SPB DATE :

12^{th} MATHS (Answer key) THIRUVALLUR DISTRICT

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I

1)	(a)	$(A^T)^2$	
2	(d)	1	
3.	(a)	$\frac{-1}{i+2}$	
4.	(d)	1	
5.	(d)	0	
6.	(d)	-4	
7.	(b)	$\frac{-q}{r}$	
8.	(b)	always consistent	(b) எப்பொழுதும் ஒருங்கமைவு உடையது
9.	(b)	3	
10.	(b)	I, II, IV	

II

11. A is a non-singular matrix of odd order

$|A| \neq 0$, $2m+1$ where $m = 0, 1, 2 \ldots$

A என்பது ஒற்றை வரிசையுடைய பூச்சியமற்றக் கோவை அணி ; $|A| \neq 0$; $2m+1$, $m = 0, 1, 2$.

தேற்றத்தின் படி $|adj A| = |A|^{2m+1-1}$

$= |A|^{2m}$

$|A|^{2m}$ என்றும் என்பது எப்பொழுதும் மிகை.

எனவே $|adj A|$ அதுவும் மிகை.

$\because$ $|A|^{2m}$ is always positive

$|adj A|$ is positive.

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12. $A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$= \frac{1}{2}\begin{bmatrix} -3 & -4 \\ -1 & -2 \end{bmatrix}$

13. $z = 5-2i, \quad w = -1+3i$

$(z+w)^2 = (4+i)^2 = 16+i^2+8i = 15+8i$

14. $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\ldots(x_n+iy_n) = a+ib$

இருபுறமும் மட்டு எடுக்க

Taking Modulus on b/s.

$|x_1+iy_1|\,|x_2+iy_2|\,|x_3+iy_3|\ldots|x_n+iy_n| = |a+ib|$

$(\sqrt{x_1^2+y_1^2})(\sqrt{x_2^2+y_2^2})\ldots(\sqrt{x_n^2+y_n^2}) = \sqrt{a^2+b^2}$

squaring on b/s

இருபுறமும் வர்க்கம் காண்க

$(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\ldots(x_n^2+y_n^2) = a^2+b^2$

எனவே நிரூபிக்கப்பட்டது.

Hence proved

15. $(2-\sqrt{3}i)$ ஒரு காரணி எனில் $(2+\sqrt{3}i)$ மற்றொரு காரணியாகும்.

$(2-\sqrt{3}i)$ is one root, another root is $2+\sqrt{3}i$

$x^2-(S.R)x+P.R = 0$ $\qquad S.R = 2+\cancel{\sqrt{3}i}+2-\cancel{\sqrt{3}i}$

$x^2-(\text{மூ.கூ})x+\text{மூ.பெ} = 0$ $\qquad = 4$

$x^2-4x+7 = 0.$ $\qquad P.R = 4-(i\sqrt{3})^2$

$= 4-3i^2$

$= 4+3$

$= 7$

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16. $i \cdot i^2 \cdot i^3 \cdots i^{40} = i^{\frac{40(40+1)}{2}}$
$= i^{20\times 41}$
$= i^{820}$
$= i^{4(205)}$
$= 1.$

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III

17. $A = \begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

$\xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix}$

$\begin{array}{cccc} 3 & -8 & 5 & 2 \\ -3 & 6 & 9 & -6 \\ \hline 0 & -2 & 14 & -4 \end{array}$

$\xrightarrow[R_3 \to R_3 + 3R_1]{R_2 \to R_2 + 2R_1} \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{bmatrix}$

$\begin{array}{cccc} 2 & -5 & 1 & 4 \\ -2 & 4 & 6 & -4 \\ \hline 0 & -1 & 7 & 0 \end{array}$

$\xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} -1 & 2 & 3 & -2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$

$\begin{array}{cccc} 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & 0 \\ \hline 0 & 0 & 0 & -4 \end{array}$

