Chapter - 1

Set, Relations and functions

Properties of set operations:

Commutative:

i) $A \cup B = B \cup A$ ii) $A \cap B = B \cap A$

Associative:

i) $(A \cup B) \cup C = A \cup (B \cup C)$

ii) $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive:

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Identity:

i) $A \cup \emptyset = A$ ii) $A \cap U = A$

Idempotent:

i) $A \cup A = A$ ii) $A \cap A = A$

Absorption:

i) $A \cup (A \cap B) = A$ ii) $A \cap (A \cup B) = A$

De Morgan laws:

i) $(A \cup B)' = A' \cup B'$

ii) $(A \cap B)' = A' \cap B'$

iii) $A - (B \cup C) = (A - B) \cap (A - C)$

iv) $A - (B \cap C) = (A - B) \cup (A - C)$

On symmetric difference:

i) $A \Delta B = B \Delta A$

ii) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

iii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

On Empty and Universal set :

i) $\emptyset' = U$ ii) $U' = \emptyset$ iii) $A \cup A' = U$

iv) $A \cap A' = \emptyset$ v) $A \cup U = U$ vi) $A \cap U = A$

On cardinality :

i) For any two finite sets A and B

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii) If A and B are disjoint finite sets

$n(A \cup B) = n(A) + n(B)$

iii) For any three sets A and B and C

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Example 1.1

Find the number of subsets of A if

$A = \{x : x = 4n+1, 2 \leq n \leq 5, n \in N\}$

Solution:

$A = \{2, 3, 4, 5\}$

$n(A) = 4$

$n(P(A)) = 2^n = 2^4 = 16$

Exercise 1.1

1) Write the following in roaster form

i) $\{x \in N : x^2 < 121$ and x is a prime$\}$

Solution:

$2^2 < 121$

$3^2 < 121$

$5^2 < 121$

$7^2 < 121$

$A = \{2, 3, 5, 7\}$

ii) The set of all positive roots of the equation $(x-1)(x+1)(x^2-1) = 0$

Solution:

$(x-1)(x+1)(x^2-1) = 0$

$(x-1) \Rightarrow x-1 = 0 \Rightarrow x = 1$

$(x+1) \Rightarrow x+1 = 0 \Rightarrow x = -1$

$(x^2-1) \Rightarrow x^2-1=0 \Rightarrow x^2-1 \Rightarrow x=\pm 1$

$A=\{1\}$

iii) $\{x \in N : 4x+9 < 52\}$

Solution:

$4x+9 < 52$

$4x < 52-9$

$4x < 43$

$x < \frac{43}{4}$

$x < 10.75$

$A=\{1,2,3,4,5,6,7,8,9,10\}$

iv) $\{x : \frac{x-4}{x+2}=3, x \in R-\{-2\}\}$

Solution:

$x-4=3(x+2)$

$x-4=3x+6$

$3x-x+6+4=0$

$2x+10=0$

$2x=-10$

$x=\frac{-10}{2}$

$x=-5$

$x=-5 \in R-\{-2\}$

$A=\{-5\}$

3) Write the set $\{-1, 1\}$ in set builder form

Solution:

$$A = \{-1, 1\}$$

$$A = \{x : x^2 - 1 = 0, x \in Z\}$$

4) Solution:

$U = \{1, 2 \ldots 10\}$

$A = \{1, 2, 3, 4\}$

$B = \{5, 6, 7\}$

$C = \{2, 3, 4, 8\}$

i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS: $A \times (B \cap C)$

$B \cap C = \{\ \}$

$A \times (B \cap C) = \{1, 2, 3, 4\} \times \{\ \}$

$= \{\ \} \longrightarrow ①$

RHS: $(A \times B) \cap (A \times C)$

$A \times B = \{1, 2, 3, 4\} \times \{5, 6, 7\}$

$= \{(1,5)(1,6)(1,7)(2,5)(2,6)(2,7)$
$(3,5)(3,6)(3,7)(4,5)(4,6)(4,7)\}$

$A \times C = \{1, 2, 3, 4\} \times \{2, 3, 4, 8\}$

$= \{(1,2)(1,3)(1,4)(1,8)(2,2)(2,3)$
$(2,4)(2,8)(3,2)(3,3)(3,4)(3,8)$
$(4,2)(4,3)(4,4)(4,8)\}$

$(A\times B)\cap(A\times C) = \{\ \} \longrightarrow ②$

From eqn ① and ②

LHS = RHS

$\boxed{A\times(B\cap C) = (A\times B)\cap(A\times C)}$

Hence verified

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ii) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

LHS: $A\times(B\cup C)$

$B\cup C = \{2,3,4,5,6,7,8\}$

$A\times(B\cup C) = \{1,2,3,4\}\times\{2,3,4,5,6,7,8\}$

$= \{(1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)$
$(2,2)(2,3)(2,4)(2,5)(2,6)(2,7)(2,8)$
$(3,2)(3,3)(3,4)(3,5)(3,6)(3,7)(3,8)$
$(4,2)(4,3)(4,4)(4,5)(4,6)(4,7)(4,8)\}$
$\longrightarrow ①$

RHS: $(A\times B)\cup(A\times C)$

$(A\times B)\cup(A\times C)$

$= \{(1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)(2,2)$
$(2,3)(2,4)(2,5)(2,6)(2,7)(2,8)(3,2)$
$(3,3)(3,4)(3,5)(3,6)(3,7)(3,8)(4,2)$
$(4,3)(4,4)(4,5)(4,6)(4,7)(4,8)\}$
$\longrightarrow ②$

From eqn ① and ②

LHS = RHS

$\boxed{A\times(B\cup C) = (A\times B)\cup(A\times C)}$

Hence verified

iv) $C-(B-A) = (C\cap A)\cup(C\cap B')$

LHS : $C-(B-A)$

$B-A = \{5,6,7\}$

$C-(B-A) = \{2,3,4,8\} - \{5,6,7\}$

$C-(B-A) = \{2,3,4,8\} \rightarrow (1)$

RHS: $(C\cap A)\cup(C\cap B')$

$C\cap A = \{2,3,4\}$

$C\cap B' = \{2,3,4,8\} \cap \{B,1,2,3,4,8,9,10\}$

$C\cap B' = \{2,3,4,8\}$

$(C\cap A)\cup(C\cap B') = \{2,3,4,8\} \rightarrow (2)$

From eqn (1) and (2)

LHS = RHS

Hence verified

$$\boxed{C-(B-A) = (C\cap A)\cup(C\cap B')}$$

iii) $(A\times B)\cap(B\times A) = (A\cap B)\times(B\cap A)$

LHS: $(A\times B)\cap(B\times A)$

$A\times B = \{1,2,3,4\}\times\{5,6,7\}$

$= \{(1,5)(1,6)(1,7)(2,5)(2,6)(2,7)$
$(3,5)(3,6)(3,7)(4,5)(4,6)(4,7)\}$

$B\times A = \{5,6,7\}\times\{1,2,3,4\}$

$= \{(5,1)(5,2)(5,3)(5,4)(6,1)(6,2)$
$(6,3)(6,4)(7,1)(7,2)(7,3)(7,4)\}$

$(A\times B)\cap(B\times A) = \{\ \} \longrightarrow (1)$

RHS: $(A\cap B)\times(B\cap A)$

$A\cap B = \{\ \}$ $B\cap B = \{\ \}$

$(A\cap B)\times(B\cap B) = \{\ \} \longrightarrow (2)$

From eqn (1) and (2)

LHR = RHS

$$\boxed{(A\times B)\cap(B\times A) = (A\cap B)\times(B\cap A)}$$

Hence verified.

vi) $(B-A) \cup C = (B \cup C) - (A-C)$

LHS: $(B-A) \cup C$

$B-A = \{\}5,6,7\}$

$(B-A) \cup C = \{5,6,7\} \cup \{2,3,4,8\}$

$(B-A) \cup C = \{2,3,4,5,6,7,8\} \longrightarrow ①$

RHS: $(B \cup C) - (A-C)$

$B \cup C = \{2,3,4,5,6,7,8\}$

$A-C = \{1\}$

$(B \cup C) - (A-C) = \{2,3,4,5,6,7,8\} \longrightarrow ②$

From eqn ① and ②

LHS = RHS

$$\boxed{(B-A) \cup C = (B \cup C) - (A-C)}$$

Hence verified

v) $(B-A) \cap C = (B \cap C) - A = B \cap (C-A)$

① $\rightarrow (B-A) \cap C = (B \cap C) - A$

LHS: $(B-A) \cap C$

$B-A = \{5,6,7\}$

$(B-A) \cap C = \{5,6,7\} \cap \{2,3,4,8\}$

$(B-A) \cap C = \{\} \longrightarrow ①$

RHS: $(B\cap C)-A$

$B\cap C = \{\}$

$(B\cap C)-A = \{\} \longrightarrow ②$

② $\Rightarrow (B\cap C)-A = B\cap(C-A)$

LHS: $(B\cap C)-A$

$B\cap C = \{\}$

$(B\cap C)-A = \{\} \longrightarrow ③$

RHS: $B\cap(C-A)$

$C-A = \{8\}$

$B\cap(C-A) = \{\} \longrightarrow ④$

③ $\Rightarrow (B-A)\cap C = B\cap(C-A)$

LHS: $(B-A)\cap C$

$B-A = \{5,6,7\}$

$(B-A)\cap C = \{\} \longrightarrow ⑤$

RHS: $B\cap(C-A)$

$C-A = \{8\}$

$B\cap(C-A) = \{\} \longrightarrow ⑥$

From eqn ①, ②, ③, ④, ⑤ and ⑥

LHS = RHS

$$\boxed{(B-A)\cap C) = (B\cap C)-A = B\cap(C-A)}$$

Hence verified

5) Justify the trueness of the statement:
"An element of a set can never be a subset of itself"

Solution:

The given statement is no true

An element of a set is a subset of itself

6) If $n[P(A)] = 1024$, $n(A \cup B) = 15$ and $n[P(B)] = 32$, then find $n(A \cap B)$

Solution:

$n[P(A)] = 1024 = 2^{10} = n(A) = 10$

$n[P(B)] = 32 = 2^{5} = n(B) = 5$

$n(A \cup B) = 15$

$n(A \cap B) = ?$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$15 = 10 + 5 - n(A \cap B)$

$n(A \cap B) = 15 - 15$

$\boxed{n(A \cap B) = 0}$

7) If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n[P(A \Delta B)]$.

Solution:

$n(A \cap B) = 3$

$n(A \cup B) = 10$

$n[P(A \Delta B)] = ?$

$A \Delta B = (A \cup B) - (A \cap B)$

$n(A \Delta B) = n(A \cup B) - n(A \cap B)$

$n(A \Delta B) = 10 - 3$

$n(A \Delta B) = 7$

$n[P(A \Delta B)] = 2^7 = 128$

8) For a set A. A×A contains 16 elements and two of its elements are (1,3) and (0,2). Find the elements of A.

Solution:

$n(A \times A) = 16$

$n(A) = 4$ elements

Elements in $A \times A = \{(1,3)(0,2)\}$

$\therefore$ possible elements of A

$A = \{0,1,2,3\}$

9) Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x,1)$ $(y,2)$ $(z,1)$ are in $A \times B$. find A and B, where x, y, z are distinct elements

Solution:

$A \times B = \{(x,1)(y,2)(z,1)\}$

$n(A) = 3$

$n(B) = 2$

$A \times B$ has 6 elements

$A = \{x, y, z\}$

$B = \{1, 2\}$

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10) If $A \times A$ has 16 elements. $S = \{(a,b) \in A \times A : a < b\}$; $(-1,2)$ and $(0,1)$ are two elements of S, then find the remaining elements of S

Solution:

$n(A \times A) = 16$

$n(A) = 4$

$S = \{(a,b) \in A \times A : a < b\}$

$A = \{-1, 0, 1, 2\}$

$A \times A = \{(-1,0)(-1,1)(-1,2)(0,1)(0,2)(1,2)\}$

Remaining elements of S

$= \{(-1,0)(-1,1)(0,2)(1,2)\}$

Example 1.8

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$n(A \cup B) = 6$

$A \cap B = \{3, 4\}$

$n(A \cap B) = \cancel{4}\,2$

$n(A \Delta B) = n(A \cup B) - n(A \cap B)$

$= 6 - 2$

$n(A \Delta B) = 4$

To find, $n[(A \cup B) \times (A \cap B) \times (A \Delta B)]$

$= 6 \times 2 \times 4 = 48$

$n[(A \cup B) \times (A \cap B) \times (A \Delta B)] = 48$

Example 1.7

$n(A) = 10$

$n(A \cap B) = 3$

$n[(A \cap B)' \cap A] = ?$

$(A \cap B)' \cap A = (A' \cup B') \cap A$

$= (A' \cap A) \cup (B' \cap A)$

$= \emptyset \cup (B' \cap A)$

$= B' \cap A$

$= A - B$

$n(A - B) = n[(A \cap B)' \cap A] = n(A) - n(A \cap B) = 10 - 3 = 7$

Example 1.4

If $x = \{1, 2, 3, \ldots 10\}$ and $A = \{1, 2, 3, 4, 5\}$ find the number of sets $B \subseteq x$. Such that $A - B = \{4\}$.

Solution:

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3, 5\}$

$C = \{6, 7, 8, 9, 10\} = 5$

Number of subsets $= 2^{(n)} = 2^5 = 32$

Example 1.5

Solution:

Given: $n(B-A) = 2n(A-B) = 4n(A\cap B)$

$n(A\cup B) = 14$

$n(A\cap B) = k$

$n[P(A)] = ?$

$n(B-A) = n(B) - n(A\cap B)$

$2n(A-B) = 2[n(A) - n(A\cap B)]$

$4n(A\cap B) =$ same

$n(B) - n(A\cap B) = 2[n(A) - n(A\cap B)] = 4n(A\cap B)$

$n(B) - k = 2n(A) - 2k = 4k$

$n(B) - k = 4k$

$n(B) = 4k + k$

$\boxed{n(B) = 5k}$

$2n(A) - 2k = 4k$

$2n(A) = 4k + 2k$

$2n(A) = 6k$

$n(A) = \frac{6k}{2}$

$\boxed{n(A) = 3k}$

$n(A \cup B) = 14$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 14$

$3k + 5k - k = 14$

$7k = 14$

$k = \frac{14}{7}$

$\boxed{k = 2}$ $n(A) = 3(2) = 6$

$n[P(A)] = 2^n = 2^6 = 64$

Example 1.9

Solution:

$n[P(\emptyset)] = 1 = 2^n = 2^0 = 1$

$n[P(P(\emptyset)] = 2^n = 2^1 = 2$

$n[P(P(P(\emptyset)] = 2^2 = 4$

$\therefore$ That is 4 element.

Example 1.6

Solution:

Given: $n(A) = m$

$n(B) = k$

A contains more elements than B, $m > k$

$$2^m - 2^k = 112$$

$$2^k\left(\frac{2^m}{2^k} - 1\right) = 112$$

$$2^k(2^{m-k} - 1) = 112$$

$$2^k(2^{m-k} - 1) = 2^4 \times 7$$

$$2^k = 2^4$$

$$\boxed{k = 4}$$

$$2^{m-k} - 1 = 7$$

$$2^{m-k} = 8$$

$$2^{m-k} = 2^8$$

$$m - k = 3$$

$$m - 4 = 3$$

$$m = 3 + 4$$

$$\boxed{m = 7}$$

The value of $m = 7$ and $k = 4$

 Relations

Let A and B be any two non empty sets

* relation R from A to B is defined as a subset of the cartesian product of A and B

Symbolically $R \subseteq A \times B$.

Types of relation

Let S be any non empty sets. Let R be a relation on S. Then

* R is said to be reflexive if a is related to a for all $a \in S$

* R is said to be symmetric if a is relative to b implies that b is related to a

* R is said to be transitive if a is related to b and b is related to c implies that a is related to c.

Equilance Relation

Let S be any set. A relation on S is said to be equivalence relation if it is reflexive, transitive and symmetric.

Exercise 1.2

1) Discuss the following relations for reflexivity, symmetrically and transitivity.

i) The relation R defined on the set of all positive integers by "mRn if m divides n"

Reflexive :

$mRm \Rightarrow m$ is divisible by $m, \forall m \in Z+$

$\therefore R$ is reflexive

Symmetry :

$mRn \rightarrow nRm \; \forall n, m \in Z+$

$2R_4 \rightarrow 4R_2 = 2/4 = 1/2 \notin Z+$

$\therefore R$ is not a symmetry

Transitive :

$mRn. \; nRp \Rightarrow mRp \; \forall \; m. n. p \in Z+$

$4R_2 \Rightarrow 4/2 = 2 \in Z+$

$2R_1 \Rightarrow 2/1 = 2 \in Z+$

$4R_1 \Rightarrow 4/1 = 4 \in Z+$

$\therefore R$ is transitive

R is a reflexive, transitive and not a symmetry

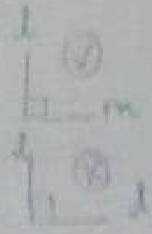
$\therefore$ Equivalance relation is not satisfied

ii) Let p denote the set of all straight lines in a plane. The relation R defined by "l R m if l is perpendicular to m"

Reflexive:

l is not ⊥r l , l R̸ l

∴ It is not reflexive

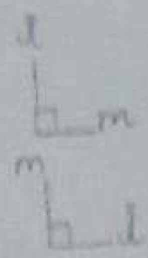


Symmetry:

l is ⊥r m, m is ⊥r l

l R m , m R l

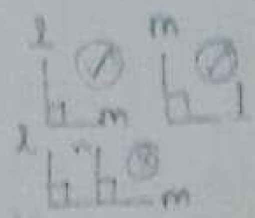
∴ It is symmetry



Transitive:

l R m , m R n , l R̸ n

∴ It is not transitive



∴ R is a symmetry but R is not a reflexive and transitive

∴ Equivalance relation is not satisfied

iii) Let A be the set consisting of all the members of a family. The relation R defined by "a R b if a is not a sister of b"

Reflexive:

A = {F, M, A, T}

F is not a sister of F , F R F

∴ It is reflexive

Symmetry :

F is not a sister of M, FRM

M is not a sister of F, MRF but

A is a sister of T, ART

∴ It is not symmetry

Transitive :

ARF, FRM but ART

∴ It is not transitive.

∴ R is reflexive but not a symmetry and transitive.

∴ Equivalance relation is not satisfied

iv) Let A be the set consisting of all the female members of a family. The relation R defined by "aRb if a is not a sister of b".

$A = \{G, M, D\}$

Reflexive :

DRD ⇒ D is not a sister of D ∀ ∈ A

∴ It is not a reflexive

Symmetry :

GRM, MRG ⇒ G is not a sister of M

∴ It is symmetry

Transitive:

GRM. MRC and GRC

G is not a sister to M but

M is a sister to C

∴ It is not transitive

∴ R is not transitive but not a reflexive and symmetry

Equivalance relation is not satified

vi) On the set of natural numbers the relation R defined by "xRy if $x+2y=1$"

Solution:

The relation R defined by

xRy if $x+2y=1$

Reflexive:

$$xRx = x+2x = 1$$
$$3x = 1$$
$$x = 1/3$$

∴ It is not reflexive

As it is empty set so it satisfies

∴ It is symmetry and transitive

∴ R is a symmetry and transitive but not a reflexive.

Equivalance relation is not satified

2) Let $x = \{a, b, c, d\}$ and $R = \{(a,a)(b,b)(a,c)\}$. Write down the minimum number of ordered pairs to be included to R make it

Solution:

$$x = \{a, b, c, d\}$$

$$R = \{(a,a)(b,b)(a,c)\}$$

i) Reflexive:

R is reflexive we must include

$(c,c)(d,d) \in R$

ii) Symmetry:

To make R is symmetry we must include $(c,a) \in R$

iii) Transitive:

R is transitive

Nothing to be added

iv) Equivalance:

R is an equivalance. we must include $(c,c)(d,d)$ and (c,a).

3) Let $A = \{a, b, c\}$ and $R = \{(a,a)(b,b)(a,c)\}$ write down the minimum number of ordered pairs to be Included to R to make it

Solution:

$$A = \{a, b, c\}$$

$$R = \{(a,a)(b,b)(a,c\}$$

i) Reflexive:

R is reflexive we must include

$(c,c) \in R$

ii) Symmetry:

To make R is symmetry we must include $(c,a) \in R$

iii) Transitive:

R is transitive

Nothing to be added

iv) Equivalance:

R is an equivalance we must include (c,c) (c,a).

41) Let P be the set of all triangles in a plane and R be the relation defined on P as aRb if a is similar to b. Prove that R is an equivalance relation

Solution:

i) Reflexive : 

aRa , a is similar to a

bRb , b is similar to b

∴ It is reflexive

ii) Symmetry : 

aRb, a is similar to b

bRa, b is similar to a

∴ It is symmetry

iii) Transitive : 

aRb. a is similar to b

bRc . b is similar to c

aRc . a is similar to c

∴ It is transitive

Three conditions are satisfied

∴ It is equivalance relation

R is an equivalance relation

Hence proved

5) On the set of natural numbers let R be the relation defined by aRb if $2a+3b=30$. Write down the relation defined by listing all the pairs. Check whether it is

Solution :

$$2a+3b=30$$
$$3b=30-2a$$
$$b=\frac{30-2a}{3}$$

$R=\{(3,\overset{8}{\underline{q}})(6,6)(9,4)(12,2)\}$

i) Reflexive :

$3\,R\,3$, $9\not R\,4$

$\therefore$ It is not reflexive

ii) Symmetry :

$3\,R\,8$ but $8\not R\,3$

$\therefore$ It is not symmetry

iii) Transitive :

aRb and bRc but $a\not R\,c$

$\therefore$ It is transitive

iv) Equivalance :

It is transitive but not a reflexive and symmetry

$\therefore$ It is not equivalance

b) Prove that the relation "friendship" is not an equivalance relation on the set of all people in chennai.

Solution :

a, b → aRb , a is a friend to b

i) Reflexive :

a$\not R$a . a is not a friend to a

b$\not R$b, b is not a friend to b

∴ It is not reflexive

ii) Symmetry :

aRb, a is friend to b

b$\not R$a, b is not a friend to a

∴ It is not a symmetry

iii) Transitive :

aRb, a is friend to b

bRc . b is friend to c

a$\not R$c , a is not a friend to c

∴ It is not transitive

Three conditions are not satisfied

It is not equivalance

∴ The relation friendship is not an equivalance relation.

Hence proved

7) Solution:

Given $a + b \le 6$

$R = \{(1,1)(1,2)(1,3)(1,4)(1,5)$
$(2,1)(2,2)(2,3)(2,4)$
$(3,1)(3,2)(3,3)$
$(4,1)(4,2)$
$(5,1)\}$

i) Reflexive : $4R4$, $5R5$

$\therefore$ It is not reflexive.

ii) Symmetric : $2R4$. $4R2$ and $5R1$. $1R5$

$\therefore$ It is symmetry.

iii) Transitive :

$1R3$, $3R2 \Rightarrow 1R2$

$5R1$, $1R4 \Rightarrow 5R4$

$\therefore$ It is not transitive

iv) Equivalance :

It is not reflexive and transitive but it is a symmetry

$\therefore$ It is not equilance

8) Solution:

Given: $A = \{a, b, c\}$

$A \times A = \{a, b, c\} \times \{a, b, c\}$

$= \{(a,a) (a,b) (a,c) (b,a) (b,b)$
$(b,c) (c,a) (c,b) (c,c)\}$

i) Reflexive :

$R = \{(a,a) (b,b) (c,c)\}$

ii) Symmetry :

$R = \{(a,b) (b,a) (a,c) (c,a) (b,c) (c,b)$
$(a,a) (a,a) (b,b) (b,b) (c,c) (c,c)\}$

iii) Transitive :

$= \{(a,b) (b,c)\} = (a,c)$

$\therefore$ The equivalance relation of smallest cardinality of A

$R = \{(a,a)(b,b) (c,c)\}$

$\therefore$ The equivalance relation of largest cardinality of A is $A \times A$

$A \times A = 9$

9) Solution:

$$\frac{m-n}{7}$$

i) Reflexive : $mRm = \frac{m-m}{7} = \frac{0}{7} = 0$

$\therefore$ It is divisible by 7

$\therefore$ It is reflexive

ii) Symmetric : mRn. nRm

$$\frac{m-n}{7} \cdot \frac{n-m}{7} \Rightarrow \frac{-(m-n)}{7}$$

$\therefore$ It is symmetry

iii) Transitive :

$$\frac{m-n}{7} = p \qquad \frac{n-r}{7} = q$$

$m-n = 7p \longrightarrow ①$

$n-r = 7q \longrightarrow ②$

$① + ② \Rightarrow m - \cancel{n} + \cancel{n} - r = 7p + 7q$

$m - r = 7(p+q)$

$$\frac{m-r}{7} = p+q$$

$\therefore$ It is transitive

$\therefore$ R is an equivalance relation

Hence proved

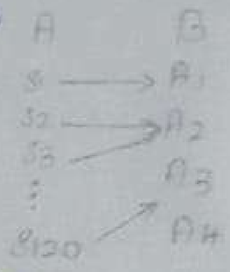
Exercise 1.3

1) Solution:

Given: 120 students in 4 sections

let, one student in one section

30 student in one section



∴ One to one function

R from A to B. It is a function

R from B to A, It is not a function

∴ Inverse relation is not a function.

