SAIVEERA ACADEMY'S GUIDE

VOL -I

(2022-2023 EDITION)

It includes

- ✓ Complete Book Back Answers
- ✓ Knowledge Based Questions
- ✓ Knowledge Based One Marks
- ✓ Additional Book Inside Questions

UNIT – 1 ELECTROSTATICS

Important Formulae

1. Electrostatic force between charges q_1 and q_2, $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21}$ (Vector form)

2. Value of k = $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$

3. Value of $\varepsilon_o = 8.854 \times 10^{-12} C^2N^{-1}m^{-2}$

4. Total charge $q = n \times e$;

5. Components of force F. $F_1 =$ F $\cos\theta$; $F_1 =$ F $\sin\theta$; $|F| = \sqrt{F_1^2 + F_2^2}$

6. Relative permittivity or Dielectric constant $\varepsilon_r = \frac{\varepsilon(\text{permittivity of medium})}{\varepsilon_0(\text{permittivity of vaccum})}$

7. Force between charges in medium

$$F_m = \frac{F_{air}}{\varepsilon_r}$$

8. Electrostatic field, $E = \frac{force}{charge} = \frac{F}{q} \Longrightarrow$

$F = qE$

9. Electric field due to a point charge

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

10. Electric dipole moment, $\vec{p} = q \times 2a\hat{\imath}$

11. (i) Electric field due to a dipole at a point on the axial line, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} (r \gg a)$

Direction of electric field : parallel to direction of dipole moment

(ii) Electric field due to a dipole at a point on the equatorial line

$\vec{E} = \frac{-1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3} (r \gg a)$

Direction of electric field : anti parallel to direction of dipole moment

12. Magnitude of torque

$\tau = \vec{p} \times \vec{E} = pE\sin\theta\ (p = q \times 2a)$

13. Electric potential at a point due to a point charge, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

14. Electric potential energy of dipole

$U = -pE\cos\theta = -\vec{p}.\vec{E}$

15. Electric potential at a point due to an electric dipole $V = \frac{p}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}$

16. Electric flux $= \frac{q}{\varepsilon_0} \Longrightarrow \phi_E = \vec{E}.\vec{A} = EA\cos\theta$

17. Electric field due to infinite long straight charged wire, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

18. Electric field due to plane sheet of charge $E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{A}\frac{1}{2\varepsilon_0}$

19. Electric field at a point between two parallel sheets of charge $E = \frac{\sigma}{\varepsilon_0}$

Electric field at a point outside two parallel sheets of charge $E = 0$

20. Electric field due to a uniformly charged sphere –

(i) at a point on the surface of the sphere, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{r}$

(ii) at a point outside the sphere

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

(iii) at a point inside the sphere $E = 0$

21. Capacitance of a conductor $C = \frac{q}{V}$

22. Work done by a charge $W = qV$

23. Surface Charge density, $\sigma = \frac{q}{A}$

Linear charge density $= \frac{q}{l}$

24. Capacitance of a parallel plate

capacitor $C = \frac{\varepsilon_0 A}{d}$

(i) with a dielectric slab, $C = \frac{\varepsilon_0 A}{\left[(d-t)+\frac{t}{\varepsilon_r}\right]}$

(ii) with the dielectric completely filled

the capacitor $C^1 = \frac{\varepsilon_0 \varepsilon_r A}{d} = C \times \varepsilon_r$

25. Energy stored in a capacitor

$E = \frac{1}{2}CV^2 \text{ or } \frac{1}{2}qV \text{ or } \frac{1}{2C}Q^2$

26. Capacitance of a spherical capacitor,

$C = 4\pi\varepsilon_0 A \text{ or } C = \frac{A}{9\times10^9}$

27. Equivalent capacitance

(i) C_1 and C_2 in series

$C_s = \frac{C_1 C_2}{C_1+C_2}$; $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

Q -same V- different

(ii) C_1 and C_2 in parallel

$C_p = C_1 + C_2$

Q - different V- same

Values and units

1. Permittivity of free space

$\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

2. $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$

3. Charge of an electron $e = 1.6 \times 10^{-19} C$

4. 1 micro farad = 10^{-6} farad

5. 1 pico farad = 10^{-12} farad

6. Permititivity of medium $\varepsilon = C^2N^{-1}m^{-2}$

7. Electric charge (q) = Coulomb (C)

8. Electric field (E) = NC^{-1} or Vm^{-1}

9. Electric potential (V) = JC^{-1} or volt

10. Electric dipole moment (p) = Cm

11. Electric potential energy (U) = Joule

12. Capacitance (C) = farad

13. Electric flux = Nm^2C^{-2}

14. Torque = Nm

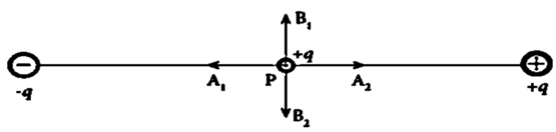
15. Relative permittivity of air = 1(no unit)

16.Linear charge density λ = C/m

17. Surface charge density σ = Cm^{-2}

Textbook One Marks Solved

1.Two identical point charges of magnitude –q are fixed as shown in the figure below. A third charge +q is placed midway between the two charges at the point P. Suppose this charge +q is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will +q be stable with respect to the displacement?



(a) A_1 and A_2 (b) B_1 and B_2 (c) both directions (d) No stable

Hint : θ between $B_1 \& B_2$ is 90°

$V = \frac{p}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}$ which implies V = 0

Therefore $W = V.q = 0$

+q will be stable along B_1 and B_2

(b) B_1 and B_2

2. Which charge configuration produces a uniform electric field?

(a) point Charge

(b) infinite uniform line charge

(c) uniformly charged infinite plane

(d) uniformly charged spherical shell

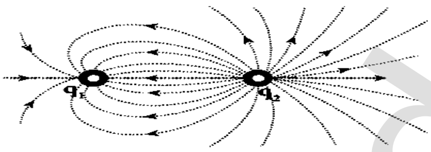
Hint : Electric field due to point charge , line charge , charged spherical are dependant on distance as they contain r term , where as electric field due to uniformly charged infinite plane ($E = \frac{\sigma}{2\varepsilon_0}$) does not contain r term

(c) uniformly charged infinite plane

3. What is the ratio of the charges $\left|\frac{q_1}{q_2}\right|$ for the following electric field line pattern?

(a) 1 /5 (b) 25/ 11

(c) 5 (d) 11/ 25



Hint : No of lines entering q_1 = 11 , No of lines entering q_2 = 25

$$\left|\frac{q_1}{q_2}\right| = \frac{11}{25}$$

(d) 11/ 25

4. An electric dipole is placed at an alignment angle of 30° with an electric field of 2 × 10^5 N C^{-1}. It experiences a torque equal to 8 N m. The charge on the dipole if the dipole length is 1 cm is

(a) 4 mC (b) 8 mC

(c) 5 mC (d) 7 mC

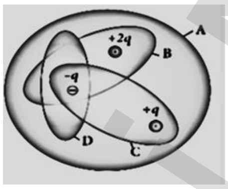
Hint : $\tau = pE\sin\theta = 2qa\,(E\sin\theta)$

Given $\theta = 30°\,, E = 2\times10^5\,, \tau = 8\,, 2a = 1\times10^{-2}m$

$$q = \frac{\tau}{2a\,E\sin\theta} = 8mC$$

(b) 8 mC

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.



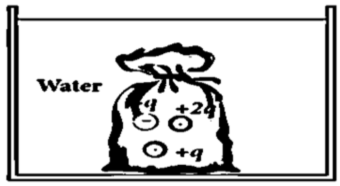
(a) D < C < B < A (b) A < B = C < D

(c) C < A = B < D (d) D > C > B > A

Hint : Net charge of A = $3q - q = 2q$ similarly net charge on B , C and D is q , 0 , -q ; Flux $\phi_E = \frac{q}{\varepsilon_0}$

(a) D < C < B < A

6. The total electric flux for the following closed surface which is kept inside water



(a) $\frac{80q}{\varepsilon_o}$ (b) $\frac{q}{40\varepsilon_o}$

(c) $\frac{q}{80\varepsilon_o}$ (d) $\frac{q}{160\varepsilon_o}$

Hint : Net charge = -q + q + 2q = 2q ε_r of water is 80

$$\phi_{net} = \frac{q}{\varepsilon_r\,\varepsilon_0} = \frac{q}{40\varepsilon_o}$$

(b) $\frac{q}{40\varepsilon_o}$

7. Two identical conducting balls having positive charges q_1 and q_2 are separated by

a center to center distance r. If they are made to touch each other and then separated to the same distance, the force between them will be

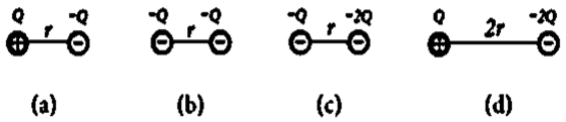
(a) less than before
(b) same as before
(c) more than before
(d) zero

