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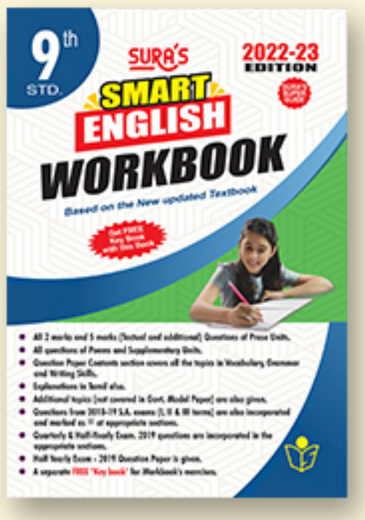

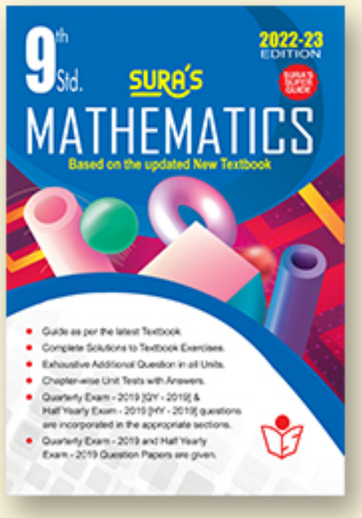

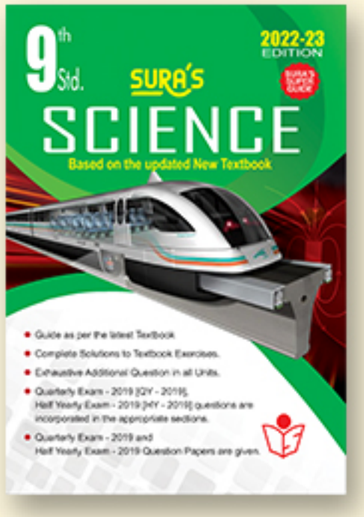

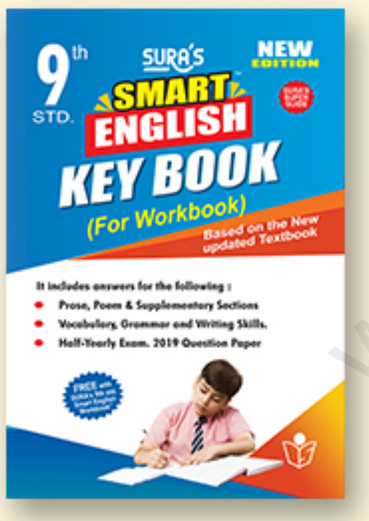

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NOTE FROM PUBLISHER

It gives me great pride and pleasure in bringing to you **Sura's Mathematics Guide** for **9th Standard**. It is prepared as per the Latest New Textbook.

This guide encompasses all the requirements of the students to comprehend the text and the evaluation of the textbook.

- Additional questions have been provided exhaustively for clear understanding of the units under study.
- Chapter-wise Unit Test are given.

In order to learn effectively, I advise students to learn the subject section-wise and practice the exercises given. It will be a teaching companion to teachers and a learning companion to students.

Though these salient features are available in this Guide, I cannot negate the indispensable role of the teachers in assisting the student to understand the subject thoroughly.

I sincerely believe this guide satisfies the needs of the students and bolsters the teaching methodologies of the teachers.

I pray the almighty to bless the students for consummate success in their examinations.

Subash Raj, B.E., M.S.
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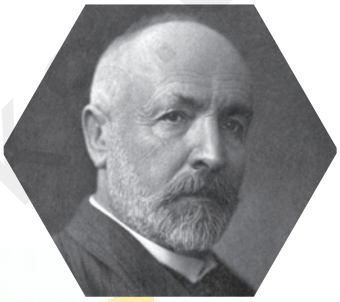
CONTENTS

1

SET LANGUAGE

1.1 Introduction

In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.



1.2 Set :

(i) A set is a well - defined collection of objects.

(ii) The objects of a set are called its members or elements.

For example,

1. The collection of all books in a District Central Library.
2. The collection of all colours in a rainbow.

1.3 Representation of a Set :

The collection of odd numbers can be described in many ways:

(1) "The set of odd numbers" is a fine description.

(2) It can be written as {1, 3, 5, …}.

(3) Also, it can be said as the collection of all numbers x where x is an odd number.

1.3.1 Descriptive Form :

In descriptive form, a set is described in words.

For Example,

(i) The set of all vowels in English alphabets.

(ii) The set of all whole numbers.

1.3.2 Set Builder Form or Rule Form :

In set builder form, all the elements are described by a rule.

For example,

(i) $A = \{x : x \text{ is a vowel in English alphabets}\}$

(ii) $B = \{x \mid x \text{ is a whole number}\}$

1.3.3 Roster Form or Tabular Form

A set can be described by listing all the elements of the set.

For example,

(i) A = {a, e, i, o, u}

(ii) B = {0,1,2,3,…}

Exercise 1.1

1. Which of the following are sets?

(i) The Collection of prime numbers upto 100.

(ii) The Collection of rich people in India.

(iii) The Collection of all rivers in India.

(iv) The Collection of good Hockey players.

Sol. (i) A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97}

As the collection of prime numbers upto 100 is known and can be counted (well defined). Hence this is a set.

(ii) The collection of rich people in India. Rich people has no definition.
Hence, it is not a set.

(iii) A = {Cauvery, Sindhu, Ganga, ...}
Hence, it is a set.

(iv) The collection of good hockey players is not a well - defied collection because the criteria for determining a hockey player's talent may vary from person to person.

Hence, this collection is not a set.

2. List the set of letters of the following words in Roster form.

(i) INDIA **(ii) PARALLELOGRAM**

(iii) MISSISSIPPI **(iv) CZECHOSLOVAKIA**

Sol. (i) A = {I, N, D, A}

(ii) B = {P, A, R, L, E, O, G, M}

(iii) C = {M, I, S, P}

(iv) D = {C, Z, E, H, O, S, L, V, A, K, I}.

3. Consider the following sets A = {0, 3, 5, 8} B = {2, 4, 6, 10} C = {12, 14, 18, 20}

(a) State whether True or false.

(i) 18 ∈ C **(ii) 6 ∉ A** **(iii) 14 ∉ C** **(iv) 10 ∈ B** **(v) 5 ∈ B** **(vi) 0 ∈ B**

(b) Fill in the blanks?

(i) 3 ∈ ____ **(ii) 14 ∈ _____** **(iii) 18 ____ B** **(iv) 4 _____ B**

Sol. (a) (i) True (ii) True (iii) False (iv) True (v) False (vi) False.

(b) (i) A (ii) C (iii) ∉ (iv) ∈

4. Represent the following sets in Roster form.

(i) A = The set of all even natural numbers less than 20. ***[QY-2019]***

(ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \leq 5\}$

(iii) C = {x : x is perfect cube, $27 < x < 216$}

(iv) $D = \{x : x \in \mathbb{Z}, -5 < x \leq 2\}$

Sol. (i) A = {2, 4, 6, 8, 10, 12, 14, 16, 18}

(ii) N = { 1, 2, 3, 4, 5}

if, $n = 1, y = \frac{1}{2n} = \frac{1}{2\times1} = \frac{1}{2}$

$n = 2, y = \frac{1}{2\times2} = \frac{1}{4}$

$n = 3, y = \frac{1}{2\times3} = \frac{1}{6}$

$n = 4, y = \frac{1}{2\times4} = \frac{1}{8}$

$n = 5, y = \frac{1}{2\times5} = \frac{1}{10}$

$\therefore B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right\}$

(iii) C = {64, 125}

(iv) D = {–4, –3, –2, –1, 0, 1, 2}

5. Represent the following sets in set builder form.

(i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.

(ii) $\mathbf{C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}}$.

(iii) D = The set of all tamil months in a year.

(iv) E = The set of an odd Whole numbers less than 9.

Sol. (i) B = {x : x is an Indian player who scored double centuries in One Day Internationals}

(ii) $C = \{x : x = \frac{n}{n+1}, n \in \mathbb{N}\}$

(iii) D = {x : x is a tamil month in a year}

(iv) E = {x : x is an odd number, $x \in \mathbb{W}$, $x < 9$, where W is the set of whole numbers}.

6. Represent the following sets in descriptive form.

(i) P = { January, June, July}

(ii) Q = {7,11,13,17,19,23,29}

(iii) R = {x : $x \in \mathbb{N}$, $x < 5$}

(iv) S = {x : x is an consonant in English alphabets}

Sol. (i) P is the set of English Months begining with J.

(ii) Q is the set of all prime numbers between 5 and 31.

(iii) R is the set of all natural numbers less than 5.

(iv) S is the set of all English consonants.

1.4. Types of sets

1.4.1 Empty Set or Null Set :

A set consisting of no element is called the empty set or null set or void set.

For example,

$A=\{x : x$ is an odd integer and divisible by $2\}$

$\therefore A=\{\ \}$ or $\varnothing$

1.4.2 Singleton Set :

A set which has only one element is called a singleton set.

For example,

$A = \{x : 3 < x < 5, x \in \mathbb{N}\}$ where $A = \{4\}$

1.4.3 Finite Set :

A set with finite number of elements is called a finite set.

For example,

1. The set of family members.
2. The set of indoor/outdoor games you play.

1.4.4 Infinite Set :

A set which is not finite is called an infinite set.

For example,

(i) {5,10,15,...} (ii) The set of all points on a line.

1.4.5 Equivalent Sets :

Two finite sets A and B are said to be equivalent if they contain the same number of elements. It is written as $A \approx B$.

If A and B are equivalent sets, then $n(A) = n(B)$.

1.4.6 Equal Sets :

Two sets are said to be equal if they contain exactly the same elements, otherwise they are said to be unequal.

In other words, two sets A and B are said to be equal, if

(i) every element of A is also an element of B

(ii) every element of B is also an element of A.

1.4.7 Subset :

Let A and B be two sets. If every element of A is also an element of B, then A is called a subset of B. We write $A \subseteq B$.

1.4.8 Proper Subset :

Let A and B be two sets. If A is a subset of B and $A \neq B$, then A is called a proper subset of B and we write $A \subset B$.

For example,

If $A=\{1,2,5\}$ and $B=\{1,2,3,4,5\}$ then A is a proper subset of B i.e. $A \subset B$.

