N K MATHS ACADEMY

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UNIT TEST 2021-22

MATHEMATICS

LESSON-7

TIME: 1.30 HRS

I. CHOOSE THE BEST ANSWER: 10X1=10

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3/\sec$.The rate of change of its radius when radius is $\frac{1}{2}cm$

(1)3 cm/s (2)2 cm/s (3) 1 cm/s (4) $\frac{1}{2}cm/s$

2. The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest is

(1) t =0 (2) $t=\frac{1}{3}$ (3)t=1 (4)t=3

3. Find the point on the curve $6y=x^3+2$ at which y-coordinate changes 8 times as fast as x-coordinate is

(1) (4, 11) (2) (4,-11) (3) (-4, 11) (4) (-4,-11)

4. The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangents is -0.25?

(1) -8 (2) -4 (3) -2 (4) 0

5. The slope of the line normal to the curve $f(x)=2\cos 4x$ at $x=\frac{\pi}{12}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

6. What is the value of the limit $\lim_{x\to 0}\left(\cot x-\frac{1}{x}\right)$?

(1) 0 (2)1 (3) 2 (4) ∞

7. The maximum slope of the tangent to the curve $y=e^x\sin x, x\in[0,2\pi]$ is at

(1) $x=\frac{\pi}{4}$ (2) $x=\frac{\pi}{2}$ (3) $x=\pi$ (4) $x=\frac{3\pi}{2}$

8. One of the closet point on the curve $x^2-y^2=4$ to the point (6, 0) is

(1) (2, 0) (2) $\left(\sqrt{5},1\right)$ (3) $\left(3,\sqrt{5}\right)$ (4) $\left(\sqrt{13},-\sqrt{3}\right)$

9. The curve $y=ax^4+bx^2$ with ab>0

(1) has no horizontal tangent (2)is concave up

(3) is concave down (4)has no points of inflection

10. The point of inflection of the curve $y=(x-1)^3$ is

(1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

II. ANSWER ANY 5 QUESTIONS: 5X2=10

11. The temperature T in Celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by $T = x(10-x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero

12. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

13. Find the slope of the tangent to the curve $y = x^4 + 2x^2 - x$ at x =1.

14. Compute the limit $\lim_{x\to a} \frac{x^n - a^n}{x-a}$

15. Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

16. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2,\infty)$

17. Find the local extremum of the function $f(x) = x^4 + 32x$

III. ANSWER ANY 5 QUESTIONS: 5X3=15

18. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100\times(1-0.1t)^2$, $0 \le t \le 10$ What is the rate at which the Person forgets the words 2 days after learning?

19. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$

20. Find the tangent and normal to $x = cost, y = 2\sin^2 t$ at $t = \frac{\pi}{3}$

21. Evaluate: $\lim_{x\to 1^+}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

22. Find the absolute extrema of the function $f(x) = 3cosx$ on the closed interval $[0, 2\pi]$

23. Find the absolute extrema of $f(x) = 3x^4 - 4x^3, [-1, 2]$

IV ANSWER ANY 3 QUESTONS: 3X5=15

24. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

25. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0, 0), (1, 1)

26. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

27 . Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm