Class : 12

Register Number

FIRST MID TERM TEST - 2022 - 23

MATHEMATICS

Time Allowed : 1.30 Hours] [Max. Marks : 45

Part - I

1. Answer all the questions by choosing the correct answer from the given 4 alternatives. 10x1=10
2. Write question number, correct option and corresponding answer.
3. Each question carries 1 mark.

1. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

(1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

2. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj\,(AB)| =$

(1) -40 (2) -80 (3) -60 (4) -20

3. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, be such that $\lambda A^{-1} = A$, then λ is

(1) 17 (2) 14 (3) 19 (4) 21

4. If $\rho(A) \neq \rho([A \mid B])$, then the system $AX = B$ of linear equations is

(1) Consistent and has a unique solution (2) Consistent

(3) consistent and has infinitely many solution (4) Inconsistent

5. If $\frac{z-1}{z+1}$, is purely imaginary, then $|z|$ is

(1) ½ (2) 1 (3) 2 (4) 3

6. If $\left| z - \frac{3\pi}{z} \right| = 2$, then the least value of $|z|$ is

(1) 1 (2) 2 (3) 3 (4) 5

7. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(1) -2 (2) -1 (3) 1 (4) 2

8. If α, β, and γ, are the zeros $x^3 + px^2 + qx + r$, then $\Sigma \frac{1}{\alpha}$ is

(1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

9. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

1) 2 2) 4 3) 1 4) ∞

10. If z lies in IV quadrant the -z lies in

(1) I quadrant (2) II quadrant 3) III quadrant (4) IV quadrant

PART – II

1. Answer any 4 questions. 2. Each question carries 2 marks. 4X2=8

3. Question number 16 is compulsory.

11. Find z^{-1}, if $z = (2 + 3i)(1 - i)$.

12. If $\omega \neq 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$.

13. Find the rank of the matrix $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

14. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of the two roots is 1.

15. Solve the equation : $x^4 - 14x^2 + 45 = 0$.

16. Find the condition on a, b, and c so that the following system of linear equations has one parameter family of solutions. $x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = c$.

PART - III

1. **Answer any four questions. 2. Each question carries 3 marks.** **4X3=12**
3. **Question number 22 is compulsory.**

17. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ verify that $A(\text{adj }A) = (\text{adj }A)A = |A|I_2$.

18. Test for consistency and if possible, solve the following systems of equations by rank method.

$2x - y + z = 2$, $6x - 3y + 3z = 6$, $4x - 2y + 2z = 4$.

19. Find the square root of $6-8i$.

20. Solve the equation $7x^3 - 43x^2 = 43x - 7$.

21. Find a polynomial equation of minimum degree with rational coefficients having $2i + 3$ as a root.

22. Show that $|3z - 5 + i| = 4$ represents a cricle, and , find its centre and radius.

PART - IV

1. **Answer All the questions. 2. Each question carries 5 marks.** **3X5= 15**

23. (a) Solve : $6x^4 - 35x^3 + 62x^2 + 35x + 6 = 0$

(Or)

(b) Solve the equation $(x-2)(x-7)(x-3)(x+2) + 19 = 0$.

24. (a) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that (i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

(Or)

(b) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$.

25. (a) By using Gaussian elimination method, balance the chemical reaction equation : $C_2H_6+O_2 \rightarrow CO_2+H_2O$.

(Or)

(b) Solve the systems of linear equations by Cramer's rule : $3x+3y-z=11$, $2x-y+2z=9$, $4x+3y+2z=25$.