1.4.9 Power set :

The set of all subsets of A is said to be the power set of the set A and is denoted as P(A)

For example,

Let $A = \{-3, 4\}$

The subsets of A are , $\varnothing$, $\{-3\}$, $\{4\}$, $\{-3, 4\}$

Then the power set of A is $P(A) = \{\varnothing, \{-3\}, \{4\}, \{-3, 4\}\}$

Exercise 1.7

Multiple Choice Questions :

1. **Which of the following is correct?**

(1) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$ (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$

(3) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$ (4) $\{7\} \nsubseteq \{1,2,3,4,5,6,7,8,9,10\}$

[Ans. (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$]

2. **The set $P = \{x \mid x \in \mathbb{Z}, -1 < x < 1\}$ is a**

(1) Singleton set (2) Power set (3) Null set (4) Subset

Hint : $P = \{0\}$ **[Ans. (1) Singleton set]**

3. **If $U = \{x \mid x \in \mathbb{N}, x < 10\}$ and $A = \{x \mid x \in \mathbb{N}, 2 \leq x < 6\}$ then $(A')'$ is**

(1) $\{1,6,7,8,9\}$ (b) $\{1,2,3,4\}$ (c) $\{2,3,4,5\}$ (d) $\{\ \}$

Hint : $(A')' = A = \{2, 3, 4, 5\}$ **[Ans. (3) $\{2,3,4,5\}$]**

4. **If $B \subseteq A$ then $n(A \cap B)$ is**

(1) $n(A - B)$ (2) $n(B)$ (3) $n(B - A)$ (4) $n(A)$

Hint : $B \subseteq A \Rightarrow A \cap B = B$ **[Ans. (2) $n(B)$]**

5. **If $A = \{x,y,z\}$ then the number of non-empty subsets of A is** ***[HY-2019]***

(1) 8 (2) 5 (3) 6 (4) 7

Hint : Number of non-empty subsets $= 2^3 - 1 = 8 - 1 = 7$ **[Ans. (4) 7]**

6. **Which of the following is correct ?**

(1) $\varnothing \subseteq \{a,b\}$ (2) $\varnothing \in \{a, b\}$ (3) $\{a\} \in \{a, b\}$ (4) $a \subseteq \{a, b\}$

Hint : Empty set is an improper subset **[Ans. (1) $\varnothing \subseteq \{a,b\}$]**

7. **If $A \cup B = A \cap B$, then**

(1) $A \neq B$ (2) $A = B$ (4) $A \subset B$ (4) $B \subset A$

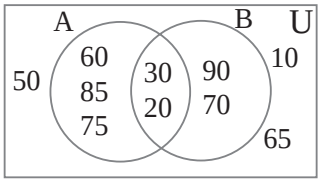
[Ans. (2) $A = B$]

8. **If $B - A$ is B, then $A \cap B$ is** ***[QY-2019]***

(1) A (2) B (3) U (4) $\varnothing$

Hint : $B - A = B \Rightarrow$ A and B are disjoint sets. **[Ans. (4) $\varnothing$]**

9. **From the adjacent diagram $n[P(A \Delta B)]$ is**



(1) 8 (2) 16

(3) 32 (4) 64

Hint : $A \Delta B = \{60, 85, 75, 90, 70\}$

$\Rightarrow n(A \Delta B) = 5$

$\Rightarrow n(P(A \Delta B)) = 2^5 = 32$ **[Ans. (3) 32]**

10. **If $n(A) = 10$ and $n(B) = 15$, then the minimum and maximum number of elements in $A \cap B$ is**

(1) 10,15 (2) 15,10 (3) 10,0 (4) 0,10

[Ans. (4) (0,10)]

11. Let A = {∅} and B = P(A) then A∩B is

(1) { ∅, {∅} } (2) {∅} (3) ∅ (4) {0}

Hint : P(A) = {∅{∅}} **[Ans. (2) {∅}]**

12. In a class of 50 boys, 35 boys play Carom and 20 boys play Chess then the number of boys play both games is *[HY-2019]*

(1) 5 (2) 30 (3) 15 (4) 10

Hint : $n(A \cup B) = n(A) + n(B) - n(A \cap B) \Rightarrow 50 = 35 + 20 - n(A \cap B) \Rightarrow n(A \cap B) = 5$

[Ans. (1) 5]

13. If U = {$x : x \in \mathbb{N}$ and $x < 10$}, A ={1, 2, 3, 5, 8} and B = {2, 5, 6, 7, 9}, then $n[(A \cup B)']$ is

(1) 1 (2) 2 (3) 4 (4) 8

Hint :
$$\begin{aligned} U &= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ A &= \{1, 2, 3, 5, 8\} \\ B &= \{2, 5, 6, 7, 9\} \\ A \cup B &= \{1, 2, 3, 5, 6, 7, 8, 9\} \\ (A \cup B)' &= \{4\}, \\ n(A \cup B)' &= 1 \end{aligned}$$

[Ans. (1) 1]

14. For any three sets P, Q and R, P−(Q∩R) is *[HY-2019]*

(1) P − (Q ∪ R) (2) (P ∩ Q) − R

(3) (P − Q) ∪ (P − R) (4) (P − Q)∩(P − R)

Hint : P – (Q ∩ R) = (P − Q) ∪ (P − R) **[Ans. (3) (P − Q)∪(P − R)]**

15. Which of the following is true?

(1) A−B = A ∩ B (2) A − B = B − A

(3) (A∪B)′ = A′∪ B′ (4) (A ∩ B)′ = A′∪ B′

Hint :
(1) (A – B) = A ∩ B ×
(2) A – B = B – A ×
(3) (A ∪ B)′ = A′ ∪ B′ ×
(4) (A ∩ B)′ = A′ ∪ B′ ✓

[Ans. (4) (A ∩ B)′ = A′∪ B′]

16. If $n(A \cup B \cup C) = 100$, $n(A) = 4x$, $n(B) = 6x$, $n(C) = 5x$, $n(A \cap B) = 20$, $n(B \cap C) = 15$, $n(A \cap C) = 25$ and $n(A \cap B \cap C) = 10$, then the value of x is

(1) 10 (2) 15 (3) 25 (4) 30

Hint :
$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C) \\ 100 &= 4x + 6x + 5x - 20 - 15 - 25 + 10 \\ 100 &= 15x - 60 + 10 \\ 100 &= 15x - 50 \\ \therefore 15x &= 100 + 50 = 150 \\ x &= 10 \end{aligned}$$

[Ans. (1) 10]

17. For any three sets A, B and C, (A − B) ∩ (B − C) is equal to *[QY-2019]*

(1) A only (2) B only (3) C only (4) ϕ

Hint : (A – B) ∩ (B – C) is equal to ϕ **[Ans. (4) ϕ]**

18. **If J = Set of three sided shapes, K = Set of shapes with two equal sides and L = Set of shapes with right angle, then J ∩ K ∩ L is**

(1) Set of isoceles triangles (2) Set of equilateral triangles

(3) Set of isoceles right triangles (4) Set of right angled triangles

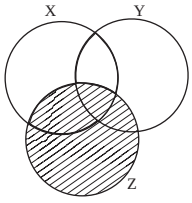
Hint : J = 

K = 

L = 

[Ans. (3) Set of isoceles right triangles**]**

19. **The shaded region in the Venn diagram is**



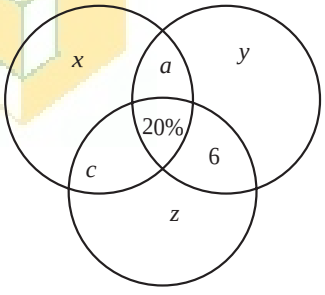
(1) Z – (X∪Y) (2) (X∪Y)∩Z (3) Z – (X∩Y) (4) Z∪(X∩Y)

Hint : Z – (X∩Y) **[Ans. (3) Z – (X∩Y)]**

20. **In a city, 40% people like only one fruit, 35% people like only two fruits, 20% people like all the three fruits. How many percentage of people do not like any one of the above three fruits?**

(1) 5 (2) 8

(3) 10 (4) 15



Hint : 40 + 35 + 20 + x = 100%

95% + x = 100%

x = 5%

[Ans. (1) 5]

TEXT BOOK ACTIVITIES

Activity - 1

1. **Discuss and give as many examples of collections from your daily life situations, which are sets and which are not sets.**

Sol. **Which are sets**

(i) Collection of pen (ii) Collection of dolls

(iii) Collection of books (iv) Collection of red flower etc.

Which are not sets

(i) Collection of good students in a class.

(ii) Collection of beautiful flowers in a garden etc.

UNIT TEST

Time : 45 Minutes **CHAPTER - 1** **Marks: 25**

Section - A

(i) Answer all the questions.

(ii) Choose the correct Answer. **5 × 1 = 5**

1. The set P = $\{x \mid x \in \mathbb{Z}, -1 < x < 1\}$ is a
   (1) Singleton set (2) Power set
   (3) Null set (4) Subset

2. Which of the following is correct ?
   (1) $\{7\} \in \{1,2,3,4,5,6,7,8,9,10\}$
   (2) $7 \in \{1,2,3,4,5,6,7,8,9,10\}$
   (3) $7 \notin \{1,2,3,4,5,6,7,8,9,10\}$
   (4) $\{7\} \notin \{1,2,3,4,5,6,7,8,9,10\}$

3. Which of the following is a correct statement?
   (1) $\varnothing \not\subseteq \{a, b\}$ (2) $\varnothing \in \{a, b\}$
   (3) $\{a\} \in \{a, b\}$ (4) $a \subseteq \{a, b\}$

4. If U = $\{x : x \in \mathbb{N}$ and $x < 10\}$, A = {1, 2, 3, 5, 8} and B = {2, 5, 6, 7, 9}, then $n[(A \cup B)']$ is
   (1) 1 (2) 2
   (3) 4 (4) 8

5. For any three sets A, B and C, $(A - B) \cap (B - C)$ is equal to
   (1) A only (2) B only
   (3) C only (4) ϕ

Section - B

(i) Answer only five of the following

(ii) However Question number 12 is compulsory. **5 × 2 = 10**

6. Represent the following sets in set builder form.
   (i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.
   (ii) C = $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4} \ldots\ldots\right\}$.

7. Which of the following sets are equivalent or unequal or equal sets?
   (i) A = The set of vowels in the English alphabets.
   B = The set of all letters in the word "VOWEL"
   (ii) C = {2, 3, 4, 5}
   D = $\{x : x \in \mathbb{W}, 1 < x < 5\}$

8. Write down the power set of the following sets.
   (i) A= $\{a, b\}$ (ii) B ={1, 2, 3}

9. If U = $\{x : x \notin \mathbb{N}, x \leq 10\}$, A = {2,3,4,8,10} and B = {1,2,5,8,10}, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

10. Write the following sets in Roster form.
    (i) C = {x : x is a prime number and a divisor of 6 }
    (ii) $x = \{x : x = 2n, n \in \mathbb{N}$ and $n \leq 5\}$

11. Which of the following sets are equivalent?
    (i) A = {2, 4, 6, 8, 10},
    B = {1, 3, 5, 7, 9}
    (ii) X = $\{x : x \in \mathbb{N}, 1 < x < 6\}$,
    Y = {x : x is a vowel in the English Alphabet}

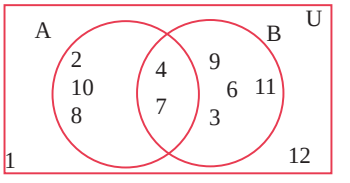
12. If A = { –2, –1, 0, 3, 4}, B = {–1, 3, 5}, find (i) A – B, (ii) B – A.

Section - C

(i) Answer only two Questions of the following.

(ii) However Question number 16 is compulsory. **2 × 5 = 10**

13. Using the given venn diagram, write the elements of
    (i) A (ii) B (iii) A∪B (iv) A∩B (v) A – B (vi) B – A (vii) A′ (viii) B′
    (ix) U



14. Let A and B be two overlapping sets and the universal set U.Draw appropriate Venn diagram for each of the following,
(i) $A \cup B$ (ii) $A \cap B$
(iii) $(A \cap B)'$ (iv) $(B - A)'$
(v) $A' \cup B'$ (vi) $A' \cap B'$
(vii) What do you observe from the diagram (iii) and (v)?

15. In an examination 50% of the students passed in mathematics and 70% of students passed in science while 10% students failed in both subjects. 300 students passed in atleast one subjects. Find the total number of students who appeared in the examination, if they took examination in only two subjects.

16. If $n(A) = 25$, $n(B) = 40$, $n(A \cup B) = 50$ and $n(B') = 25$, find $n(A \cap B)$ and $n(U)$.

ANSWERS

SECTION - A

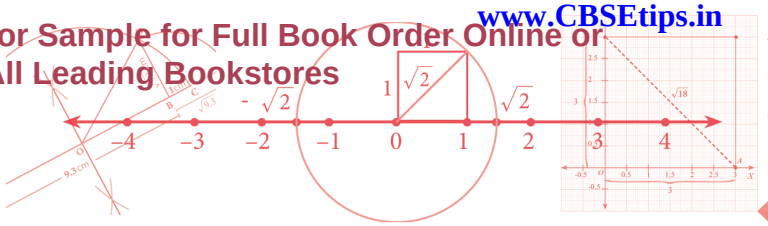
1. (1) Singleton
2. (2) $7 \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
3. (1) $\varnothing \not\subseteq \{a, b\}$
4. (1) 1
5. (4) ϕ

SECTION - B

6. $\{x : x$ is an Indian player who scored double (i) in one day international$\}$
(ii) $\{x : x = \frac{n}{n+1}, n \in \mathbb{N}\}$
7. (i) Equivalent sets
(ii) Unequal sets
8. (i) $\{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1,3\}, \{1, 2, 3\}\}$
9. Thus verified
10. (i) $\{2, 3\}$;(ii) $\{2, 4, 8, 16, 32\}$
11. (i) Equivalent sets
(ii) Not equivalent set
12. (i) $\{-2, 0, 4\}$ (ii) $\{5\}$

SECTION - C

13. (i) $\{2, 4, 7, 8, 10\}$
(ii) $\{3, 4, 6, 7, 9, 11\}$
(iii) $\{2, 3, 4, 6, 7, 8, 9, 10, 11\}$
(iv) $\{4, 7\}$
(v) $\{2, 8, 10\}$
(vi) $\{3, 6, 9, 14\}$
(vii) $\{1, 3, 6, 9, 11, 12\}$
(viii) $\{1, 2, 8, 10, 12\}$
(ix) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
14. ***Refer Sura's Guide Exercise No.1.3, Q. No.7***
15. 1000
16. 15; 65



2 REAL NUMBERS

Introduction

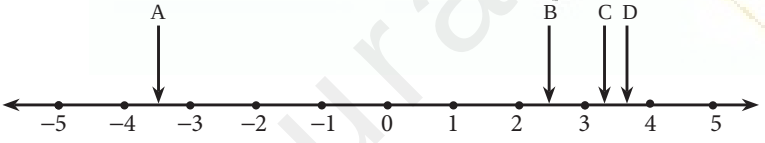
2.1 Number, numbers, everywhere! you have learnt baout many types of numbers so far. Now is the time to extend the ideas further.

2.2. Rational Numbers :

A rational number is a fraction indicating the quotient of two integers, excluding division by zero.

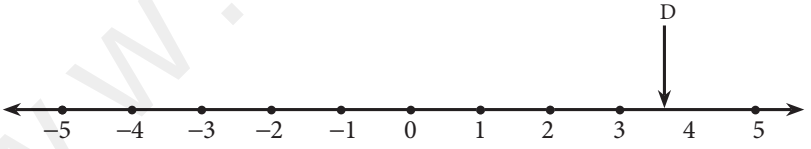
Exercise 2.1

1. Which arrow best shows the position of $\frac{11}{3}$ on the number line?



Sol. $\frac{11}{3} = 3.666..... = 3.7$ (nearly)

$\therefore$ D arrow best shows the position of $\frac{11}{3}$ on the number line.



2. Find any three rational numbers between $\frac{-7}{11}$ and $\frac{2}{11}$.

Sol. Three rational numbers between $\frac{-7}{11}$ and $\frac{2}{11}$ are $\frac{-6}{11}, \frac{-5}{11}, \frac{-4}{11}, ..., \frac{-1}{11}$

3. Find any five rational numbers between

(i) $\frac{1}{4}$ and $\frac{1}{5}$ (ii) 0.1 and 0.11 (iii) –1 and –2

Sol. (i) $a = \frac{1}{4}, b = \frac{1}{5}$

Let q_1, q_2, q_3, q_4 and q_5 be five rational numbers, $q_1 = \frac{1}{2}(a+b)$

$$= \frac{1}{2}\left(\frac{1}{4}+\frac{1}{5}\right) = \frac{1}{2}\left(\frac{5+4}{20}\right) = \frac{1}{2}\left(\frac{9}{20}\right) = \frac{9}{40}$$

$$q_2 = \frac{1}{2}(a+q_1) = \frac{1}{2}\left(\frac{1}{4}+\frac{9}{40}\right) = \frac{1}{2}\left(\frac{10+9}{40}\right) = \frac{1}{2}\left(\frac{19}{40}\right) = \frac{19}{80}$$

$$q_3 = \frac{1}{2}(a+q_2) = \frac{1}{2}\left(\frac{1}{4}+\frac{19}{80}\right) = \frac{1}{2}\left(\frac{20+19}{80}\right) = \frac{1}{2}\left(\frac{39}{80}\right) = \frac{39}{160}$$

$$q_4 = \frac{1}{2}(a+q_3) = \frac{1}{2}\left(\frac{1}{4}+\frac{39}{160}\right) = \frac{1}{2}\left(\frac{40+39}{160}\right) = \frac{1}{2}\left(\frac{79}{160}\right) = \frac{79}{320}$$

$$q_5 = \frac{1}{2}(a+q_4) = \frac{1}{2}\left(\frac{1}{4}+\frac{79}{320}\right) = \frac{1}{2}\left(\frac{80+79}{320}\right) = \frac{1}{2}\left(\frac{159}{320}\right) = \frac{159}{640}$$

Hence five rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$ are $\frac{9}{40}, \frac{19}{80}, \frac{39}{160}, \frac{79}{320}, \frac{159}{640}$

(ii) The rational numbers between 0.1 and 0.11 are 0.101, 0.102, 0.103, ...0.109.

(iii) – 1 and – 2

Let q_1, q_2, q_3, q_4 and q_5 be five rational numbers.

$$q_1 = \frac{1}{2}(a+b) = \frac{1}{2}((-1)+(-2)) = \frac{1}{2}(-3) = \frac{-3}{2}$$

$$q_2 = \frac{1}{2}(a+q_1) = \frac{1}{2}\left(-1+\frac{-3}{2}\right) = \frac{1}{2}\left(\frac{-2+(-3)}{2}\right) = \frac{1}{2}\left(\frac{-5}{2}\right) = \frac{-5}{4}$$

$$q_3 = \frac{1}{2}(a+q_2) = \frac{1}{2}\left(-1-\frac{5}{4}\right) = \frac{1}{2}\left(\frac{-4+(-5)}{4}\right) = \frac{1}{2}\left(\frac{-9}{4}\right) = \frac{-9}{8}$$

$$q_4 = \frac{1}{2}(a+q_3) = \frac{1}{2}\left(-1+\frac{-9}{8}\right) = \frac{1}{2}\left(\frac{(-8)+(-9)}{8}\right) = \frac{1}{2}\left(\frac{-17}{8}\right) = \frac{-17}{16}$$

$$q_5 = \frac{1}{2}(a+q_4) = \frac{1}{2}\left(-1+\frac{-17}{16}\right) = \frac{1}{2}\left(\frac{(-16)+(-17)}{16}\right) = \frac{1}{2}\left(\frac{-33}{16}\right) = \frac{-33}{32}$$

The five rational numbers between –1 and –2 are $\frac{-3}{2}, \frac{-5}{4}, \frac{-9}{8}, \frac{-17}{16}, \frac{-33}{32}$.

2.3 Irrational numbers :

A number having a non-terminating and non-recurring decimal expansion is called an irrational number. (i.e) it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

For example, $\sqrt{2}, \sqrt{3}$, e, π .....

(i.e) The square root of every positive number but not a perfect square number is an irrational number.

Exercise 2.2

1. Express the following rational numbers into decimal and state the kind of decimal expansion.

(i) $\frac{2}{7}$ **(ii)** $-5\frac{3}{11}$ **(iii)** $\frac{22}{3}$ **(iv)** $\frac{327}{200}$

Sol. **(i)** $\frac{2}{7}$

$\frac{2}{7} = 0.\overline{285714}$,

Non-terminating and recurring decimal expansion

$$\begin{array}{r} 0.2857142 \\ 7\overline{)20} \\ 14 \\ \hline 60 \\ 56 \\ \hline 40 \\ 35 \\ \hline 50 \\ 49 \\ \hline 10 \\ 7 \\ \hline 30 \\ 28 \\ \hline 20 \\ 14 \\ \hline 6 \end{array}$$

(ii) $-5\frac{3}{11} = \frac{-58}{11}$

$-5\frac{3}{11} = \frac{-58}{11}$

$-5\frac{3}{11} = -5.\overline{27}$

Non-terminating and recurring decimal expansion.

$$\begin{array}{r} 5.27 \\ 11\overline{)58} \\ 55 \\ \hline 30 \\ 22 \\ \hline 80 \\ 77 \\ \hline 3 \\ \vdots \end{array}$$

(iii) $\frac{22}{3}$

$\frac{22}{3} = 7.\bar{3}$,

Non-terminating and recurring decimal expansion.

$$\begin{array}{r} 7.3 \\ 3\overline{)22} \\ 21 \\ \hline 10 \\ 9 \\ \hline 1 \\ \vdots \end{array}$$

(iv) $\frac{327}{200}$

$\frac{327}{200} = 1.635$, Terminating decimal expansion.

$$\begin{array}{r} 1.635 \\ 200\overline{)327} \\ 200 \\ \hline 1270 \\ 1200 \\ \hline 700 \\ 600 \\ \hline 1000 \\ 1000 \\ \hline 0 \end{array}$$

2. **Express $\frac{1}{13}$ in decimal form. Find the length of the period of decimals.** *[QY-2019]*

Sol.

$$13\overline{)100}\ 0.076923$$

$$\begin{array}{r} 91 \\ \hline 90 \\ 78 \\ \hline 120 \\ 117 \\ \hline 30 \\ 26 \\ \hline 40 \\ 39 \\ \hline 1 \\ \vdots \end{array}$$

$\frac{1}{13} = 0.\overline{076923}$ has the length of the period of decimals = 6.

