2. COMPLEX NUMBER.

Introduction :

General Formula : $z = a + ib$

eg : $z = -2 + i\sqrt{3}$.

$x^2 - 4 = 0$

$x^2 = +4$

$x = \pm\sqrt{4}$

$x = \pm 2$

$x = +2, -2$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$= \pm\sqrt{4} \times \sqrt{-1}$ → Represent i

$\boxed{x = \pm 2i}$

$\boxed{x = 2i, -2i.}$

$\Rightarrow i = \sqrt{-1}$ $\Rightarrow \sqrt{-1} = i$

$\Rightarrow i^2 = -1$

$\Rightarrow i^3 = i^2 \times i^1 = (-1)i = -i$

$\Rightarrow i^4 = i^2 \cdot x\, i^2$

$= (-1)(-1) = 1$

$\boxed{\text{General Formula : } i^{4n} = 1}$ $n = 0, 1, 2, 3 \ldots$

Ex : $i^{4(0)} = i^0 = 1$

$i^{4(2)} = i^8 = 1$

$i^{4(8)} = i^4 \times i^4 = 1.$

NOTE :

$$\boxed{\Rightarrow i^4 + i^5 + i^6 + i^7 = 0}$$

$$\Rightarrow \cancel{1} + \cancel{i} - \cancel{1} - \cancel{i} = 0$$

$$\boxed{\Rightarrow i^{-101} + i^{-102} + i^{-103} + i^{-104} = 0}$$

$$\Rightarrow i + i + i + i = 0$$

$\therefore$ The continuous four terms $= 0$.

Example : 2.1

i) i^7

$\Rightarrow i^4 \times i^3$

$\Rightarrow -i$

ii) i^{1729}

$\Rightarrow i^{1728} \times i^1$

$\Rightarrow i$

iii) $i^{-1924} + i^{2018}$

$\Rightarrow 1 + (i^{2016} \times i^2)$

$\Rightarrow 1 - 1$

$\Rightarrow 0$ //

iv) $\sum_{n=1}^{102} i^n$

$\Rightarrow (i^1 + i^2 \ldots i^{100}) + i^{101} + i^{102}$

$\Rightarrow (i^{100} \times i) + (i^{100} \times i^2)$

$\Rightarrow i + (-1)$

$\Rightarrow -1 + i$ //

v) $i\, i^2 i^3 \ldots i^{40}$

$\Rightarrow i^{(1+2+3+\ldots 40)}$

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$\Rightarrow i^{\left(\frac{\cancel{40}^{20} \times 41}{\cancel{2}}\right)}$

$\Rightarrow i^{820}$

$\Rightarrow 1$ //

EXERCISE: 2.1

1) Simplify the following:-

1. $i^{1947} + i^{1950}$

$\Rightarrow i^{1944} \times i^3 + i^{1948} \times i^2$

$\Rightarrow 1 \times -i + 1 \times -1$

$\Rightarrow -i + -1$

$\Rightarrow -i - 1$

2. $i^{1948} - i^{-1869}$

$\Rightarrow i^{1948} - i^{-1868} \times i^{-1}$

$\Rightarrow 1 - (-i)$

$\Rightarrow 1 + i$

3. $\sum_{n=1}^{12} i^n$

$\Rightarrow (i^1 + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + (i^9 + i^{10} + i^{11} + i^{12})$

$\Rightarrow 0$

4. $i^{59} + \frac{1}{i^{59}}$

$\Rightarrow i^{59}$

$\Rightarrow i^{59} \times i^3 = -i$

$\Rightarrow$ |||ly $\frac{1}{i^{59}} = i$

$\Rightarrow i^{59} + \frac{1}{i^{59}} = -i + i$

$= 0$

5. $i\, i^2\, i^3 \ldots i^{2000}$

$\Rightarrow i^{(1+2+3\ldots+2000)}$

$\Rightarrow i^{\frac{\cancel{2000}^{1000} \times 2001}{\cancel{2}}}$

$\Rightarrow i^{2001000}$

$\Rightarrow 1$

6. $\sum_{n=1}^{10} i^{n+50}$

$\Rightarrow (i^{51} + i^{52} + \ldots + i^{58}) + i^{59} + i^{60}$

$\Rightarrow 0 + (i^{56} \times i^3) + i^{60} \Rightarrow -i + 1 \Rightarrow 1 - i$

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Complex Number:

$$z = a + ib$$

Ex: $-2 + i$, $5i$, $\sqrt{2} - i\sqrt{3}$

$Re(z) = a$

$Im(z) = b$

Add:

$z_1 = a + ib$

$z_2 = c + id$

$z_1 + z_2 = (a + c) + i(b + d)$

Eg: $z_1 = 1 + i$,

$z_2 = -2 + 3i$

$z_1 + Z_2 = -1 + 4i$

$\Rightarrow$

Subt:

$Z_1 - Z_2 = (a - c) + i(b - d)$

Multi:

$Z_1 Z_2 = (a + ib)(c + id)$

$= ac + iad + ibc + i^2 bd$

$= ac + iad + ibc - bd$

$= (ac - bd) + i\,(ad + bc)$

$z_1 z_2 = (2 - i)(1 + 2i)$

$= 2 + 4i - i - i^2 2$

$= 2 + 4i - i + 2$

$= 4 + 3i$

Conjugate ;

$z = a + ib$

$\bar{z} = a - ib$

$$\frac{1}{z} = \frac{1}{a+ib} \times \frac{a-ib}{a-ib}$$

$(a+b)$ $(a-b) = a^2 - b^2$

$$= \frac{a-ib}{a^2 - i^2 b^2}$$

$$= \frac{a-ib}{a^2+b^2} \Rightarrow \left(\frac{a}{a^2+b^2}\right) + i\left(\frac{-b}{a^2+b^2}\right)$$

NOTE : $(a-b)(a+b) = a^2 + b^2$

$(1+i)(1-i) = 1 + 1 = 2$

$(2-3i)(2+3i) = 4 + 9 = 13$

Ex : $(1-2i)(2+i)$

$\Rightarrow (6-3i)(6+3i)$

$\Rightarrow 36 + 64 = 100$

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EXERCISE : 2.2.

1. Evaluate the following if $z = 5-2i$ and $w = -1+3i$.

i) $z + w$

$= (5-2i) + (-1+3i)$

$= 4 + i.$

ii) $z - iw$

$\Rightarrow (5-2i) - i(-1+3i)$

$\Rightarrow 5-2i+i+3$

$\Rightarrow 8-i$

iii) $2z + 3w$

$\Rightarrow 2(5-2i) + 3(-1+3i)$

$\Rightarrow 10-4i+3-9i$

$\Rightarrow 13-13i$

$\Rightarrow 13(1-i)$

iv) zw

$\Rightarrow (5-2i)\cdot(-1+3i)$

$\Rightarrow -5+15i+2i+6$

$\Rightarrow 1+17i.$

v) $z^2 + 2\cancel{w}zw + w^2$

$z^2 = zz \Rightarrow (5-2i)(5-2i)$

$\Rightarrow 25 - 10i - 10i - 4$

$z^2 = 21 - 20i$

$w^2 = ww \Rightarrow (-1+3i)(-1+3i)$

$\Rightarrow (1 - 3i - 3i - 9)$

$w^2 \Rightarrow -8 - 6i$

$2zw = (5-2i)(-1+3i)$

$\Rightarrow -5 + 15i + 2i + 6$

$\Rightarrow 1 + 17i$

$\Rightarrow 2(1+17i)$

$\Rightarrow 2 + 34i$

$z^2 + 2zw + w^2$

$(21-20i) - (-8-6i) + (2+34i)$

$\Rightarrow 15 + 8i$

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vi) $(z+w)^2$

$\Rightarrow ((5-2i) + (-1+3i))^2$

$\Rightarrow (4+i)^2$

$\Rightarrow (4+i)(4+i)$

3. Find the values of the real numbers x and y, if the complex numbers

$$(3-i)x-(2-i)y+2i+5 \text{ and } 2x+(-1+2i)y+3+2i$$

are equal.

$$3x-ix-2y+iy+2i+5 = 2x-y+2iy+3+2i$$

$$(3x-2y+5)+i(-x+y+2) = (2x-y+3)+i(2y+2)$$

Equ Real and Imaginary

Re $\Rightarrow$ $3x-2y+5 = 2x-y+3$

$$x-y = -2 \longrightarrow (1)$$

Im $\Rightarrow$ $-x+y+\cancel{2} = 2y+\cancel{2}$

$$-x-y = 0 \longrightarrow (2)$$

(1)+(2) $-\cancel{2}y = -\cancel{2} \Rightarrow \boxed{y=1}$

Sub $y=1$ in (2)

$$-x-1 = 0$$

$$\boxed{x=-1}$$

$$\boxed{\text{Sol: } x=-1,\ y=1.}$$