Hint : We all know that force is directly proportional to charge

When they are separated , the magnitude of charge will be increased

(c) more than before

8. Rank the electrostatic potential energies for the given system of charges in increasing order.



(a) 1 = 4 < 2 < 3 (b) 2= 4 < 3< 1
(c) 2 = 3 < 1 < 4 (d) 3 < 1 < 2 < 4

Hint : U $= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

For a : U= $\frac{1}{4\pi\varepsilon_0}\frac{-Q^2}{r}$, For b : U= $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r}$

For c : U= $\frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{r}$, For d : U= $\frac{1}{4\pi\varepsilon_0}\frac{-Q^2}{r}$

(a) 1 = 4 < 2 < 3

9. An electric field $\vec{E} = 10x\hat{\imath}$ exists in a certain region of space. Then the potential difference V = $V_o - V_A$, where Vo is the potential at the origin and V_A is the potential at x = 2 m is:

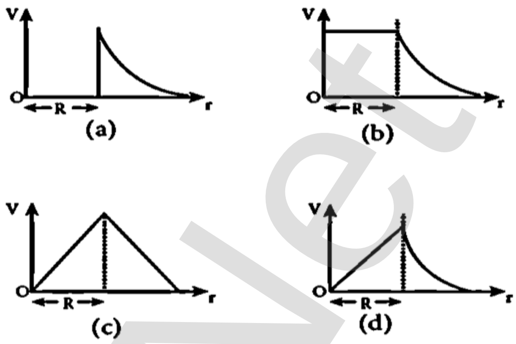
(a) 10 V (b) – 20 V
(c) +20 V (d) -10 V

Hint : $E = -\frac{dv}{dx}$ $dv = -10x\,dx$

On integration $V_o - V_A$ = 20J

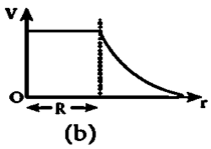
(c) +20 J

10. A thin conducting spherical shell of radius R has a charge Q which is uniformly distributed on its surface. The correct plot for electrostatic potential due to this spherical shell is



Hint : For option b potential decreased outside the spherical shell as distance increases

(b)



11. Two points A and B are maintained at a potential of 7 V and -4 V respectively. The work done in moving 50 electrons from A to B is

(a) 8.80×10^{-17} J (b) -8.80×10^{-17} J
(c) 4.40×10^{-17} J (d) 5.80×10^{-17} J

Hint : $Potenial\ V = 7 - (-4) = 11V$

$w = q.V = (ne)V$

$= 50 \times 1.6 \times 10^{-19} \times 11$

$= 8.8 \times 10^{-17} J$

(a) 8.80×10^{-17} J

12. If voltage applied on a capacitor is increased from V to 2V, choose the correct conclusion.

(a) Q remains the same, C is doubled
(b) Q is doubled, C doubled
(c) C remains same, Q doubled
(d) Both Q and C remain same

Hint : $Q = CV$ i.e $Q \propto V$ when V changes Q changes

But $C = \frac{\varepsilon_0 A}{d}$ (It does not contain any V term)

(c) C remains same, Q doubled

13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?

(a) Capacitance (b) Charge

(c) Voltage (d) Energy density

Hint : Area A changes to 2A , distance d changes to 2d

Capacitance $C = \frac{\varepsilon_0 A}{d} = C = \frac{\varepsilon_0 2A}{2d} = \frac{\varepsilon_0 A}{d}$ (same)

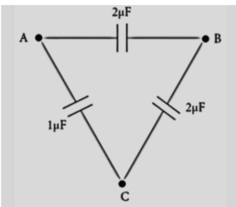
Charge $Q \propto V$ V remains same so Q remains same

Potential $V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} = \frac{Q \times 2d}{\varepsilon_0 2A} = \frac{Qd}{\varepsilon_0 A}$ (same

$U_E = \frac{1}{2}\varepsilon_o E^2 Ad = \frac{1}{2}\varepsilon_o E^2 \times 2A \times 2d$ (changes by four times)

(d) Energy density

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the points A and C is



(a) 1μF (b) 2 μF

(c) 3 μF (d)1/4 μF

Hint :

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{2} = 1$

$C_p = C_1 + C_s = 1 + 1 = 2$μF

(b) 2 μF

15.Two metallic spheres of radii 1 cm and 3 cm are given charges of -1 x 10^{-2} C and 5 x 10^{-2} C respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is

(a) 3 × 10^{-2} C

(b) 4 × 10^{-2} C

(c) 1 × 10^{-2} C

(d) 2 × 10^{-2} C

Hint : $V = \frac{KQ}{r}$; $V_1 = \frac{KQ_1}{3}$ $V_2 = \frac{KQ_2}{1}$

$V_1 = V_2$ which implies $3Q_2 = Q_1$

$Q_1 + Q_2 = 4 \times 10^{-2} C$

On solving

(a) 3 × 10^{-2} C

Creative One Marks Solved

1. Three point charges each +q are placed at the corner of an equilateral triangle of side a the electric field at the circumference coil be

a) $\frac{q}{4\pi\varepsilon_0 r}$ b) $\frac{q}{4\pi\varepsilon_0 r^2}$

c) q/r^2 **d) Zero**

2. If electric field is through an area 3 m^2 saying in 1/2 plane then the electric flux due to the electric field in a region is given by $\vec{E} = 3\vec{\imath} + 4\vec{\jmath}\ NC^{-1}\ is$ _____ units

a) 3 **b) 9**

c) 18 d) 6

Solution:

$$\phi_E = \vec{E}.\vec{A} = (3\vec{\imath} + 4\vec{\jmath}).3i = 9$$

3. Two charged conducting spheres of radii $R_1\ and\ R_2$ separated by a large distance are connected by a wire. The ratio of the charges on them is

a) $\frac{R_2}{R_1}$ **b) $\frac{R_1}{R_2}$**

c) $\frac{R_1^2}{R_2^2}$ d) $\sqrt{\frac{R_1}{R_2}}$

Solution:

Since the two spheres are connected they have the same potential.

$$\frac{q_1}{4\pi\varepsilon_0 R_1} = \frac{q_2}{4\pi\varepsilon_0 R_2}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

4. The effective capacitance of a system in which n identical capacitors having capacitance 'C' are connected in series is

a) nC b) $\frac{C}{n^2}$

c) $\boldsymbol{\frac{C}{n}}$ d) n^2C

5. The equivalent capacitance between A and B in the following circuit is

a) $38\ \mu F$ **b)** $\boldsymbol{19\ \mu F}$

c) $9\ \mu F$ d) $11\mu F$

Solution:

$C_p = C_1 + C_2 + C_3$

$\quad = 3 + 3 + 13 = 19\ \mu F$

6. A charged oil drop is suspended in uniform field of $3 \times 10^4\ V/m$ So that it neither falls nor rises. If mass of the charge is 9.9×10^{-15} kg then its charge is _________$[g = 10\ m/s^2]$

a) $6.6 \times 10^{-18}\ C$ **b)** $\boldsymbol{3.3 \times 10^{-18}\ C}$

c) $13.2 \times 10^{-18}\ C$ d) $1.65\times 10^{-18}\ C$

Solution:

In steady state

Electric force on drop = weight of drop

$qE = mg$

$$q = \frac{mg}{E} = \frac{9.9 \times 10^{-15} \times 10}{3 \times 10^4}$$

$$= \mathbf{3.3 \times 10^{-18}\ C}$$

7. Consider a neutral conducting sphere. A positive charge is placed outside the sphere. The net charge on sphere is then

a) negative charge is distributed uniformly over the surface of the sphere.

b) negative charge is not distributed uniformly over the surface of the sphere.

c) negative charge appears only at the point charg

d) Zero

Solution:

When a point charge is placed outside the neutral conducting sphere, the net charge is zero.

8. Two capacitors of equal capacitance are first connected in parallel and then in series. The ratio of the total capacities in two cases would be

a) 1:2 b) 1:4

c) 4:1 d) 2:1

Solution:

$C_p = 2\ C$

$C_s = \frac{C}{2}$

$$\frac{C_p}{C_s} = \frac{2C}{\frac{C}{2}} = 4$$

$C_p/C_s = 4:1$

9. Van de Graff generator works on the principle of :

(a) electromagnetic induction and action at points

(b) electrostatic induction and action at points

(c) electrostatic induction only

(d) action at points only

10. In the figure the distance of the point from A where the electric field is zero is

a) 20 cm **b) 36 cm**

c) 10 cm d) 54 cm

Solution:

$$\frac{1}{4\pi\varepsilon_0} \times \frac{(10 \times 10^{-6})}{x^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{(20 \times 10^{-6}) \times 1}{(0.8 - x)^2}$$

$\therefore 2x^2 = (0.8 - x)^2$

$1.414x = 0.8 - x$

$x = \frac{0.8}{2.2} = 0.36\,m = \mathbf{36\ cm}$

11. A long hollow, conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.

a) A potential difference appears between two cylinders when a charge density is given to the inner cylinder.

b) A potential difference appears between two cylinders when a charge density is given to the outer cylinder.

c) No potential difference between two cylinders when a line charge is kept along the axis of the cylinder.

d) No potential difference between two cylinders when same charge density is given to both cylinders.

Solution:

Electric field between two cylinders is

$$E = \frac{q}{2\pi\varepsilon_0 rL}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 r} \qquad [q = \lambda L]$$

12. A bullet of mass of 2 g has a charge of $2\mu C$. The potential difference through which it must be accelerated from rest to acquire a speed of $10ms^{-1}$ is

a) 5V b) 50 V

c) 5 kV **d) 50 k V**

Solution:

$$W = KE$$

$$qV = \frac{1}{2}mv^2$$

$$V = \frac{m^2v^2}{2q} = \frac{2\times10^{-3}\times10^2}{2\times2\times10^{-6}} = \frac{10^5}{2} = 50\ kV$$

13. A point charge +q is placed at the origin O as shown in the figure. Work done in taking another point charge –Q from the point A to another point B along the straight line path AB is

a) $\frac{qQ}{4\pi\varepsilon_0 a^2}\left(\frac{a}{\sqrt{2}}\right)$ b) $\left(\frac{-qQ}{4\pi\varepsilon_0}\right)\sqrt{2a}$

c) $\left(\frac{qQ}{4\pi\varepsilon_0 a^2}\right)\sqrt{2a}$ **d) Zero**

Solution:

Initial PE, $E_i = \frac{q(-Q)}{4\pi\varepsilon_0 a}$:Final PE, $E_f = \frac{q(Q)}{4\pi\varepsilon_0 a}$

Work done – change in PE $=E_f - E_i = 0$

14. Three capacitors each of capacitance C and of break down voltage V are joined in series. The capacitance and breakdown voltage of the combination will be

a) 3C, V **b) $\frac{C}{3}$, 3V**

c) 3C, 3V d) $\frac{C}{3}$, $\frac{V}{3}$

Solution:

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$C_s = \frac{C}{3}$$

$$V_s = V_1 + V_2 + V_3 = V + V + V = 3V$$

$$\therefore V_s = 3V$$

15. The work done in placing a charge $8\times10^8\ C$ on a capacitor of capacitance $100\ \mu F$ is

a) $16\times10^{-32}\,J$ b) $4\times10^{-16}\,J$

c) $64\times10^{-32}\,J$ d) $32\times10^{-16}\,J$

Solution:

$$W = Q\times V = Q = \frac{Q}{C} = \frac{Q^2}{C}$$

$$W = \frac{(8\times10^{-18})^2}{100\times10^{-6}} = \mathbf{64\times10^{-32}\,J}$$

16. An electric dipole is placed at angle of 30° with an electric field of

$2 \times 10^5\ NC^{-1}$. Its length is 2 cm and if it experiences a torque about 4 Nm then its charge is _________ mC

a) 8 **b) 2** c) 5 d) 7

Solution:

$2l = 2 \times 10^{-2} m \qquad \theta = 30°$

$\tau = 4\ Nm$

$\tau = PE\sin\theta = (q \times 2l) \times E \times \sin\theta$

$$q = \frac{z}{(2l) \times E \times \sin\theta} = \frac{4}{2 \times 10^{-2} \times 2 \times 10^5 \times \sin 30°} = 2 \times 10^{-3} = 2\ mC$$

17. In a region potential is $V = (6x - 8xy - 8y + 6yz)V$ in V and $x, y\ and\ z$ are in m. The electric force experienced by a charge of $2C$ situated at (1,1,1) is

a) 24 N **b) $4\sqrt{35}$ N**

c) $6\sqrt{5}$ N d) 30 N

Solution:

$$\vec{E} = -\left[\left(\frac{\partial V}{\partial x}\right)\hat{\imath} + \left(\frac{\partial V}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial V}{\partial z}\right)\hat{k}\right]$$

$V = 6x - 8xy - 8y + 6yz$

At point (1,1,1)

$$E_y = -\frac{\partial V}{\partial y} = -[-8x - 8 + 6z]$$
$$= -(-8 - 8 + 6) = +10$$

At point (1,1,1)

$$E_z = -\frac{\partial V}{\partial z} = -[+6y] = -6(1) = -6$$

$$|\vec{F}| = q|\vec{E}| = 2\sqrt{(2)^2 + (10)^2 + (-6)^2}$$
$$= 2\sqrt{4 + 100 + 36}$$
$$= 2\sqrt{140} = 4 \times \sqrt{35}\ N$$
$$\vec{F} = 4\sqrt{35}\ N$$

18. A force of 0.01 N is exerted on a charge of $1.2 \times 10^{-5} C$ at a certain point. The electric field at that point is

a) $5.3 \times 10^4 NC^{-1}$ b) $8.3 \times 10^4 NC^{-1}$

c) $5.3 \times 10^{-4} NC^{-1}$ **d) $8.3 \times 10^2 NC^{-1}$**

Solution:

$$E = \frac{F}{q} = \frac{0.01}{1.2 \times 10^{-5}} = 8.3 \times 10^2 NC^{-1}$$

19. The electric field intensity at a point 20 cm away from a charge of $2 \times 10^{-5} C$ is

a) $4.5 \times 10^6\ NC^{-1}$ b) $3.5 \times 10^5 NC^{-1}$

c) $3.5 \times 10^6\ NC^{-1}$ d) $4.5 \times 10^5\ NC^{-1}$

Solution:

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9 \times 2\times10^{-5}}{(0.2)^2} = 4.5 \times 10^6\ NC^{-1}$$

20. How many electrons will have a charge of one coulomb?

a) 6.25×10^{18} b) 6.25×10^{19}

c) 1.6×10^{18} d) 1.6×10^{19}

Solution:

Number of electron, $n = \frac{q}{e} = \frac{1}{1.6\times10^{-19}} = 6.25 \times 10^{18}$

21. The ratio of the force between two charges in air and that in a medium of dielectric constant K is

a) $K: 1$ b) $1: K$ c) $K^2: 1$ d) $1: K^2$

22. At a large distance (r), the electric field due to a dipole varies as

a) $\frac{1}{r}$ b) $\frac{1}{r^2}$ **c) $\frac{1}{r^3}$** d) $\frac{1}{r^4}$

23. Two isolated, charged conducting spheres of radii $R_1\ and\ R_2$ produce the same electric field near their surfaces. The ratio of electric potentials on their surfaces is

a) $\frac{R_1}{R_2}$ b) $\frac{R_2}{R_1}$ c) $\frac{R_1^2}{R_2^2}$ d) $\frac{R_2^2}{R_1^2}$

24. A 1 μF c capacitor is placed in parallel with a 2 μF capacitor across a 100 V supply. The total charge on the system is

a) $\frac{100}{3}\mu C$ b) $100\ \mu C$

c) $150\ \mu C$ **d) $300\ \mu C$**

Solution:

Equivalent capacitor $= 1 + 2 = 3\ \mu F$

Total charge, $q = CV = 3 \times 100 = 300\ \mu C$

25. A parallel plate capacitor of capacitance 100 μF is charged to 500 V. The plate separation is then reduce to half

its original value. Then the potential on the capacitor becomes

a) 250 V b) 500 V c) 1000 V d) 2000 V

Solution:

Here, $C' = 2C$, since the charge remains the same.

$q = C'V' = CV \quad => V' = \frac{CV}{2C} = \frac{500}{2} = 250\ V$

26. A point charge q is rotating around a charge Q in a circle of radius r. The work done on it by the coulomb forces is

a) $2\pi rq$ b) $2\pi Qq$ c) $\frac{Q}{2\varepsilon_0 r}$ **d) Zero**

27. Capacitance of a parallel plate capacitor can be increased by

a) increasing the distance between the plates
b) increasing the thickness of the plates
c) decreasing the thickness of the plates
d) decreasing the distance between the plates

28. Two charges are placed in a vacuum at a distance d apart. The force between them is F. if a medium of dielectric constant 2 is introduced between them, the force will now be

a) 4 F b) 2 F c) $\frac{F}{2}$ **d) $\frac{F}{4}$**

29. An electric charge is placed at the centre of a cube of side a. The electric flux through one of its faces will be

a) $\frac{q}{6\varepsilon_0}$ b) $\frac{q}{\varepsilon_0 a^2}$ c) $\frac{q}{4\pi\varepsilon_0 a^2}$ d) $\frac{q}{\varepsilon_0}$

Solution:

According to Gauss's law, the electric flux through the cube is $\frac{q}{\varepsilon_0}$. since there are six faces, the flux through one face is $\frac{q}{6\varepsilon_0}$.

