the lamp post $\Rightarrow$ NL = h.

ΔMLN, ΔMGR are similar triangles

[$\because \angle M$ is common $\angle BGL = \angle NLG = 90°$ by AA criteria]



$\therefore \frac{GR}{LN} = \frac{MG}{ML}$

$\frac{1.25}{h} = \frac{2.5}{6.6}$

$1.25 \times 6.6 = 2.5 \times h$

$h = \frac{1.25 \times 6.6}{2.5}$

$h = \frac{\cancel{125}^{5}}{\cancel{100}_{4\,2}} \times \frac{\cancel{66}^{33}}{\cancel{10}} \times \frac{\cancel{10}}{\cancel{25}_{5}}$

$= \frac{33}{10} = 3.3$ m

$\therefore$ Height of the lamp post is 3.3 m.

3. A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.

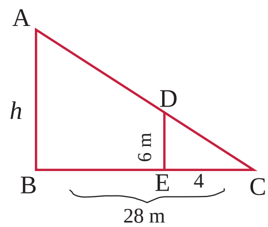
Sol. Let AB = h m be the height of the tower,

DE = 6 m be the length of the stick

Also, BC = 28 m be the shadow of the tower and

EC = 400 cm = 4 m be the shadow of the stick

In the picture ΔABC, ΔDEC are similar triangles.



In ΔABC and ΔDEC, $\angle C$ is common and $\angle ABC = \angle DEC = 90^o$.

$\therefore$ By AA criteria, ΔABC and ΔDEC are similar

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EC}$$

$$\Rightarrow \frac{h}{6} = \frac{28}{4}$$

$$\Rightarrow \frac{h}{6} = 7$$

$$\Rightarrow h = 6 \times 7$$

$$\Rightarrow h = 42 \text{ m}$$

$\therefore$ Height of a tower = 42 m.

4. Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that PT × TR = ST × TQ. **[PTA - 6]**

Sol. Consider ΔPQT and ΔTSR,

$$\angle P = \angle S = 90^o \text{ and}$$

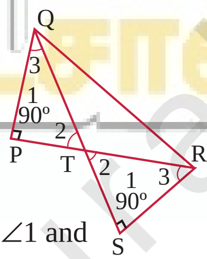
$$\angle PTQ = \angle STR$$

[Vertically opposite angles]

$\therefore$ By AA criteria of similarity

ΔPQT is similar to $\Delta\ TSR$,

$\therefore \Delta\ PQT \sim \Delta\ TSR$



When two triangles are similar, their corresponding sides are proportional.

In Δ PQT, side lying between $\angle 1$ and $\angle 2$ is PT

In ΔTSR, side lying between $\angle 1$ and $\angle 2$ is TS ... (1)

Similarly in ΔPQT, side lying between $\angle 2$ and $\angle 3$ is QT and in ΔTSR, side lying between $\angle 2$ and $\angle 3$ is TR ... (2)

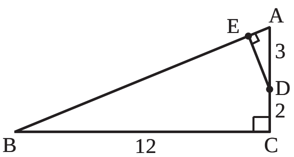
From (1) and (2) [$\because \Delta PQT \sim \Delta\ TSR$]

$$\Rightarrow PT.TR = TS.QT$$

$$\Rightarrow PT \times TR = ST \times TQ$$

Hence proved.

5. In the adjacent figure, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.



Sol. Given Δ ABC is right angled at C and $DE \perp AB$.

From the right angle ΔABC,

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 5^2 + 12^2 = 25 + 144$$

$$AB = \sqrt{169} = 13$$

$$AD = 3 \text{ ; } DC = 2 \text{ ; } BC = 12$$

In ΔABC and ADE,

$$\angle AED = \angle ACB = 90^o$$

$\angle A$ is common.

$\therefore$ By AA similarly, Δ ABC ~ Δ ADE

$$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

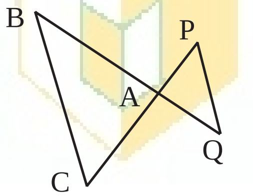
$$\frac{3}{13} = \frac{AE}{5} = \frac{DE}{12}$$

$$\frac{AE}{5} = \frac{3}{13} \Rightarrow AE = \frac{3 \times 5}{13} = \frac{15}{13}$$

$$\frac{DE}{12} = \frac{3}{13} \Rightarrow DE = \frac{3 \times 12}{13} = \frac{36}{13}$$

$$\text{Thus } AE = \frac{15}{13} \text{ ; } DE = \frac{36}{13}$$

6. In the adjacent figure, $\Delta ACB \sim \Delta APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.



Sol. $\Delta ABC \sim \Delta APQ$

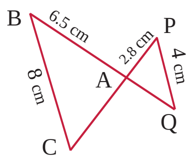
$\Rightarrow$ Their corresponding sides are proportional

$$\frac{AB}{AQ} = \frac{BC}{PQ} = \frac{CA}{AP}$$

$$\Rightarrow \frac{6.5}{AQ} = \frac{8}{4} = \frac{CA}{2.8}$$

$$\Rightarrow \underset{\text{I}}{\frac{6.5}{AQ}} = \underset{\text{II}}{\frac{2}{1}} = \underset{\text{III}}{\frac{CA}{2.8}}$$

From II and III



$$\Rightarrow CA = 2 \times 2.8$$

$$\Rightarrow CA = 5.6 \text{ cm}$$

From I and II

$$\Rightarrow 6.5 = 2\ AQ$$

$$\Rightarrow AQ = \frac{6.5}{2}$$

$$\Rightarrow AQ = 3.25 \text{ cm}$$

Chapter 6

TRIGONOMETRY

FORMULAE TO REMEMBER

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
- $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$
- $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
- $\sec\theta = \dfrac{1}{\cos\theta}$
- $\sin(90° - \theta) = \cos\theta$
- $\text{cosec}\,(90° - \theta) = \sec\theta$
- $\cos(90° - \theta) = \sin\theta$
- $\sec(90° - \theta) = \text{cosec}\,\theta$
- $\tan(90° - \theta) = \cot\theta$
- $\cot(90° - \theta) = \tan\theta$
- $\cos^2\theta + \sin^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \text{cosec}^2\theta$

EXERCISE 6.1

1. Prove the following identities.

(i) cot θ + tan θ = sec θ cosec θ

(ii) tan⁴ θ + tan² θ = sec⁴ θ – sec² θ

Sol. (i) L.H.S $= \dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}$

$$\left[\because \cot\theta=\frac{\cos\theta}{\sin\theta} \text{ and } \tan\theta=\frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} = \sec\theta\ \text{cosec}\theta = \text{R.H.S}$$

(ii) L.H.S = $\tan^2\theta(\tan^2\theta+1)$

[Taking $\tan^2\theta$ as common]

$= \tan^2\theta(\sec^2\theta) = (\sec^2\theta-1)(\sec^2\theta)$

Hint: $1+\tan^2\theta=\sec^2\theta$
$\tan^2\theta=\sec^2\theta-1$

$= \sec^4\theta-\sec^2\theta = \text{R.H.S}$

2. Prove the following identities

(i) $\dfrac{1-\tan^2\theta}{\cot^2\theta-1}=\tan^2\theta$

(ii) $\dfrac{\cos\theta}{1+\sin\theta}=\sec\theta-\tan\theta$ **[Qy - 2019]**

Sol. (i) L.H.S $= \dfrac{1-\tan^2\theta}{\cot^2\theta-1} = \dfrac{1-\tan^2\theta}{\dfrac{1}{\tan^2\theta}-1}$

$$\left[\because \cot^2\theta=\frac{1}{\tan^2\theta}\right]$$

$$= \frac{1-\tan^2\theta}{\dfrac{1-\tan^2\theta}{\tan^2\theta}} = \cancel{1-\tan^2\theta}\times\frac{\tan^2\theta}{\cancel{1-\tan^2\theta}}$$

$= \tan^2\theta$ = RHS Hence proved.

(ii) L.H.S $= \dfrac{\cos\theta}{1+\sin\theta}$

(Multiplying Numerator and denominator by $(1-\sin\theta)$)

$$= \frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta-\cos\theta\sin\theta}{1-\sin^2\theta}$$

$$= \frac{\cos\theta-\cos\theta\sin\theta}{\cos^2\theta} = \frac{\cos\theta}{\cos^2\theta}-\frac{\cos\theta\sin\theta}{\cos^2\theta}$$

$$= \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta} = \sec\theta-\tan\theta = \text{R.H.S}$$

3. Prove the following identities

(i) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}=\sec\theta+\tan\theta$ **[Sep.-2020]**

(ii) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

Sol. (i) L.H.S $= \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}\times\dfrac{1+\sin\theta}{1+\sin\theta}}$

[Rationalising the denominator]

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$[\because \sin^2\theta+\cos^2\theta=1 \Rightarrow \cos^2=1-\sin^2\theta]$

$$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta} = \sec\theta+\tan\theta = \text{R.H.S}$$

(ii) L.H.S $= \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}\times\dfrac{\sqrt{1+\sin\theta}}{1+\sin\theta}$

[Rationalising the denominator]

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$= \sec\theta+\tan\theta$ ... (1)

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\times\frac{\sqrt{1-\sin\theta}}{1-\sin\theta}$$

[Rationalising the denominator]

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}$$

$= \sec\theta-\tan\theta$ ... (2)

LHS = (1) + (2) $\Rightarrow \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$= \sec\theta+\tan\theta+\sec\theta-\tan\theta$

$= 2\sec\theta$ = R.H.S.

Hence proved

4. Prove the following identities

(i) $\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$

(ii) $(\sin\theta+\sec\theta)^2+(\cos\theta+\text{cosec}\theta)^2 = 1+(\sec\theta+\text{cosec}\theta)^2$

Sol. (i) L.H.S $= \sec^6\theta = (\sec^2\theta)^3 = (1+\tan^2\theta)^3$

$= (\tan^2\theta+1)^3$

$[(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

[Here $a=\tan^2\theta$, $b=1$]

$(\tan^2\theta)^3+3(\tan^2\theta)^2\times1+3\times\tan^2\theta\times1^2+1$

$= \tan^6\theta + 3\tan^2\theta \times (\sec^2\theta - 1) + 3\tan^2\theta + 1$

$[\because \tan^2\theta = \sec^2\theta - 1]$

$= \tan^6\theta + 3\tan^2\theta\sec^2\theta - \cancel{3\tan^2\theta} + \cancel{3\tan^2\theta} + 1$

$= \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$ = R.H.S

(ii) L.H.S $= (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2$

$= \sin^2\theta + 2\sin\theta\sec\theta + \sec^2\theta + \cos^2\theta + 2\cos\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta$

$= 1 + \sec^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\sec\theta + 2\cos\theta\operatorname{cosec}\theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\left[\because \sec\theta = \frac{1}{\cos\theta}; \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$

$= 1 + \sec^2\theta + \operatorname{cosec}^2\theta + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= 1 + \sec^2\theta + \operatorname{cosec}^2\theta + 2\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$

$= 1 + \sec^2\theta + \operatorname{cosec}^2\theta + 2\times\left(\frac{1}{\sin\theta\cos\theta}\right)$

$= 1 + (\sec^2\theta + \operatorname{cosec}^2\theta + 2\sec\theta\operatorname{cosec}\theta)$