3. **Express the rational number $\frac{1}{33}$ in recurring decimal form by using the recurring decimal expansion of $\frac{1}{11}$. Hence write $\frac{71}{33}$ in recurring decimal form.**

Sol. The recurring decimal expansion of $\frac{1}{11} = 0.09090909\ldots = 0.\overline{09}$

$$\frac{1}{33} = \frac{1}{11} \times \frac{1}{3}$$
$$= 0.\overline{09} \times \frac{1}{3}$$
$$= 0.\overline{03}$$

$\therefore \frac{1}{33} = 0.03030303\ldots = 0.\overline{03}$ Also, $\frac{71}{33} = 2\frac{5}{33} = 2 + \frac{5}{33}$

$$= 2 + \left(5 \times \frac{1}{33}\right)$$
$$= 2 + (5 \times 0.\overline{03})$$
$$= 2 + (5 \times 0.030303\cdots)$$
$$= 2 + 0.151515\cdots\cdots$$
$$= 2.151515\cdots\cdots = 2.\overline{15}$$

4. **Express the following decimal expression into rational numbers.**

(i) $0.\overline{24}$ **(ii)** $2.\overline{327}$ **(iii)** -5.132

(iv) $3.1\overline{7}$ **(v)** $17.2\overline{15}$ **(vi)** $-21.21\overline{37}$

Sol. **(i)** $0.\overline{24}$

Let $x = 0.\overline{24} = 0.24242424\cdots\cdots$ ...(1)

(Here period of decimal is 2, multiply equation (1) by 100)

$100x = 24.242424.....$ ....(2)

(2) – (1)

$$100x - x = (24.242424....) - (0.242424....)$$
$$99x = 24$$
$$x = \frac{24}{99}$$

(ii) $\mathbf{2.\overline{327}}$

Let $x = 2.327327327.....$ ...(1)

(Here period of decimal is 3, multiply equation (1) by 1000)

$$1000x = 2327.327.....$$ ...(2)

(2) – (1)

$$1000x - x = (2327.327327...) - (2.327327...)$$
$$999x = 2325$$
$$x = \frac{2325}{999}$$

(iii) $\mathbf{-5.132}$

$$x = -5.132 = \frac{-5132}{1000} = \frac{-1283}{250}$$

(iv) $\mathbf{3.1\overline{7}}$

Let $x = 3.1777.....$ ...(1)

(Here the repeating decimal digit is 7, which is the second digit after the decimal point, multiply equation (1) by 10)

$$10x = 31.7777.....$$ ...(2)

(Now period of decimal is 1, multiply equation (2) by 10)

$$100x = 317.7777.....$$ ...(3)

(3) – (2)

$$100x - 10x = (317.777....) - (31.777....)$$
$$90x = 286$$
$$x = \frac{286}{90} = \frac{143}{45}$$

(v) $\mathbf{17.2\overline{15}}$

Let $x = 17.2151515.....$ ...(1)

(Here the repeating decimal digit is 15, which is the second digit after the decimal point, so multiply (1) by 10)

$$10x = 172.151515.....$$ ...(2)

Now period of decimal is 2, multiply equation (2) by 100

$$1000x = 17215.1515....$$ ... (3)

(3) – (2)

$$\Rightarrow \quad 990x = 17043$$
$$x = \frac{\overset{5681}{\cancel{17043}}}{\underset{330}{\cancel{990}}} = \frac{5681}{330}$$

(vi) $\mathbf{-21.213\overline{7}}$

Let $x = -21.213\overline{7} = -21.2137777....$ ...(1)

(Here the repeating decimal digit is 7, which is the fourth digit after the decimal point, so multiply (1) by 1000)

$$1000\,x = -21213.7777....$$ ...(2)

Now period of decimal is 1, multiply equation (2) by 10

$$10000\,x = -21213.777....$$ ...(3)

(3) – (2)

$9000x = -190924$

$x = \frac{-190924}{9000}$

5. Without actual division, find which of the following rational numbers have terminating decimal expansion.

(i) $\frac{7}{128}$ **(ii)** $\frac{21}{15}$ **(iii)** $4\frac{9}{35}$ **(v)** $\frac{219}{2200}$

Sol. **(i)** $\frac{7}{128}$

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

So $\frac{7}{128} = \frac{7}{2^7 5^0}$

This of the form $\frac{P}{2^m \times 5^n}$ Where P = 7, m = 7, n = 0

So $\frac{7}{128}$ has a terminating decimal expansion.

(ii) $\frac{21}{15}$

$\frac{21}{15} = \frac{\cancel{3} \times 7}{5 \times \cancel{3}} = \frac{7}{5} = \frac{7}{2^0 5^1}$. This is of the form $\frac{P}{2^m \times 5^n}$

5	15
3	3
	1

3	21
	7

Where P = 7, m = 0, n = 1

So $\frac{21}{15}$ has a terminating decimal expansion.

(iii) $4\frac{9}{35}$

$4\frac{9}{35} = \frac{149}{35}$

5	35
7	7
	1

$\frac{49}{35} = \frac{149}{5^1 7^1}$ ∴ This is **not** of the form $\frac{P}{2^m 5^n}$

So, $4\frac{9}{35}$ has a non-terminating recurring decimal expansion.

(v) $\frac{219}{2200}$

2	2200
2	1100
2	550
5	275
5	55
	11

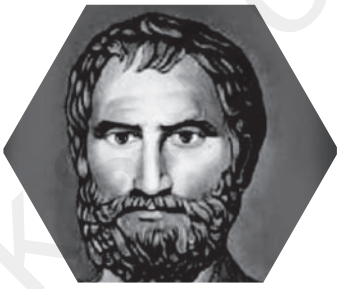
$\frac{219}{2200} = \frac{219}{2^3 5^2 11^1}$

This is not of the form $\frac{P}{2^m 5^n}$

So, $\frac{219}{2200}$ has a non-terminating recurring decimal expansion.

4 GEOMETRY

4.1 Introduction



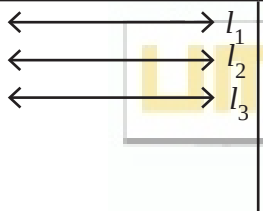
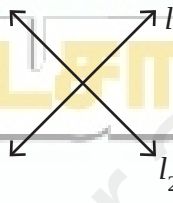
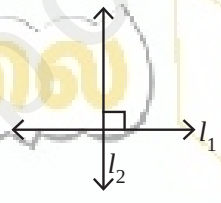
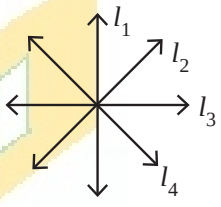
We describe different shapes by their properties.

Parallel lines : Two or more lines lying in the same plane that never meet.

Intersecting lines : Two lines which meet at a common point.

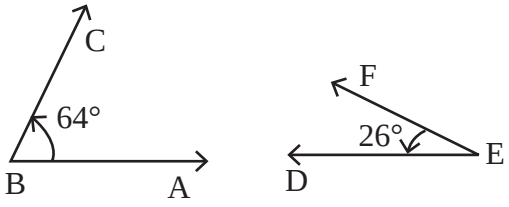
Perpendicular lines : Two lines which intersect each other at right angle.

Concurrent lines : Three or more lines passing through the same point.

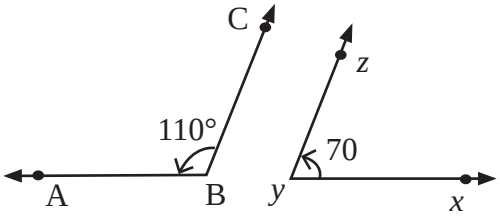
Parallel Lines	Intersecting Lines	Perpendicular Lines	Concurrent Lines
			
$l_1 \parallel l_2 \parallel l_3$		$l_1 \perp l_2$	

Types of angles :

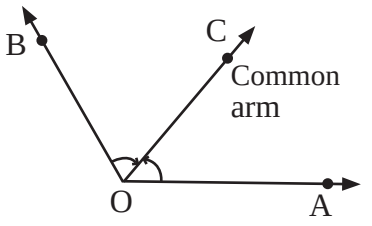
Acute angles



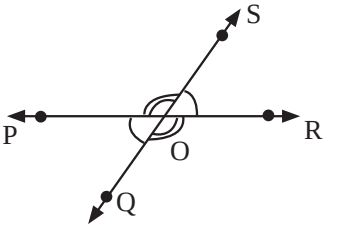
Obtuse angles



Adjacent Angles

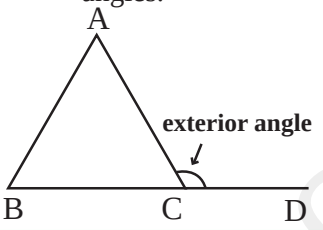


Opposite Angles

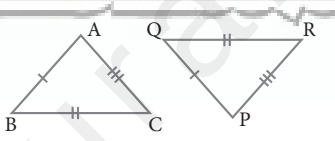
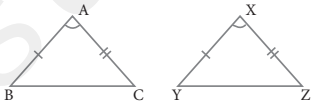
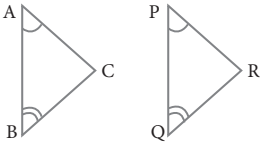
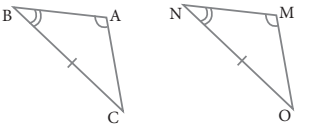
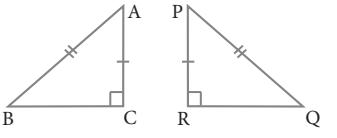


Transversal : A line which intersects two or more lines at a distinct points is called a transversal of lines.

