COMMON QUARTERLY EXAMINATION -2022

half

12th Standard

Date : 27-Sep-22

Maths

Reg.No. :

Exam Time : 03:00:00 Hrs Total Marks : 90

I CHOOSE THE CORRECT ANSWER 20x 1 = 20

1) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) adj A = $|A|A^{-1}$ (b) adj(AB) = (adj A)(adj B) (c) det A^{-1} = $(\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2) If ρ (A) = ρ([A| B]) , then the system AX = B of linear equations is

(a) consistent and has a unique solution (b) consistent (c) consistent and has infinitely many solution (d) inconsistent

3) If A is a matrix of order m $\times$ n, then ρ (A) is ________

(a) m (b) n (c) ≤ min (m,n) (d) ≥ min (m,n)

4) Cramer's rule is applicable only when ______

(a) $\Delta \neq 0$ (b) $\Delta = 0$ (c) $\Delta = 0, \Delta_x = 0$ (d) $\Delta_x = \Delta_y = \Delta_z = 0$

5) If |z| = 1, then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

6) The product of all four values of $\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) -2 (b) -1 (c) 1 (d) 2

7) The complex numbers z_1, z_2, and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is __________

(a) of area zero (b) right angled isosceles (c) equilateral (d) obtuse-angle isosceles

8) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

(a) a ≥ 0 (b) a > 0 (c) a < 0 (d) a ≤ 0

9) The polynomial $x^3 + 2x + 3$ has

(a) one negative and two imaginary zeros (b) one positive and two imaginary zeros (c) three real zeros (d) no zeros

10) If $ax^2 + bx + c = 0$, a, b, c $\in$ R has no real zeros, and if a + b + c < 0, then __________

(a) c > 0 (b) c< 0 (c) c=0 (d) c≥0

11) If the product of the roots of $3x^4 - 4x^3 + 2x^2 + x + a = 0$ is 21, then the value of a is ____________

(a) 7 (b) -7 (c) -63 (d) 63

12) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}tan^{-1}\left(\frac{3}{5}\right)$ (d) $tan^{-1}\left(\frac{1}{2}\right)$

13) If $|x| \leq 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

(a) $\tan^{-1}x$ (b) $\sin^{-1}x$ (c) 0 (d) π

14) If x > 1, then $2tan^{-1}x+sin^{-1}\left(\frac{2x}{1+x^2}\right)$ _______

(a) $4\tan^{-1}x$ (b) 0 (c) $\frac{\pi}{2}$ (d) π

15) If x + y = k is a normal to the parabola $y^2 = 12x$, then the value of k is

(a) 3 (b) -1 (c) 1 (d) 9

16) The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a + b)x - 4 = 0$, then the value of (a+b) is

(a) 2 (b) 4 (c) 0 (d) -2

17) In an ellipse $5x^2 + 7y^2 = 11$, the point (4, -3) lies ________ the ellipse

(a) on (b) outside (c) inside (d) none

18) Find the centre and vertices of the hyperbola $11x^2 - 25y^2 + 22x + 250y - 889 = 0$

(a) centre: (-1, 5), vertices: (1, -10), (1, 0) (b) centre: (-1, 5), vertices: (-1, 0), (-1, 10)

(c) centre: (--1, 5), vertices: (-6, 5), (4, 5) (d) centre: (-1, 5), vertices: (-4, -5), (6, -5)

19) If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

20) Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

21) Let $\vec{a}$, $\vec{b}$and $\vec{c}$be three non- coplanar vectors and let $\vec{p}, \vec{q}, \vec{r}$ be the vectors defined by the relations $\vec{P} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ Then the value of $\left(\vec{a} + \vec{b}\right).\vec{p} + \left(\vec{b} + \vec{c}\right).\vec{q} + (\vec{c} + \vec{a}).\vec{r} =$ ___________

(a) 0 (b) 1 (c) 2 (d) 3

22) If $\vec{a}$ and $\vec{b}$ include an angle 120^o and their magnitude are 2 and $\sqrt{3}$ then $\vec{a}.\vec{b}$ is equal to ____________

(a) $\sqrt{3}$ (b) $-\sqrt{3}$ (c) 2 (d) $-\frac{\sqrt{3}}{2}$

II ANSWER ANY 7 QUESTION Q.NO 30 COMPLUSARY 10 x 2 = 20

23) If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

24) If A is symmetric, prove that then adj A is also symmetric.

25) Simplify the following
$i\, i^2 i^3 \ldots i^{2000}$

26) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4 = 0$, form a cubic equation whose roots are, 2α, 2β, 2γ

27) It is known that the roots of the equation $x^3- 6x^2- 4x + 24 = 0$ are in arithmetic progression. Find its roots.

28) Find the principal value of
$\sec^{-1}(\frac{2}{\sqrt{3}})$

29) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
$y^2 = -8x$

30) Find the equation of the parabola. if the curve ie open leftward, vertex is (2,1) and passing through the point (1, 3)

31) Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.

32) Find the value of
$tan^{-1}(tan\frac{5\pi}{4})$

III ANSWER ANY 7 QUESTION Q.NO 40 COMPLUSARY 10 x 3 = 30

33) If A and B are any two non-singular square matrices of order n , then adj(AB) = (adj B)(adj A).

34) If $2cos\alpha = x + \frac{1}{x}$ and $2cos\,\beta = y + \frac{1}{y}$, show that $x^m y^n + \frac{1}{x^m y^n} = 2cos(m\alpha + n\beta)$

35) Solve the equation $z^3 + 27 = 0$

36) If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma\frac{1}{\beta\gamma}$ in terms of the coefficients.

