KKC12M

Kanyakumari District Common Examinations

Common First Mid Term Test - August 2022

Standard 12

MATHEMATICS

Time: 1.30 Hrs. Marks: 45

Section - A

I. **Choose the correct answer:** **10×1=10**

1) If $A=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adjB|}{|C|}=$

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{1}{4}$ d) 1

2) If A^TA^{-1} is symmetric, then $A^2=$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3) If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ then adj(adj A) is

a) $\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ b) $\begin{bmatrix}6 & -6 & 8\\ 4 & -6 & 8\\ 0 & -2 & 2\end{bmatrix}$ c) $\begin{bmatrix}-3 & 3 & -4\\ -2 & 3 & -4\\ 0 & 1 & -1\end{bmatrix}$ d) $\begin{bmatrix}3 & -3 & 4\\ 0 & -1 & 1\\ 2 & -3 & 4\end{bmatrix}$

4) If $A=\begin{bmatrix}5 & 0 & 1\end{bmatrix}$ then the rank of AA^T is

a) 1 b) 2 c) 3 d) 0

5) $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ is

a) 0 b) 1 c) −1 d) i

6) If $|z-2+i|\le 2$, then the greatest value of $|z|$ is

a) $\sqrt{3}-2$ b) $\sqrt{3}+2$ c) $\sqrt{5}-2$ d) $\sqrt{5}+2$

7) If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni)=x+iy$ then $2.5.10\ldots\ldots(1+n^2)$ is

a) 1 b) i c) x^2+y^2 d) $1+n^2$

8) If $-\bar{z}$ lies in the 3rd quadrant, then z lies in the

a) 1st quadrant b) 2nd quadrant c) 3rd quadrant d) 4th quadrant

9) A zero of x^3+64 is

a) 0 b) 4 c) 4i d) −4

10) The polynomial x^3-Kx^2+9x has three real zeros if and only if, K satisfies

a) $|K|\le 6$ b) $K=0$ c) $|K|>6$ d) $|K|\ge 6$

Section - B

II. **Answer any 3 questions: [Question No. 15 is compulsory]** **3×2=6**

11) Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

12) Find the square root of $-6+8i$.

13) Find the modulus and argument of -3.

14) If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

15) Solve the following system of linear equation by matrix inversion method $2x+5y=-2; x+2y=-3$.

Section - C

III. Answer any 3 questions: [Question No. 20 is compulsory] **3×3=9**

16) If $A=\begin{bmatrix}8 & -4\\ -5 & 3\end{bmatrix}$ verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_2$.

17) Find the rank of the matrix $\begin{bmatrix}3 & -8 & 5 & 2\\ 2 & -5 & 1 & 4\\ -1 & 2 & 3 & -2\end{bmatrix}$ by row reduction method.

18) Simplify: $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$

19) Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

20) If $\omega \neq 1$ is a cube root of unity, show that $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6 = 128$.

Section - D

IV. Answer all the questions. **4×5=20**

21) a) Solve the following systems of equations by Cramer's rule:

$$\frac{1}{x}+\frac{2}{y}-\frac{1}{z}=1;\ \frac{2}{x}+\frac{4}{y}+\frac{1}{z}=5;\ \frac{3}{x}-\frac{2}{y}-\frac{2}{z}=0.$$

(OR)

b) A boy is walking along the path $y=ax^2+bx+c$ through the points (-6, 8) (-2, -12) and (3, 8). He wants to meet his friend at P(7, 60). Will he meet his friend? (use Gaussian elimination method)

22) a) Investigate for what values of λ and μ the system of linear equations $x+2y+z=7; x+y+\lambda z=\mu; x+3y-5z=5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$ show that $x^2+y^2=1$.

23) a) Show that $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is purely real.

(OR)

b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle incribed in the circle $|z| = 2$, if $z_1 = 1+i\sqrt{3}$ then find z_2 and z_3.

24) a) If $2+i$ and $3-\sqrt{2}$ are roots of the equation

$x^6-13x^5+62x^4-126x^3+65x^2+127x-140 = 0$

(OR)

b) Solve the equation $6x^4-35x^3+62x^2-35x+6 = 0$.