$\therefore \rho(A) = 3.$

18. $|\text{adj} A| = -36 + 72 = 36$
$\sqrt{|\text{adj} A|} = \sqrt{36} = \pm 6$

$A^{-1} = \pm \frac{1}{\sqrt{|\text{adj} A|}} (\text{adj} A)$
$= \pm \frac{1}{6} \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$

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19. A என்பது பூச்சியக் கோவை அணி

A is ~~non~~ singular matrix

$|A| = 0$

$$A = \begin{pmatrix} 4 & -3 & -1 \\ 2 & 4 & a \\ 3 & -5 & -4 \end{pmatrix}$$

$$\begin{vmatrix} 4 & -3 & -1 \\ 2 & 4 & a \\ 3 & -5 & -4 \end{vmatrix} = 0$$

$(-64 - 9a + 10) - (-12 - 20a + 24) = 0$

$-54 - 9a - 12 + 20a = 0$

$11a = 66$

$a = \frac{66}{11}$

$\boxed{a = 6}$

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20. $\overset{z_1}{\sqrt{3} + i}, \quad \overset{z_2}{\sqrt{2} + i\sqrt{2}}, \quad \overset{z_3}{1 + i\sqrt{3}}$

Question

$|z_1 - z_2| = |\sqrt{3} + i - \sqrt{2} - i\sqrt{2}|$

$= |(\sqrt{3} - \sqrt{2}) + i(1 - \sqrt{2})|$

$= \sqrt{(\sqrt{3} - \sqrt{2})^2 + (1 - \sqrt{2})^2}$

$= \sqrt{3 + 2 - 2\sqrt{6} + 1 + 2 - 2\sqrt{2}}$

$= \sqrt{8 - 2\sqrt{6} - 2\sqrt{2}}$

$|z_2 - z_3| = |\sqrt{2} + i\sqrt{2} - 1 - i\sqrt{3}|$

$= |(\sqrt{2} - 1) + i(\sqrt{2} - \sqrt{3})|$

$= \sqrt{(\sqrt{2} - 1)^2 + (\sqrt{2} - \sqrt{3})^2}$

$= \sqrt{2 + 1 - 2\sqrt{2} + 2 + 3 - 2\sqrt{6}}$

$= \sqrt{8 - 2\sqrt{2} - 2\sqrt{6}}$

$|z_3 - z_1| = |(1 + i\sqrt{3} - \sqrt{3} - i)|$

$= |(1 - \sqrt{3}) + i(\sqrt{3} - 1)| = \sqrt{(1 - \sqrt{3})^2 + (\sqrt{3} - 1)^2}$

$= \sqrt{1 + 3 - 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}$

$= \sqrt{8 - 4\sqrt{3}}$

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21. $p(x) = 9x^9 + 2x^5 - x^4 - 7x^2 + 2$

$+ \quad + \quad - \quad - \quad +$

1 2

$P(-x) = -9x^9 - 2x^5 - x^4 - 7x^2 + 2$

$- \quad - \quad - \quad - \quad +$

1

$2 + 1 = 3$

$9 - 3 = 6.$

∴ atleast 6 imaginary roots.

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22. Sol:

$|6-8i| = \sqrt{36+64}$
$= \sqrt{100}$
$= 10$

2). 8 மார்க்

Applying sq. root formula.

$\sqrt{6-8i} = \pm\left(\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right)$

$= \pm\left(\sqrt{\frac{16}{2}}\,8 - i\sqrt{\frac{4}{2}}\,2\right)$

$= \pm\,(2\sqrt{2} - i\sqrt{2})$

IX 23(a) Sol:

$3x - y + 2z = 8$
$x + y + z = 2$
$2x + y - z = -1$

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$$\Delta = \left|\begin{array}{ccc} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{array}\right| \begin{array}{cc} 3 & -1 \\ 1 & 1 \\ 2 & 1 \end{array}$$