Exterior angle property : If a side of a triangle is stretched, the exterior angle so formed is equal to the sum of the two remote interior angles.



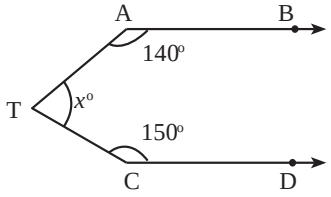
Congruent triangles : Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of another triangle.

Rule	Diagrams	Reason
SSS		AB = PQ BC = QR AC = PR ΔABC≅ ΔPQR
SAS		AB = XY ∠BAC = ∠YXZ AC = XZ ΔABC≅ ΔXYZ
ASA		∠A = ∠P AB = PQ ∠B = ∠Q ΔABC≅ ΔPQR
AAS		∠A = ∠M ∠B = ∠N BC = NO ΔABC≅ ΔMNO
RHS		∠ACB = ∠PRQ = 90°(R) AB = PQ hypotenuse (H) AC = PR (S) ΔABC ≅ ΔPQR

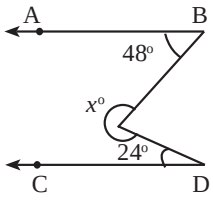
Exercise 4.1

1. In the figure, AB is parallel to CD, find *x*

(i)



(ii)



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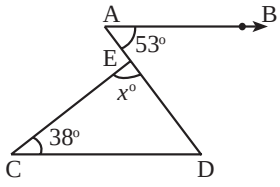
(iii)



Sol. (i) From the figure

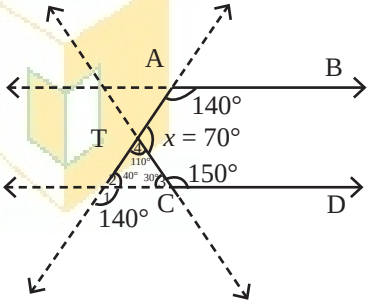
$\angle 1 = 140^\circ$ ($\because$ corresponding angles are equal)

$\angle 2 = 40^\circ$ ($\because \angle 1 + \angle 2 = 180^\circ \Rightarrow \angle 2 = 180^\circ - \angle 1 = 180^\circ - 140^\circ = 40^\circ$)

$\angle 3 = 30^\circ$ ($\because \angle 3 + 150^\circ = 180^\circ$)

$\angle 4 = 110^\circ$ ($\because \angle 2 + \angle 3 + \angle 4 = 180^\circ$)

$\therefore \angle x = 70^\circ$ ($\because \angle 4 + \angle x = 180^\circ$)



(ii) From the figure

$\angle 1 = 48^\circ$

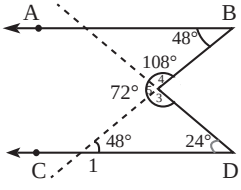
$\angle 3 = 108^\circ$ ($\angle 1 + 24^\circ + \angle 3 = 180^\circ$)

$\angle 4 = 108^\circ$ (If two lines are intersect, then the vertically the opposite angles are equal)

$\angle 5 = 72^\circ$ ($\because \angle 3 + \angle 5 = 180^\circ$)

$\therefore \angle 3 + \angle 4 + \angle 5 = 108^\circ + 108^\circ + 72^\circ$

$x = 288^\circ$



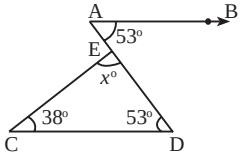
(iii) From the figure

$\angle D = 53^\circ$ ($\because \angle B$ and $\angle D$ are alternate interior angles)

Sum of the three angles of a triangle is 180°

$\angle x^\circ = 180^\circ - (38^\circ + 53^\circ)$

$= 180^\circ - 91^\circ = 89^\circ$



2. The angles of a triangle are in the ratio 1: 2 : 3, find the measure of each angle of the triangle.

Sol. Let the angles be x, $2x$ and $3x$ respectively.

Sum of the three angles of a triangle $= 180°$

$\therefore \quad x + 2x + 3x = 180^\circ$

$$6x = 180^o \Rightarrow x = \frac{180^o}{6}$$

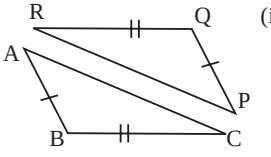
$$\therefore \quad x = 30^o$$

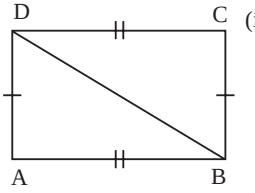
$$2x = 2 \times 30 = 60^o$$

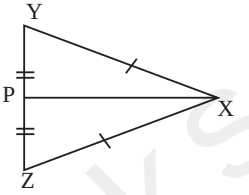
$$3x = 3 \times 30 = 90^o$$

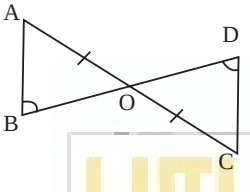
The 3 angles of the triangle are 30°, 60°, 90°.

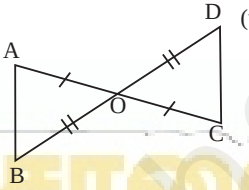
3. Consider the given pairs of triangles and say whether each pair is that of congruent triangles. If the triangles are congruent, say 'how'; if they are not congruent say 'why' and also say if a small modification would make them congruent:

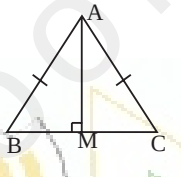
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) 

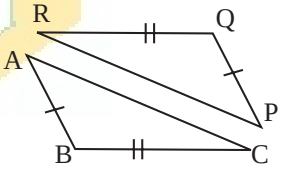
Sol. (i) Consider ΔPQR and ΔABC

Given, RQ = BC

PQ = AB

ΔABC is not congruent to ΔPQR

If PR = AC, then ΔABC ≅ ΔPQR

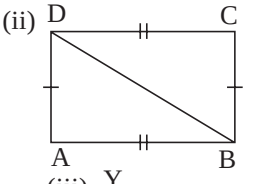


(ii) Consider ΔABD and ΔBCD for the triangles to be congruent.

Given, AB = DC

AD = BC and AB is common side.

∴ By SSS rule ΔABD ≅ ΔBCD.



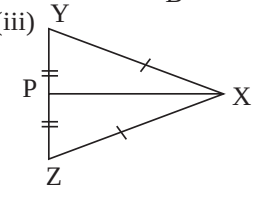
(iii) Consider ΔPXY and ΔPXZ,

Given, XY = XZ

PY = PZ

and PX is common

∴ By SSS rule ΔPXY ≅ ΔPXZ.



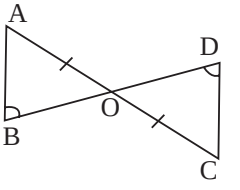
(iv) Consider ΔOAB and ΔODC,

Given, OA = OC

∠ABO = ∠ODC

and ∠AOB = ∠DOC [vertically opposite angles]

∴ By AAS rule, ΔOAB ≅ ΔODC.



(v) Consider ΔAOB and ΔDOC,

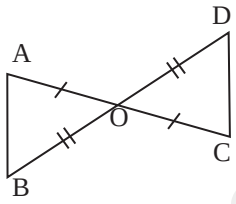
Given, AO = OC

OB = OD

and ∠AOB = ∠DOC [vertically opposite angles]

∴ By SAS rule, ΔAOB ≅ ΔDOC.

(v)



(vi) Consider ΔAMB and ΔAMC,

Given, AB = AC

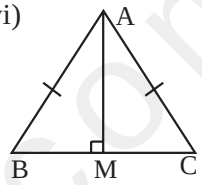
∠AMB = ∠AMC = 90°

∴ AM is common.

∴ By RHS rule

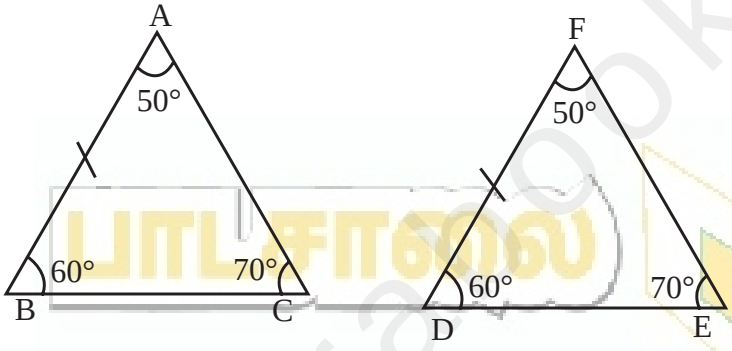
ΔAMB ≅ ΔAMC.

(vi)



4. ΔABC and ΔDEF are two triangles in which AB=DF, ∠ACB=70°, ∠ABC=60°; ∠DEF=70° and ∠EDF=60°. Prove that the triangles are congruent.

Sol.



In ΔABC, ∠ACB = 70°, ∠ABC = 60°

∴ ∠BAC = 180° – (70° + 60°)

= 180° – 130° = 50°

In ΔDEF, ∠DEF = 70°, ∠EDF = 60°

∴ ∠DFE = 180° – (70° + 60°)

= 180° – 130° = 50°

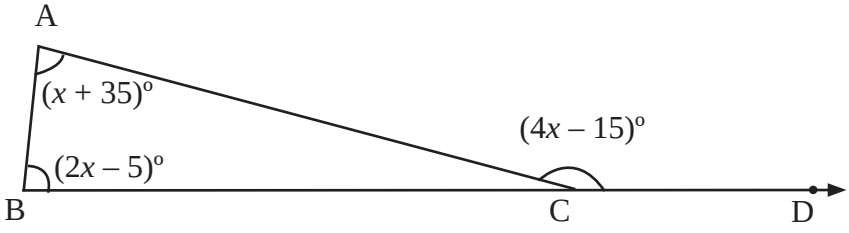
∠A = ∠F

AB = DF

and ∠B = ∠D

∴ By ASA rule ΔABC ≡ ΔFDE

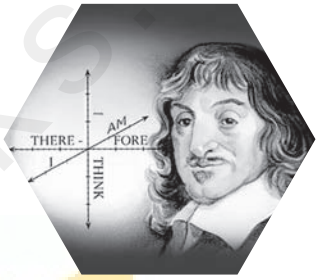
5. Find all the three angles of the ΔABC ***[QY-2019]***