37) Find the value of the expression in terms of x, with the help of a reference triangle.
$\sin(\cos^{-1}(1-x))$

38) Simplify $sec^{-1}\left(sec\left(\frac{5\pi}{3}\right)\right)$

39) The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

40) Find the length of the chord intercepted by the circle $x^2 + y^2 - 2x - y + 1 = 0$ and the line x - 2y = 0

41) Find the equation of the plane passing through the intersection of the planes 2x + 3y −z + 7 = 0 and and x +y −2z + 5 = 0 and is perpendicular to the plane x +y −3z −5 = 0.

42) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{p}, \vec{q}, \vec{r}$ are any two systems of three vectors, and if $\vec{p} = x_1\vec{a} + y_1\vec{b} + z_1\vec{c}$

$\vec{q} = x_2\vec{a} + y_2\vec{b} + z_2\vec{c}$, and, $\vec{r} = x_3\vec{a} + y_3\vec{b} + z_3\vec{c}$ then $[\vec{p}, \vec{q}, \vec{r}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}]$

III ANSWER THE ALL QUESTION 7x 5 = 35

43) a) Find all the roots $(2 - 2i)^{\frac{1}{3}}$ and also find the product of its roots.

(OR)

b) A straight line passes through the point (1, 2, −3) and parallel to $4\hat{i} + 5\hat{j} - 7\hat{k}$. Find
(i) vector equation in parametric form
(ii) vector equation in non-parametric form
(iii) Cartesian equations of the straight line.

44) a) Prove that $\tan^{-1} x + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(OR)

b) Find the acute angle between the following lines
2x = 3y = −z and 6x = − y = −4z.

45) a) Find the equations of tangent and normal to the parabola $x^2+6x+4y+5 = 0$ at (1, -3) .

(OR)

b) Find the equation of the tangent at t = 2 to the parabola $y^2 = 8x$. (Hint: use parametric form)

46) a) If the system of equations px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution and p ≠ a, q ≠ b, r ≠ c, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

(OR)

b) Determine the values of λ for which the following system of equations x + y + 3z = 0, 4x + 3y + λz = 0, 2x + y + 2z = 0 has
(i) a unique solution
(ii) a non-trivial solution

47) a) Show that the equation $x^3 + qx + r = 0$ has two equal roots if $27r^2 + 4q^3 = 0$.

(OR)

b) Find the equation of the plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x - y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, -1)

48) a) If $z_1 = 2 + 5i$, $z_2 = -3 - 4i$, and $z_3 = 1 + i$, find the additive and multiplicate inverse of z_1, z_2 and z_3

(OR)

b) If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_2 + z_2z_3| = 6$

49) a) Solve the equations:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

(OR)

b) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

$y^2 - 4y - 8x + 12 = 0$

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5) If |z| = 1, then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

6) The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

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13) If $|x| \leq 1$, then $2\tan^{-1} x - \sin^{-1} \frac{2x}{1+x^2}$ is equal to

(a) $\tan^{-1}x$ (b) $\sin^{-1}x$ **(c) 0** (d) π

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(c) centre: (--1, 5), vertices: (-6, 5), (4, 5) (d) centre: (-1, 5), vertices: (-4, -5), (6, -5)

19) If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(a) $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = 1$ (b) $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = -1$ **(c)** $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = \mathbf{0}$ (d) $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = 2$

20) Distance from the origin to the plane 3x − 6y + 2z + 7 = 0 is

(a) 0 **(b) 1** (c) 2 (d) 3

21) Let $\vec{a}$, $\vec{b}$and $\vec{c}$be three non- coplanar vectors and let $\vec{p},\vec{q},\vec{r}$ be the vectors defined by the relations $\vec{P} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ Then the value of $\left(\vec{a}+\vec{b}\right).\vec{p} + \left(\vec{b}+\vec{c}\right).\vec{q} + (\vec{c}+\vec{a}).\vec{r}$= ___________

(a) 0 (b) 1 (c) 2 **(d) 3**

22) If $\vec{a}$ and $\vec{b}$ include an angle 120° and their magnitude are 2 and $\sqrt{3}$ then $\vec{a}.\vec{b}$ is equal to ____________

(a) $\sqrt{3}$ **(b)** $-\sqrt{3}$ (c) 2 (d) $-\frac{\sqrt{3}}{2}$

II ANSWER ANY 7 QUESTION Q.NO 30 COMPLUSARY 10 x 2 = 20

23) If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

Answer : We first find adj A. By definition, we get adj A =

$\begin{bmatrix} +M_{11} & -M_{12} \\ -M_{21} & +M_{22} \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -c & a \end{bmatrix}$.

Since A is non-singular, |A| = ad - bc ≠ 0.

As $A^{-1} = \frac{1}{|A|}$ adj A, we get A_{-1} = $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

24) If A is symmetric, prove that then adj A is also symmetric.

Answer : Suppose A is symmetric. Then, A^T = A and so, by theorem (vi), we get adj(A^T) = (adj A)^T ⇒ adj A = (adj A)^T ⇒ adj A is symmetric.

25) Simplify the following

$i\, i^2 i^3 \ldots i^{2000}$

Answer : $i\, i^2\, i^3 \ldots i^{2000}$

$=i^{1+2+3+\ldots+2000}$

$=i^{\frac{2000\times 2001}{2}}$

$[\therefore 1+2+3+\ldots n = \frac{n(n+1)}{2}]$

$= i^{1000 \times 2001}$

$= i^{2001000}$

$= 1$

[∴ 2001000 is divisible by 4 as its last two digits are divisible by 4]

26) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4 = 0$, form a cubic equation whose roots are, 2α, 2β, 2γ

Answer : The roots of $x^3+2x2+3x+4 = 0$ are ∝, β, ɤ

$\therefore \propto+\beta+ɤ$ = -co-efficient of x^2 = -2 ...(1)

∝β + βɤ + ɤ∝ = co-effficient of x = 3 ....(2)

-∝βɤ = +4 ⇒ ∝βɤ = -4 ...(3)

Form a cubic equation whose roots are 2∝, 2β, 2ɤ

2∝+2β+2ɤ = 2(∝+β+ɤ) = 2(-2) = -4 [from (1)]

4∝β+4βɤ+4ɤ∝ = 4(∝β+βɤ+ɤ∝) = 4(3) = 12 [from (2)]

(2∝)(2β)(2ɤ) = 8(∝βɤ) = 8(-4) = -32 [from (3)]

∴ The required cubic equation is

x^3-(2∝+2β+2ɤ)x^2 + (2∝β+2βɤ+2ɤ∝)x - (2∝)(2β)(2ɤ) = 0

⇒ $x^3+(-4)x^2+12x+32 = 0$

⇒ $x^3+4x^2+12x+32 = 0$

27) It is known that the roots of the equation $x^3- 6x^2- 4x + 24 = 0$ are in arithmetic progression. Find its roots.

Answer : Let the roots be a−d, a, a+d.

Then the sum of the roots is 3a which is equal to 6 from the given equation.

Thus 3a = 6 and hence a = 2.

The product of the roots is $a^3- ad^2$ which is equal to −24 from the given equation.

Substituting the value of a, we get $8-2d^2 = -24$ and hence d = ±4.

If we take d = 4 we get −2, 2, 6 as roots and if we take d = −4, we get 6, 2, −2 as roots (same roots given in reverse order) of the equation.

28) Find the principal value of

$\sec^{-1}(\frac{2}{\sqrt{3}})$

Answer : $\sec^{-1}(\frac{2}{\sqrt{3}})$

Let $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \frac{2}{\sqrt{3}} = sec\theta \Rightarrow cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow cos\theta = cos\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = \frac{\pi}{6}$

$\therefore sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

29) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

$y^2 = -8x$

Answer : $y^2 = -8x$

The given parabola is left open parabola

and 4a = 8 ⇒ a = 2

(a) Vertex is (0, 0)

⇒ h = 0, k = 0

(b) focus is (h - a, 0 + k)

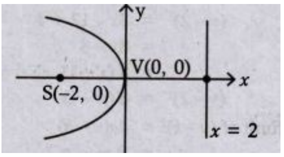
⇒ (0-2, 0 + 0)

⇒ (-2, 0)

(c) Equation of directrix is x = h + a

⇒ x = 0 + 2 ⇒ x = 2

(d) Length of latus rectum is 4a = 8.



30) Find the equation of the parabola. if the curve ie open leftward, vertex is (2,1) and passing through the point (1, 3)

Answer : Since the curve is open leftward, the required equation of the parabola is

$(y-k)^2=-4a(x-h)$

Given vertex (h, k) = (2, 1)

$\therefore (y-1)^2=-4a(x-2)$ ....(2)

Since this pass through (1, 3) we get

$(3-1)^2=-4a(1-2)$

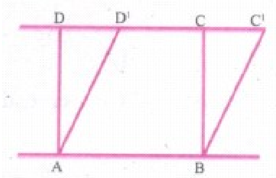
$4=-4a(-1)$

$a=1$

$\therefore (1) \Rightarrow (y-1)^2=-4(x-2)$

which is required equation of the parabola

31) Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.

Answer : 

Let ABCD be the given parallelogram and ABC[1] D[1] be the new parallelogram with same base AB and between the same parallel lines AB and DC.

∴ Vector area of parallelogram

ABCD = $\vec{AB} \times \vec{AD}$

= $\vec{AB} \times (\vec{AD^1} + D^1\vec{D})$

[By Δ law of addition is ΔADD[1]]

= $(\vec{AB} \times \vec{AD^1}) + (\vec{AB} \times D^1\vec{D})$ [∵ vector product is distributive]

= $(\vec{AB} \times \vec{AD})$ +0 [∵ $\vec{AB}$ and $\vec{DD^1}$ are parallel]

= Vector area of parallelogram ABC[1]D[1]

∴ Area of parallelogram ABCD = Area of parallelogram ABC[1]D[1].

Hence, the parallelogram on the same base andabetween the same parallels are equal in area.

32) Find the value of

$tan^{-1}(tan\frac{5\pi}{4})$

Answer : $tan^{-1}(tan\frac{5\pi}{4})$

= $tan^{-1}\left(tan\left(\pi + \frac{\pi}{4}\right)\right)$

= $tan^{-1}\left(tan\frac{\pi}{4}\right)$ $[\because tan\,(\pi + \theta = tan\theta)]$

= $\frac{\pi}{4}\varepsilon\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

III ANSWER ANY 7 QUESTION Q.NO 40 COMPLUSARY 10 x 3 = 30

33) If A and B are any two non-singular square matrices of order n , then adj(AB) = (adj B)(adj A).