$= (-3 \cancel{-2} + \cancel{2}) - (4 + 3 + 1)$

$= -3 - 8$

$= -11$

$\neq 0$

Cramer's rule is applicable

கிராமரின் விதி பயன்படும்.

$$\Delta_1 = \left|\begin{array}{ccc} 8 & -1 & 2 \\ 2 & 1 & 1 \\ -1 & 1 & -1 \end{array}\right| \begin{array}{cc} 8 & -1 \\ 2 & 1 \\ -1 & 1 \end{array}$$

$= (-8 + 1 + 4) - (\cancel{-2} + 8 + \cancel{2})$

$= -3 - 8$

$= -11$

$$\Delta_2 = \left|\begin{array}{ccc} 3 & 8 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & -1 \end{array}\right| \begin{array}{cc} 3 & 8 \\ 1 & 2 \\ 2 & -1 \end{array}$$

$= (-6 + 16 - 2) - (\cancel{8} - 3 - \cancel{8})$

$= 8 + 3$

$= 11$

$$\Delta_3 = \left|\begin{array}{ccc} 3 & -1 & 8 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{array}\right| \begin{array}{cc} 3 & -1 \\ 1 & 1 \\ 2 & 1 \end{array}$$

$= (-3 - 4 + 8) - (16 + 6 + 1)$

$= 1 - 23$

$= -22$

$x = \dfrac{\Delta_1}{\Delta} = \dfrac{-11}{-11} = 1$; $y = \dfrac{\Delta_2}{\Delta} = \dfrac{11}{-11} = -1$

$z = \dfrac{\Delta_3}{\Delta} = \dfrac{-22}{-11} = 2.$

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23(b) $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

P.T. $(F(\alpha))^{-1} = F(-\alpha)$

Sol:

LHS: $(F(\alpha))^{-1} = \dfrac{1}{|F(\alpha)|} \operatorname{adj} F(\alpha)$

$|F(\alpha)| = \cos^2\alpha - 0 + \sin^2\alpha$

$= 1$

$\neq 0.$

$\therefore (F(\alpha))^{-1}$ exists

$(F(\alpha))^{-1}$ செல்லும் (மதிப்புண்டு) (அ) (காண இயலும்)

$\operatorname{adj} F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ +\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

$(F(\alpha))^{-1} = \dfrac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \rightarrow (1)$

RHS:

$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \rightarrow (2)$

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① & ② $\Rightarrow$

$$LHS = RHS$$

$(F(\alpha))^{-1} = F(-\alpha)$ என நிரூபிக்கப்பட்டது

Hence proved.

24 a) $2x + 3y + 5z = 9$

$7x + 3y - 5z = 8$

$2x + 3y + \lambda z = \mu$

(i) யாதொரு தீர்வும் பெற்றிராது (no solution)

(ii) ஒரே ஒரு தீர்வைப் பெற்றிருக்கும் (unique solution)

(iii) எண்ணிக்கையற்ற தீர்வுகளை பெற்றிருக்கும் (Infinitely many sol.)

Sol:

$$[A, B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$$

$$\begin{array}{rrrr} 14 & 6 & -10 & 16 \\ \underline{14} & \underline{21} & \underline{35} & \underline{63} \\ 0 & -15 & -45 & -47 \end{array}$$

~~14 21 35 56~~

$$\sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{bmatrix} \quad \begin{array}{l} \\ R_2 \to 2R_2 - 7R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

(i) $\lambda = 5, \mu \neq 9$

$\rho(A) = 2, \quad \rho(A, B) = 3$

$\rho(A) \neq \rho(A, B)$

யாதொரு தீர்வும் இல்லை (no solution)

(ii) $\lambda \neq 5, \mu \neq 9$

$\rho(A) = \rho(A, B) = 3$

ஒரே ஒரு தீர்வைப் பெற்றிருக்கும் (unique solution)

(iii) $\lambda = 5, \mu = 9$; $\rho(A) = \rho(A, B) = 2 < 3$

எண்ணிக்கையற்ற தீர்வுகளை பெற்றிருக்கும் (Infinitely many solution).

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24 (b) $\left(\frac{19-7i}{9-i}\right)^{12} + \frac{20-5i}{7-6i}$

தீர்வு :

$$\frac{19-7i}{9+i} \times \frac{9+i}{9+i} = \frac{19(9)+19i-63i+7i^2}{81-i^2}$$

$$= \frac{171-82i-7}{81+1}$$

$$= \frac{164-82i}{82}$$

$$= \frac{\cancel{82}(2-i)}{\cancel{82}}$$

$$= 2-i$$

$$\frac{20-5i}{7-6i} \times \frac{7+6i}{7+6i} = \frac{140+120i-35i-30i^2}{49+36}$$

$$= \frac{170+85i}{85}$$

$$= \frac{\cancel{85}(2+i)}{\cancel{85}}$$

$$= 2+i$$

$$z = (2-i)^{12} + (2+i)^{12}$$

$$\bar{z} = (\overline{2-i})^{12} + (\overline{2+i})^{12}$$

$$= (2+i)^{12} + (2-i)^{12}$$

$$= z$$

$\therefore$ z is a real.

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25(a) Sol:

$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$$

$$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x^2-1) - iy(x-1) + iy(x+1) - i^2y^2}{(x+1)^2 - i^2y^2}$$

$$= \frac{x^2-1 - \cancel{ixy} + iy + \cancel{ixy} + iy + y^2}{(x+1)^2 + y^2}$$

$$= \frac{(x^2+y^2-1) + i2y}{(x+1)^2+y^2}$$

$$\arg\left[\frac{z-1}{z+1}\right] = \tan^{-1}\left[\frac{2y}{x^2+y^2-1}\right] = \pi/2$$

$$\frac{2y}{x^2+y^2-1} = \tan\frac{\pi}{2}$$

$$\frac{2y}{x^2+y^2-1} = \frac{1}{0}$$

$$x^2+y^2-1 = 0$$

$$\therefore \boxed{x^2+y^2=1}$$

Hence proved.

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(b) Sol:

$$z^3 + 27 = 0.$$

$$z^3 = -27$$

$$= 3^3(-1)$$

$$z = 3(-1)^{1/3}$$

$$z = 3[\cos(2k\pi+\pi) + i\sin(2k\pi+\pi)]^{1/3}$$

$$= 3\left[\cos\left[\frac{2k\pi+\pi}{3}\right] + i\sin\left[\frac{2k\pi+\pi}{3}\right]\right]$$

$$k = 0, 1, 2.$$

$k=0$, $z = 3\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

$k=1$, $z = 3\left[\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right]$
$= -3$

$k=2$,
$z = 3\left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right]$

26(a) Question wrong: $x^4 - 14x^2 = 45 = 0$

correct Question is

$$x^4 - 14x^2 + 45 = 0$$

put $x^2 = y$

$$y^2 - 14y + 45 = 0$$

$$(y-9)(y-5) = 0$$

45: −9, −5

$y - 9 = 0$	$y - 5 = 0$
$x^2 - 9 = 0$	$x^2 - 5 = 0$
$x^2 = 9$	$x^2 = 5$
$\boxed{x = \pm 3}$	$\boxed{x = \pm\sqrt{5}}$

26(b) sol:

$\frac{1}{3}$	6	−5	−38	−5	6
	0	2	−1	−13	−6
3	6	−3	−39	−18	0
	0	18	45	18	
	6	15	6	0	

$$6x^2 + 15x + 6 = 0$$

36: 12, 3; $\frac{12}{6} = 2$, $\frac{3}{6} = \frac{1}{2}$

$$x = -2, -\frac{1}{2}$$

∴ The roots are $\frac{1}{3}, 3, -2, -\frac{1}{2}$.

∴ தீர்வு, (மூலங்கள்) $\frac{1}{3}, 3, -2, -\frac{1}{2}$

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