30. Total electric flux coming out of a unit positive charge put in air is

a) ε_0 **b) ε_0^{-1}** c) $(4\pi\varepsilon_0)^{-1}$ d) $4\pi\varepsilon_0$

31. A point Q lies on the perpendicular bisector of an electric dipole of dipole moment P. if the distance of Q from the dipole is r, then the electric field at Q is proportional to

a) p^{-1} and r^{-2} b) p and r^{-2}
c) p and r^{-3} d) p^2 and r^{-3}

32. A particle of charge q is placed at rest in a uniform electric field E and then released. The kinetic energy attained by the particle after moving a distance y is

a) qE^2y b) q^2Ey c) qEy^2 **d) qEy**

Solution:

Force on the particle $= qE$

KE = work done by the force $= F.y = qEy$

33. Twenty seven water drops of the same size are charged to the same potential. If they are combined to form a big drop, the ratio of potential of the big drop to that of a small drop is

a) 3 b) 6 **c) 9** d) 27

Solution:

$$V' = n^{2/3}V \quad => \frac{V'}{V} = (27)^{2/3} = 9$$

34. A point charge +q is placed at the midpoint of a cube of side l. The electric flux emerging from the cube is

a) $\frac{q}{\varepsilon_0}$ b) $\frac{6ql^2}{\varepsilon_0}$ c) $\frac{q}{6\varepsilon_0 l^2}$ d) $\frac{C^2V^2}{2}$

35. If the electric field in a region is given by $\vec{E} = 5\hat{\imath} + 4\hat{\jmath} + 9\hat{k}$. then the electric flux through a surface of area 20 units lying in the y – z plane will be

a) 20 un its b) 80 units **c) 100 units** d) 180 units

Solution:

The area vector $\vec{A} = 20\,\hat{\imath}$; $\vec{E} = (5\hat{\imath} + 4\hat{\jmath} + 9\hat{k})$

Flux $(\phi) = \vec{E}.\vec{A} = 5 \times 20 = 100$ *units*

36.An electric dipole is placed in a uniform electric field with its axis parallel to the field. It experiences

a) only a net force

b) neither a net force nor a torque
c) both a net force and torque
d) only a torque

37. The work done in moving 4μC charge from one point to another in an electric field is 0.012J. The potential difference between them is
a) 3000 V b)6000 V c)30 V d) 48 x 10^3 V
Solution:
$V = \frac{w}{q} = \frac{0.012}{4\times10^6} = 3000$

38.The electric field outside the two oppositely charged place sheets each of charge density σ is
a) $\frac{\sigma}{2\varepsilon_0}$ b) $-\frac{\sigma}{\varepsilon_0}$ c) $\frac{\sigma}{\varepsilon_0}$ **d) zero**
Solution:
$E = \frac{\sigma}{\varepsilon_0} + \left(-\frac{\sigma}{\varepsilon_0}\right) = 0$

39. Torque on a dipole in a uniform electric field is maximum when angle between P and E is
a)0° **b) 90°** c) 45° d) 180°
Solution:
$\tau = pE\sin\theta$
$\sin\theta$ will be maximum when it is 90°

40.Potential energy of two equal negative point charges of magnitude 2μC placed 1 m apart in air is
a) 2 J b) 0.36 J c)4 J **d) 0.036 J**
Solution:
$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{9\times10^9 \times 2\times10^{-6}\times2\times10^{-6}}{1} = .036$

41.A hollow metallic spherical shell carrying an electric charge produces no electric field at points
a) on the surface of the sphere
b) inside the sphere
c) at infinite distance from the centre of the sphere
d) outside the sphere

42. Electric potential energy of an electric dipole in an electric field is given as
a) pEsin θ b) –pEsin θ
c) –pEcos θ d) pEcos θ

43. The direction of electric field at a point on the equatorial line due to an electric dipole is
a) along the equatorial line towards the dipole
b) along the equatorial line away from the dipole
c) parallel to the axis of the dipole and opposite to the direction of dipole moment
d) parallel to the axis of the dipole and in the direction of dipole moment.

43.The number of electric lines of force originating from a charge of 1 micro coulomb is
a) 1.129 x 10^5 b) 1.6 x 10^{-1}
c) 6.25 x 10^{18} d) 8.85 x 10^{-12}
Solution:
$N = \frac{q}{\varepsilon_0} = 1\times10^{-6}\times1.29\times10^{11}$ =1.129 x 10^5

44. An electric dipole place at an angle θ in a non- uniform electric field experiences
a) neither a force nor a torque
b) torque only
c) both force and torque
d) force only

45. A capacitor of capacitance 6 μF is connected to a 100 V battery. The energy stored in the capacitor is
a) 30 J b) 3J **c) 0.03 J** d) 0.06 J
Solution:
$E = \frac{1}{2}CV^2 = \frac{1}{2}\times6\times10^{-6}\times100^2 = 0.03$ J

46.The capacitance of a parallel Plate capacitor increases from 5μF to 60 μF when a dielectric is filled between the plates. The dielectric constant of dielectric is

a) 65 b) 55 **c) 12** d) 10

Solution:

$C_m = C \times \varepsilon_r$

$\varepsilon_r = \frac{C_m}{C} = \frac{60}{5} = 12$

47. The magnitude of the force acting on a charge of 2×10^{-10} C placed in a uniform electric field of $10 Vm^{-1}$ is

a) 2×10^{-9} N b) 4×10^{-9} N

c) 2×10^{-10} N d) 4×10^{-10} N

Solution:

$F = qE = 2 \times 10^{-10} \times 10 = 2 \times 10^{-9}$

48.Electric potential energy (U) of two point charges is

a) $q_1q_2 / 4\pi \varepsilon_o r^2$ **b) $q_1q_2 / 4\pi \varepsilon_o r$**

c) pE cosθ d) pE sinθ

49. The capacitance of a parallel plate capacitor increases from 5 μF to 50 μF when a dielectric is filled between the plates. The permittivity of the dielectric is

a) $8.854 \times 10^{-12} C^2N^{-1}m^{-2}$ **b) $8.854 \times 10^{-11} C^2N^{-1}m^{-2}$**

c) 12 d) 10

Solution:

$\varepsilon = \varepsilon_o \varepsilon_r = \varepsilon_o \frac{C_m}{C} = 8.854 \times 10^{-12} \times \frac{50}{5} = 8.854 \times 10^{-11}$

50. The negative gradient of potential is

a) electric force b) torque

c) electric current **d) electric field intensity**

Solution:

$$E = -\frac{dV}{dx}$$

51.When a point charge of 6μC is moved between two points in an electric field, the work done is1.8×10^{-5} J. The potential difference between the two points is

a) 1.08 V b) 1.08 mV **c) 3V** d) 30 V

Solution:

$V = \frac{w}{q} = \frac{1.8 \times 10^{-5}}{6 \times 10^{-6}} = 3V$

52. The electric field intensity at a distance r due to infinitely long straight charged wire is directly proportional to

a) r **b) 1/r** c) r^2 d) $1/r^2$

Solution:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

53.The ratio of electric potential at points 10 cm and 20 cm from the centre of an electric dipole along its axial line is

a) 1 : 2 b) 2 : 1 c) 1 : 4 **d) 4 : 1**

Solution:

$V = \frac{p}{4\pi\varepsilon_0} \frac{\cos\theta}{r^2}$

$\frac{V_1}{V_2} = \frac{r_2^2}{r_1^2} = \frac{20^2}{10^2} = 4:1$

54. The intensity of the electric field that produces a force of 10^{-5} N on a charge of 5μC is

a) $5 \times 10^{-11} NC^{-1}$ b) $50 NC^{-1}$

c) $2 NC^{-1}$ d) $0.5 NC^{-1}$

Solution:

$E = \frac{F}{q} = \frac{10^{-5}}{5 \times 10^{-6}} = 2$

55.The unit of the number of electric lines of force passing through a given area is

a) no unit b) NC^{-1}

c) Nm^2C^{-1} d) Nm

56. Two point charges +q and –q are placed at points A and B respectively separated by a small distance. The electric field intensity at the midpoint O of AB

a) is zero

b) acts along AB

c)acts along BA

d) acts perpendicular to AB

57.The work done in rotating the dipole through an angle of 90° is :

a) zero b) – PE **c) PE** d) 2PE

Solution:

Work done $= PE(1-\cos\theta)$

$\theta = 90°$ w$= PE$

58. The total flux over a closed surface enclosing a charge q (in $Nm^2 C^{-1}$)

a) 8πq b) 9×10^9q

c) $36\pi \times 10^9$q d) 8.854×10^{-12} q

Solution:

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{q}{1/4\pi \times 9 \times 10^9} = 36\pi \times 10^9 q$$

59.What must be the distance between two equal and opposite point charges (say +q and –q) for the electrostatic force between them to have a magnitude of 16 N?

a) $\frac{q}{16}\sqrt{K}$metre **b) $\frac{q}{4}\sqrt{K}$metre**

c) 4 kq metre d) 4k / q metre

Solution:

$$F = K\frac{q_1 q_2}{r^2} = \frac{Kq^2}{4}$$

$$r = \frac{q}{4}\sqrt{K}$$

Short Answers Questions

Book Back

1.What is Quantisation of charges?