$= 1 + (\sec\theta + \operatorname{cosec}\theta)^2$ = R.H.S

5. Prove the following identities

(i) $\sec^4\theta(1 - \sin^4\theta) - 2\tan^2\theta = 1$

(ii) $\dfrac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$

Sol. (i) L.H.S $= \sec^4\theta(1 - \sin^4\theta) - 2\tan^2\theta$

$= \frac{1}{\cos^4\theta}(1 - \sin^4\theta) - 2\tan^2\theta$

$= \frac{1}{\cos^4\theta} - \frac{\sin^4\theta}{\cos^4\theta} - 2\tan^2\theta$

$= \frac{1}{\cos^4\theta} - \frac{\sin^4\theta}{\cos^4\theta} - \frac{2\sin^2\theta}{\cos^2\theta}$

$= \frac{1 - \sin^4\theta - 2\sin^2\theta\cos^2\theta}{\cos^4\theta}$

$[\because$ Taking $\cos^4\theta$ as LCM$]$

$= \frac{1 + \cos^4\theta - \cos^4\theta - \sin^4\theta - 2\sin^2\theta\cos^2\theta}{\cos^4\theta}$

[Adding and subtracting $\cos^4\theta$ in the numerator]

$= \frac{1 + \cos^4\theta - [\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta]}{\cos^4\theta}$

[Taking (–) common for the last 3 terms]

$= \frac{1 + \cos^4\theta - (\sin^2\theta + \cos^2\theta)^2}{\cos^4\theta}$

$[\because (a+b)^2 = a^2 + 2ab + b^2$ where $a = \sin^2\theta$, $b = \cos^2\theta]$

$= \frac{\cancel{1} + \cos^4\theta - \cancel{(1)}}{\cos^4\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{\cos^4\theta}{\cos^4\theta} = 1$ = RHS Hence proved

(ii) LHS

$\frac{\cot\theta - \cos\theta}{\cot\theta + \cos\theta} = \frac{\frac{\cos\theta}{\sin\theta} - \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$

$= \frac{\cancel{\cos\theta}\left(\frac{1}{\sin\theta} - 1\right)}{\cancel{\cos\theta}\left(\frac{1}{\sin\theta} + 1\right)} = \frac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$ = R.H.S

$\left[\because \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$

6. Prove the following identities

(i) $\dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B} = 0$

(ii) $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \dfrac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

Sol. (i) LHS $= \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$

$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A + \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)} = 0$ = R.H.S

(ii) $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \dfrac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

LHS $= \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$

Chapter 7

MENSURATION

FORMULAE TO REMEMBER

- CSA of a right circular cylinder = $2\pi rh$ sq. units.
- TSA of a right circular cylinder = $2\pi r\ (htr)$ sq. unit.
- CSA of the hollow cylinder = 2π Rh + $2\pi rh$.
- CSA of right circular cone = πrl.
- TSA of a right circular cone = $\pi r\ (l + r)$.
- CSA of a hemisphere = $2\pi r^2$.
- TSA of a hemisphere = $3\pi r^2$.
- CSA of hemisphere = $2\pi\ (R^2 + r^2)$.
- CSA of a frustum = $\pi(R + r)l$.
- TSA of a frustum = $\pi\ (R + r)l + \pi R^2 + \pi r^2$.

EXERCISE 7.1

1. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Sol. Given $r : h = 5 : 7$

$\Rightarrow r = 5x$ and

$h = 7x$

CSA of a cylinder $= 2\pi rh$

$\therefore 5500 = 2 \times \frac{22}{7} \times 5x \times 7x$

$22\cancel{0}x^2 = 550\cancel{0}$

$x^2 = \frac{550}{22} = 25$

$\therefore x = 5$

$\therefore$ Radius $= 5 \times 5 = 25$ cm $[\because r = 5x$ and $x = 5]$

Height $= 7 \times 5 = 35$ cm.

2. A solid iron cylinder has total surface area of 1848 sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.

Sol. Given C.S.A. $= \frac{5}{6}$ T.S.A

TSA $= 1848$ m^2

$2\pi r(h + r) = 1848$ m^2

$2\pi rh + 2\pi r^2 = 1848$ m^2

$\frac{5}{6} \times 1848 + 2\pi r^2 = 1848 \quad [\because 2rh = \frac{5}{6} \times \text{TSA}]$

$1540 + 2\pi r^2 = 1848$

$2\pi r^2 = 1848 - 1540 = 308$

$2 \times 22 \times r^2 = 308$

$\Rightarrow r^2 = 308 \times \frac{1}{2} \times \frac{7}{22}$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ m.

$2\pi rh = \frac{5}{6} \times 1848$

$2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times h = \frac{5}{6} \times 1848$

$\Rightarrow h = 35$ m

$\therefore$ Radius $r = 7$ m, Height $= 35$ m.

3. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.

Sol. Given R = 16 cm, $h = 13$ cm, Thickness = 4 cm

$r = R -$ thickness $= 16 - 4 = 12$ cm $\Rightarrow r = 12$ cm.

Total surface area of hollow cylinder $= 2\pi(R + r)(R - r + h)$ sq. units.

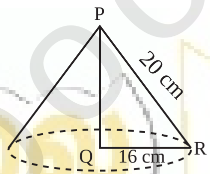
T.S.A $= 2 \times \frac{22}{7}(16 + 12)(16 - 12 + 13)$

$= \frac{44}{\cancel{7}}\left(\cancel{28}^{4}\right)(17)$

T.S.A $= 2992$ sq.cm

4. A right angled triangle PQR where ∠Q = 90° is rotated about QR and PQ. If QR = 16 cm and PR = 20 cm, compare the curved surface areas of the right circular cones so formed by the triangle.

Sol. When it is rotated about PQ the C.S.A of the cone formed $= \pi rl$. [Here $r = 16$, $l = 20$]



$= \frac{22}{7} \times 16 \times 20$

$= \frac{7040}{7} = 1005.71$ cm^2

When it is rotated about QR CSA of the cone formed.

$= \pi rl$. Here $h = 16$, $l = 20$

here $r = \sqrt{l^2 - h^2} = \sqrt{20^2 - 16^2}$

$= \sqrt{400 - 256} = \sqrt{144} = 12$ cm

CSA $= \pi rl = \frac{22}{7} \times 12 \times 20 = \frac{5280}{7}$

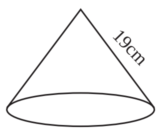
$= 754.28$ cm^2.

$1005.71 > 754.28$.

$\therefore$ CSA of the cone when rotated about its PQ is larger.

5. 4 persons live in a conical tent whose slant height is 19 m. If each person require 22 m^2 of the floor area, then find the height of the tent.

Sol. Given $l = 19$ m



Base area of the cone

$= \pi r^2 =$ sq units.

$\pi r^2 = 4 \times 22$ m^2

$\frac{22}{7} \times r^2 = 88$

$[\because$ Area for 4 persons $= 4 \times 22]$

$r^2 = \cancel{88}^{4} \times \frac{7}{\cancel{22}}$

Hint:

$$\begin{array}{r|l} & 18.25 \\ 1 & 33300 \\ & 1 \\ 28 \times 8 & 233 \\ & 224 \\ 362 \times 2 & 900 \\ & 724 \\ & 17600 \end{array}$$

$$r^2 = 28 \text{ m}^2$$
$$l = 19 \text{ m}$$
$$h = \sqrt{l^2 - r^2} \quad [\because l^2 = h^2 + r^2]$$
$$= \sqrt{19^2 - 28}$$
$$= \sqrt{361 - 28} = \sqrt{333}$$
$$h = 18.25 \text{ m.}$$

Hence, height of the tent = 18.25 m.

6. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm.

Sol. Given $r = 5$ cm

$h = 12$ cm

$\Rightarrow l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{25 + 144}$

$\Rightarrow l = \sqrt{169}$

$\Rightarrow l = 13$ cm

Required no. of caps $= \dfrac{\text{Area of the paper}}{\text{Area of 1 cap}}$

Area of 1 cap $= \pi r l$

$= \dfrac{22}{7} \times 5 \times 13$

$$= \frac{5720}{\pi r l} = \frac{5720}{\frac{22}{7} \times 5 \times 13}$$
$$= \frac{5720}{\frac{1430}{7}} = 572\cancel{0} \times \frac{7}{143\cancel{0}}$$
$$= \frac{4004}{143} = 28 \text{ caps}$$

Hence, 28 birthday caps can be made.

7. The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone. **[PTA - 2]**

Sol. Let r_1 , r_2 be the radii of two right circular cones and h_1, h_2 be their heights respectively.

Given $r_1 : r_2 = 1 : 3 \Rightarrow r_1 = 1(x)$ and $r_2 = 3(x)$ where x is some positive number.

Here $r_1 = x$ is the radius of the smaller cone ... (1)

Also, it is given that $h_1 = 3$ (radius of smaller cone) and $h_2 = 3$ (radius of smaller cone)

$\Rightarrow h_1 = 3x$ and $h_2 = 3x$ [using (1)]

Now let l_1, l_2 be the slant height of two right circular cones.

$$l_1 = \sqrt{h_1^2 + r_1^2} = \sqrt{(3x)^2 + x^2} = \sqrt{9x^2 + x^2}$$
$$= \sqrt{10x^2} = x\sqrt{10} \text{ and}$$
$$l_2 = \sqrt{h_2^2 + r_2^2}$$
$$= \sqrt{9x^2 + 9x^2} = \sqrt{18x^2} = \sqrt{9 \times 2 \times x^2}$$
$$\Rightarrow l_2 = 3x\sqrt{2}$$

Let CSA_1 and CSA_2 be their respective curved surface areas.

$\therefore CSA_1 = \pi r_1 l_1 = \pi (x)(x)\sqrt{10} = \pi x^2\sqrt{10}$ ... (3)

and $CSA_2 = \pi r_2 l_2 = \pi (3x)(3x\sqrt{2}) = 9x^2\pi\sqrt{2}$ ... (4)

$\therefore$ Ratio of CSA of two right circular cones

$$= \frac{CSA_1}{CSA_2} = \frac{\cancel{\pi}\, \cancel{x^2} \sqrt{10}}{9\cancel{x^2}\, \cancel{\pi}\sqrt{2}}$$

[Dividing (3) by (4)]

$$= \frac{\sqrt{10}}{9\sqrt{2}} = \frac{\cancel{\sqrt{2}} \times \sqrt{5}}{9 \times \cancel{\sqrt{2}}} = \frac{\sqrt{5}}{9}$$

$\Rightarrow CSA_1 : CSA_2 = \sqrt{5} : 9$

8. The radius of a sphere increases by 25%. Find the percentage increase in its surface area. **[Govt. MQP - 2019]**

Sol. Let r be the radius of the sphere and r' be the radius of the new sphere

$r' = r + 25\%r$

[$\because$ radius is increased by 25%]

$$r' = r + \frac{25}{100}r = r + 0.25r$$
$$= r(1 + 0.25) = 1.25\,r$$

Old surface area $= 4\pi r^2$

New surface area $= 4\pi(r')^2$ ... (1)

$= 4\pi(1.25r)^2$

$= 4\pi(1.25)(1.25)r^2$

$= 4(1.5625)r^2$ ... (2)

Change in surface area $= 4\pi(1.5625)r^2 - 4\pi r^2$ [(2) – (1)]

$= 4\pi r^2 (1.5625 - 1)$

[Taking $4\pi r^2$ as common]

$= 4\pi r^2 (0.5625)$

$\therefore$ Percentage increase in surface area

$$= \frac{\text{Change in surface area} \times 100}{\text{old area}}$$

$$= \frac{\cancel{4\pi r^2}(.5625) \times 100}{\cancel{4\pi r^2}} = 0.5625 \times 100 = 56.25\%$$

Hence, the percentage increase in surface area = 56.25%

9. The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per cm^2 .

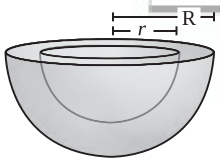
Sol. External diameter D = 28 cm

Internal diameter d = 20 cm

$\therefore$ R $= \frac{28}{2} = 14$ cm, $r = \frac{20}{2} = 10$ cm.

T.S.A of the hemispherical vessel $= \pi(3R^2 + r^2)$

$= \frac{22}{7}(3 \times 14^2 + 10^2)$



$= \frac{22}{7}(3 \times 196 + 100)$

$= \frac{22}{7} \times 688$

$= \frac{15136}{7} = 2162.28$ cm^2

Cost of painting for 1 cm^2 = ₹ 0.14

Cost of painting for 2162.28 cm^2

= 2162.28 × 0.14= ₹ 302.72

10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Sol. Here given that R = 12 cm

r = 6 cm

h = 8 cm

$l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{8^2 + (12-6)^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$l = \sqrt{100} = 10$ cm

$\therefore$ CSA of the frustum

$= \pi(R + r)l$

$= \frac{22}{7}(12 + 6)10 = \frac{220}{7} \times 18$

$= \frac{3960}{7} = 565.71$ cm^2 ... (1)

Area of the top part $= \pi r^2$

$= \frac{22}{7} \times 6 \times 6$

$= \frac{792}{7} = 113.14$ cm^2 ... (2)

$\therefore$ The total area to be painted

= 565.71 + 113.14 [(1) + (2)]

= 678.85 cm^2

$\therefore$ The cost of painting for 1 cm^2 = ₹ 2

$\therefore$ Cost of painting for 678.85 cm^2 = 2 × 678.85

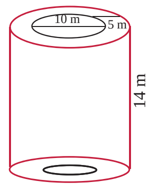
= ₹ 1357.72

Hence, the cost of painting the lamp is = ₹ 1357.72

EXERCISE 7.2

1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.

Sol. Given inner diameter = 10 m



$\Rightarrow$ Inner radius $= \frac{10}{2} = 5$ m

and height (h) = 14 m

Volume of the cylinder = Volume of the earth taken out

$= \pi r^2 h = \frac{22}{\cancel{7}} \times 5 \times 5 \times \overset{2}{\cancel{14}}$

= 1100 m^3 ... (1)

The earth spread all around the well

= Volume of hollow cylinder

$= \pi(R^2 - r^2)$

Outer radius (R) = inner radius + width

= 5 + 5 = 10 m

r = 5m

and h = ? [not given]

$\therefore$ The earth spread all around the well

$= \frac{22}{7}(10^2 - 5^2)$

$$= \frac{22}{7}(100 - 25)\,h$$

$$= \frac{22}{7} \times 75 \times h \qquad \ldots (2)$$

It is also given that the earth taken out = The earth spread all around the well.

Equating (1) and (2) we get,

$$1100 = \frac{22}{7} \times 75 \times h$$

$$\Rightarrow \quad \frac{\overset{2}{\cancel{\overset{44}{\cancel{1100}}}}\times 7}{\cancel{22} \times \underset{3}{\cancel{75}}} = h \quad \Rightarrow h = \frac{14}{3}$$

$$\Rightarrow \quad h = 4.666 \text{ m}$$

Hence, height of the embankment = 4.67 m

2. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass? **[Sep. - 2020]**

Sol.

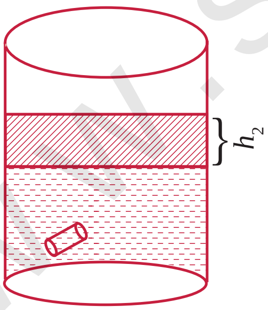
For the cylindrical metal	For the water raised which is in cylindrical form
$r_1 = 5$	diameter = 20 cm
$h_1 = 4$	$r_2 = 10$ cm
	$h_2 = ?$

By Archimedes's principle,

Volume of the water raised

= Volume of the cylindrical metal

$$\Rightarrow \quad \cancel{\pi} r_2^2 h_2 = \cancel{\pi} r_1^2 h_1$$



$$\Rightarrow \quad (10)^2 h_2 = (5)^2 \times 4$$

$$\Rightarrow \quad 100 h_2 = (25)\,4$$

$$\Rightarrow \quad h_2 = \frac{(25)(4)}{100} = \frac{100}{100} = 1 \text{ cm}$$

Hence, height of the raised water in the glass = 1 cm

3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.

Sol. Circumference of the base of the cone = 484 cm

Height = 105 cm

$$\therefore 2\pi r = 484 \Rightarrow 2 \times \frac{22}{7} \times r = 484$$

$$\Rightarrow \quad r = \overset{11}{\cancel{\overset{121}{\cancel{\overset{242}{\cancel{484}}}}}} \times \frac{1}{\cancel{2}} \times \frac{7}{\underset{1}{\cancel{\underset{11}{\cancel{22}}}}} = 77 \text{ cm}$$

$$\Rightarrow \quad r = 77 \text{ cm}$$

$$\therefore \text{Its volume} = \frac{1}{3}\pi r^2 h \text{ cubic units}$$

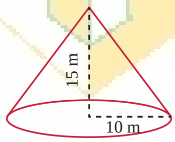
$$= \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{11}{\cancel{77}} \times 77 \times \overset{35}{\cancel{105}}$$

$$= 652190 \text{ cm}^3.$$

$\therefore$ The required volume of the cone = 652190 cm^3.

4. A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.

Sol.



Given $r = 10$ m

$h = 15$m

Volume of the cone = $\frac{1}{3}\pi r^2 h$ cu. units.

Volume of the given conical

Container = $\frac{1}{3} \times \pi \times 10 \times 10 \times 15 = 500\pi \text{ m}^3$

To empty 25 m^3, the time taken = 1 minutes

To empty 500π m^3 the time taken = $\dfrac{500 \times \frac{22}{7} \times 1}{25}$

$$\Rightarrow \frac{\overset{20}{\cancel{500}} \times 22}{7 \times \cancel{25}} \Rightarrow \frac{440}{7} = 62.857 \text{ minutes}$$

$\cong$ 63 minutes.(approx.)

Hence, the conical container will be emptied in 63 minutes.

Chapter 8

STATISTICS AND PROBABILITY

FORMULAE TO REMEMBER

- Range R = L – S.
- Coefficient of range = $\frac{L-S}{L+S}$, L - Largest value, S-smallest value standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$.
- Variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_1-\bar{x})^2}{n}$
- Assumed mean method $\sigma = c \times \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$

EXERCISE 8.1

1. Find the range and coefficient of range of the following data.
(i) 63, 89, 98, 125, 79, 108, 117, 68 [Sep.-2020]
(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Sol. Range R = L – S.

Co-efficient of range $= \frac{L-S}{L+S}$

L – Largest value, S – Smallest value.

(i) 63, 89, 98, 125, 79, 108, 117, 68.

Here L = 125

S = 63

$\therefore$ R = L – S = 125 – 63 = 62

Co-efficient of range $= \frac{L-S}{L+S} = \frac{125-63}{125+63}$

$= \frac{62}{188} = 0.33$

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4

S = 13.6

R = L – S = 61.4 – 13.6 = 47.8

Co-efficient of range $= \frac{L-S}{L+S} = \frac{47.8}{75} = 0.64$

2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value. [Hy - 2019]

Sol. Given range = 36.8 and

the smallest value = 13.4 (S)

the largest value = L = R + S [$\because$ Range = L – S]

= 36.8 + 13.4 = 50.2

3. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Sol. Here the largest value = 650

The smallest value = 400

$\therefore$ Range = L – S = 650 – 400 = 250

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages completed by them.

Sol. $\bar{x} = \frac{\Sigma x}{n} = \frac{275}{8} = 34.3$

x_i	x_i^2
32	1020
35	1225
37	1369
30	900
33	1089
36	1296
35	1225
37	1369
$\Sigma x_i = 275$	$\Sigma x_i^2 = 9497$

Standard deviation:

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{\frac{9497}{8} - \left(\frac{275}{8}\right)^2}$$

$$= \sqrt{\frac{9497 \times 8 - 275 \times 275}{8 \times 8}} = \sqrt{\frac{75976 - 75625}{64}} = \sqrt{\frac{351}{64}}$$

$\sigma = \sqrt{5.48}$

$\therefore$ Standard deviation $\sigma = 2.34$

5. Find the variance and standard deviation of the wages of 9 workers given below:
₹ 310, ₹ 290, ₹ 320, ₹ 280, ₹ 300, ₹ 290, ₹ 320, ₹ 310, ₹ 280.

Sol. Arrange in ascending order we get,
280, 280, 290, 290, 300, 310, 310, 320 and 320

$\bar{x} = \frac{\Sigma x}{n} = \frac{2700}{9} = 300$

x	$d = x - \bar{x}$	d^2
280	–20	400
280	–20	400
290	–10	100
290	–10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
$\Sigma x = 2700$	0	$\Sigma d^2 = 2000$

$$\text{Variance} = \frac{\Sigma d^2}{n} = \frac{2000}{9} = 222.22$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{222.22} = 14.91$$

6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Sol.

x	$d = x - \bar{x}$	d^2
(1 + 1) = 2	–11	121
(2 + 2) = 4	–9	81
(3 + 3) = 6	–7	49
(4 + 4) = 8	–5	25
(5 + 5) = 10	–3	9
(6 + 6) = 12	–1	1
(7 + 7) = 14	1	1
(8 + 8) = 16	3	9
(9 + 9) = 18	5	25
(10 + 10) = 20	7	49
(11 + 11) = 22	9	81
(12 + 12) = 24	11	121
156		572

$$\bar{x} = \frac{\Sigma x}{n} = 13$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{572}{12}} = \sqrt{47.66} \simeq 6.9$$

7. Find the standard deviation of first 21 natural numbers. **[PTA - 6]**

Sol. Standard deviation of first n natural number is

$$\sigma = \sqrt{\frac{n^2 - 1}{12}}$$

∴ Standard deviation of first 21 natural numbers

$$\sigma = \sqrt{\frac{21^2 - 1}{12}} = \sqrt{\frac{441 - 1}{12}} = \sqrt{\frac{440}{12}} = \sqrt{36.67}$$

$$\sigma = 6.06$$

∴ Standard deviation of first 21 natural numbers $\sigma = 6.06$

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Sol. If the standard deviation of a data is 4.5 and each value of the data decreased by 5, the new standard deviation does not change and it is also 4.5.

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation. **[PTA - 1]**

Sol. If the standard deviation of a data is 3.6, and each of the data is divided by 3 then the new standard deviation is also divided by 3.

∴ The new standard deviation $= \frac{3.6}{3} = 1.2$

The new variance = (standard deviation)^2

$= \sigma^2 = 1.2^2 = 1.44$

10. The rainfall recorded in various places of five districts in a week are given below. Find its standard deviation.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Sol.

Rainfall x_i (mm)	No. of places f_i	$f_i x_i$	$d = x - \bar{x}$	d_i^2	$f_i d_i^2$
45	5	225	–11	121	605
50	13	650	–6	36	468
55	4	220	–1	1	4
60	9	540	4	16	144
65	5	325	9	81	405
70	4	280	14	196	784
	N = 40	$\Sigma x_i f_i = 2240$		$\Sigma f_i d_i^2 = 2410$	

$$\text{mean, } \bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{2240}{40} = 56$$

∴ Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{2410}{40}} = \sqrt{60.25} = 7.76$$

11. In a study about viral fever, the number of people affected in a town were noted as. Find its standard deviation.

Age in years	0 - 10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Sol. Let the assumed mean A = 35, C = 10

Age (X)	No. of people affected f_i	Mid (value) x_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
0–10	3	5	–30	–3	–9	9	27
10–20	5	15	–20	–2	–10	4	20
20–30	16	25	–10	–1	–16	1	16
30–40	18	35	0	0	0	0	0
40–50	12	45	10	1	12	1	12
50–60	7	55	20	2	14	4	28
60–70	4	65	30	3	12	9	36
	N = 65			$\Sigma x_i f_i = 3$			139

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 10 \times \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2} = 10 \times \sqrt{2.138 - (0.046)^2}$

$= 10 \times \sqrt{2.138 - 0.002116} = 10 \times \sqrt{2.136}$

$= 10 \times 1.46 = 14.6$ $\therefore$ Standard deviation = 14.6

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Sol. Assumed mean A = 30.5, C = 4

Diameter class interval X	Mid value x_i	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
20.5–24.5	22.5	15	–8	–2	–30	4	60
24.5–28.5	26.5	18	–4	–1	–18	1	18
28.5–32.5	30.5	20	0	0	0	0	0
32.5–36.5	34.5	16	4	1	16	1	16
36.5–40.5	38.5	8	8	2	16	4	32
40.5–44.5	42.5	7	12	3	21	9	63
		N = 84			5		189

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 4 \times \sqrt{\frac{189}{84} - \left(\frac{5}{84}\right)^2} = 4 \times \sqrt{2.25 - (0.059)^2}$

$= 4 \times \sqrt{2.25 - 0.0035} = 4 \times \sqrt{2.2465} = 4 \times 1.498$

$\cong 5.99 = 6$

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation. **[PTA - 5]**

Time taken (sec)	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Number of students	6	8	17	10	9

Sol. Assumed mean A = 11, C = 1

Time Taken X	Mid value x_i	No. of Students f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
8.5–9.5	9	6	–2	–2	–12	4	24
9.5–10.5	10	8	–1	–1	–8	1	8
10.5–11.5	11	17	0	0	0	0	0
11.5 – 12.5	12	10	1	1	10	1	10
12.5 – 13.5	13	9	2	2	18	4	36
		N = 50			8		78

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 1 \times \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2} = 1 \times \sqrt{1.56 - (0.16)^2}$

$= 1 \times \sqrt{1.56 - 0.0256} = 1 \times \sqrt{1.534} = 1 \times 1.238$

$= 1.238 = 1.24$

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

Sol. Given n = 100, $\bar{x} = 60$, $\sigma = 15$

$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \Sigma x = \bar{x} \times n$

$\therefore$ $\Sigma x = \bar{x} \times n = 60 \times 100 = 6000$

Correct Σx = Incorrect Σx – Wrong value + Correct value

Correct $\Sigma x = 6000 + 45 + 72 - 40 - 27 = 6117 - 67$

Correct Σx = 6050

n = 100

Correct $\bar{x}$ = Correct $\frac{\Sigma x}{n} = \frac{6050}{100} = 60.5$

Unit Test

Time : 45 Minutes **Marks : 25**

SECTION - A

(5 × 1 = 5)

1. Which of the following is not a measure of dispersion?
(A) Range (B) Standard deviation
(C) Arithmetic mean (D) Variance

2. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is
(A) 3.5 (B) 3 (C) 4.5 (D) 2.5

3. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
(A) $\frac{3}{10}$ (B) $\frac{7}{10}$ (C) $\frac{3}{9}$ (D) $\frac{7}{9}$

4. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is
(A) 5 (B) 10 (C) 15 (D) 20

5. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?
(A) $\frac{1}{5}$ (B) $\frac{3}{10}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$

SECTION - B **(5 × 2 = 10)**

1. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
2. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.
3. A coin is tossed thrice. What is the probability of getting two consecutive tails?
4. The standard deviation of some temperature data in degree celsius (°C) is 5. If the data were converted into degree Fahrenheit (°F) then what is the variance?
5. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

SECTION - C **(2 × 5 = 10)**

1. In a study about viral fever, the number of people affected in a town were noted as

Age in years	0 - 10	10 - 20	20 - 30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Find its standard deviation.

2. The frequency distribution is given below.

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a variance of 160. Determine the value of k.

Answers

SECTION - A

1. (C) Arithmetic mean
2. (A) 3.5
3. (B) $\frac{7}{10}$
4. (C) 15
5. (D) $\frac{4}{5}$

SECTION - B

1. Refer Sura's Guide Exercise 8.1, Q.No.6
2. Refer Sura's Guide Exercise 8.2, Q.No.6
3. Refer Sura's Guide Exercise 8.3, Q.No.4
4. Refer Sura's Guide unit Exercise Q.No.4
5. Refer Sura's Guide unit Exercise Q.No.8

SECTION - C

1. Refer Sura's Guide Exercise No.8.1, Q.No.11
2. Refer Sura's Guide Unit Exercise Q.No.3

10th STD

GOVT. SUPPLEMENTARY EXAMINATION
September 2021

Reg. No.

PART - III

Mathematics (With Answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 100

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

Note : This question paper contains **four** parts.

Part - I

Note : (1) Answer **all** the questions. **14 × 1 = 14**
(2) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer

1. If $n(A\times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is :
(a) 1 (b) 2 (c) 3 (d) 6

2. The sum of the exponents of the prime factors in the prime factorization of 1729 is :
(a) 1 (b) 2 (c) 3 (d) 4

3. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is :
(a) 3 (b) 5 (c) 8 (d) 11

4. The square root of $\dfrac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to :
(a) $\dfrac{16}{5}\left|\dfrac{x^2z^4}{y^2}\right|$ (b) $16\left|\dfrac{y^2}{x^2z^4}\right|$
(c) $\dfrac{16}{5}\left|\dfrac{y}{xz^2}\right|$ (d) $\dfrac{16}{5}\left|\dfrac{xz^2}{y}\right|$

5. Graph of a linear equation is a ________.
(a) Straight line (b) Circle
(c) Parabola (s) Hyperbola

6. The G.C.D. of a^m, a^{m+1}, a^{m+2} is :
(a) a^m (b) a^{m+1}
(c) a^{m+2} (d) 1

7. If in ΔABC, DE||BC, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then, the length of AE is :
(a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 cm

8. How many tangents can be drawn to the circle from an exterior point?
(a) one (b) two (c) infinite (d) zero

9. The area of a triangle formed by the points (–5, 0), (0, 5) and (5, 0) is :
(a) 0 sq. units (b) 25 sq. units
(c) 5 sq. units (d) none of these

10. The perimeter of a triangle formed by the points (0, 0), (1, 0) and (0, 1) is :
(a) $\sqrt{2}$ (b) 2 (c) $2+\sqrt{2}$ (d) $2-\sqrt{2}$

11. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$ then, the angle of elevation of the sun has measure :
(a) 45º (b) 30º (c) 90º (d) 60º

12. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be :
(a) 12 cm (b) 10 cm (c) 13 cm (d) 5 cm

13. The total surface area of a hemisphere is how many times the square of its radius?
(a) π (b) 4π (c) 3π (d) 2π

14. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is :
(a) $\dfrac{3}{10}$ (b) $\dfrac{7}{10}$ (c) $\dfrac{3}{9}$ (d) $\dfrac{7}{9}$

Part - II

Note : Answer **any 10** questions. Question No.**28** is **compulsory**. **10 × 2 = 20**

15. If A = {1, 3, 5} and B = {2, 3} then show that $n(A \times B) = n\{B\}$.

16. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "is square of a number" on A. Write ℝ as a subset of A × A. Also, find the domain and range of ℝ.

17. Find the number of terms in the A.P.
3, 6, 9, 12, ..., 111.

18. If $3 + k$, $18 - k$, $5k + 1$ are in A.P. then find k.

19. Determine the quadratic equations, whose sum and product of roots are –9 and 20.

20. Determine the nature of the roots for the quadratic equations $15x^2 + 11x + 2 = 0$

21. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC.
If $\dfrac{AD}{DB} = \dfrac{3}{4}$ and AC = 15 cm, find AE.

22. Show that the points (–3, –4) , (7, 2) and (12, 5) are collinear.

23. Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$.

24. Find the intercepts made by the line $3x - 2y - 6 = 0$ on the coordinate axes.

25. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

26. Find the volume of a cylinder whose height is 2 m and base area is 250 m^2.

27. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

28. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Part - III

Note : Answer **any 10** questions. Question No.**42** is **compulsory**. **10 × 5 = 50**

29. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that
$A \times (B \cap C) = (A \times B) \cap (A \times C)$

30. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

31. Find the HCF of 396, 504, 636.

32. Solve :
$x + y + z = 5;\ 2x - y + z = 9;\ x - 2y + 3z = 16.$

33. Find the square root of
$64x^2 - 16x^3 + 17x^2 - 2x + 1.$

34. State and prove Pythagoras Theorem.

35. Show that in a triangle, the medians are concurrent.

36. Find the equation of the median of ΔABC through A where the vertices are A(6, 2), B(–5, –1) and C(1, 9).

37. If the points P(–1, –4), Q(b, c) and R(5, –1) are collinear and if $2b + c = 4$. then find the values of b and c.

38. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3}$ = 1.732).

39. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

40. A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.

41. Two dice are rolled. Find the probability that the sum of outcome is (i) equal to 4 (ii) greater than 10 (iii) less than 13.

42. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then prove that $c^2 = a^2(1 + m^2)$.

Part - IV

Note : Answer the following questions. **2 × 8 = 16**

43. (a) Construct a ΔPQR whose base PQ = 4.5 cm, ∠R = 35° and the median from R to RG is 6 cm.

(OR)

(b) Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw two tangents PA and PB to the circle and measure their lengths.

44. (a) Draw the graph of $x^2 + x - 12 = 0$ and state the nature of their solution.

(OR)

(b) Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$.

*****

ANSWERS

Part - I

1.	(c) 3	**2.**	(c) 3
3.	(d) 11	**4.**	(d) $\frac{16}{5}\left\lvert\frac{xz^2}{y}\right\rvert$
5.	(a) straight line	**6.**	(a) a^m
7.	(a) 1.4 cm	**8.**	(b) two
9.	(b) 25 sq. units	**10.**	(c) $2 + \sqrt{2}$
11.	(d) 60°	**12.**	(a) 12 cm
13.	(c) 3π	**14.**	(b) $\frac{7}{10}$

Part - II

15.
$$A = \{1, 3, 5\}\ B = \{2, 3\}$$
$$n(A) = 3;\ n(B) = 2$$
$$\therefore\ n(A) \times n(B) = 3 \times 2 = 6 \qquad ...(1)$$
$$A \times B = \{(1, 2)\ (1, 3)\ (3, 2)\ (3, 3)\ (5, 2)\ (5, 3)\}$$
$$\therefore\ n(A \times B) = 6 \qquad ...(2)$$
From (1) and (2)
$$n(A \times B) = n(A) \times n(B)$$