12th Maths (I Mid Term -2022)

Answer Key

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PART-I

1	(1) A^{-1}	6	(1) 1
2	(2) -80	7	(2) -1
3	(3) 19	8	(1) $-\frac{q}{r}$
4	(4) Inconsistent	9	(3) 1
5	(2) 1	10	(3) III quadrant

PART-II

11. Sol:

$$z = 5+i$$

$$z^{-1} = \frac{1}{z}$$

$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{5-i}{25+1}$$

$$= \frac{5}{26} - i\frac{1}{26}$$

12. **Sol:**

$$1+\omega+\omega^2 = 0$$
$$1+\omega = -\omega^2$$
$$\omega+\omega^2 = -1$$
$$1+\omega^2 = -\omega.$$

$$\therefore \omega^3 = 1$$

LHS:

$$= \frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$$

$$= \frac{\omega(a+b\omega+c\omega^2)}{\omega(b+c\omega+a\omega^2)} + \frac{\omega^2(a+b\omega+c\omega^2)}{\omega^2(c+a\omega+b\omega^2)}$$

$$= \frac{a\omega+b\omega^2+c}{b\omega+c\omega^2+a} + \frac{a\omega^2+b\omega^3+c\omega}{c\omega^2+a+b\omega}$$

$$= \frac{a\omega+b\omega^2+c+a\omega^2+b+c\omega}{a+b\omega+c\omega^2}$$

$$= \frac{a(\omega+\omega^2)+b(\omega^2+1)+c(1+\omega)}{a+b\omega+c\omega^2}$$

$$= \frac{-a-b\omega-c\omega^2}{a+b\omega+c\omega^2}$$

$$= \frac{-\cancel{[a+b\omega+c\omega^2]}}{\cancel{[a+b\omega+c\omega^2]}}$$

$$= -1$$

$= RHS$

$\therefore$ Hence proved.

15. Sol:

$$x^4 - 14x^2 + 45 = 0$$

Let $y = x^2$

$$y^2 - 14y + 45 = 0$$

$$(y-9)(y-5) = 0$$

$y = 9$ (or) $y = 5$

$x^2 = 9$ $x^2 = 5$

$x = \sqrt{9}$ $x = \sqrt{5}$

$\boxed{x = \pm 3}$ $\boxed{x = \pm\sqrt{5}}$

16. Sol:

$$x + y + z = a$$

$$x + 2y + 3z = b$$

$$3x + 5y + 7z = c.$$

$$[A;B] = \begin{bmatrix} 1 & 1 & 1 & a \\ 1 & 2 & 3 & b \\ 3 & 5 & 7 & c \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-a \\ 0 & 2 & 4 & c-3a \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-a \\ 0 & 0 & 0 & c-a-2b \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\rho(A) = 2 = \rho(A, B)$$

$\therefore$ $c - a - 2b = 0$

$$c = a + 2b.$$

13. Sol:

$$A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} \quad R_3 \leftrightarrow R_1$$

$$\begin{array}{cccc} 3 & -8 & 5 & 2 \\ -3 & 6 & 9 & -6 \\ \hline 0 & -2 & 14 & -4 \end{array} \qquad \begin{array}{cccc} 2 & -5 & 1 & 4 \\ -2 & 4 & 6 & -4 \\ \hline 0 & -1 & 7 & 0 \end{array}$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + 3R_1 \end{array}$$

$$\begin{array}{cccc} 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & 0 \\ + & - & - & \\ \hline 0 & 0 & 0 & -4 \end{array}$$

$$\sim \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$\therefore$ No. of non-zero rows are 3.

$\rho(A) = 3$

14. Sol:

$3x^3 - 16x^2 + 23x - 6 = 0$

$\alpha\beta = 1$ (Given) $\Rightarrow \beta = \frac{1}{\alpha}$

$\alpha+\beta+\gamma = \frac{-b}{a}$; $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$; $\alpha\beta\gamma = \frac{-d}{a}$