- ✓ The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.
- ✓ **q = ne** n is any integer (0, ±1, ±2, ±3, ±4……….).
- ✓ This is called quantisation of electric charge.

2. Write down coulomb's law in vector form & mention what each term represents

According to Coulomb, the force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

$\hat{r}_{21}$ - the unit vector directed from charge q_2 to charge q_1

r^2 - the distance between charges

k - proportionality constant

3. What are the difference between coulomb force and gravitational force?

Gravitational force	Coulomb force
Force between two masses is always attractive	Force between two charges can be attractive or repulsive, depending on the nature of charges
Force between two masses is independent of the medium	Force between the two charges depends on nature of the medium
The value of the gravitational constant **G =** 6.626×10^{-11} Nm^2Kg^{-2}	The value of the constant k in Coulomb law is **k = 9×10^9 Nm^2C^{-2}**
Force between two point masses is the same whether two	Force between two point charges will change with respect to motion

masses are at rest or in motion.	

4. Write a short note on Superposition principle

The total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \ldots \vec{F}_{1n}$$

5. Define Electric Field

The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by $\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

Quantity ; Vector quantity

Unit ; NC^{-1}

6.What is meant by Electric field lines

Electric field vector are visualized by the concept of electric field lines . They form a set of continuous lines which represent the electric field in some region of space.

7. The electric field never intersect. Justify

If some charge placed in intersection point then it has to move in two different direction at the same time, which is physically impossible .Hence Electric field lines do not intersect

8. Define electric dipole. Give the expression for the magnitude of its electric dipole moment and its direction

Two equal and opposite charges separated by a small distance constitute an electric dipole.

Dipole moment p = 2qa

Direction of dipole moment : from -q to +q

9.Write the general definition of electric dipole moment for a collection of point charge

It is product of any one of charges of dipole and distance(2d) between them

$$\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i$$

Quantity ; Vector quantity **Unit :** Cm

10. Define electrostatic potential

The electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$ **Unit :** V or JC^{-1}

11. What is an equipotential surface ?

An equipotential surface is a surface on which all the points are at the same potential

12. Write about Properties of equipotential surfaces

(i) The work done to move a charge q between any two points A and B is zero

(ii) The electric field is normal to an equipotential surface.

13.Give the relation between Electric field and electric potential

$dV = -E\,dx$

$$E = -\frac{dV}{dx}$$

Electric field is negative gradient of electric potential

14.Define electrostatic potential energy

It is defined as work done in bringing the various charges to their respective positions from infinitely large mutual separation

Unit : Joule

15.Define Electric Flux

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux

$$\Phi_E = EA\cos\theta$$

Unit : Nm^2C^{-1} **Quantity :** scalar

16.What is meant by electrostatic energy density?

The energy stored per unit volume of space is defined as energy density

$$U_E = \frac{U}{VOLUME} = \frac{1}{2}\varepsilon_o E^2$$

17.Define electrostatic shielding

Consider a cavity inside the conductor.Whatever the charges at the surfaces and whatever the electrical disturbances outside, the electric field inside the cavity is zero. A sensitive electrical instrument which is to be protected from external electrical disturbance is kept inside this cavity. This is called electrostatic shielding.

18. What is polarization?

Polarisation is defined as the total dipole moment per unit volume of the dielectric

$$\vec{p} = \chi \vec{E}_{ext}$$

χ is a constant called the electric susceptibility which is a characteristic of each dielectric.

19.What is dielectric strength?

The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength

20.Define capacitance

The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors.

$$C = \frac{Q}{V}$$

Unit : coulomb per volt or farad (F)

21.Write about action of points or corona discharge ? (Mar 20)

When the electric field near the edge is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called action at points or corona discharge.

Book inside

1.State Coulomb's law

It states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

2. What is dielectric?

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms.

Examples : Ebonite, glass and mica

3.What is Non-polar molecules?

A non-polar molecule is one in which centres of positive and negative charges coincide. It has no permanent dipole moment. **Examples** : Hydrogen (H_2), oxygen (O_2), and carbon dioxide (CO_2)

4. What is Polar molecules

In polar molecules, the centres of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment.

Examples : H_2O, N_2O, HCl, NH_3.

5. Define electrostatic induction

Charging without actual contact is called electrostatic induction

6.Why it is always safer to sit inside a bus than in open ground or under a tree ?

The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

7. Define Gauss's law

Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_0}$$

8. What are two kind of electric field?

Uniform electric field will have the same direction and constant magnitude at all points in space. **Non-uniform electric field** will have different directions or different magnitudes or both at different points in space

9.Define one coulomb

One coulomb is a quantity of charge which when placed at a distance of one metre in air from equal and opposite charge experiences a repulsive force of 9×10^9

$r = 1m$, $F = 9\times10^9 N$, $q_1 = q_2 = 1C$

10. When is electric potential energy (i) maximum (ii) minimum

- ✓ The potential energy is maximum when the dipole is aligned anti-parallel ($\theta = \pi$) to the external electric field and
- ✓ The potential energy is minimum when the dipole is aligned parallel ($\theta = 0$) to the external electric field.

11.What is called triboelectric charging?

Charging the objects through rubbing is called triboelectric charging

12.Give reason to support water acts as good solvent

When common salt (NaCl) is taken in water , the electrostatic force between Na and Cl ions is reduced due to high relative permittivity of water

13.What is dielectric breakdown?

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown.

14.What is fringing field?

For finite sized plates, the electric field is not strictly uniform between the plates. At both edges, the electric field is bent outwards which is called as fringing field

15. State the application of corona discharge

(i) Lightning arrester

(ii) Van de Graff generator

16. Give the applications of capacitors

They are used in

(i) digital camera to emit flashlight

(ii) heart defibrillator to retrieve the normal heart function

(iii) ignition system of automobile engines to eliminate sparking

(iv) to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

17.Explain the working of computer keyboard keys

- ✓ They are constructed using capacitors with a dielectric
- ✓ When the key is pressed, the separation between the plates decreases leading to an increase in the capacitance. This in turn triggers the electronic circuits in the computer to identify which key is pressed.

18. Sometimes we notice that the ceiling fan does not start rotating as soon as it is switched on. But when we rotate the blades, it starts to rotate as usual. Why it is so?

To rotate any object, there must be a torque applied on the object. For the ceiling fan, the initial torque is given by the capacitor widely known as a condenser. If the condenser is faulty, it will not give sufficient initial torque to rotate the blades when the fan is switched on.

19.Give the principle of Van de Graff Generator

(i) Electrostatic induction

(ii) Action at points

Long Answer Question

Book back

1.Discuss the basic properties of electric charge

(i) Electric charge

- ✓ Electric charge is intrinsic and fundamental property of particles..
- ✓ The SI unit of charge is coulomb.

(ii) Conservation of charges

- ✓ Charges are neither created or nor destroyed but can only be transferred from one object to other.
- ✓ This is called conservation of total charges

✓ The total electric charge in the universe is constant and charge can neither be created nor be destroyed.

(iii) Quantisation of charges

The charge q on any object is equal to an integral multiple of this fundamental unit of charge e.

q = ne n is any integer (0, ±1, ……..)

2.Explain in detail about Coulomb's law & its various aspects

It states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between the two point charges.

$$F \propto \frac{q_1 q_2}{r^2}$$

The direction of forces is along the line joining two charges

$$F = k\frac{q_1 q_2}{r^2}$$

where $k = \frac{1}{4\pi\varepsilon_o}$ $k = 9\times10^9\ Nm^2C^{-2}$

ε_0 – Permittivity of free space and its value is $8.854\times10^{-12}\ C^2N^{-1}m^{-2}$

- One coulomb is defined as quantity of charges which when placed at a distance of 1m in air or vaccum from an equal and similar charge experiences a repulsive force of 9×10^9 in vaccum

In medium of permittivity

- $F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$
- $\varepsilon > \varepsilon_0$,
- Force between two point charges in a medium other than vacuum is always less than that in vacuum

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

Gravitational force	Electrostatic force or coulomb force
Force between two masses is always attractive	Force between two charges can be attractive or repulsive, depending on the nature of charges
Force between two masses is independent of the medium	Force between the two charges depends on nature of the medium
The value of the gravitational constant **$G = 6.626\times10^{-11}\ Nm^2Kg^{-2}$**	The value of the constant k in Coulomb law is **$k = 9\times10^9\ Nm^2C^{-2}$**
Force between two point masses is the same whether two masses are at rest or in motion.	Force between two point charges will change with respect to motion

3.Define Electric field and its various aspect

The charge q creates an electric field in the surrounding space. The electric field at the point P at a distance r from the point charge q is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_o} = \frac{kq}{r^2}\hat{r}$$

Unit : NC^{-1} **Quantity :** Vector

Important aspects of Electric field

- ✓ If charge q is positive -electric field points away from the source charge . If q is negative - electric field points towards the source charge q.
- ✓ Force experienced by test charge placed at point P is Eq_o.
- ✓ From equation of electric field. its depends only on the source charge q & independent on charge q_o.
- ✓ The electric field is a vector quantity, at every point in space, this field has unique direction and magnitude .
- ✓ Distance r decreases , Electric field Increases .
- ✓ The test charge is made sufficiently small such that it will not modify the electric field of the source charge.
- ✓ The expression $E = \frac{F}{q_o} = \frac{kq}{r^2}$ is valid only for point charges.