2) Solution:

Given: To write the values f at -4, 1, -2, 7, 0

$$f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \quad -3, -4, -5, -6 \ldots \\ x+4 & \text{if } -3 < x < -2 \quad -2.1, -2.2, -2.3 \ldots \\ x^2-x & \text{if } -2 \le x < 1 \quad -2, -1, 0 \\ x-x^2 & \text{if } 1 \le x < 7 \quad 1, 2, 3, 4, 5, 6 \\ 0 & \text{if otherwise} \end{cases}$$

$f(-4) \Rightarrow f(x) = -x+4$

$f(-4) = -(-4)+4$

$f(-4) = 4+4$

$\boxed{f(-4) = 8}$

$f(1) \Rightarrow f(x) = x - x^2$

$f(1) = 1 - 1^2$

$f(1) = 1 - 1$

$\boxed{f(1) = 0}$

$f(-2) \Rightarrow f(x) = x^2 - x$

$f(-2) = (-2)^2 - (-2)$

$f(-2) = 4 + 2$

$\boxed{f(-2) = 6}$

$f(7) \Rightarrow f(x) = 0$

$\boxed{f(7) = 0}$

$f(0) \Rightarrow f(x) = x^2 - x$

$f(0) = 0^2 - 0$

$\boxed{f(0) = 0}$

$\therefore f(-4) = 8$

$f(1) = 0$

$f(-2) = 6$

$f(7) = 0$

$f(0) = 0$

3) Solution:

Given: To write the values of $f = -3, 5, 2, -1, 0$

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\alpha, 0) \quad -1, -2, -3 \ldots \\ x^2 + 3x - 2 & \text{if } x \in (3, \alpha) \quad 4, 5, 6, 7 \ldots \\ x^2 & \text{if } x \in (0, 2) \quad 1 \\ x^2 - 3 & \text{if otherwise} \end{cases}$$

$f(x) = x^2 + x - 5 \rightarrow (f(-3)$

$f(-3) = (-3)^2 + (-3) - 5$

$f(-3) = 9 - 8$

$\boxed{f(-3) = 1}$

$f(5) \Rightarrow f(x) = x^2 + 3x - 2$

$f(5) = 5^2 + 3(5) - 2.$

$f(5) = 25 + 15 - 2$

$f(5) = 25 + 13$

$\boxed{f(5) = 38}$

$f(2) \Rightarrow f(x) = x^2 - 3$

$f(2) = 2^2 - 3$

$f(2) = 4 - 3$

$\boxed{f(2) = 1}$

$f(-1) \Rightarrow f(x) = x^2 + x - 5$

$f(-1) = (-1)^2 + (-1) - 5$

$f(-1) = 1 - 5 - 1$

$f(-1) = 1 - 6$

$\boxed{f(-1) = -5}$

$f(0) \rightarrow f(x) = x^2 - 3$

$\boxed{f(0) = -3}$

$\therefore \ f(-3) = 1$

$f(5) = 38$

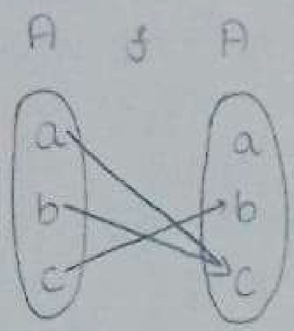
$f(2) = 1$

$f(-1) = -5$

$f(0) = -3$

4) Solution:

i) If $A = \{a, b, c\}$ and $f = \{(a,c)(b,c)(c,b)\}$; $(f : A \rightarrow A)$

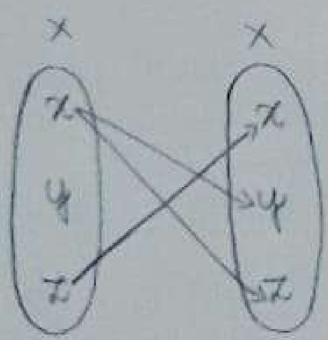


It is a function

It is not one to one, c - 2 pre image

It is not onto, A - no pre image

ii) If $x = \{x, y, z\}$ and $f = \{(x,y)(x,z)(z,x)\}$; $(f : x \rightarrow x)$.



It is not a function

x - 2 image

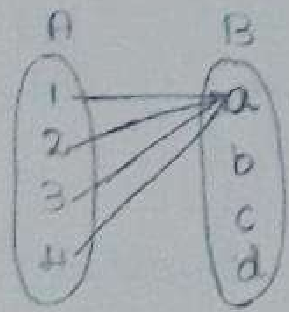
y - no image

5) Solution:

Given: $A = \{1, 2, 3, 4\}$

$B = \{a, b, c, d\}$

i) neither one to one nor onto



$R = \{(1,a)(2,a)(3,a)(4,a)\}$

$\therefore$ It is neither one to one nor onto

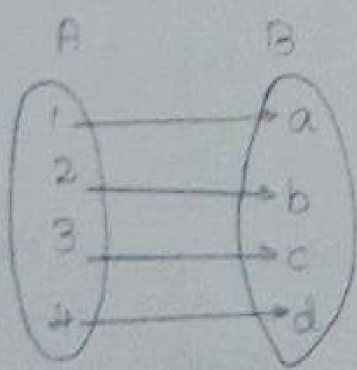
ii) not one to one but onto

Impossible

iii) one to one but not onto

Impossible

iv) One to one and onto



$R = \{(1,a)(2,b)(3,c)(4,d)\}$

$\therefore$ It is one to one and onto function

6) Solution:

Given: To find the domain of $\frac{1}{1-2\sin x}$

$$1-2\sin x = 0$$
$$-2\sin x = -1$$
$$\sin x = \frac{1}{2}$$
$$\sin x = \sin 30^\circ$$
$$\sin x = \sin \frac{\pi}{6}$$
$$\sin x = \sin y \quad (y = \frac{\pi}{6}$$

$$x = n\pi + (-1)^n \frac{\pi}{6} \quad (n \in Z)$$

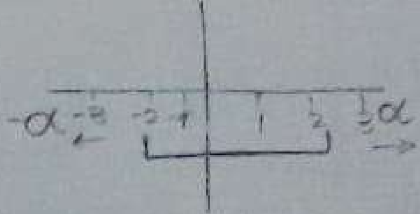
Domain of $f : R - \left\{ n\pi + (-1)^n \frac{\pi}{6} \right\}, n \in z$

7) Solution:

Given: To find the largest possible domain of real valued function

$$f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$$

$$x^2 - 9 > 0$$
$$x^2 > 9$$
$$x > \pm 3$$



$$(-\alpha, -3) \cup (\alpha, 3)$$

$$4 - x^2 = 0$$
$$x^2 = 4$$
$$x = \pm 2 \quad [-2, +2]$$

$$x = \{ \quad \}$$

8) Solution:

Given: $\frac{1}{2\cos x - 1}$

$-1 \leq \cos x \leq 1$

$\times$ by 2 $\quad -2 \leq 2\cos x \leq 2$

add -1 $\quad -2-1 \leq 2\cos x - 1 \quad 2-1$

$-3 \leq 2\cos x - 1 \leq 1$

Reciprocal, $\frac{1}{-3} \leq \frac{1}{2\cos x - 1} \leq 1$

Range $\Rightarrow -\frac{1}{3}$ and 1

Range $\left(-\alpha, -\frac{1}{3}\right] \cup [1, \alpha)$

9) Solution:

Given: $xy = -2$

To find domain and Range

Domain: $xy = -2$

$y = -\frac{2}{x}$

$f(x) = \frac{-2}{x}$

$\boxed{\text{domain} = R - \{0\}}$

Range: $xy = -2$

$x = \frac{-2}{y}$

$\boxed{\text{Range} = R - \{0\}}$

10) Solution:

Given: $f, g : R \to R$

$f(x) = |x| + x$

$g(x) = |x| - x$

To find $g \circ f$ and $f \circ g$

$f(x) = |x| + x$

$$f(x) = \begin{cases} 2x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$g(x) = |x| - x$

$$g(x) = \begin{cases} 0 & x \geq 0 \\ -2x & x < 0 \end{cases}$$

$$g \circ f = g(f) = \begin{cases} g(2x) = 0 & x \geq 0 \\ g(0) = 0 & x < 0 \end{cases}$$

$$f \circ g = f(g) = \begin{cases} f(0) = 0 & x \geq 0 \\ f(-2x) = 0 & x < 0 \end{cases}$$

$f \circ g = 0$

$g \circ f = 0$

11) Solution:

Given: f, g, h are real valued function defined on R to prove $(f+g) \circ h = f \circ h + g \circ h$ what can you say about $f \circ (g+h)$?

$(f+g) \circ h = f \circ h + g \circ h$ $\quad f \circ h$ $\quad g \circ h$

$f(h(x))$ $\quad g(h(x))$

LHS: $(f+g)\, h(x) = f(h(x)) + g(h(x))$

$$\boxed{(f+g) \circ h = f \circ h + g \circ h}$$

Hence proved

$fo(g+h)$

let, $f(x) = x^2$

$g(x) = x$

$h(x) = 1$

LHS $fo(g+h) = fo(x+1) \rightarrow fo(x+1)$

$f(x)$

$= (x+1)^2$

$= x^2+2x+1 \longrightarrow ①$

$fog + foh = f(g(x)) + f(h(x))$

$= f(x) + f(1)$

$= x^2+1 \longrightarrow ②$

$\therefore$ From ① and ②

$fo(g+h) \neq fog + foh$

12) Solution:

Given $f: R \rightarrow R$ is defined by $f(x) = 3x-5$

To prove that f is a bijection and find its inverse

Bijection $\rightarrow$ one-one and onto

One - One:

$x = y$

$f(x) = f(y)$

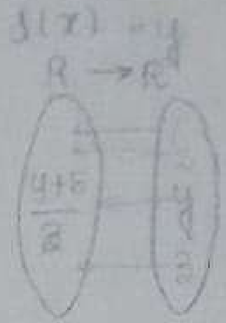
$3x - \cancel{5} = 3y - \cancel{5}$

$\cancel{3}x = \cancel{3}y$

$\boxed{x = y}$

$\therefore$ It is one one function

Onto:



$f(x) = 3x - 5$

Let $y = 3x - 5$

$y + 5 = 3x$

$$\boxed{x = \frac{y+5}{3}}$$

$f(x) = 3x - 5$

$f(x) = 3\left(\frac{y+5}{3}\right) - 5$

$f(x) = y \cancel{+5} \cancel{-5}$

$$\boxed{f(x) = y}$$

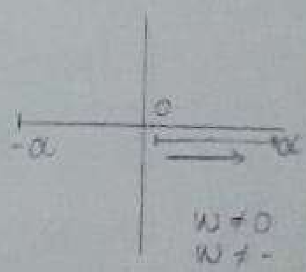
∴ It is an onto function

Inverse : $f(x') = \frac{x+5}{3}$

13) Solution:

Given: weight of muscles of a man = x, $w(x) = 0.35x$. To determine its domain.

$$\boxed{\text{Domain} = x > 0}$$



14) Solution:

Given: To graph the function and determine if it is one - one. The distance of an object falling is a function of time t and can be expressed as $s(t) = -16t^2$

$$s(t) = -16t^2$$

$$s(t_1) = s(t_2)$$

$$-16t_1^2 = -16t_2^2$$

$$t_1^2 = t_2^2$$

$$t_1 = \sqrt{t_2^2}$$

$$t_1 = \pm t_2$$

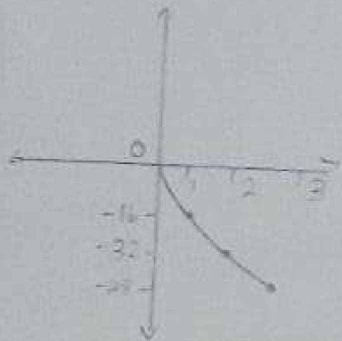
$\Rightarrow t_1 = t_2$ and $t_1 = -t_2$ (not possible)

$$\boxed{t_1 = t_2}$$

$\therefore$ It is one-one function

$$s(t) = -16t^2$$

x	1	2	3	4
y	-16	-32	-48	-64



15) Solution:

Given: base cost $= C$

fuel surcharges in rupee $= S$

Both C and S are function of mileage m

$C(m) = 0.4m + 50$ and $S(m) - 0.03m$.

To determine a function for total cost of a ticket in terms of mileage and find the airfare for flying 1600 miles

$$C(m) = 0.4m + 50$$

$$S(m) = 0.03m$$

$$T(m) = C(m) + S(m)$$

$T(m) = 0.4m + 50 + 0.03m$

$T(m) = 0.43m + 50$

Airfare for flying 1600 miles

$T(1600) = 0.43(1600) + 50$

$T(1600) = 688 + 50$

$\boxed{T(1600) = 738}$

16) Solution:

Given: $A(x) = 30.000 + 0.04x$

$S(x) = 25000 + 0.05x$

$(A+S)x = A(x) + S(x)$

$= 30.000 + 0.04x + 25000 + 0.05x$

$= 55.000 + 0.09x$

$(A+S)(1.50.00.000)$

$= 55.000 + 0.09(1.50.00.000)$

$= 55.000 + 13.50.000$

$= 14.05.000$ R

17) Solution:

Given: Singapore dollar

$= 1.23x$ (US dollor)

Indian rupee

$= 50.50y$ (Sing dollar)

Number of American dollars $= x$

US dollar $- x$

Singapore dollar $- y$

$$g \circ f = g \circ (f(x))$$
$$= g(1.23x)$$
$$= 50.50(1.23x)$$
$$= 62.115x$$

(18) Solution:

Given: particular meal - Rs.100

Menu price = x ₹

Number of customers - $D(x) = 200 - x$

cost of $(200-x) \times$ particular meal $= (200-x)(100)$

Total menu price $= (200-x)x$

$$\text{Profit} = \text{M.P} - \text{cost}$$
$$= (200-x)x - (200-x)100$$
$$= (200-x)(x-100)$$

$$(200-x)(x-100) = 200x - 20000 - x^2 + 100x$$
$$= -x^2 + 100x + 200x - 20000$$
$$= -x^2 + 300x - 20000$$
$$\text{Profit} = x^2 - 300x + 20000$$

19) Solution:

Given: $y = \frac{5x}{9} - \frac{160}{9}$

To find the inverse of this equation and determine whether the inverse is function

$y = \frac{5x}{9} - \frac{160}{9}$

$9y = 5x - 160$

$9y + 160 = 5x$

$x = \frac{9y+160}{5}$

Inverse $\rightarrow f(x)' = \frac{9x+160}{5}$

$\therefore$ The inverse is a function

20) Solution:

Given: $f(x) = 3x - 4$

Let $y = 3x - 4$

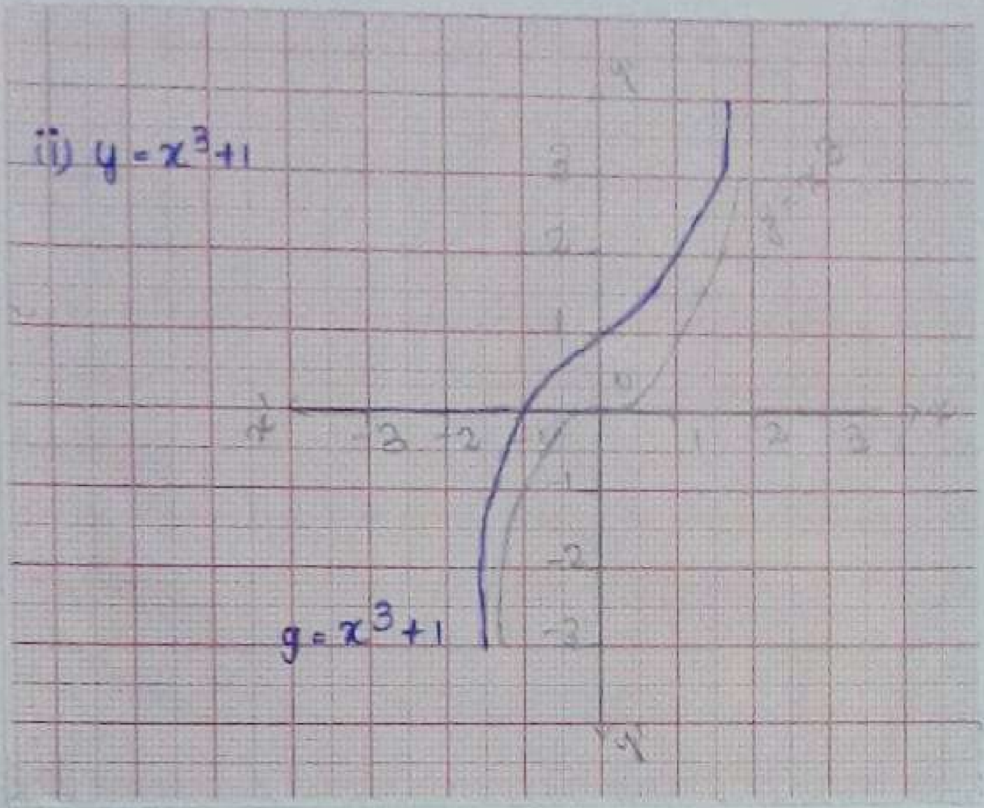
$y + 4 = 3x$

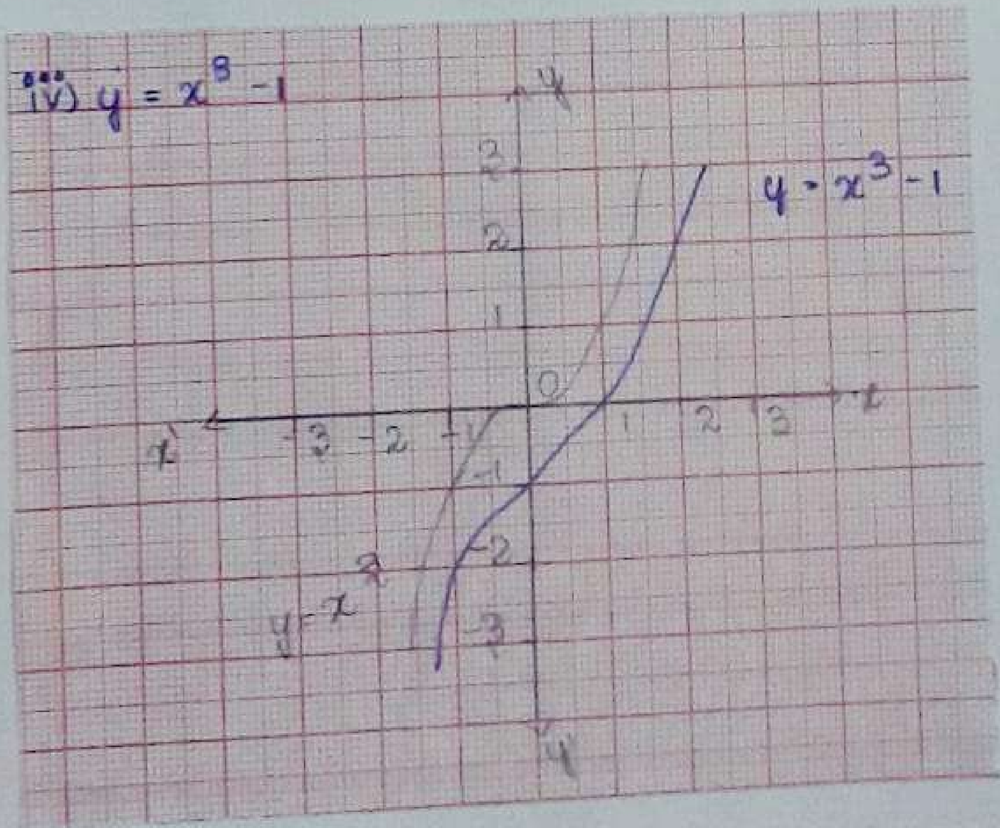
$x = \frac{y+4}{3}$

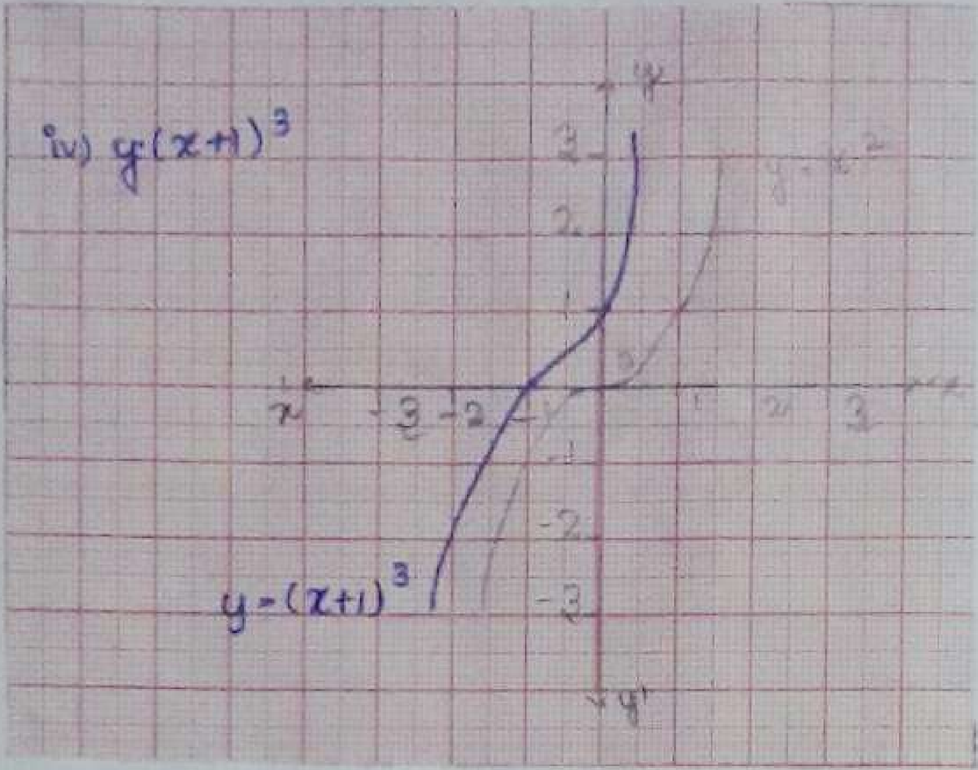
$f'(x) = \frac{x+4}{3}$

Inverse $\Rightarrow f'(x) = \frac{x+4}{3}$

$\therefore$ The inverse is a function.