Answer : Replacing A by AB in adj(A) = $|A|A^{-1}$ we get

$\text{adj}(AB) = |AB|(AB\,|)^{-1} = \left(|B|B^{-1}\right)\left(|A|A^{-1}\right) = \text{adj}(B)\,\text{adj}(A)$

34) If $2cos\alpha = x + \frac{1}{x}$ and $2cos\,\beta = y + \frac{1}{y}$, show that $x^my^n + \frac{1}{x^my^n} = 2cos(m\alpha + n\beta)$

Answer : Given 2cos α = x+$\frac{1}{x}$

⇒ 2cos α = $\frac{x^2+1}{x}$

⇒ x^2+1 = 2xcos α

⇒ x^2-2x cos α+1 = 0

⇒ $\frac{2cos\alpha\pm\sqrt{(-2cos\alpha)^2-4(1)(1)}}{2}$

= $\frac{2cos\alpha\pm\sqrt{4cos^2\alpha-4}}{2}$ $\left[\because \frac{b\pm\sqrt{b^2-4ac}}{2a}\right]$

= $\frac{2cos\alpha\pm\sqrt{-sin^2\alpha}}{2}$

= $\frac{2cos\alpha\pm isin\alpha}{2}$ [∵ $sin^2α+cos^2α = 1$]

⇒ x^2 = cos α ± sin α

Also, 2cos β = y+$\frac{1}{y}$

⇒ 2cos β = $\frac{y^2+1}{y}$

⇒ y^2-2y cos β+1 = 0

⇒ $\frac{2cos\beta\pm\sqrt{(-2cos^2\beta^2-4(1)(1)}}{2}$

= $\frac{2cos\beta\pm\sqrt{4cos^2\beta-4}}{2} = \frac{2cos\beta\pm 2isin\beta}{2}$

⇒ y = cosβ ± i sinβ

$$x^my^n+\frac{1}{x^my^n}=2cos(m\alpha+n\beta)$$

x^my^n = (cos α + i sin mα) (cos nβ + i sin nβ)

cos(mα+nβ)+i sin(mα+nβ)

$\frac{1}{x^my^n}$ = cos(mα+nβ)-i sin(mα+nβ)

$$\therefore x^my^n+\frac{1}{x^my^n}=\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)+\cos(m\alpha+n\beta)-i\sin(m\alpha+n\beta)$$

= 2cos(mα+nβ)

35) Solve the equation z^3+ 27 = 0

Answer : z^3 = -27 = $(-1 \times 3)^3$ = -1 × 33

z = $(-1)^{\frac{1}{3}} \times 3^{3\times\frac{1}{3}} = (-1)^{\frac{1}{3}} \times 3$

∴ z = 3$[cos\pi+isin\pi]^{\frac{1}{3}}$

[∵ cos π = -1 and sin π = 0]

= 3$\left[cos\frac{1}{3}(2k\pi+\pi)isin\frac{1}{3}(2k\pi+\pi)\right]$

k = 0, 1, 2

When k = 0,

z = 3$\left[cos\frac{1}{3}(\pi)isin\frac{1}{3}(\pi)\right] = 3cos\frac{\pi}{3}$

When k = 1

z = 3$\left[cos\frac{1}{3}(3\pi)isin\frac{1}{3}(3\pi)\right]$

= 3[cos π + i sin π] = 3(-1+0)

When k = 2

z = 3$\left[cos\frac{1}{3}(5\pi)isin\frac{1}{3}(5\pi)\right] = 3\left[cos5\frac{\pi}{3}\right]$

Hence, the roots are 3 cis$\frac{\pi}{3}$, -3, 3 c is 5$\frac{\pi}{3}$

36) If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma\frac{1}{\beta\gamma}$ in terms of the coefficients.

Answer : Since α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, we have

$\Sigma_1\ \alpha + \beta + \gamma = -p$ and $\Sigma_3\ \alpha\beta\gamma = -r$

$\Sigma\frac{1}{\beta\gamma} = \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{-p}{-r} = \frac{p}{r}$.

37) Find the value of the expression in terms of x, with the help of a reference triangle.

$\sin(\cos^{-1}(1-x))$

Answer : $\sin(\cos^{-1}(1-x))$

we know that $cos^{-1}x = sin^{-1}\left(\sqrt{1-x^2}\right)$ if $0 \le x \le 1$

$\therefore cos^{-1}(1-x) = sin^{1}\sqrt{1-(1-x)^2}\ [\because 0 \le x \le 1]$

= $sin^{-1}\left(\sqrt{1-(1+x^2-2x)}\right)$

= $sin^{-1}\left(\sqrt{1-1-x^2+2x}\right) = sin^{-1}\left(\sqrt{2x-x^2}\right)$

$\therefore sin\left(cos^{-1}(1-x)\right) = sin\left(sin^{-1}\left(\sqrt{2x-x^2}\right)\right)$

= $\sqrt{2x-x^2}$

38) Simplify $sec^{-1}\left(sec\left(\frac{5\pi}{3}\right)\right)$

Answer : $sec^{-1}\left(sec\left(\frac{5\pi}{3}\right)\right)$

Note that $\frac{5\pi}{3}$ is not in $[0, \pi]\backslash\{\frac{\pi}{2}\}$, the principal range of $\sec^{-1} x$.

we write $\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$.

Now, $\sec\left(\frac{5\pi}{3}\right) = sec\left(2\pi - \frac{\pi}{3}\right) = sec\left(\frac{\pi}{3}\right)$ and $\frac{\pi}{3} \in [0,\pi]\ \backslash\{\frac{\pi}{2}\}$

Hence, $\sec^{-1}\left(sec\left(\frac{5\pi}{3}\right)\right) = sec^{-1}\left(sec\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

39) The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

Answer : Equation of the parabola is $y = \frac{1}{32}x^2$

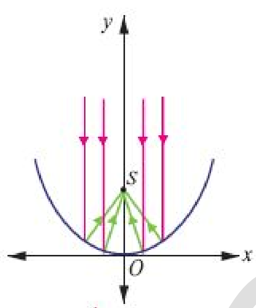
That is $x^2 = 32y$; the vertex is (0, 0)

= 4 (8)y

$\Rightarrow a = 8$

So the heating tube needs to be placed at focus (0, a)

Hence the heating tube needs to be placed 8 units above the vertex of the parabola.



40) Find the length of the chord intercepted by the circle $x^2 + y^2 - 2x - y + 1 = 0$ and the line $x - 2y = 0$

Answer : To find the end points of the chord, solve the equations of the circle and the line.

Substitute x = 2y + 1 in the equation of the circle

$(2y+1)^2+y^2-2(2y+1)-y+1=0$

$4y^2+4y+1+y^2-4y-2-y+1=0$

$5y^2-y=0$

$\therefore y(5y-1)=0$

y = 0 (or) $y=\frac{1}{5}$

$\Rightarrow x=1$ (or) $x=\frac{7}{5}$

The two end points are (1, 0) and $\frac{7}{5},\frac{1}{5}$

Length of the chord $=\sqrt{\left(1-\frac{7}{5}\right)+\left(0-\frac{1}{5}\right)^2}$

$=\sqrt{\frac{4}{25}+\frac{1}{25}}=\frac{1}{\sqrt{5}}$ units

41) Find the equation of the plane passing through the intersection of the planes 2x + 3y −z + 7 = 0 and and x +y −2z + 5 = 0 and is perpendicular to the plane x +y −3z −5 = 0.

Answer : The equation of the plane passing through the intersection of the planes 2x + 3y−z + 7 = 0 and x + y− 2z + 5 = 0 is (2x + 3y −z + 7) +λ (x + y −2z + 5) = 0 or (2 + λ)x+ (3 + λ)y+ (−1 − 2λ) z+ (7 + 5λ) = 0

since this plane is perpendicular to the given plane x+y−3z−5 = 0, the normals of these two planes are perpendicular to each other.

Therefore, we have (1)(2 + λ) + (1)(3 + λ) + (−3)(−1 − 2λ)z = 0

which implies that λ = −1.

Thus the required equation of the plane is

(2x + 3y − z + 7)−(x + y −2z + 5) = 0

⇒ x + 2y + z + 2 = 0

42) If $\vec{a},\vec{b},\vec{c}$ and $\vec{p},\vec{q},\vec{r}$ are any two systems of three vectors, and if $\vec{p}=x_1\vec{a}+y_1\vec{b}+z_1\vec{c}$

$\vec{q}=x_2\vec{a}+y_2\vec{b}+z_2\vec{c},$ and, $\vec{r}=x_3\vec{a}+y_3\vec{b}+z_3\vec{c}$ then $[\vec{p},\vec{q},\vec{r}]=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}[\vec{a},\vec{b},\vec{c}]$

Answer : If $\vec{a},\vec{b},\vec{c}$ are non-coplanar and

$\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}\neq 0$

then the three vectors $\vec{p}=x_1\vec{a}+y_1\vec{b}+z_1\vec{c},\quad \vec{q}=x_2\vec{a}+y_2\vec{b}+z_2\vec{c},$ and $,\vec{r}=x_3\vec{a}+y_3\vec{b}+z_3\vec{c}$ are also non-coplanar.

III ANSWER THE ALL QUESTION 7x 5 = 35

43) a) Find all the roots $(2-2i)^{\frac{1}{3}}$ and also find the product of its roots.

Answer : Let 2-2i = r(cosθ + isinθ)

r = $\sqrt{2^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

The principal value α =tan$^{-1}\left|\frac{y}{x}\right| = tan^{-1}\left|\frac{-z}{z}\right||$

= tan^{-1}(1) = $\frac{\pi}{4}$

Since the complex number 2 - 2i lies in the quadrant

θ = -α = -$\frac{\pi}{4}$

∴ 2-2i = $2\sqrt{2}\left[cos\left(-\frac{\pi}{4}\right)+isin\left(\frac{\pi}{4}\right)\right]^{\frac{1}{3}}$

∴ $(2\sqrt{2})^{\frac{1}{3}}\left[cos\left(-\frac{\pi}{4}\right)+isin\left(\frac{\pi}{4}\right)\right]^{\frac{1}{3}}$

= $8^{\frac{1}{6}}\left[cos\frac{1}{3}\left(2k\pi-\frac{\pi}{4}\right)+isin\frac{1}{3}\left(2k\pi-\frac{\pi}{4}\right)\right]$

k = 0, 1, 2

The roots are

∴ When k = 0, $8^{\frac{1}{6}}cis\left(-\frac{\pi}{12}\right)$

when k = 1, $8^{\frac{1}{6}}cis\left(\frac{7\pi}{12}\right)$

when k = 2, $8^{\frac{1}{6}}cis\left(\frac{15\pi}{12}\right)$

∴ The product of the root

= $8^{\frac{1}{6}}cis\left(-\frac{\pi}{12}+\frac{7\pi}{12}+\frac{15\pi}{12}\right)$

= $8^{\frac{1}{6}}cis\left(\frac{21\pi}{12}\right) = 8^{\frac{1}{6}}cis\left(\frac{7\pi}{12}\right)$

= $8^{\frac{1}{6}}cis\left(2\pi-\frac{\pi}{4}\right) = 8^{\frac{1}{6}}cis\left(-\frac{\pi}{4}\right)$