5

Coordinate Geometry

5.1 Introduction

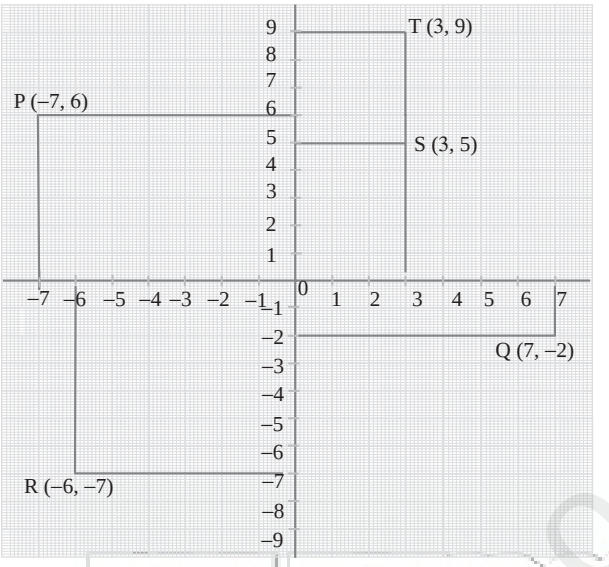


1. The X-co-ordinate is called the abscissa and Y-co-ordinate is called the ordinate. We call the meeting point of the axes (0, 0), the origin.
2. For any point P on the x axis, the value of y co-ordinate (ordinate) is zero,i.e. P(x, 0)
3. For any point Q on the y axis, the value of x coordinate (abscissa) is zero,i.e. Q (0, y)
4. $(x, y) \neq (y, x)$ unless $x = y$.
5. A plane with the rectangular co-ordinate system is called the cartesian plane.
6. A line parallel to x axis, the y-co-ordinates are equal.
7. Distance between two points is given by the formula $\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ if P(x_1, y_1), Q(x_2, y_2) are points.
8. The distance of a point P(x_1, y_1) from the origin O(0, 0) is OP = $\sqrt{x_1^2 + y_1^2}$.
9. Arithmetic introduced us to the world of numbers and operations on them.
10. Algebra taught us how to work with unknown values and find them using equations.
11. Geometry taught us to describe shapes by their properties.
12. Co-ordinate geometry will teach us how to use numbers and algebraic equations for studying geometry and beautiful integration of many techniques in one place.

Exercise 5.1

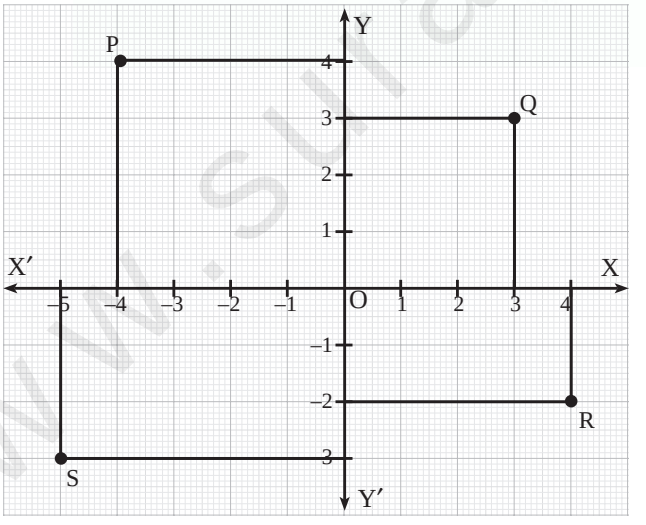
1. Plot the following points in the coordinate system and identify the quadrants P(–7, 6), Q(7, –2), R(–6, –7), S(3, 5) and T(3, 9).

Sol.



(i) P(–7,6) = II Quadrant
(ii) Q(7, –2) = IV Quadrant
(iii) R (–6, –7) = III Quadrant
(iv) S(3, 5) = I Quadrant
(v) T(3, 9) = I Quadrant

2. Write down the abscissa and ordinate of the following from the given figure.
(i) P (ii) Q (iii) R (iv) S



Sol.

(i) P (–4, 4)
Abscissa is –4
Ordinate is 4.

(ii) Q (3, 3)
Abscissa is 3
Ordinate is 3.

(iii) R (4, –2)

Abscissa is 4

Ordinate is –2.

(iv) S (–5, –3)

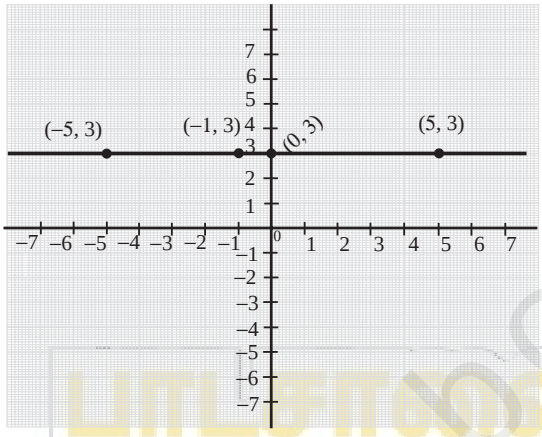
Abscissa is –5

Ordinate is –3.

3. Plot the following points in the coordinate plane and join them. What is your conclusion about the resulting figure?

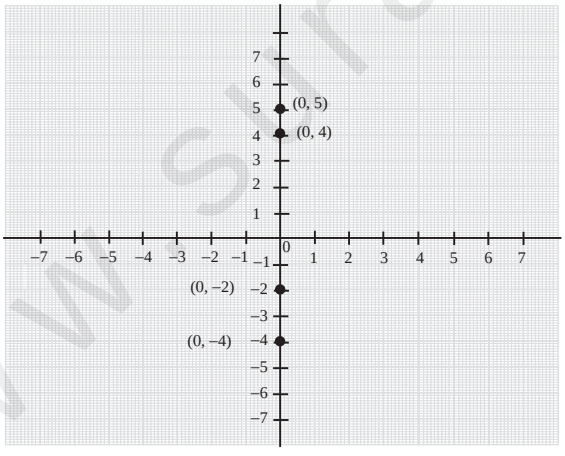
(i) (–5, 3) (–1, 3) (0, 3) (5, 3) **(ii) (0, –4) (0, –2) (0, 4) (0, 5)**

Sol. (i)



When we join the points, we see that they lie on a straight line parallel to ***x***-axis.

(ii)

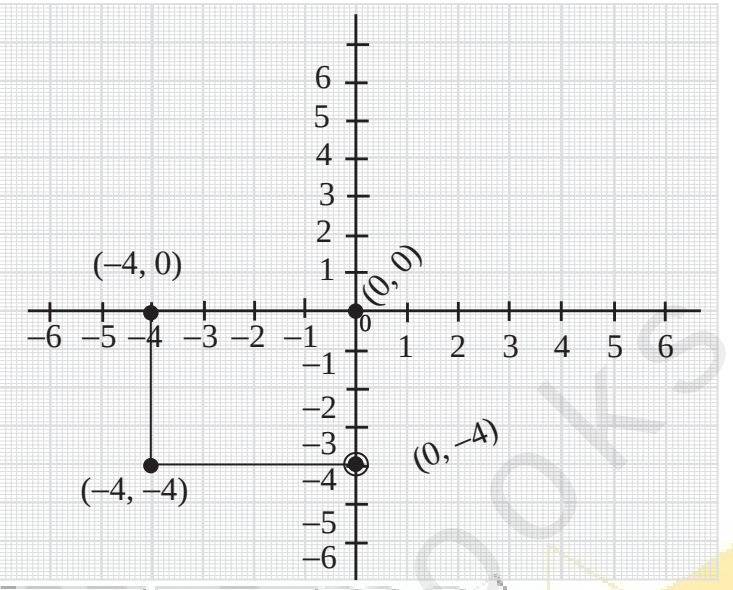


When we join the points, we see that they lie on a straight line which is ***y***-axis.

4. **Plot the following points in the coordinate plane. Join them in order. What type of geometrical shape is formed?**

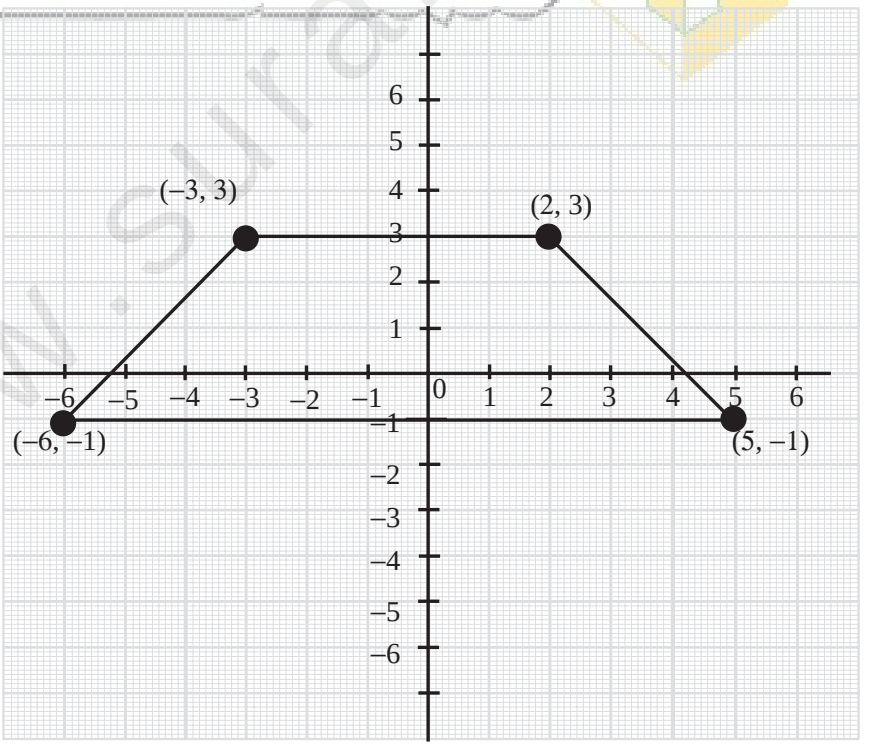
(i) (0,0) (–4,0) (–4,–4) (0,–4) **(ii) (–3,3) (2,3) (–6,–1) (5,–1)**

Sol. (i) (0,0) (–4,0) (–4,–4) (0,–4)



The type of geometrical shape is square.

(ii) (–3,3) (2,3) (–6,–1) (5,–1)



The type of geometrical shape is Trapezium.

INTRODUCTION

5.3 Distance between any Two Points :

5.3.1 Distance between Two Points on the Coordinate Axes :

Points on x – axis: If two points lie on the x- axis, then the distance between them is equal to the difference between the x- coordinates.

Points on y – axis: If two points lie on y-axis then the distance between them is equal to the difference between the y-coordinates.

5.3.2 Distance Between Two Points Lying on a Line Parallel to Coordinate Axes:

Consider the points A(x_1, y_1) and B(x_2, y_1). Since the y - coordinates are equal the points lie on a line parallel to x- axis. From A and B draw AP and BQ perpendicular to x- axis respectively.