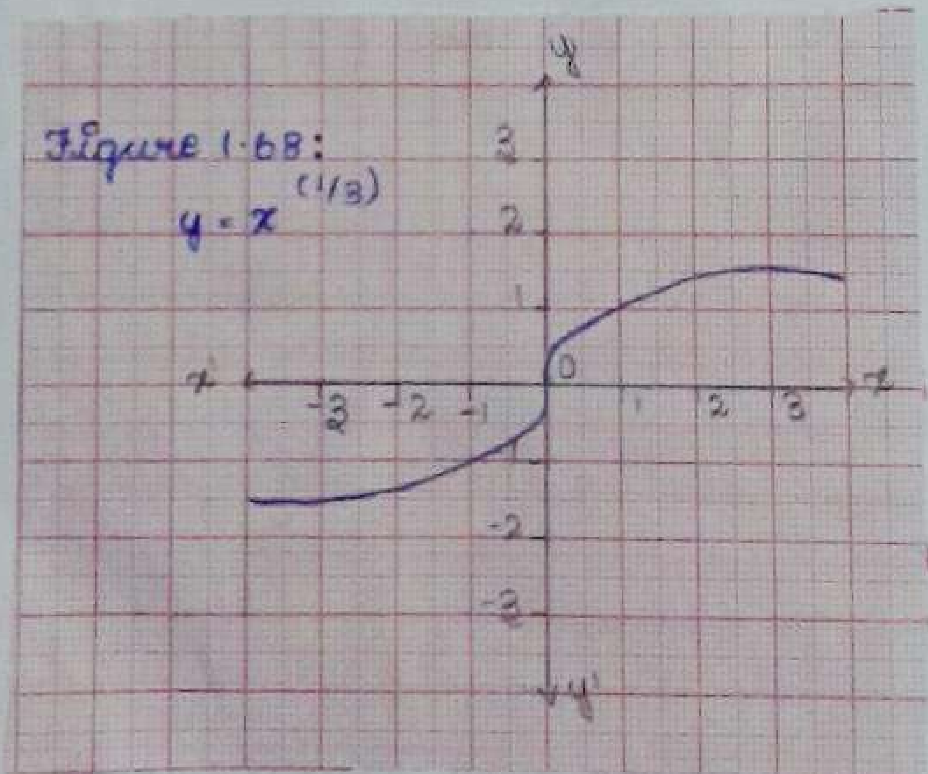

ii) y = x^3+1
y = x^3+1


iv) y = x^3 - 1
y = x^3 - 1
y = x^3

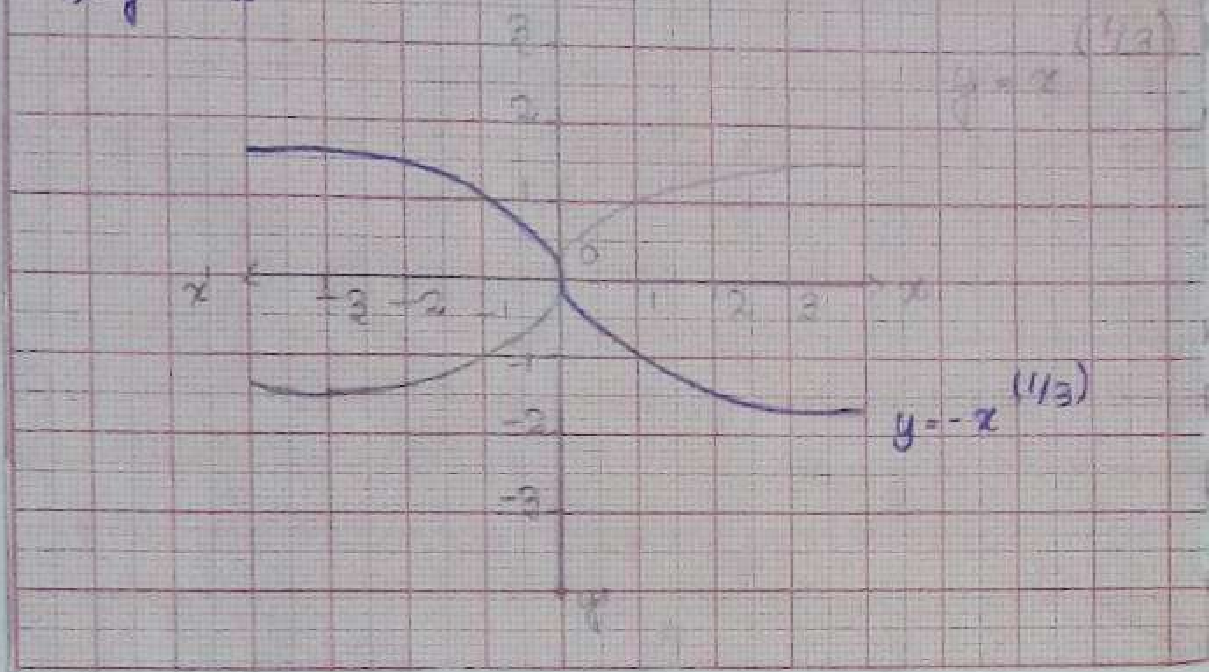


2) For the curve $y = x^{(1/3)}$ in figure 1.68, draw

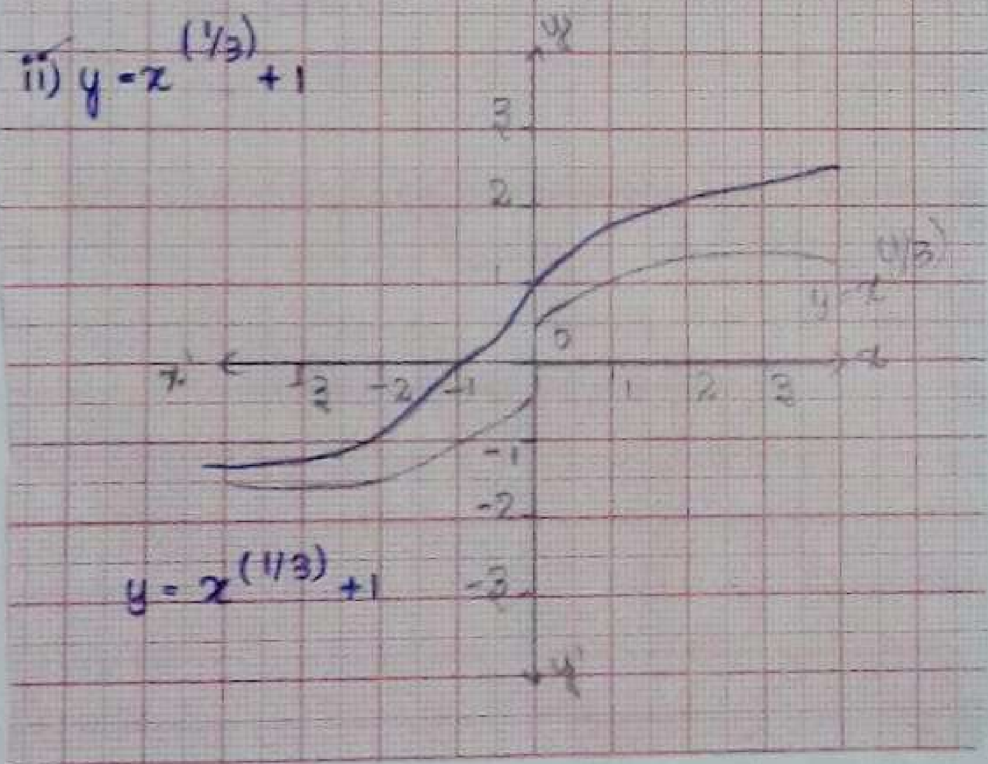
i) $y = -x^{(1/3)}$ ii) $y = x^{(1/3)} + 1$ iii) $y = x^{(1/3)} - 1$ iv) $y = (x+1)^{(1/3)}$

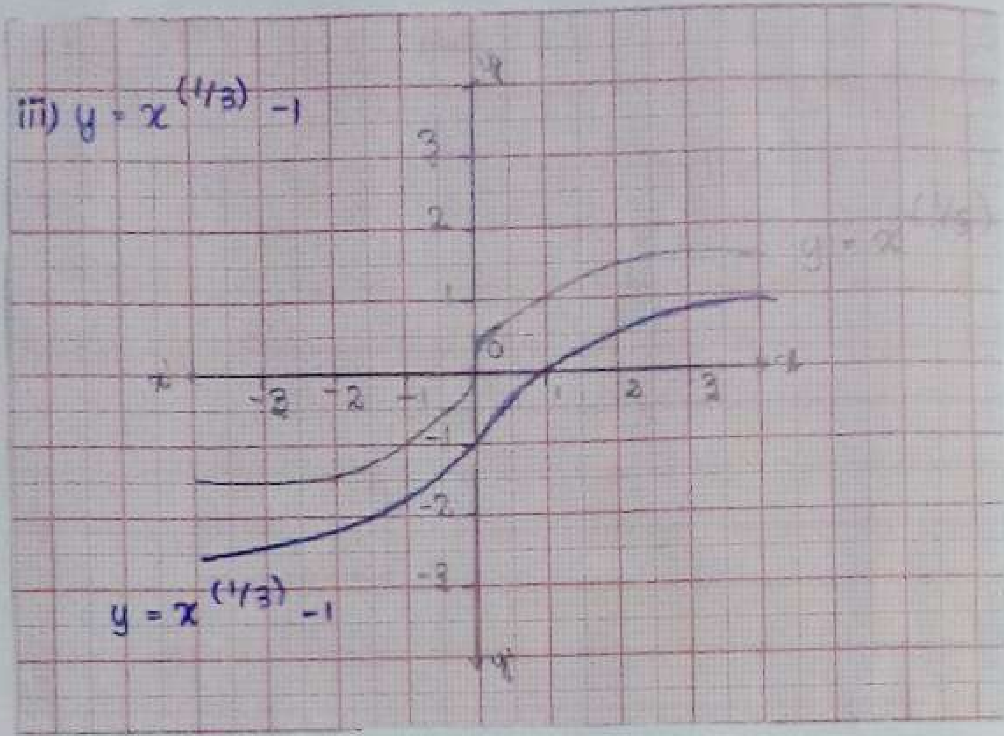


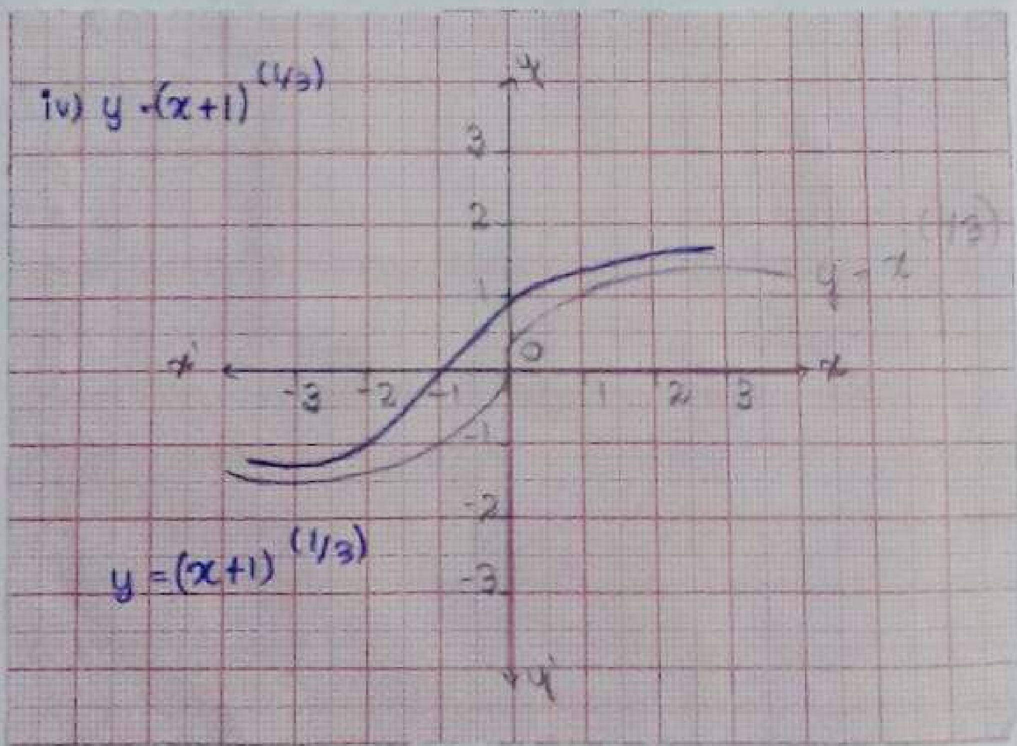
i) $y = -x^{(1/3)}$



ii) $y = x^{(1/3)} + 1$




iii) $y = x^{(1/3)} - 1$
y
3
2
1
0
x'
x
-3
-2
-1
1
2
3
-1
-2
-3
y'
$y = x^{(1/3)} - 1$


iv) $y = (x+1)^{(1/3)}$
y
3
2
1
0
x'
x
-3
-2
-1
1
2
3
-1
-2
-3
y'
$y = (x+1)^{(1/3)}$

3) Solution:

Given: $f(x) = x^3$

$g(x) = x^{1/3}$

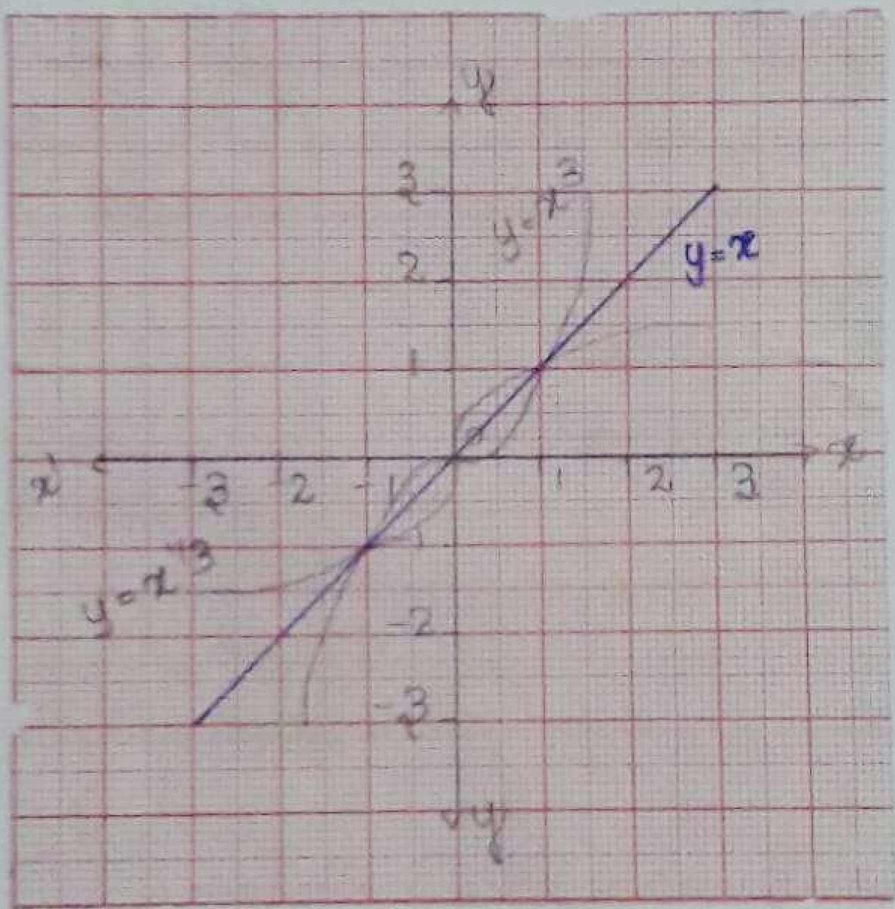
To find $f \circ g$

$f \circ g = f(g(x))$

$= f(x^{1/3})$

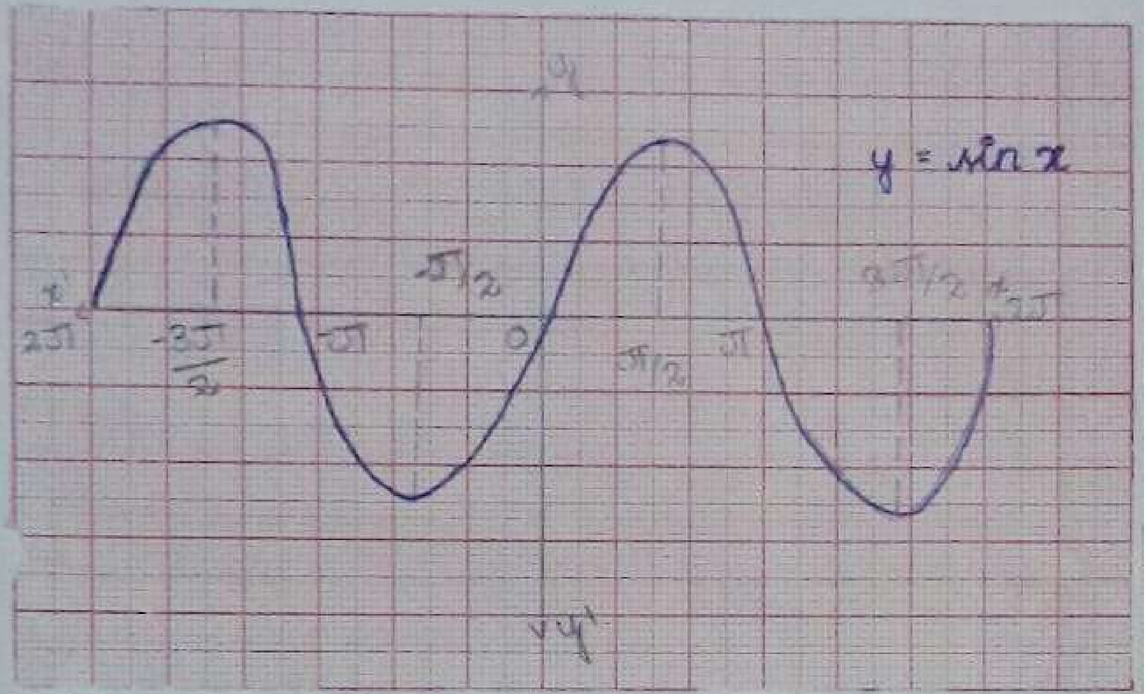
$= (x^{1/3})^3$

$\boxed{y = x}$



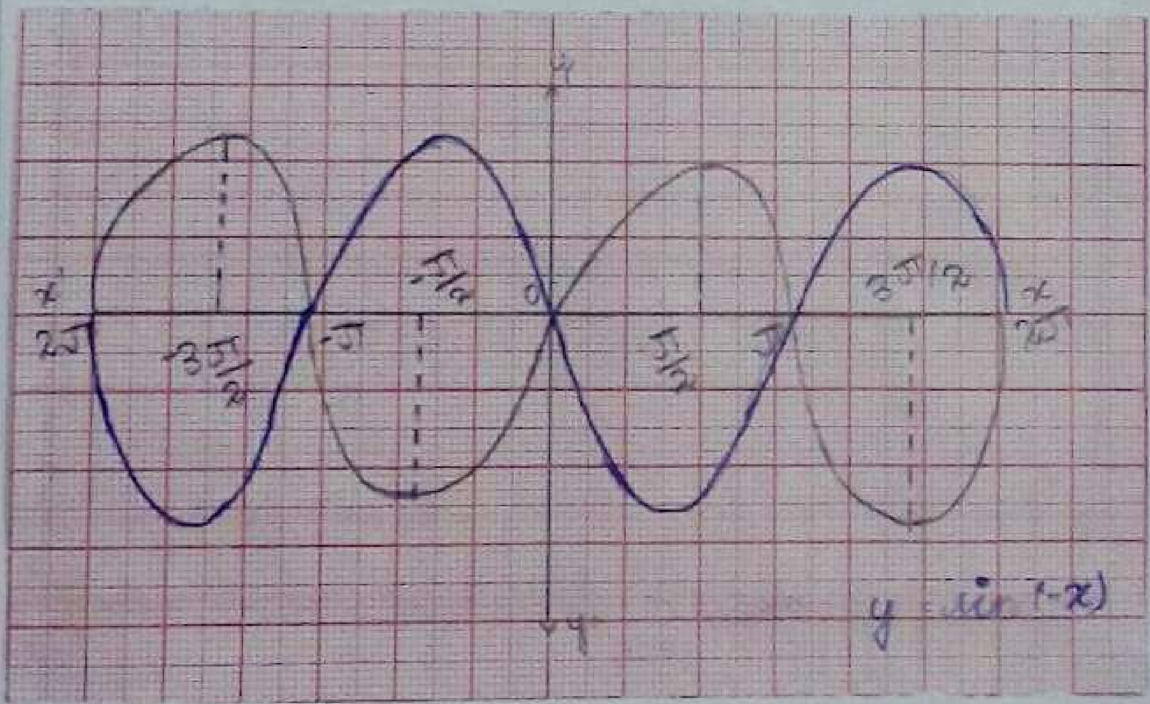
5) Solution:

From the curve $y = \sin x$.



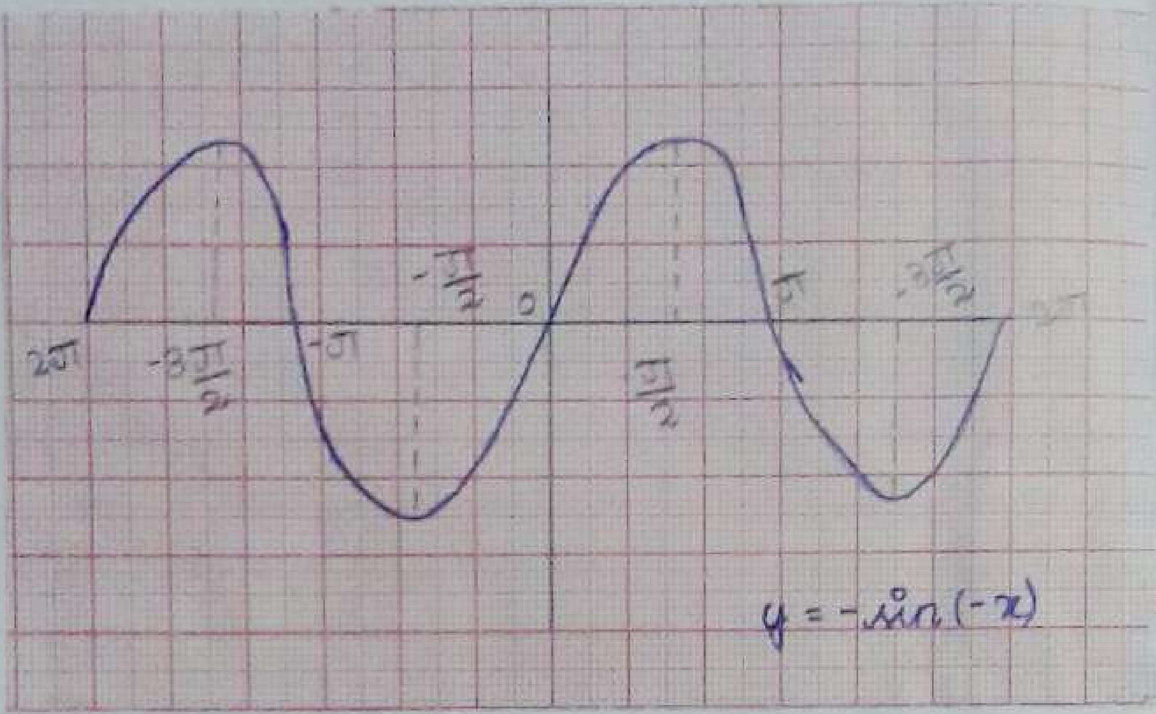
Graph the functions.

i) $y = \sin(-x)$



ii) $y = -\sin(-x)$

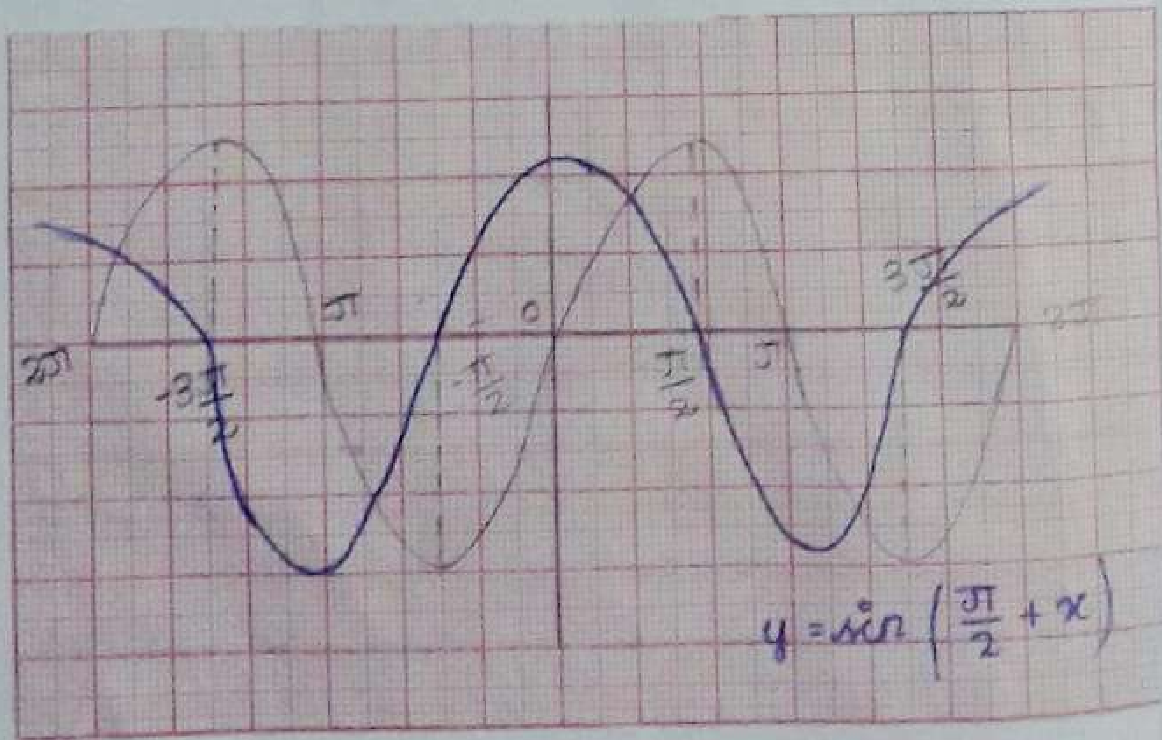
$y = \sin x$



iii) $y = \sin\left(\frac{\pi}{2} + x\right)$ which is $\cos x$

The graph of $y = f(x+c)$, $c > 0$

causes the shift to the left.



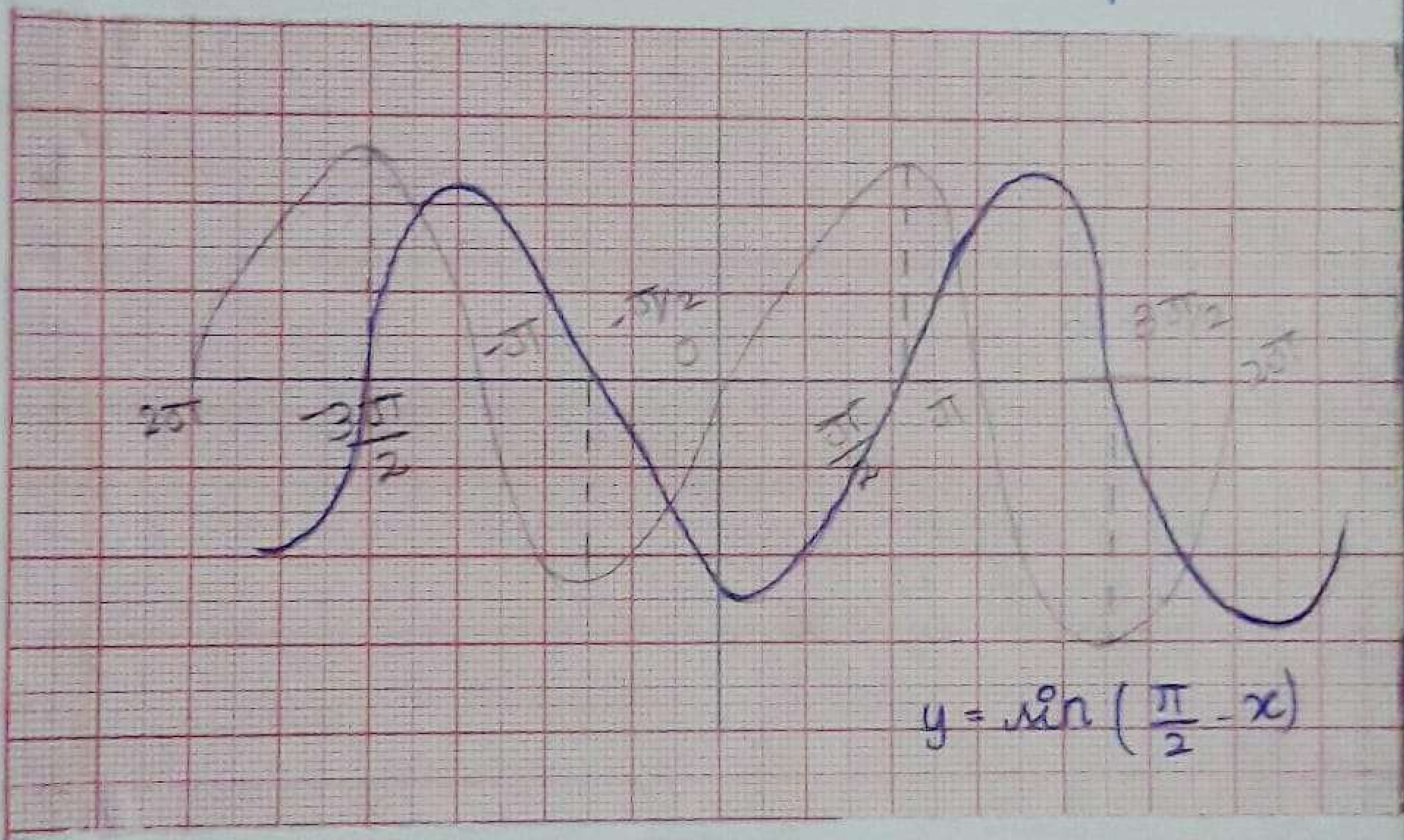
iv) $y = \sin\left(\frac{\pi}{2} - x\right)$ which is also $\cos x$ (refer trignometry)

$$y = \sin\left(\frac{\pi}{2} - x\right)$$

$$y = \sin\left(x - \frac{\pi}{2}\right)$$

The graph of $y = f(x-c),\ c > 0$

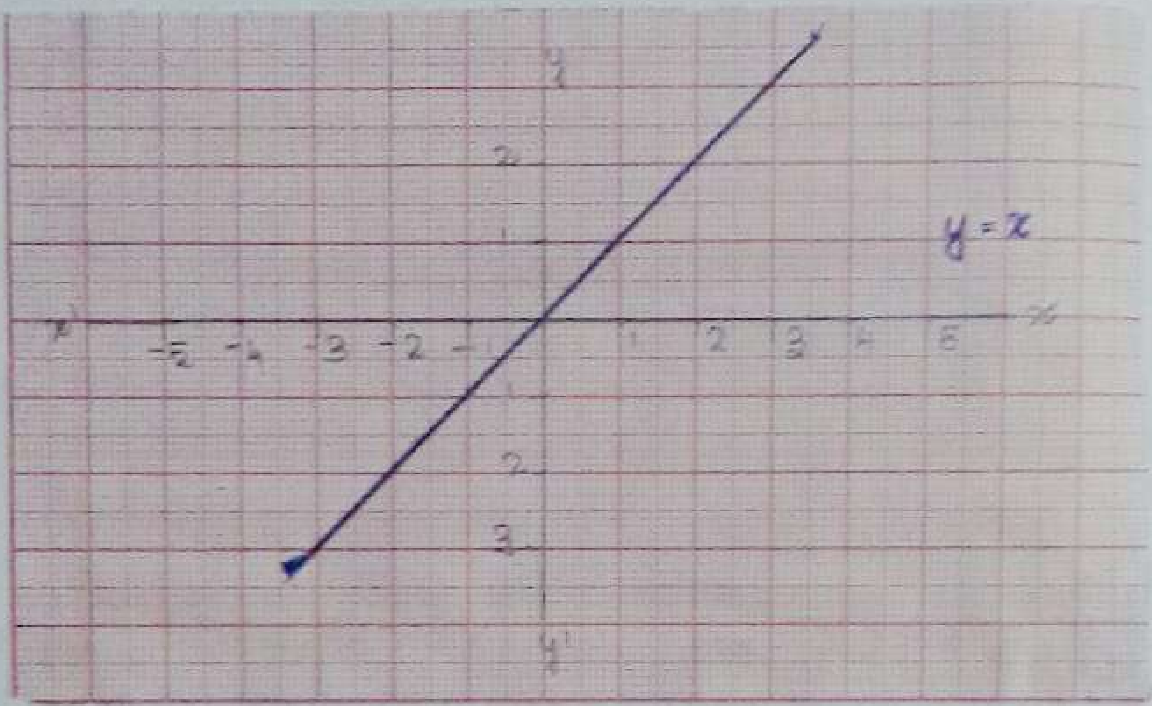
causes the shift to the right.