$= \frac{+16}{3}$ $\qquad = \frac{23}{3}$ $\qquad = \frac{6}{3}$

$= 2.$

$\boxed{\gamma = 2}$

$\alpha + \frac{1}{\alpha} = \frac{16}{3} - 2$

$\frac{\alpha^2+1}{\alpha} = \frac{10}{3}$

$3\alpha^2 - 10\alpha + 3 = 0$

$\therefore \alpha = 1/3, 3$

$\therefore$ the roots are $3, 1/3, 2$

15. **Sol:**

$$x^4 - 14x^2 + 45 = 0$$

Let $y = x^2$

$$y^2 - 14y + 45 = 0$$

$$(y-9)(y-5) = 0$$

$y = 9$ (or) $y = 5$

$x^2 = 9$ $x^2 = 5$

$x = \sqrt{9}$ $x = \sqrt{5}$

$\boxed{x = \pm 3}$ $\boxed{x = \pm\sqrt{5}}$

16. **Sol:**

$$x + y + z = a$$

$$x + 2y + 3z = b$$

$$3x + 5y + 7z = c.$$

$$[A;B] = \begin{bmatrix} 1 & 1 & 1 & a \\ 1 & 2 & 3 & b \\ 3 & 5 & 7 & c \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-a \\ 0 & 2 & 4 & c-3a \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-a \\ 0 & 0 & 0 & c-a-2b \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\rho(A) = 2 = \rho(A, B)$$

$$\therefore c - a - 2b = 0$$

$$c = a + 2b.$$

PART-III

17. **Sol:**

$$A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$$

$$|A| = 24 - 20$$
$$= 4$$

$$A(\text{adj}A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$(\text{adj}A)A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$|A| I_2 = 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$\therefore A(\text{adj}A) = (\text{adj}A)A = I_2$

$\therefore$ Hence proved.

18. **Sol:**

$$2x - y + z = 2$$
$$6x - 3y + 3z = 6$$
$$4x - 2y + 2z = 4$$

$$\begin{bmatrix} 2 & -1 & 1 \\ 6 & -3 & 3 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

$$[A, B] = \begin{bmatrix} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

The above matrix is in echelon form

$\rho(A, B) = \rho(A) = 1 <$ no. of unknowns)

$\therefore$ System of equations is consistent and has infinitely many solution.

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$2x - y + z = 2.$$

Let

$z = t, \quad y = s \quad (t, s \in R).$

$$2x - s + t = 2$$

$$2x = 2 + s - t$$

$$x = \frac{1}{2}(2 + s - t)$$

$\therefore$ the solution is

$$x = \frac{1}{2}(2 + s - t)$$

$$y = s$$

$$z = t \qquad s, t \in R$$

19. **Sol:**

$$|6-8i| = \sqrt{36+64}$$

$$= \sqrt{100}$$

$$= 10$$

Applying sq. root formula,

$$\sqrt{6-8i} = \pm\left[\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right]$$

$$= \pm\left[\sqrt{\cancel{\frac{16}{2}}\,8} - i\sqrt{\cancel{\frac{4}{2}}\,2}\right]$$

$$= \pm\,(2\sqrt{2} - i\sqrt{2})$$

20. **Sol:**

$$7x^3 - 43x^2 = 43x - 7$$

$$7x^3 - 43x^2 - 43x + 7 = 0$$

$$\begin{array}{r|rrrr}
-1 & 7 & -43 & -43 & 7 \\
 & 0 & -7 & 50 & -7 \\
\hline
7 & 7 & -50 & +7 & \boxed{0} \\
 & 0 & 49 & -7 & \\
\hline
 & 7 & -1 & \boxed{0} &
\end{array}$$

49

$-\frac{49}{7}$, $-\frac{1}{7}$

$7x^2 - 50x + 7 = 0$

$(x-7)(x-1/7) = 0$

$x = 7,\ x = 1/7$

$\therefore$ the roots are $-1, 7, \frac{1}{7}$

21. **Sol:**

$2i+3$, another root $3-2i$

$\therefore\ x^2 - (S.R)x + P.R = 0$

$\therefore\ x^2 - 6x + 13 = 0$

22. Sol:

$$|3z-5+i| = 4.$$

$$3\left|z - \frac{5-i}{3}\right| = 4$$

$$\left|z - \frac{5-i}{3}\right| = \frac{4}{3}$$

$$|z - z_0| = r$$

$\therefore$ centre $= \left(\frac{5}{3}, \frac{-1}{3}\right)$

radius $= \frac{4}{3}$

PART-IV

23(a) Sol:

$$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$$

3	6	-35	62	-35	+6
	0	18	-51	33	-6
2	6	-17	11	-2	0
	0	12	-10	2	
	6	-5	1	0	

6

$-3 \quad -2$

$\frac{-3}{6} \quad \frac{-2}{6}$

2

$$6x^2 - 5x + 1 = 0$$

$$(x - 1/2)(x - 1/3) = 0$$

$$x = 1/2, 1/3.$$

$\therefore$ The roots are $2, 1/2, 3, 1/3$.