Now distance of AB = Distance between PQ = $|x_2 - x_1|$

Simimarly, consider A(x_1, y_1) and B(x_1, y_2) parallel to y axis, then the distance between the two points is $|y_2 - y_1|$.

5.3.3 Distance Between the Two Points on a Plane :

Let P(x_1, y_1) and Q(x_2, y_1) be two points in the Cartesian plane (or xy – plane), at a distance 'd' apart such that d = PQ.

5.3.4 Properties of Distances :

We have already seen that distance (A,B) = distance (B,A) for any points A, B on the plane. What other properties have you noticed ? In case you have missed them, here are some:

Distance (A,B) = 0 exactly when A and B denote the identical point:A = B.

Distance (A,B) >0 for any two distinct points A and B.

Exercise 5.2

1. Find the distance between the following pairs of points.

(i) **(1, 2) and (4, 3)** **(ii)** **(3,4) and (– 7, 2)**

(ii) **(a, b) and (c, b)** **(iv)** **(3,– 9) and (–2, 3)**

Sol. We know that distance,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(i) (1, 2) and (4, 3)

$$d = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ units}$$

6 TRIGONOMETRY

6.1 Introduction

i) $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

ii) $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

iii) $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

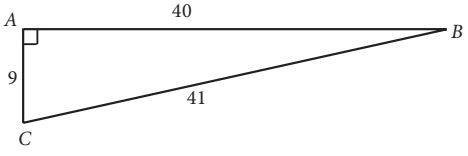
iv) $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{opposite side}}$

v) $\text{secant}\,\theta = \dfrac{\text{hypotenuse}}{\text{adjacent side}}$

vi) $\text{cotangent}\,\theta = \dfrac{\text{adjacent side}}{\text{opposite side}}$

Exercise 6.1

1. From the given figure, find all the trigonometric ratios of angle B.



Sol. $\sin B = \dfrac{9}{41}$

$\cos B = \dfrac{40}{41}$

$\tan B = \dfrac{9}{40}$

$\text{cosec}\,B = \dfrac{1}{\sin B} = \dfrac{41}{9}$

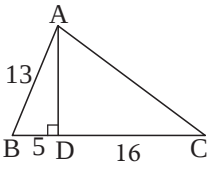
$\sec B = \dfrac{1}{\cos B} = \dfrac{41}{40}$

$\cot B = \dfrac{1}{\tan B} = \dfrac{40}{9}$

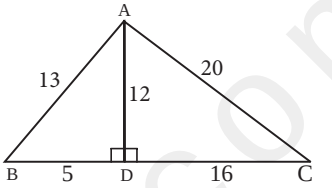
2. **From the given figure, find the values of**

(i) sin B (ii) sec B (iii) cot B

(iv) cos C (v) tan C (vi) cosec C



Sol. From the figure



By the pythagoras theorem,

$AD = \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$

$= \sqrt{144} = 12$

$AC = \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256} = \sqrt{400} = 20$

(i) $\sin B = \frac{12}{13}$

(ii) $\sec B = \frac{1}{\cos B}$

$= \frac{1}{\frac{5}{13}} = \frac{13}{5}$

(iii) $\cot B = \frac{1}{\tan B} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$

(iv) $\cos C = \frac{\cancel{16}^{4}}{\cancel{20}_{5}} = \frac{4}{5}$

(v) $\tan C = \frac{\cancel{12}^{3}}{\cancel{16}_{4}} = \frac{3}{4}$

(vi) $\operatorname{cosec} C = \frac{1}{\sin C} = \frac{1}{\frac{12}{20}} = \frac{\cancel{20}^{5}}{\cancel{12}_{3}} = \frac{5}{3}$

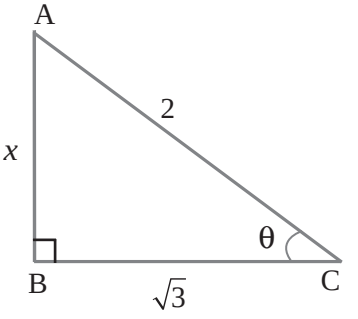
3. **If $2\cos\theta = \sqrt{3}$, then find all the trigonometric ratios of angle θ.**

Sol.

$$\text{If } 2\cos\theta = \sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

By the Pythagoras theorem,



$$x = \sqrt{2^2 - \sqrt{3}^2} = \sqrt{4-3} = \sqrt{1} = 1$$

$$\therefore \sin\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

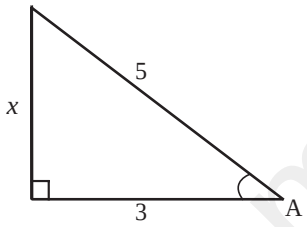
$$\operatorname{cosec}\theta = 2$$

$$\sec\theta = \frac{2}{\sqrt{3}}$$

$$\cot\theta = \sqrt{3}$$

4. **If $\cos A = \frac{3}{5}$, then find the value of $\frac{\sin A - \cos A}{2 \tan A}$**

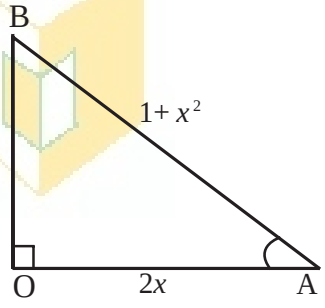
Sol. By the Pythagoras theorem,



$$x = \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9}$$
$$= \sqrt{16} = 4$$
$$\sin A = \frac{4}{5}$$
$$\cos A = \frac{3}{5}$$
$$\tan A = \frac{4}{3}$$
$$\therefore \frac{\sin A - \cos A}{2 \tan A} = \frac{\frac{4}{5} - \frac{3}{5}}{2 \times \frac{4}{3}} = \frac{\frac{1}{5}}{2 \times \frac{4}{3}} = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$$

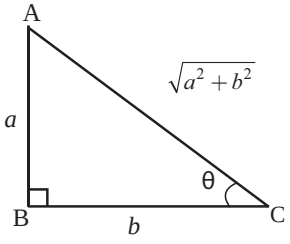
5. **If $\cos A = \frac{2x}{1 + x^2}$, then find the values of sin A and tan A in terms of x.**

Sol. By the pythagoras theorem, $(AB)^2 = (OA)^2 + (OB)^2$



$$(1 + x^2)^2 = (2x)^2 + OB^2$$
$$(OB)^2 = (1 + x^2)^2 - (2x)^2 = 1 + x^4 + 2x^2 - 4x^2$$
$$= 1 + x^4 - 2x^2$$
$$(OB)^2 = (1 - x^2)^2$$
$$OB = (1 - x^2)$$
$$\therefore \sin A = \frac{1 - x^2}{1 + x^2}$$
$$\therefore \tan A = \frac{1 - x^2}{2x}$$

6. **If $\sin \theta = \frac{a}{\sqrt{a^2 + b^2}}$, then show that $b \sin \theta = a \cos \theta$.**



Sol. $\sin \theta = \frac{a}{\sqrt{a^2 + b^2}}$,

$\cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$

$b \sin \theta = b \times \frac{a}{\sqrt{a^2 + b^2}} = \frac{ab}{\sqrt{a^2 + b^2}}$ ...(1)

$a \cos \theta = a \times \frac{b}{\sqrt{a^2 + b^2}} = \frac{ab}{\sqrt{a^2 + b^2}}$ ...(2)

(1) = (2) $\Rightarrow$ LHS = R.H.S.

Hence proved.

By the Pythagoras theorem

$(BC)^2 = (AC)^2 - (AB)^2$

$= (\sqrt{a^2 + b^2})^2 - a^2$

$= a^2 + b^2 - a^2$

$= b^2$

$BC = b$

7. **If 3 cot A = 2, then find the value of $\frac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$.**

Sol.

$$3\cot A = 2$$

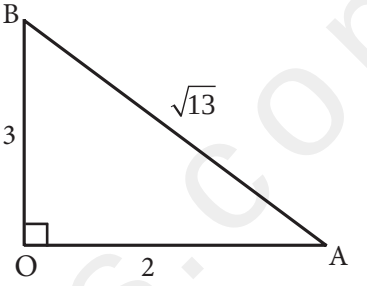
$$\cot A = \frac{2}{3}$$

$$\cot A = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

$$\tan A = \frac{\text{Opp. side}}{\text{Adj. side}} = \frac{3}{2}$$

$$\sin A = \frac{3}{\sqrt{13}}$$

$$\cos A = \frac{2}{\sqrt{13}}$$



By Pythagoras theorem
In ΔOAB,
$(AB)^2 = (OA)^2 + (OB)^2$
$= 2^2 + 3^2 = 4 + 9$
$= 13$
$AB = \sqrt{13}$

$$\therefore \frac{4\sin A - 3\cos A}{2\sin A + 3\cos A} = \frac{\left(4\times\frac{3}{\sqrt{13}}\right) - \left(3\times\frac{2}{\sqrt{13}}\right)}{\left(2\times\frac{3}{\sqrt{13}}\right) + \left(3\times\frac{2}{\sqrt{13}}\right)}$$

$$= \frac{\frac{12}{\sqrt{13}} - \frac{6}{\sqrt{13}}}{\frac{6}{\sqrt{13}} + \frac{6}{\sqrt{13}}} = \frac{\frac{6}{\sqrt{13}}}{\frac{12}{\sqrt{13}}} = \frac{6}{\cancel{\sqrt{13}}} \times \frac{\cancel{\sqrt{13}}}{12} = \frac{\cancel{6}}{\cancel{12}_{2}} = \frac{1}{2}$$

8. **If $\cos\theta : \sin\theta = 1 : 2$, then find the value of $\frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$.**

Sol.

$$\cos\theta : \sin\theta = 1 : 2$$

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{2}$$

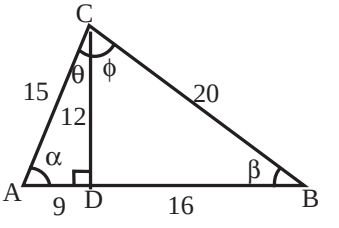
$$\therefore \cos\theta = \frac{1}{2}\sin\theta$$

$$\sin\theta = 2\cos\theta$$

$$\therefore \frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta} = \frac{\left(\cancel{8}^{4}\times\frac{1}{\cancel{2}}\sin\theta\right) - 2\sin\theta}{\left(\cancel{4}^{2}\times\frac{1}{\cancel{2}}\sin\theta\right) + 2\sin\theta} = \frac{4\sin\theta - 2\sin\theta}{2\sin\theta + 2\sin\theta} = \frac{\cancel{2}^{1}\sin\theta}{2\cancel{2}\sin\theta} = \frac{1}{2}$$

$$\therefore \frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta} = \frac{1}{2}$$