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + \dots\dots\dots + \frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right)$$

$$\approx \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP}$$

Two kinds of the electric field: uniform (constant) electric field and non-uniform electric field.

Uniform electric field - same direction and constant magnitude at all points in space.

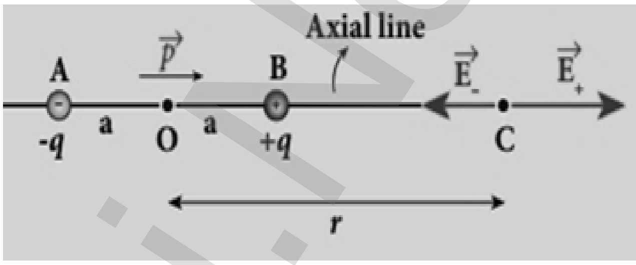
Non-uniform electric field - different directions or different magnitudes or both at different points in space.

The electric field created by a point charge is basically a non uniform electric field.

4.Calculate the electric field due to a dipole on axial and equatorial plane

Electric field due to an electric dipole on the axial line

- ✓ Consider an electric dipole placed on x axis.
- ✓ A point C is located at a distance of r from the midpoint O of the dipole along the axial line.



The electric filed at a point C due to +q

$\vec{E}_+ = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}$ along BC

$\hat{p}$ – Direction is -q to +q and along BC

$\vec{E}_+ = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}\hat{p}$ ....(1)

The electric filed at a point C due to -q

$\vec{E}_- = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}$ along CA

$\hat{p}$ – Direction is -q to +q and directed opposite to field

$\vec{E}_- = -\frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}\hat{p}$ ....(2)

Total electric field at C calculated using super position principle

$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$

$= \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}\hat{p}$

$= \frac{q}{4\pi\varepsilon_o}\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p}$

r >> a

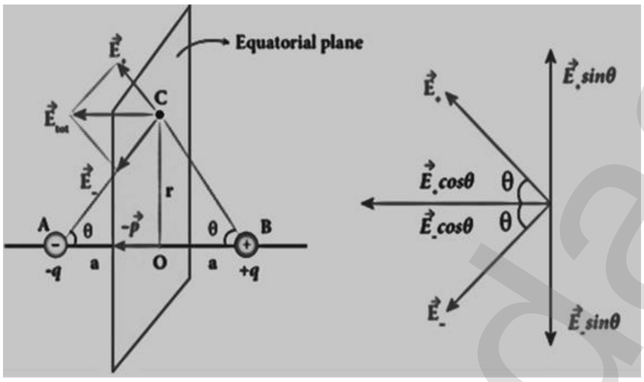
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}\left(\frac{4aq}{r^3}\right)\hat{p}$

$= \frac{1}{4\pi\varepsilon_o}\left(\frac{2\vec{p}}{r^3}\right)$ **since** $2aq\hat{p} = \vec{p}$

The direction of electric field is along the direction of the dipole moment

Electric field due to an electric dipole on the equatorial plane

- ✓ Consider point C is located at a distance of r from the midpoint O of the dipole on the equatorial plane.
- ✓ C is equidistant from +q & -q , the magnitude of electric field of +q & -q are the same
- ✓ Direction of $\vec{E}_+$ along BC
- ✓ Direction of $\vec{E}_-$ along CA
- ✓ $\vec{E}_+$ & $\vec{E}_-$ resolved into two components : One component parallel to dipole and perpendicular to it
- ✓ Perpendicular components $|\vec{E}_+|\sin\theta$ & $|\vec{E}_-|\sin\theta$ are oppositely directed so cancel each other .



$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\hat{p} - |\vec{E}_-|\cos\theta\hat{p}$$

$$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{2q\cos\theta}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{2qa}{(r+a)^{\frac{3}{2}}}\hat{p} \quad \textbf{since } \cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{\vec{p}}{(r+a)^{\frac{3}{2}}} \quad \textbf{since } 2aq\hat{p} = \vec{p}$$

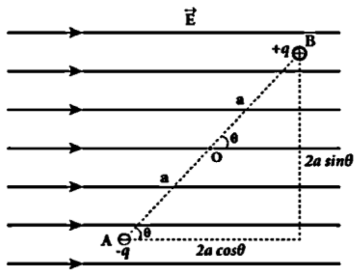
$$r >> a \quad (r+a)^{\frac{3}{2}} = r^3$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\left(\frac{\vec{p}}{r^3}\right)$$

The direction of electric field acts opposite to the direction of the dipole moment

5.Derive an expression for Torque experienced by a dipole due to a uniform electric field

- ✓ Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field.
- ✓ The charge +q will experience a force q $\vec{E}$ in the direction of the field charge –q will experience a force -q$\vec{E}$ in a direction opposite to the field.
- ✓ Since the external field is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque.
- ✓ This torque tends to rotate the dipole.



Torque on the dipole about the point O

$$\vec{\tau} = \vec{OA} \times (-q\vec{E}) + \vec{OB} \times (q\vec{E})$$

The magnitude of total torque

$$\vec{\tau} = |\vec{OA}|(-q\vec{E})\sin\theta + |\vec{OB}|(q\vec{E})\sin\theta$$

$$= qE\,2a\sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\text{since } p = 2qa)$$

a)When $\theta = 0 \quad \tau = 0$

The dipole moment of dipole parallel to electric field – No torque

b)When θ = **90°** **τ = pE**

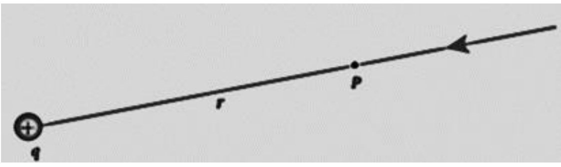
Dipole moment perpendicular to electric filed , torque is maximum

c)When $\theta = \mathbf{180°}$ $\boldsymbol{\tau} = \mathbf{0}$

Dipole moment anti parallel to electric filed , torque is zero

6.Derive an expression for electrostatic potential due to point charge

- ✓ Consider a positive charge q kept fixed at the origin.
- ✓ Let P be a point at distance r from the charge q.



Electric potential at point P

$$V = \int_{\infty}^{r}(-\vec{E}).d\vec{r} = -\int_{\infty}^{r}(\vec{E}).d\vec{r}$$

Electric field due to positive charge q

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$

$$V = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}.d\vec{r}$$

$$d\vec{r} = dr\hat{r} \qquad \hat{r}.\hat{r} = 1$$

$$V = -\frac{1}{4\pi\varepsilon_o}\int_{\infty}^{r}\frac{q}{r^2}.dr$$

After the integration

$$V = -\frac{1}{4\pi\varepsilon_o}q\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

$\boldsymbol{V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}}$ which is potential at a point due to a point charge

7.Derive an expression for electrostatic potential due to an electric dipole

- ✓ Consider two equal and opposite charges separated by a small distance 2a The point P is located at a distance r from the midpoint of the dipole.
- ✓ Let θ be the angle between the line OP and dipole axis AB.
- ✓ r_1 be the distance of point P from +q and r_2 be the distance of point P from –q.