= $8^{\frac{1}{6}}\left[cos\left(-\frac{\pi}{4}\right)+isin\left(-\frac{\pi}{4}\right)\right]$

= $8^{\frac{1}{6}}\left[cos\left(\frac{\pi}{4}\right)+isin\left(\frac{\pi}{4}\right)\right]$

= $8^{\frac{1}{6}}\left[\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right] = 2^{3\times\frac{1}{6}}\left(\frac{1-i}{\sqrt{2}}\right) = 2^{1/2}\left(\frac{1-i}{\sqrt{2}}\right)$

= 1-i

(OR)

b) A straight line passes through the point (1, 2, −3) and parallel to $4\hat{i}+5\hat{j}-7\hat{k}$. Find

(i) vector equation in parametric form

(ii) vector equation in non-parametric form

(iii) Cartesian equations of the straight line.

Answer : The required line passes through (1, 2, −3). So, the position vector of the point is $\hat{i}+2\hat{j}-3\hat{k}$.

Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=4\hat{i}+5\hat{j}-7\hat{k}$. Then, we have

(i) vector equation of the required straight line in parametric form is $\vec{r}=\vec{a}+t\vec{b}$, t ∈ R

Therefore, $\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+t(4\hat{i}+5\hat{j}-7\hat{k})$, t ∈ R

(ii) vector equation of the required straight line in non-parametric form is $(\vec{r}-\vec{a})\times\vec{b}=\vec{0}$

Therefore, $\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+t(4\hat{i}+5\hat{j}-7\hat{k})=\vec{0}$

(iii) Cartesian equations of the required line are $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}$

Here, (x_1, y_1, z_1) = (1, 2, −3) and direction ratios of the required line are proportional to 4, 5, −7 .

Therefore, Cartesian equations of the straight line are $\frac{x-1}{4}=\frac{y-2}{5}=\frac{z+3}{-7}$

44) a) Prove that tan^{-1} x + tan^{-1} z = tan$^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

Answer : We know that $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\therefore LHS = tan^{-1}(x) + tan^{-1}(y) + tan^{-1}(z)$

= $tan^{-1}\left(\frac{x+y}{1-xy}\right) + tan^{-1}(z)$

= $tan^{-1}\left(\frac{\frac{x+y}{1-xy}+z}{1-z\left(\frac{x+y}{1-xy}\right)}\right)$ by(1)

= $tan^{-1}\left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{(1-xy)-z(x+y)}{1-xy}}\right)$

= $tan^{-1}\left(\frac{\frac{x+y+z-xyz}{1-xy}}{\frac{1-xy-zx-zy}{1-xy}}\right)$

$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy} \times \frac{1-xy}{1-xy-yz-zx}\right)$

= $tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = RHS$

Hence proved.

(OR)

b) Find the acute angle between the following lines

2x = 3y = −z and 6x = − y = −4z.

Answer : $a_1 = \frac{1}{2}, a_2 = \frac{1}{6}, b_1 = \frac{1}{3}, b_2 = \frac{-1}{1}, c_1 = -1$

$c_2 = \frac{1}{-4}$

$a_1a_2 + b_1b_2 + c_1c_2 = \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = \frac{1-4+3}{12} = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ or 90°

45) a) Find the equations of tangent and normal to the parabola $x^2+6x+4y+5 = 0$ at (1, -3) .

Answer : Equation of parabola is $x^2+6x+4y+5 = 0$.

$x^2+6x+9-9+4y + 5 = 0$

$(x + 3)^2 = -4(y-1)$ ... (1)

Let X = x+3,Y = y −1

Equation (1) takes the standard form

$X^2 = -4Y$

Equation of tangent is $XX_1 = -2(Y+ Y_1)$

At (1, -3) $X_1 = 1+3 = 4$; $y_1 = -3-1 = -4$

Therefore, the equation of tangent at (1,-3) is

(x +3)4 = -2(y-1- 4)

2x +6 = −y+5.

2x+y+1 = 0

Slope of tangent at(1, -3) is -2, so slope of normal at (1, -3) is $\frac{1}{2}$

Therefore, the equation of normal at (1, -3) is given by y + 3 = $\frac{1}{2}$(x-1)

2y + 6 = x -1

x-2y-7 = 0 .

(OR)

b) Find the equation of the tangent at t = 2 to the parabola $y^2 = 8x$. (Hint: use parametric form)

Answer : Equation of the parabola is $y^2 = 8x$

$\therefore 4a = 8 \Rightarrow a = 2$

Equation of tangent to the parabola in parametric form is $yt = x + at^2$

When t = 2, the equation of tangent is

$y(2) = x + 2(2)^2 \Rightarrow 2y = x + 8$

$\Rightarrow x - 2y + 8 = 0$ is the required equation of tangent.

46) a) If the system of equations px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution and p ≠ a, q ≠ b, r ≠ c, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

Answer : Assume that the system px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution.

So, we have $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ in the above equation,

we get $\begin{vmatrix} p & b & c \\ a-p & q-b & c \\ a-p & b & r-c \end{vmatrix} = 0$. That is, $\begin{vmatrix} p & b & c \\ -(p-a) & q-b & c \\ -(p-a) & b & r-c \end{vmatrix} = 0$.

Since p ≠ a, q ≠ b, r ≠ c, we get $(p-a)(q-b)(r-c) \begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$.

So, we have $\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$.

Expanding the determinant, we get $\frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$.

That is, $\frac{p}{p-a} + \frac{q-(q-b)}{q-b} + \frac{r-(r-c)}{r-c} = 0$

$\Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 2$.

(OR)

b) Determine the values of λ for which the following system of equations x + y + 3z = 0, 4x + 3y + λz = 0, 2x + y + 2z = 0 has

(i) a unique solution

(ii) a non-trivial solution

Answer : x + y + 3z = 0, 4x + 3y + λz = 0, 2x + y + 2z = 0

Reducing the augmented matrix to row - echelon form we get,

$$[A|0] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda - 2 & 0 \end{array}\right]$$

$$\underset{R_3 \to R_3 - 4R_1}{\xrightarrow{R_2 \to R_2 - 2R_1}} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda - 8 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & not \quad zero & 0 \end{array}\right]$$

Case (i) when λ ≠ 8

$$[A|0] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Here $\rho(A) = 3$, $\rho([A|0]) = 3$

∴ $\rho(A) = \rho([A|0]) = 3$ = the number of unknowns

∴ The given system is consistent and has unique solution.

Case (ii) when λ = 8

Here $\rho(A) = 2$, $\rho([A|0]) = 2$

∴ $\rho(A) = \rho([A|0]) = 2 < 3$ the number of unknowns,

∴ The system is consistent and has non-trivial solutions.

47) a) Show that the equation $x^3 + qx + r = 0$ has two equal roots if $27r^2 + 4q^3 = 0$.

Answer : Let α, β, γ be the roots of the equation

$f(x) = x^3 + qx + r = 0$

$\Sigma_1 = \alpha + \beta + \gamma = 0$ ....(1)

$\sum_2 = \alpha\beta + \beta\gamma + \gamma\alpha = +q$ ....(2)

$\sum_3 = \alpha\beta\gamma = -r$ ....(3)

Given $\alpha = \beta$

(1) $\Rightarrow 2\alpha + \gamma = 0$

$\gamma = -2\alpha$

(2) $\Rightarrow a^2 + a\gamma + a\gamma = q$

$\alpha^2 + 2a\gamma = q$

$a^2 + 2a(-2a) = q$

$a^2 - 4a^2 = q$

$-3a^2 = q$

$a^2 = \frac{-q}{3}$

(3) $\Rightarrow \alpha^2 \cdot \gamma = r$

$\alpha^2 \cdot (-2a) = r$

$-2a^3 = r$

Taking square on both sides,

$4\left(a^2\right)^3 = r^2$

$4\left(\frac{-q}{3}\right)^3 = r^2$

$\frac{4(-q)^3}{27} = r^2$

$-4q^3 = 27r^2$

$27r^2 + 4q^3 = 0$

(OR)

b) Find the equation of the plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x - y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, -1)

Answer : Given equation of planes are x + 2y + 3z = 2 and x-y+z+11 = 3

The Cartesian equation of a plane which passes through the line of intersection of the planes is

$(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2z - d_2) = 0$

$\therefore$ The required equation of the plane is

$(x + 2y + 3z - 2) + \lambda(x + y + z + 8) = 0$

$x(\lambda + 1) + y(2 + \lambda) + z(3 + \lambda) - 2 + 8\lambda = 0$

The distance from (3, 1, -1) to this plane is $\frac{2}{\sqrt{3}}$

$\therefore \frac{3(\lambda+1)+1(2+\lambda)-1(3+\lambda)-2+8\lambda}{\sqrt{(\lambda+1)^2+(2+\lambda)^2+(3+\lambda)^2}} = \frac{2}{\sqrt{3}}$

$\frac{3\lambda + \cancel{3} + \cancel{2} + 2\lambda - \cancel{3} - \lambda - \cancel{2} + 8\lambda}{\sqrt{\lambda^2 + 1 + 2\lambda + 4 + \lambda^2 + 4\lambda + 9 + \lambda^2 + 6\lambda}} = \frac{2}{\sqrt{3}}$

$\Rightarrow \frac{12\lambda}{\sqrt{3\lambda^2+12\lambda+14}} = \frac{2}{\sqrt{3}}$

Squaring on both sides

$\frac{\lambda^2}{3\lambda^2+4\lambda+14} = \frac{1}{3}$

$3\lambda^2 = 3\lambda^2 + 4\lambda + 14$

$4\lambda = -14$

$\lambda = \frac{-7}{2}$

Putting

$\lambda = \frac{-7}{2}$ in (1)

The required equation

$(x + 2y + 3z - 2) - \frac{7}{2}(x - y + z - 3) = 0$

$2x + 4y + 6z - 4 - 7x + 7y - 7z + 21 = 0$

$-5x + 11y - z + 17 = 0$

$5x - 11y + z - 17 = 0$

48) a) If z_1 = 2 + 5i, z_2 = -3 - 4i, and z_3 = 1 + i, find the additive and multiplicate inverse of z_1, z_2 and z_3

Answer : Given $z_1 = 2 + 5i$, $z_2 = -3 - 4i$ and $z_3 = 1 + i$

Additive inverse of z_1 is

$-z_1 = -(2 + 5i)$

$= -2 - 5i$

Multiplicative inverse of z_1 is

$\frac{1}{z_1} = \frac{1}{2+5i} \times \frac{2-5i}{2-5i}$

[Multiply and divide by the conjugate of denominator]