9. **From the given figure, prove that $\theta + \phi = 90°$. Also prove that there are two other right angled triangles. Find sin α, cos β and tan ϕ**



Sol. In Δ ABC

$$(AC)^2 = 15^2 = 225 \quad \ldots (1)$$

$$(BC)^2 = 20^2 = 400 \quad \ldots (2)$$

$$(AB)^2 = (9 + 16)^2$$

$$= (25)^2 = 625 \quad \ldots (3)$$

EXERCISE 6.3

1. **Find the value of cos 19° 59′ + tan 12° 12′ + sin 49° 20′.**

Sol. cos 19° 59′+ tan 12° 12′ + sin 49° 20′ = 0.9398 + 0.2162 + 0.7585 = 1.9145

2. **Given that sin α = $\frac{1}{\sqrt{2}}$ and tan β = $\sqrt{3}$. Find the value of α + β.**

Sol.
$$\alpha = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$
$$\beta = \tan^{-1}\left(\sqrt{3}\right) = 60°$$
$$\alpha + \beta = 105°$$

3. **Find the value of $\frac{\cos 63° 20'}{\sin 26° 40'}$**

Sol. $\frac{\cos 63° 20'}{\sin 26° 40'} = \frac{0.4488}{0.4488} = 1$

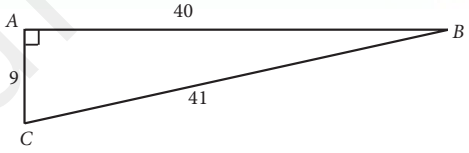
UNIT TEST

Time : 45 Minutes **CHAPTER - 6** **Marks: 25**

Part - A

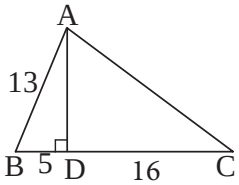
I. **Answer any 5 questions** **5 × 2 = 10**

1. From the given figure find all the trigonometric ratios of angle B.



2. From the given figure, find the values of

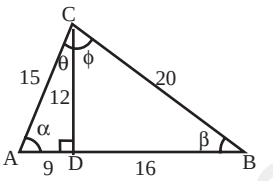
(i) sin B (ii) sec B (iii) cot B (iv) cos C (v) tan C (vi) cosec C



3. If cos A = $\frac{3}{5}$, then find the value of $\frac{\sin A - \cos A}{2 \tan A}$

4. If cos A = $\frac{2x}{1+x^2}$, then find the values of sin A and tan A in terms of x.

5. If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, then show that $b \sin\theta = a \cos\theta$.



6. If $3 \cot A = 2$, then find the value of $\frac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$

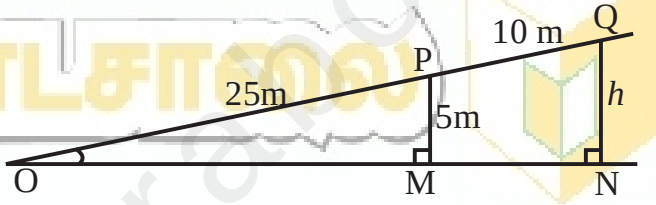
7. From the given figure, prove that $\theta + \phi = 90°$. Also prove that there are two other right angled triangles. Find $\sin\alpha$, $\cos\beta$ and $\tan\phi$

Part - B

II. Answer any 3 questions **3 × 5 = 15**

8. If $\cos\theta : \sin\theta = 1 : 2$, then find the value of $\frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$

9. A boy standing at point O finds his kite flying at a point P with distance OP = 25m. It is at a height of 5m from the ground. When the thread is extended by 10m from P, it reaches a point Q. What will be the height QN of the kite from the ground? (use trigonometric ratios)



10. Verify the following equalities:

(i) $\sin^2 60° + \cos^2 60° = 1$

(ii) $1 + \tan^2 30° = \sec^2 30°$

(iii) $\cos 90° = 1 - 2\sin^2 45° = 2\cos^2 45° - 1$

(iv) $\sin 30° \cos 60° + \cos 30° \sin 60° = \sin 90°$

11. Find the value of the following

(i) $\frac{\tan 45°}{\operatorname{cosec} 30°} + \frac{\sec 60°}{\cot 45°} - \frac{5\sin 90°}{2\cos 0°}$

(ii) $(\sin 90° + \cos 60° + \cos 45°) \times (\sin 30° + \cos 0° - \cos 45°)$

(iii) $\sin^2 30° - 2\cos^3 60° + 3\tan^4 45°$

7 MENSURATION

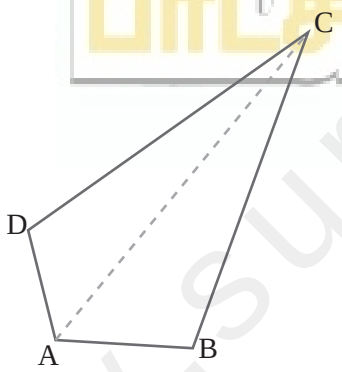
7.1 Introduction

- If a, b, c, are the sides of a triangle, then the area of a triangle

  $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units.

  Where $s = \frac{a+b+c}{2}$, 's' is the semi - perimeter (that is half of the perimeter) of the triangle.

- Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD



Exercise 7.1

1. Using Heron's formula, find the area of a triangle whose sides are

(i) 10 cm, 24 cm, 26 cm (ii) 1.8 m, 8 m, 8.2 m

Sol. (i) sides : 10 cm, 24 cm, 26 cm

Using Heron's formula

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units

$$s = \frac{a+b+c}{2} = \left(\frac{10+24+26}{2}\right)\text{cm} = \frac{60}{2} = 30 \text{ cm}$$

$$\therefore \text{Area} = \sqrt{30(30-10)(30-24)(30-26)}$$

$$= \sqrt{30\times20\times6\times4} = \sqrt{600\times24} = \sqrt{14400} = 120 \text{ sq. cm}$$

(ii) Sides : 1.8 m, 8 m, 8.2 m

$$s = \frac{1.8+8+8.2}{2} = \frac{18}{2} = 9$$

$$\therefore \text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

$$= \sqrt{9(9-1.8)(9-8)(9-8.2)} = \sqrt{9\times7.2\times1\times0.8}$$

$$= \sqrt{51.84} = 7.2 \text{ sq.m}$$

2. The sides of the triangular ground are 22 m, 120 m and 122 m. Find the area and cost of leveling the ground at the rate of ₹ 20 per m^2.

Sol. Sides : 22 m, 120 m, 122 m

Using Heron's formula

$$s = \frac{22+120+122}{2} = \frac{264}{2} = 132 \text{ m}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{132(132-22)(132-120)(132-122)}$$

$$= \sqrt{132\times110\times12\times10}$$

$$= \sqrt{1742400} = \sqrt{11\times11\times3\times3\times2\times\times2\times2\times2\times10\times10}$$

$$= 11\times3\times2\times2\times10 = 1320 \text{ m}^2$$

2	1742400
2	871200
2	435600
2	217800
9	108900
11	12100
11	1100
10	100
	10

Cost of levelling 1 m^2 = ₹ 20

∴Cost of levelling 1320 m^2 = 1320 × 20 = ₹ 26400

3. The perimeter of a triangular plot is 600 m. If the sides are in the ratio 5:12:13, then find the area of the plot.

Sol. We are given that the sides are in the ratio 5 : 12 : 13

Let the ratio be x

So, sides are $5x$, $12x$ and $13x$

Perimeter of a triangular plot = Sum of all sides = $5x + 12x + 13x = 30x$

The perimeter of a triangular plot is 600 m

$$\text{So, } 30x = 600 \text{ m}$$

$$x = \frac{600}{30} = 20 \text{ m}$$

$$\text{Sides} = 5x = 5(20) = 100 \text{ m}$$

$$12x = 12\,(20) = 240 \text{ m}$$

$$13x = 13(20) = 260 \text{ m}$$

$$\text{Area of triangular field} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

$$s = \frac{100+240+260}{2}$$

$$= \frac{600}{2} = 300 \text{ m}$$

$$\text{Area} = \sqrt{300(300-100)(300-240)(300-260)}$$

$$= \sqrt{300\times200\times60\times40}$$

$$= \sqrt{60000\times2400}$$

$$= \sqrt{144000000}$$

$$= 12000 \text{ sq.m}$$

4. Find the area of an equilateral triangle whose perimeter is 180 cm.

Sol. Perimeter of an equilateral triangle = 180 cm

$$\therefore \text{One side } (a) = \frac{180}{3} = 60 \text{ cm}$$

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4}a^2 \text{ sq. units} = \frac{\sqrt{3}}{4}\times 60 \times 60$$

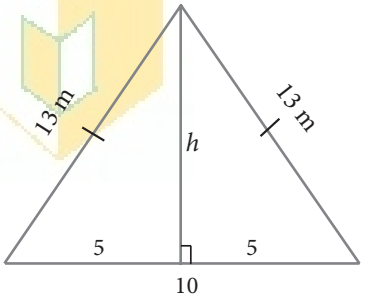
$$= 900\sqrt{3} \text{ m}^2 = 900 \times 1.732 = 1558.8 \text{ cm}^2$$

5. An advertisement board is in the form of an isosceles triangle with perimeter 36m and each of the equal sides are 13 m. Find the cost painting it at ₹ 17.50 per square metre.

Sol. Perimiter of an is isosceles triangle = 36 m

$$\therefore 13+13+x = 36$$

$$x = 36-13-13=10\text{m}$$



Let the height be 'h' and base be 'b'

$$h = \sqrt{13^2-5^2} = \sqrt{169-25}$$

$$= \sqrt{144} = 12 \text{ m}$$

$\therefore$ Area of the triangular board

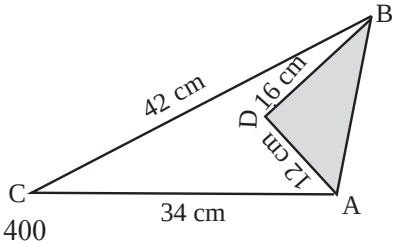
$$= \frac{1}{2}\times b\,h$$

$$= \frac{1}{2}\times 10\times 12 = 60 \text{ m}^2$$

Cost of painting 1m^2 = ₹ 17.50

$\therefore$ Cost of painting 60m^2 = 60×17.50 = ₹ 1050

6. Find the area of the unshaded region.



Sol. By the Pythagoras theorem

$$AB^2 = AD^2 + DB^2$$

$$= (12)^2 + (16)^2 = 144 + 256 = 400$$

$$AB = 20 \text{ cm.}$$