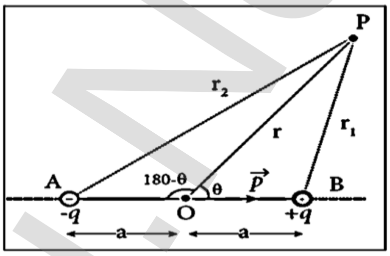


Figure 1.26 Potential due to electric dipole

Potential at P due to charge +q $= \frac{1}{4\pi\varepsilon_o}\frac{q}{r_1}$

Potential at P due to charge -q $= -\frac{1}{4\pi\varepsilon_o}\frac{q}{r_2}$

Total potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_o}q\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\text{............(1)}$$

By the cosine law for triangle BOP

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$= r^2\left(1+\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right)$$

$r >> a$; $\frac{a^2}{r^2}$ can be neglected

$$r_1 = r\left(1-\frac{2a}{r}\cos\theta\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$

$\frac{a}{r} \ll 1$ Using binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left(1+\frac{a}{r}\cos\theta\right)\text{............(2)}$$

Similarly applying the cosine law for triangle AOP,

$r_2^2 = r^2 + a^2 - 2ra\cos(180-\theta)$

$= r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$

$r >> a$; $\frac{a^2}{r^2}$ can be neglected

$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$

Using binomial theorem

$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)$……….(3)

Sub (3) & (2) in (1)

$V = \frac{1}{4\pi\varepsilon_o} q\left[\frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) - \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)\right]$

$V = \frac{1}{4\pi\varepsilon_o}\frac{2aq}{r^2}\cos\theta$

$\boldsymbol{V = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}\cos\theta}$ **(since $p = 2qa$)**

$p\cos\theta = \vec{p}.\hat{r}$

The electric potential at a point P due to an electric dipole is given by

$= \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}.\hat{r}}{r^2} (r \gg a)$

Special cases

Case (i) If the point P lies on the axial line of the dipole on the side of +q, then θ = 0

$\boldsymbol{V = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}}$

Case (ii) If the point P lies on the axial line of the dipole on the side of –q, then θ = 180°, then

$\boldsymbol{V = -\frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}}$

Case (iii) If the point P lies on the equatorial line of the dipole, then θ = 90°. Hence $V = 0$

8.Obtain an expression for potential energy due to collection of three point charges which are separated by finite distances

The electric potential at a point at a distance *r* from point charge q_1 is given by

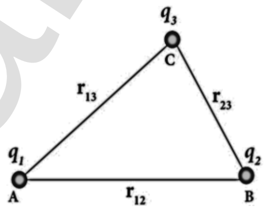
$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$$

This potential *V* is the work done to bring a unit positive charge from infinity to the point. Now if the charge q_2 is brought from infinity to that point at a distance *r* from q_1, the work done is the product of q_2 and the electric potential at that point.

$$W = q_2 V$$

This work done is stored as the electrostatic potential energy U

$$U = q_2V = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r}$$



Three charges are arranged in the following configuration

i)Bringing a charge q_1 from infinity to the point A requires no work, because there are no other charges already present in the vicinity of charge q_1

ii) To bring the second charge q_2 to the point B, work must be done against the electric field created by the charge q_1 So the work done on the charge q_2 is $W = q_2 V_{1B}$. Here V_{1B} is the electrostatic potential due to the charge q_1 at point *B*.

$$U = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r_{12}}$$

iii) Similarly to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is = q_3 (V_{1C} + V_{2C}). Here V_{1C} is the electrostatic potential due to charge q_1 at point C and V_{2C} is the electrostatic potential due to charge q_2 at point C.

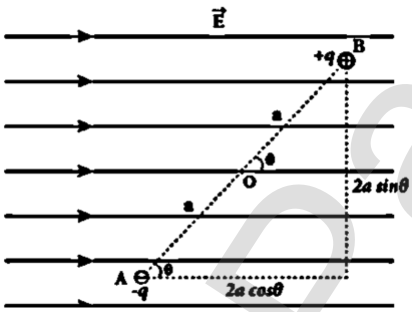
$$U = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

iv) Total electrostatic potential energy for the system of charges q_1 , q_2 , q_3 is

$$U = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_2}{r_{12}}\right)$$

9.Derive an expression for electrostatic potential energy of the dipole in a uniform electric field

- ✓ Consider a dipole placed in the uniform electric field $\vec{E}$ .A dipole experiences a torque when kept in an uniform electric field $\vec{E}$.
- ✓ This torque rotates the dipole to align it with the direction of the electric field.
- ✓ To rotate the dipole (at constant angular velocity) from its initial angle θ′ to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole



The work done by the external torque to rotate the dipole from angle θ′ to θ at constant angular velocity

$$W = \int_{\theta'}^{\theta} \tau_{ext}\, d\theta \quad ....(1)$$

$$\tau = pE\sin\theta \quad ....(2)$$

substituting (2) in (1) in above equation

$$W = \int_{\theta'}^{\theta} pE\sin\theta\, d\theta$$

$$W = pE(\cos\theta' - \cos\theta)$$

This work done is equal to the potential energy difference between the angular positions and θ′.

$$U(\theta) - U(\theta') = -pE\cos\theta + -pE\cos\theta'$$

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

- ✓ **θ = 180** dipole aligned **antiparallel** to field U is maximum
- ✓ **θ = 0** dipole aligned **parallel** to field U is minimum

10.Obtain Gauss law from Coulomb's law

Gauss's law

Gauss's law states that if a charge Q is enclosed by an arbitrary closed surface, then the total electric flux Φ_E through the closed surface is

$$\Phi_E = \frac{Q_{encl}}{\varepsilon_0}$$

A positive point charge Q is surrounded by an imaginary sphere of radius r electric flux through the closed surface of sphere

$$\phi_E = \oint \vec{E}\cdot d\vec{A}\cos\theta \quad ....(1)$$

The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and $\theta = 0^0$

$$\phi_E = \oint E.dA \quad ....(2)$$ since cos0° = 1

E is uniform on the surface of the sphere

$$\phi_E = E\oint .dA \quad ....(3)$$

Sub $\oint . dA = 4\pi r^2$ in (3)

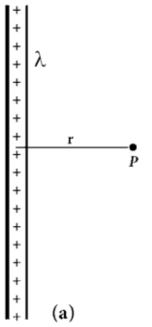
Therefore $\phi_E = 4\pi r^2 E$

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

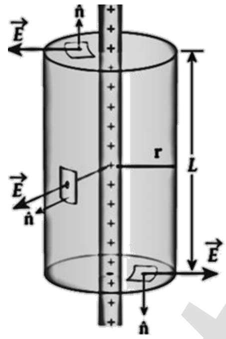
$\phi_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \times 4\pi r^2$

$\phi_E = \frac{Q}{\varepsilon_0}$

11.Obtain expression for electric filed due to an infinitely long charged wire



- ✓ Consider an infinitely long straight wire having uniform linear charge density λ.
- ✓ Let P be a point located at a perpendicular distance r from the wire.
- ✓ The electric field at the point P can be found using Gauss law. We choose two small charge elements A_1 and A_2 on the wire which are at equal distances from the point P.
- ✓ The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r.
- ✓ Charged wire possesses a cylindrical symmetry of radius r and length L.



$\phi_E = \oint \vec{E} . d\vec{A}$

$= \oint \vec{E} . d\vec{A} + \oint \vec{E} . d\vec{A} + \oint \vec{E} . d\vec{A}$

Curved Surface **Top surface** **Bottom surface**

Since $\vec{E}$ *and* $d\vec{A}$ are right angles to each other , the electric flux through the top and bottom surface is zero

Flux through the curved surface = $\oint E.dA \cos\theta$

$\phi_E = \oint E.dA$ [since $\theta = 0$ $\cos 0 = 1$]

= E(2πrl) ……(1) [since $\oint dA = 2\pi rl$]

The net charge enclosed by Gaussian surface is

Q = λl

By Gauss law $\phi_E = \frac{Q}{\varepsilon_0}$ **…………(2)**

Equating (1) & (2)

$E(2\pi rl) = \frac{Q}{\varepsilon_0}$

Applying Q value

$E(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

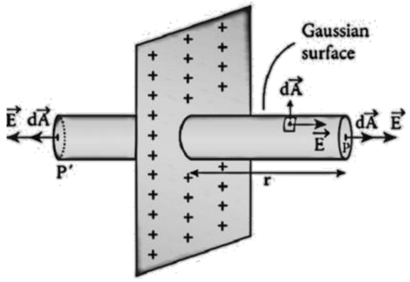
Direction of electric field is radially outward if line charge is positive and inward , if the line charge is negative

In vector form $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{r}$

12. Obtain expression for electric filed due to an charged infinitely plane sheet

- ✓ Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet.

- ✓ Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points.
- ✓ A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.



$\phi_E = \oint \vec{E}\,.d\vec{A}$

$= \oint \vec{E}\,.d\vec{A} + \oint \vec{E}\,.d\vec{A} + \oint \vec{E}\,.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

Curved surface **P** **P′**

The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P′ .Then

$\phi_E = \oint \vec{E}\,.d\vec{A} + \oint \vec{E}\,.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

P P'

Since the magnitude of the electric field at these two equal surfaces is uniform, E is taken out of the integration and Q_{encl} = σ A

$2E \int dA = \frac{Q_{encl}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$ [since Q_{encl} = σ A]

The total area of surface either at P or P′

$\int dA = A$

$2EA = \frac{\sigma A}{\varepsilon_0}$

$$E = \frac{\sigma A}{2\varepsilon_0}$$

In vector $\vec{E} = \frac{\sigma A}{2\varepsilon_0}\hat{n}$ $\hat{n}$ *is* outward unit vector normal to the plane.