23(b). **Sol:**

$$(x-2)\ (x-7)\ (x-3)\ (x+2) = 0$$

$$(x-2)\ (x-3)\ (x-7)\ (x+2) = 0$$

$$(x^2-5x+6)\ (x^2-5x-14) = 0$$

put

$$x^2-5x = y$$

$$(y+6)\ (y-14) = 0$$

$$y=-6 \quad (\text{or}) \quad y = 14$$

$x^2-5x+6=0$	$x^2-5x-14=0$
$x = 3, 2$	$x = 7, -2.$
$6 \to -3, -2$	$14 \to -7, 2$

24(a) **Sol:**

$z = x+iy$, if $Im\left(\frac{2z+1}{iz+1}\right)$

S.T. locus of z is $2x^2+2y^2+x-2y=0$

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$$

$$\Rightarrow \frac{2x+i2y+1}{ix-y+1} = \frac{(2x+1)+i2y}{(1-y)+ix.}$$

$$\because Im\left(\frac{a+ib}{c+id}\right) = i\left(\frac{bc-ad}{c^2+d^2}\right)$$

$$a=2x+1,\ b=2y,\ c=1-y,\ d=x$$

$$Im\left(\frac{2z+1}{iz+1}\right) = i\left(\frac{2y(1-y)-(2x+1)x}{(1-y)^2+x^2}\right)$$

$$0 = \frac{2y-2y^2-2x^2-x}{(1-y)^2+x^2}$$

$$-2x^2-2y^2-x+2y=0$$

$$\therefore\ 2x^2+2y^2+x-2y=0$$

$\therefore$ Hence proved.

25(a) Sol:

$$x_1 C_5H_8 + x_2 O_2 \longrightarrow x_3 CO_2 + x_4 H_2O$$

Carbon:

$$5x_1 - x_3 = 0$$

Hydrogen:

$$8x_1 - 2x_4 = 0 \Rightarrow 3x_1 - x_4 = 0$$

Oxygen:

$$2x_2 - 2x_3 - x_4 = 0.$$

$$(A, B) = \left[\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \leftrightarrow R_2]{} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \to 4R_3 - 5R_1]{} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array}\right]$$

$\therefore\ \rho(A) = \rho(A, B) = 3 < 4 =$ No. of unknowns.

$\therefore$ System is consistent and has infinite no. of solutions

$4x_1 - x_4 = 0$

$2x_2 - 2x_3 - x_4 = 0$

$-4x_3 + 5x_4 = 0$

Let $x_4 = t$

$x_3 = \frac{5t}{4}$, $x_2 = \frac{7t}{4}$, $x_1 = \frac{t}{4}$

$t = 4$, then

$x_1 = 1$; $x_2 = 7$; $x_3 = 5$; $x_4 = 4$

$\therefore$ the Balanced equation is

$C_3H_8 + 7O_2 \longrightarrow 5CO_2 + 4H_2O$.

25(b) Sol:

$3x + 3y - z = 11$

$2x - y + 2z = 9$

$4x + 3y + 2z = 25$

$$\begin{bmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix} = -24 + 12 - 10 = -22$$

$$\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix} = -88 + 96 - 52 = -44$$

$$\Delta_2 = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix} = -96+44-14 = -66$$

$$\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix} = -156-42+110 = -88$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-44}{-22} = 2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-66}{-22} = 3$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-88}{-22} = 4$$

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