Sri Ramakrishna Garden H. Sr. Sec. School

Time: 3 hrs. XII - Mathematics Marks: 90

Part - A

20 × 1 = 20

I. Answer All the questions:

1. If $|adj(adj A)| = |A|^9$, then the order of the square matrix A is
   a) 3 b) 4 c) 2 d) 5

2. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, then $9I_2 - A =$
   a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$.

3. If $A^T A^{-1}$ is symmetric, then $A^2 =$
   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$.

4. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adj A) = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$, then $K =$
   a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1.

5. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is
   a) 1 b) 2 c) 3 d) 4.

6. If $P(A) = P(A|B)$, then the system $AX = B$ of linear equations is
   a) Consistent and has a unique solution b) consistent
   c) consistent and has intinitely many solution d) inconsistent

7. If A is a non-singular square matrise of order n, then $|adj A| =$
   a) $|A|^{n+1}$ b) $|A|^{n-1}$ c) $|A|^n$ d) $|A|$.

8. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
   a) 0 b) 1 c) −1 d) i.

9. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
   a) $\frac{1}{2}$ b) 1 c) 2 d) 3.

10. If $|z - \frac{3}{z}| = 2$, then the least value of $|z|$ is
    a) 1 b) 2 c) 3 d) 5.

11. The solution of the equation $|z| - z = 1 + 2i$ is
    a) $\frac{3}{2} - 2i$ b) $-\frac{3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$.

12. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
    a) $\frac{1}{2}$ b) 1 c) 2 d) 3

13. The Principal argument of $\frac{3}{-1+i}$ is

a) $\frac{-5\pi}{6}$ b) $\frac{-2\pi}{3}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{2}$.

14. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

a) -2 b) -1 c) 1 d) 2.

15. The value of $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{19}$ is

a) 1 b) -1 c) 0 d) i

16. A zero of x^3+1 is

a) 0 b) $+1$ c) $+i$ d) -1.

17. A Polynomial equation in x of degree n always has

a) n distinct roots b) n real roots c) n complex roots d) atmost n roots.

18. If α, β and γ are the zeros of x^3+px^2+qx+r, then $\sum\frac{1}{\alpha}$ is

a) $-\frac{q}{r}$ b) $-\frac{p}{q}$ c) $\frac{q}{r}$ d) $-\frac{q}{p}$.

19. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

a) 2 b) 4 c) 1 d) ∞.

20. The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r (-1)^r x^r$ is

a) 0 b) n c) $<n$ d) r.

Part-B.

$7\times 2 = 14$

II Answer Any 7 Questions :- (Q.no 30 is compulsory).

21. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

22. If $\text{adj}(A) = \begin{bmatrix}0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

23. Find the rank of the matrix $\begin{bmatrix}3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2\end{bmatrix}$.

24. Solve. $5x+2y=3$; $3x+2y=5$, using matrix inversion method.

25. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

26. If $|z|=3$, show that $7 \le |z+6-8i| \le 13$.

27. Simplify. $\sum_{n=1}^{16} i^{n+50}$.

28. Find z^{-1}, if $z = (2+3i)(1-i)$.

29. Find the a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ as a root.

30. If α and β are the roots of the quadratic equation $17x^2+13x-73=0$, construct a quadratic equation whose roots are $\alpha+1$ and $\beta+1$.

Part-C

$7 \times 3 = 21$

III Answer Any 7 Questions:- (Q.No 40 is compulsory).

31. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

32. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

33. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$.

34. Find the values of the real numbers x and y, if the Complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal.

35. Find the square roots of $-6+8i$.

36. Solve the equation $x^3+27=0$.

37. Solve the equation $x^3-9x^2+14x+24=0$ if it is given that two of its roots are in the ratio 3:2.

38. Solve the equation $x^4-14x^2+45=0$.

39. Show that the polynomial $9x^9+2x^5-x^4-7x^2+2$ has atleast Six imaginary roots.

40. Prove that the complex numbers $3+3i$, $-3-3i$, $-3\sqrt{3}+3\sqrt{3}i$ are the vertices of an equilateral triangle in the Complex plane.

Part – D

7×5 = 35

IV Answer All the Questions:-

11. a) Solve, $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$; $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$; $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$. by cramer's rule. (OR)

b) Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3 \neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$.

12. a) Solve, $2x+3y-z=9$, $x+y+z=9$, $3x-y-z=-1$ by matrix inversion method. (OR)

b) Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

13. a) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2=1$. (OR)

b) Solve. $(x-5)(x-7)(x+6)(x+4)=504$.

14. a) If $z=x+iy$ is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$. (OR)

b) A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. will he meet his friend? (use Gaussian elimination method).

15. a) If $w \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2w, 1-2w^2$. (OR)

b) Find the value of k for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have

i) No solution ii) Unique solution iii) infinitely many solution.

16. a) If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$ S.T $|9z_1z_2+4z_1z_3+z_2z_3|=6$.

(OR)

b) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

17. a) Solve the equations $6x^4-25x^3+62x^2-25x+6=0$ (OR)

b) If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, S.T $x^2+y^2+3x-3y+2=0$.

——— All the Best ———