$= \frac{2-5i}{2^2-(5i)^2} = \frac{2-5i}{4-25^2} = \frac{2-5i}{4+25}$

$(z_1)^{-1} = \frac{1}{29}(2- 5i)$ $[\therefore i^2 = -1]$

Additive inverse of z_2 is

$-z_2 = -(3 - 4i)$

$= 3 + 4i$

Multiplicative inverse of z_2 is

$\frac{1}{z_2} = \frac{1}{-3-4i} \times \frac{-3+4i}{-3+4i}$

$= \frac{-3+4i}{(-3)^2-(4i)^2}$

$= \frac{-3+4i}{9-16i^2} = \frac{-3+4i}{9+16}$

$(z_2)^{-1} = \frac{1}{25}(-3 + 4i)$

Additive inverse of z_3 is

$-z_3 = -(1 + i)$

$= -1- i$

Multiplicative inverse of z_3 is

$\frac{1}{z_3} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1^2-(i^2)}$

$= \frac{1-i}{1+i}$

$(z_3)^{-1} = \frac{1}{2}(1 - i)$

(OR)

b) If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_2 + z_2z_3| = 6$

Answer : Given $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, $|z_1 + z_2 + z_3| = 1$

$|z_1|^2 = 1^2 \Rightarrow z_1 \overline{z_1} = 1 \Rightarrow z_1 = \frac{1}{\overline{z_1}}$

$|z_2|^2 = 4 \Rightarrow z_2 \overline{z_2} = 1 \Rightarrow z_2 = \frac{4}{\overline{z_2}}$

$|z_3|^2 = 9 \Rightarrow z_3 \overline{z_3} = 1 \Rightarrow z_3 = \frac{9}{\overline{z_3}}$

$\therefore \left|9, \frac{1}{\overline{z_1}}. \frac{4}{\overline{z_2}} + 4. \frac{1}{\overline{z_1}}. \frac{9}{\overline{z_3}} + \frac{4}{\overline{z_2}}. \frac{9}{\overline{z_3}}\right|$

$\left|\frac{36}{\overline{z_1\, z_2}} + \frac{36}{\overline{z_1\, z_3}} + \frac{36}{\overline{z_2\, z_3}}\right| = \left|36\left(\frac{\overline{z_3} + \overline{z_2} + \overline{z_1}}{\overline{z_1\, z_2\, z_3}}\right)\right|$

$[\because |\overline{z_1} + \overline{z_2} + \overline{z_3}| = |\overline{z_1 + z_2 + z_3}|]$

$= \frac{36|\overline{z_1 + z_2 + z_3}|}{|\overline{z_1}||\overline{z_2}||\overline{z_3}|} = 36 \frac{\overline{|\overline{z_1} + \overline{z_2} + \overline{z_3}|}}{|\overline{z_1}||\overline{z_2}||\overline{z_3}|}$

$[\because |\overline{z_1}| = |z_1|, |\overline{z_2} = |z21|, |\overline{z_3}| = |\overline{z_3}|]$

$= \frac{36(1)}{1(2)(3)} = \frac{36}{6} = 6$

$\therefore |9z_1 + z_2 + 4z_1z_3 + z_2z_3| = 6$

49) a) Solve the equations:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Answer : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

This equation is type I even degree reciprocal equation.

Hence, it can be rewritten as

$6\left(x^2 + \frac{1}{x}\right) - 35(x + \frac{1}{x}) + 62 = 0 \ldots (1)$

putting $x + \frac{1}{x} = y$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

$\therefore$ (1) becomes as,

$\Rightarrow 6(y^2 - 2) - 35y + 62 = 0$

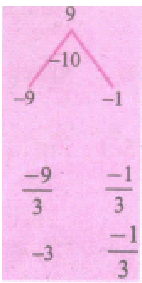
$\Rightarrow 6y^2 - 12 - 35y + 62 = 0$

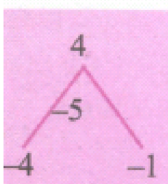
$\Rightarrow 6y^2 - 35y + 50 = 0$

$\Rightarrow (3y - 10)(2y - 5) = 0$

$\Rightarrow y = \frac{10}{3}, \frac{5}{2}$

Case (i) when $y = \frac{10}{3}, x + \frac{1}{x} = \frac{10}{3}$





$\Rightarrow \frac{x^2+1}{x} = \frac{10}{3}$

$\Rightarrow 3x^2 - 10x + 3 = 10x$

$\Rightarrow 3x^2 - 10x + 3 = 0$

$\Rightarrow (x - 3)(3x - 1) = 0$

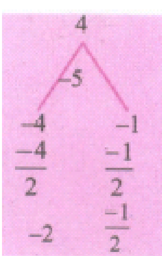
$\Rightarrow x = 3, \frac{1}{3}$

Case (ii) when $y = \frac{5}{2}, x + \frac{1}{x} = \frac{5}{2} \Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow 2x^2 + 2 = 5x \Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow (x - 2)(2x - 1) = 0$

Hence the roots are $2, \frac{1}{2}, 3, \frac{1}{3}$



(OR)

b) Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2 - 4y - 8x + 12 = 0$

Answer : $y^2 - 4y - 8x + 12 = 0$

$y^2 - 4y = 8x - 12$

Adding 4 both sides, we get,

$y - 4y + 4 = 8x - 12 + 4 = 8x - 8$

$\Rightarrow (y - 2)^2 = 8(x - 1)$

This is a right open parabola and latus rectum is $4a = 8 \Rightarrow a = 2$.

(a) Vertex is $(1, 2) \Rightarrow h = 1, k = 2$

(b) focus is $(h + a, 0 + k)$

$\Rightarrow (1 + 2, 0 + 2)$

$\Rightarrow (3, 2)$

(c) Equation of directrix is $x = h - a$

$\Rightarrow x = 1-2$

$\Rightarrow x = -1$

(d) Length of latus rectum is $4a = 8$ units.