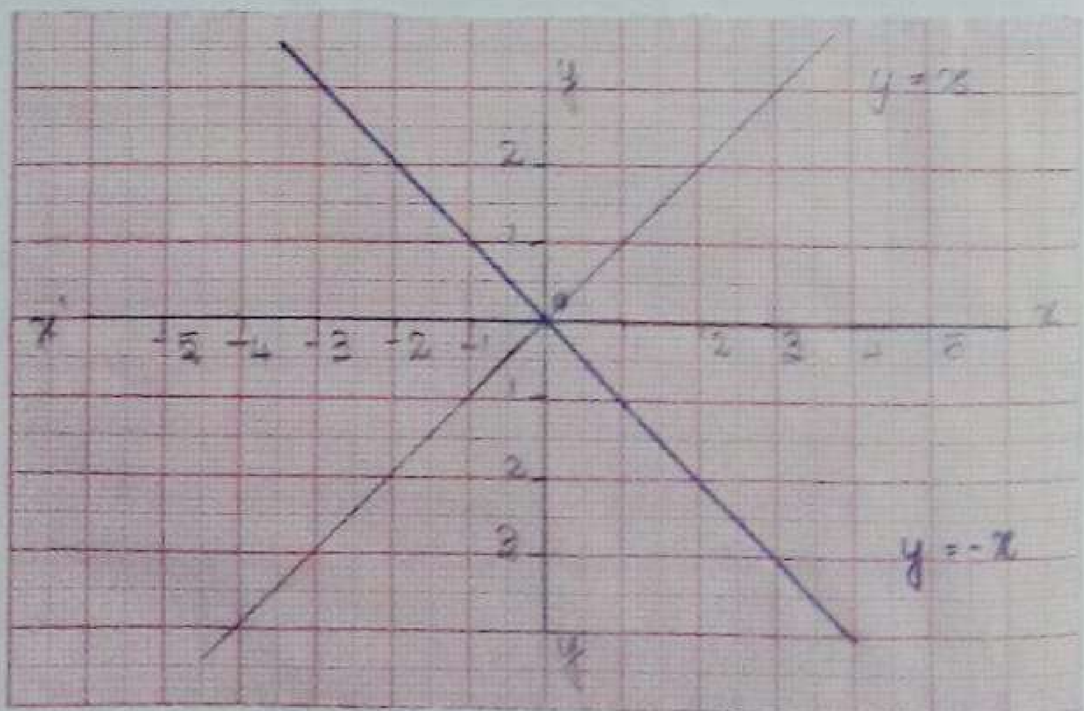
b) From the curve $y = x$, draw i) $y = -x$ ii) $y = 2x$

iii) $y = x + 1$ iv) $y = \frac{1}{2}x + 1$ v) $2x + y + 3 = 0$

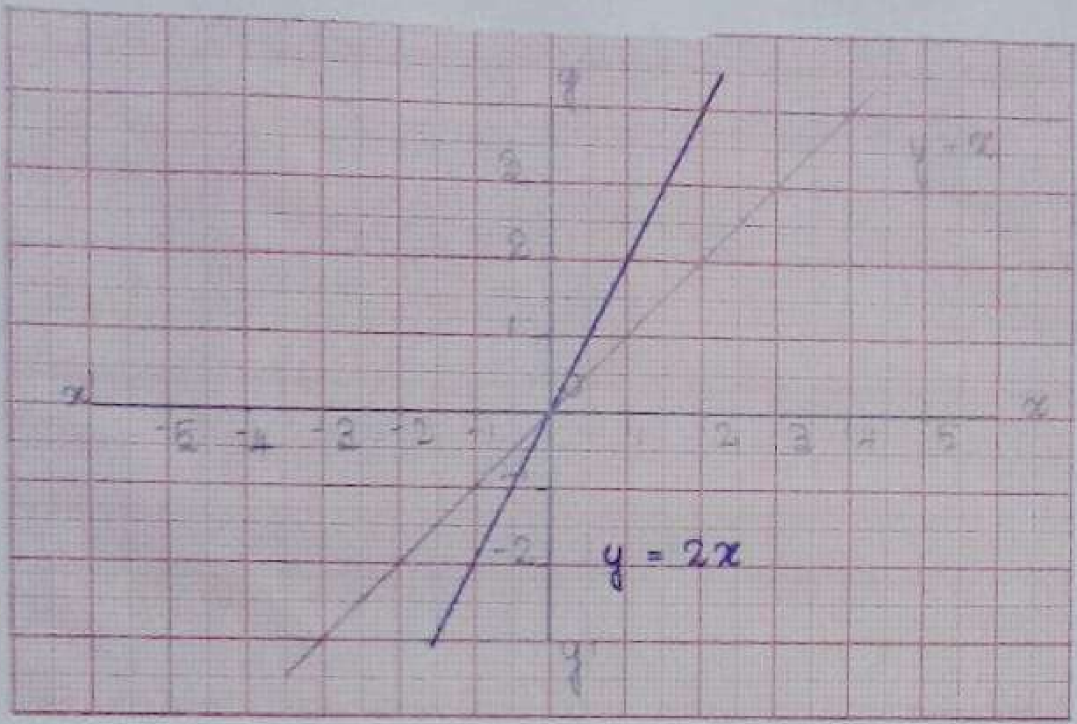
$y = x$



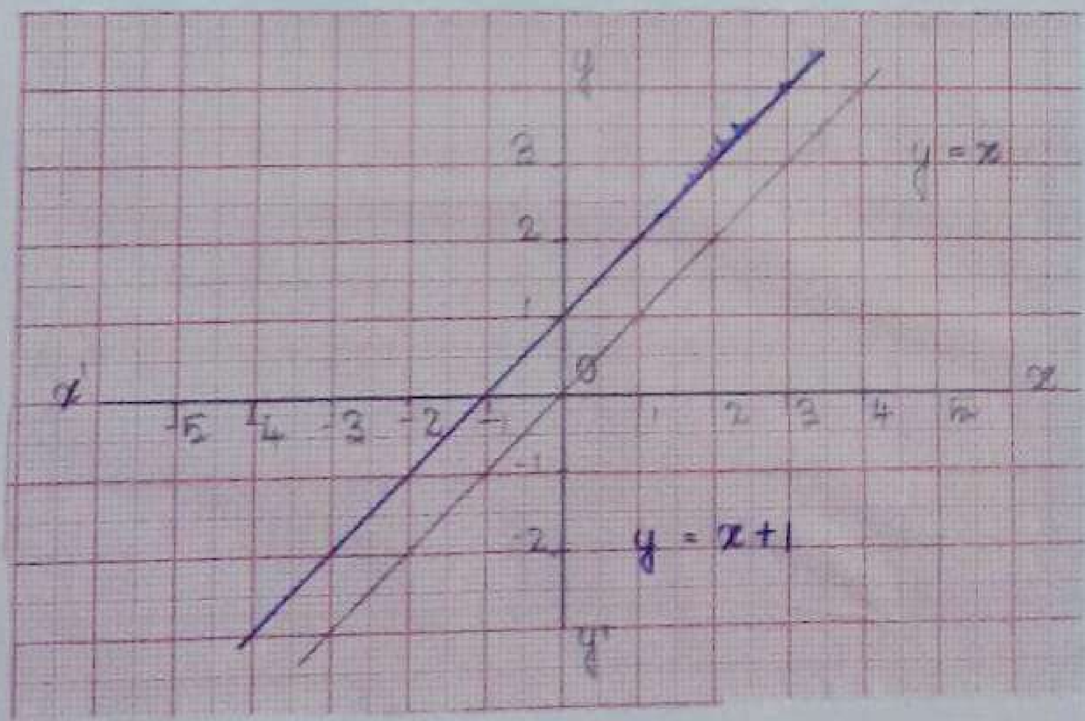
i) $y = -x$



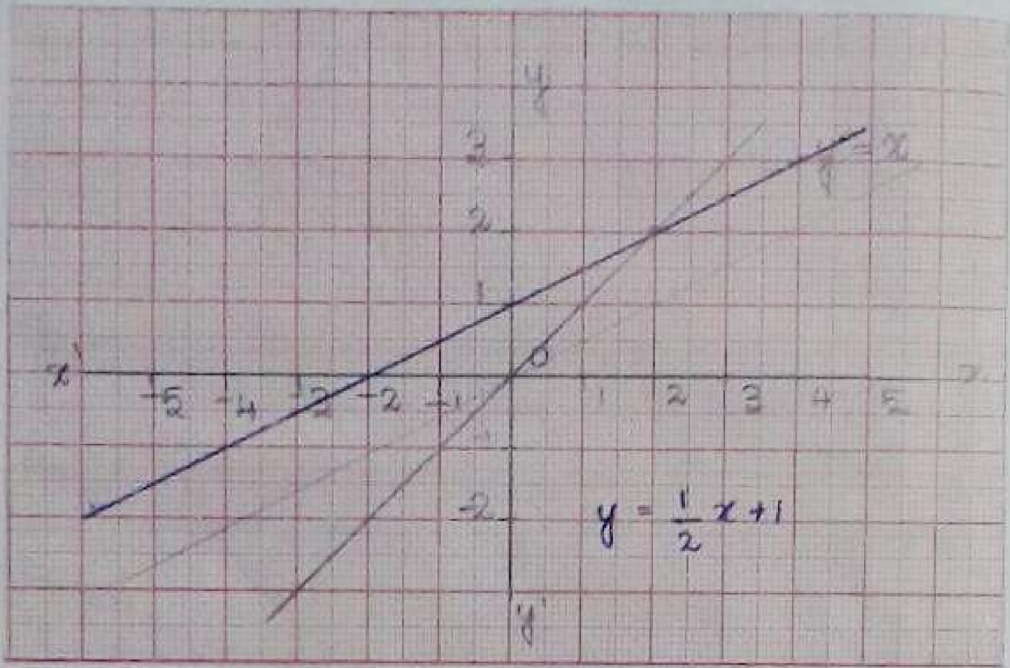
ii) $y = 2x$



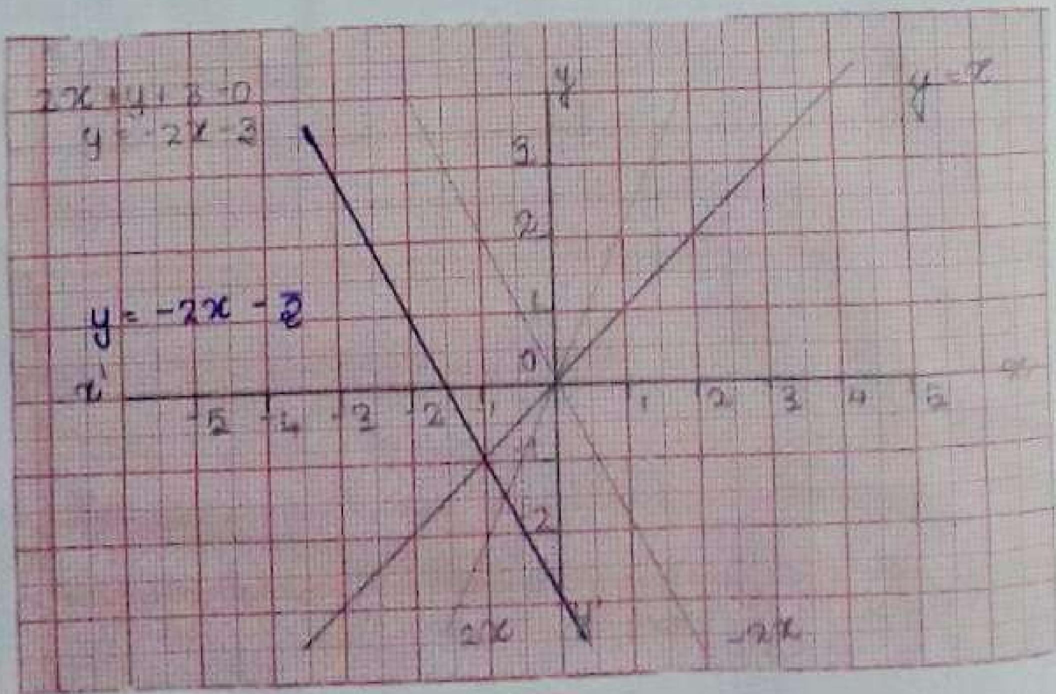
iii) $y = x + 1$



iv) $y = \frac{1}{2}x + 1$



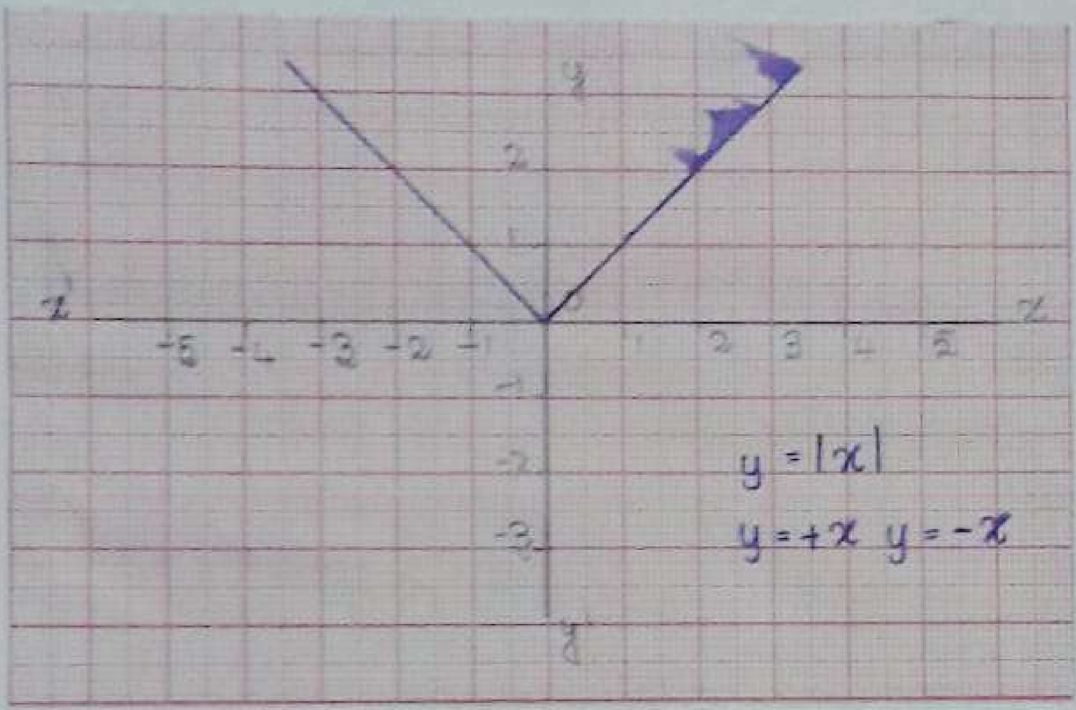
v) $2x + y + 3 = 0$



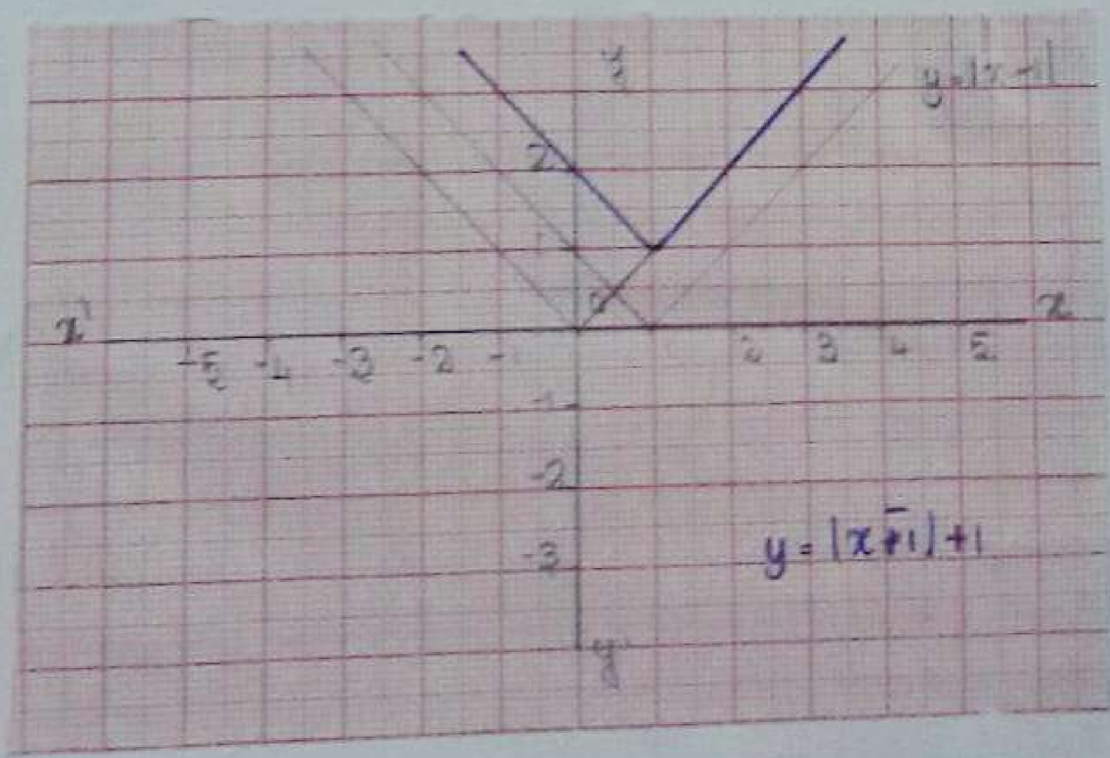
7) From the curve $y = |x|$, draw

i) $y = |x-1|+1$ ii) $y = |x+1|-1$ iii) $y = |x+2|-3$

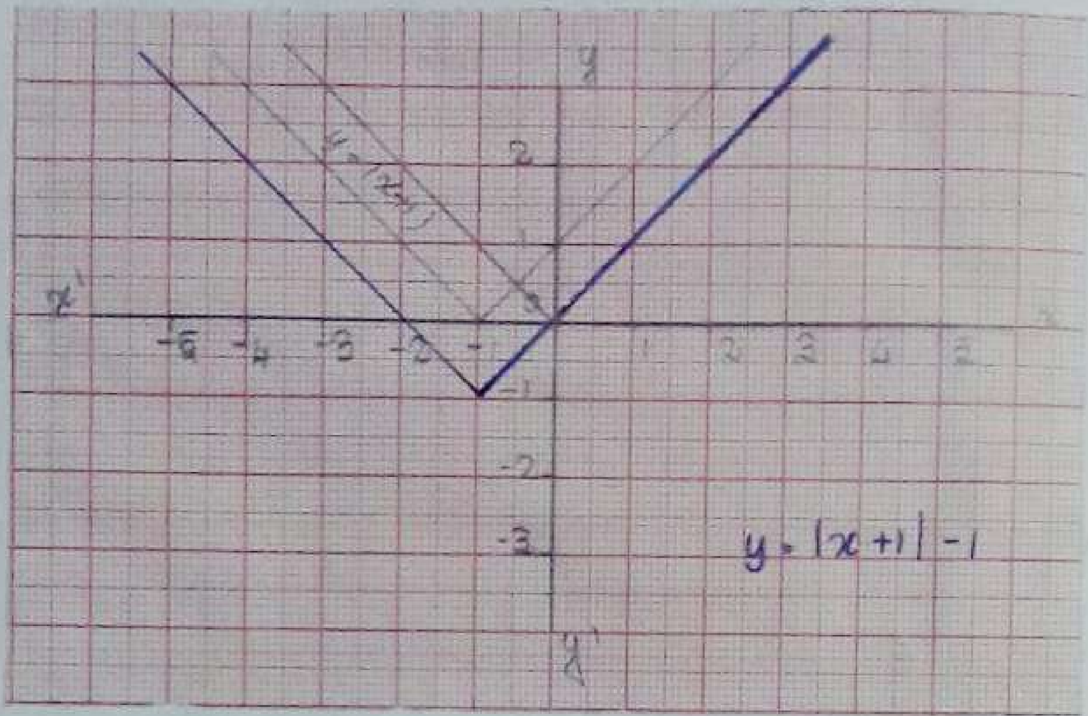
$y = |x|$



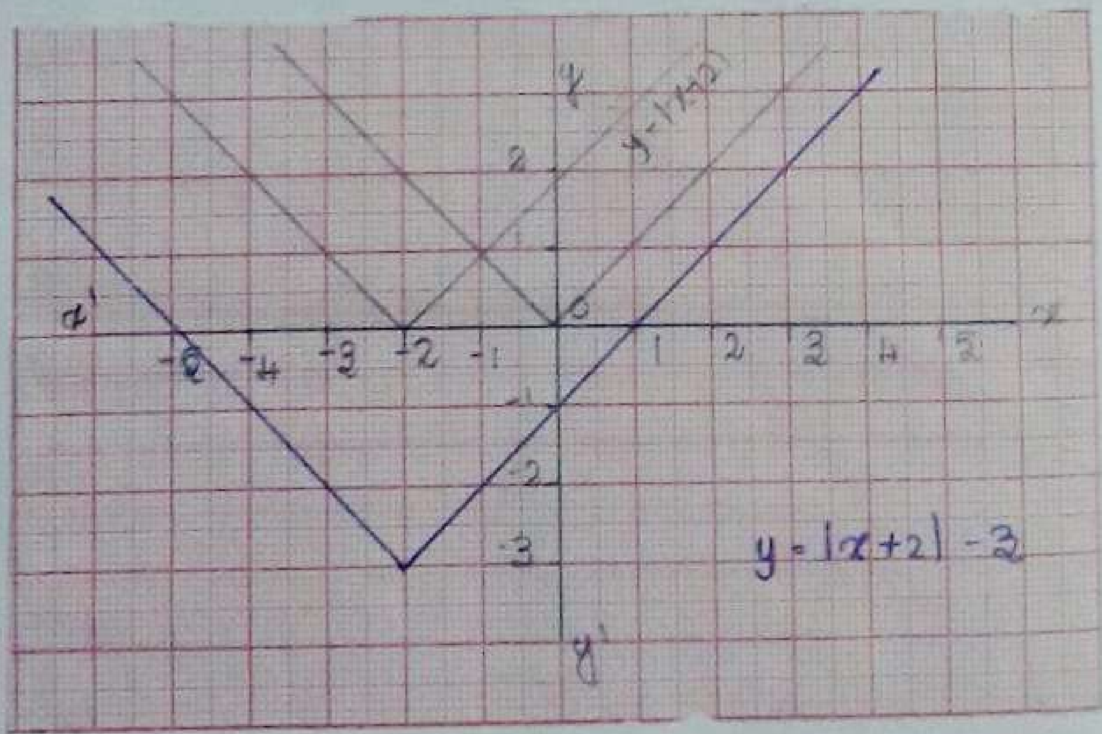
i) $y = |x-1|+1$



ii) $y = |x+1| - 1$

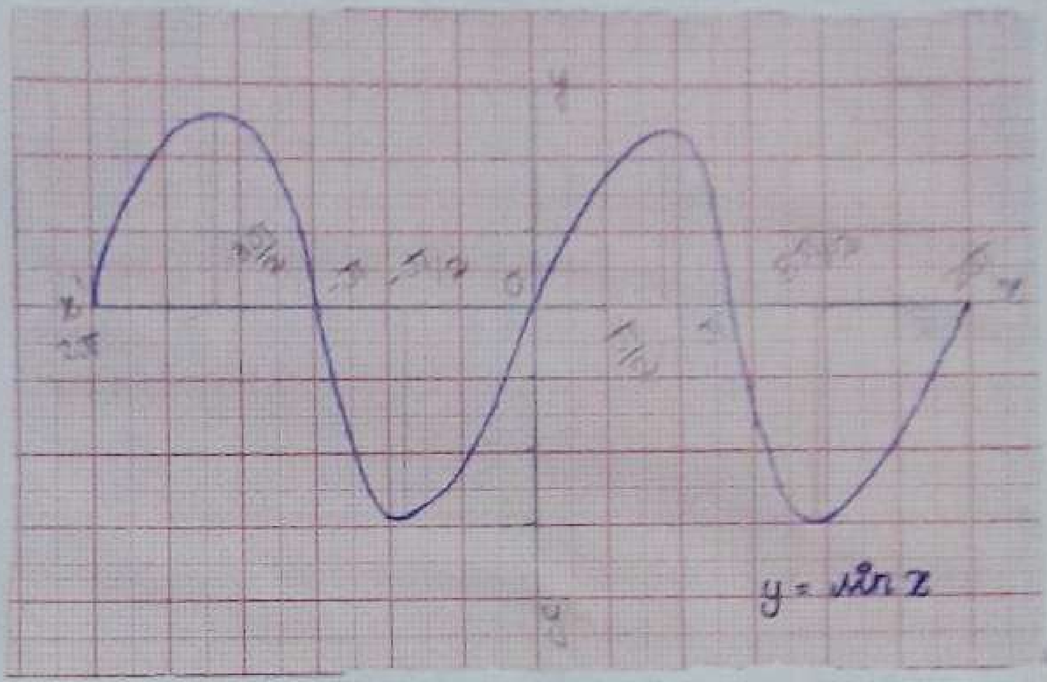


iii) $y = |x+2| - 3$



8) From the curve $y = \sin x$, draw $y = |\sin|x||$
(Hint : $\sin(-x) = -\sin x$)

$y = \sin x$



$y = \sin|x|$