The electric field due to an infinite plane sheet of charge depends on the surface charge density and **is independent of the distance r.**

13.Obtain expression for electric field due to uniformly charged spherical shell

Consider a uniformly charged spherical shell of radius R and total charge Q. The electric field at points outside and inside the sphere is found using Gauss law.

Case (a) At a point outside the shell (r > R)

- ✓ Consider a point P outside the shell at a distance r from the center .
- ✓ The charge is uniformly distributed on the surface of the sphere (spherical symmetry).
- ✓ Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0.
- ✓ Consider a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q.

Applying Gauss law

$\oint \vec{E}\,.d\vec{A} = \frac{Q}{\varepsilon_0}$

Magnitude of $\vec{E}$ is the same at all points due to the spherical symmetry of the charge distribution.

$E \oint dA = \frac{Q}{\varepsilon_0}$

Gaussian Surface

$\oint dA$ = Total area of Gaussian surface $= 4\pi r^2$

Gaussian

Surface

$$E.4\pi r^2 = \frac{Q}{\varepsilon_0}$$

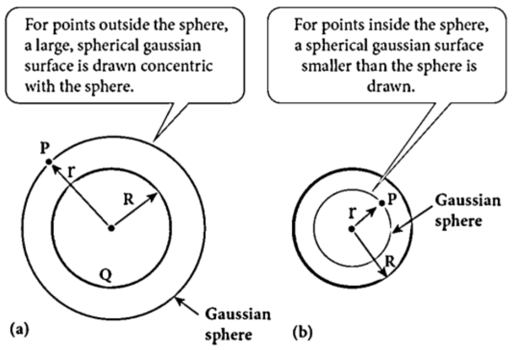
$$\boldsymbol{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

In vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

Case(b) : At a point on the surface of the spherical shell (r = R)

The electric field at a point on the spherical shell (r = R) is given by

$$\boldsymbol{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{r}}$$



Case(c) : At a point inside the spherical shell (r < R)

Consider a point P inside the shell at a distance r from the center.

Applying Gauss law

$$\oint \vec{E}\,.d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E.4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Gaussian surface encloses no charge, So Q = 0.

Hence **E= 0**

The electric field due to the uniformly charged spherical shell is zero at all points inside the shell

14.Discuss the various properties of conductors in electrostatic equilibrium

- ✓ The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.
- ✓ There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.
- ✓ The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density at that point.
- ✓ The electrostatic potential has the same value on the surface and inside of the conductor.
- ✓ Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential

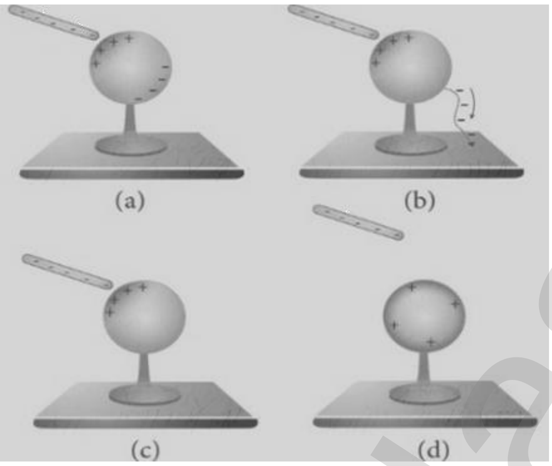
15.Explain the process of electrostatic induction

Charging without actual contact is called electrostatic induction.

a) Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it. The negative charge of the rod repels the electrons in the conductor to the opposite side. As a result, positive charges are induced near the region of the charged rod while negative charges on the farther side. Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero. Once the charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

b) Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding.

c) When the grounding wire is removed from the conductor, the positive charges remain near the charged rod



d) Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor . By this process, the neutral conducting sphere becomes positively charged

16.Explain dielectric in detail and how an electric field is induced inside a dielectric

- ✓ In dielectric, which has no free electrons, when the external electric field is applied . the field only realigns the charges so that an internal electric field is produced.
- ✓ The magnitude of the internal electric field is smaller than that of external electric
- ✓ field. Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with magnitude less than that of the external electric field.
- ✓ Let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor)
- ✓ The uniform electric field between the plates acts as an external electric field which polarizes the dielectric placed between plates. The positive charges are induced on one side surface and negative charges are induced on the other side of surface.
- ✓ But inside the dielectric, the net charge is zero even in a small volume. So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors.

17.Obtain the expression for capacitance for a parallel plate capacitor

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d

The electric field between two infinite parallel plates is uniform and is given by

$E = \frac{\sigma}{\varepsilon_0}$ ....(1)

σ – Surface charge density on the plates ($\sigma = \frac{Q}{A}$)

The electric field between the plates is

$E = \frac{Q}{A\varepsilon_0}$ ....(2)

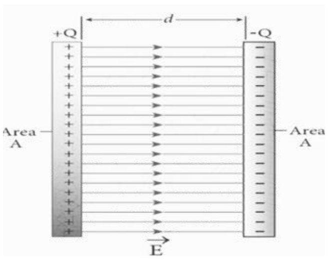
Since the electric field is uniform, the electric potential between the plates having separation d is

$V = Ed = \frac{Qd}{A\varepsilon_0}$ ....(3)

Capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} = \frac{\varepsilon_0 A}{d}$$

$C \alpha A \quad C \alpha \frac{1}{d}$



18.Obtain the expression for energy stored in parallel plate capacitor

- ✓ Capacitor not only stores the charge but also it stores energy.
- ✓ When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate.
- ✓ To transfer the charge, work is done by the battery.
- ✓ This work done is stored as electrostatic potential energy in the capacitor.
- ✓ To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = V\,dQ$$

$$V = \frac{Q}{C}$$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C}$$

This work done is stored as electrostatic potential energy (U_E) in the capacitor

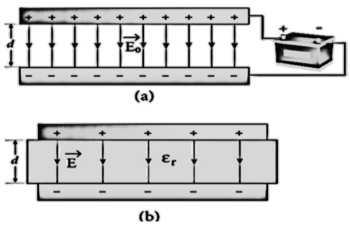
$U_E = \frac{Q^2}{2C} = CV^2$

$U_E \, \alpha \, C \qquad U_E \, \alpha \, V^2$

19.Explain in detail effect of a dielectric placed in parallel plate capacitor

The dielectric can be inserted into the plates in two different ways. (i) when the capacitor is disconnected from the battery. (ii) when the capacitor is connected to the battery.

(i) When the capacitor is disconnected from the battery



- ✓ Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d.
- ✓ The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0.

The capacitance of the capacitor without the dielectric is

$C_0 = Q_0 / V_0$

- ✓ The battery is then disconnected from the capacitor and the dielectric is inserted between the plates
- ✓ The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified

E = $\frac{E_0}{\varepsilon_r}$

E_0 - electric field inside the capacitors when there is no dielectric

ε_r – relative permeability of the dielectric

$\varepsilon_r > 1$, the electric field $E < E_0$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected.

Hence the new potential difference is

$V = Ed = \frac{\varepsilon_0 d}{\varepsilon_r} = V_0 / \varepsilon_r$

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases. Thus new capacitance in the presence of a dielectric is

$$C = \frac{Q_o}{V} = \varepsilon_r \frac{Q_o}{V_o} = \varepsilon_r C_o$$

$\varepsilon_r > 1$, we have $C > C_o$.

Thus insertion of the dielectric constant ε_r increases the capacitance.

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon A}{d}$$

The energy stored in the capacitor before the insertion of a dielectric is given by

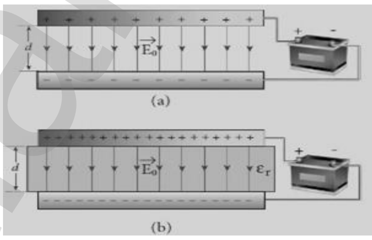
$U_0 = \frac{Q_0^2}{2C_0}$

After the dielectric is inserted, the charge remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r}$$

Since **$\varepsilon_r > 1$ we get $U < U_0$**. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

ii) When the battery remains connected to the capacitor



- ✓ Consider battery of voltage V_0 remains connected to the capacitor when the dielectric is inserted into the capacitor
- ✓ The potential difference V_0 across the plates remains constant.
- ✓ When dielectric is inserted, the charge stored in the capacitor is increased by a factor ε_r.

$Q = \varepsilon_r Q_o$

Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q_o}{V} = \varepsilon_r \frac{Q_o}{V_o} = \varepsilon_r C_o$$

$$C_o = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon A}{d}$$

The energy stored in the capacitor before the insertion of a dielectric is given by

$U_0 = \frac{1}{2}C_o V_o^2$

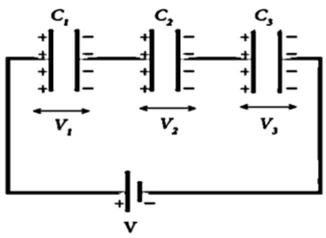
After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$$U = \frac{1}{2}CV_o^2 = \frac{1}{2}\varepsilon_r C V_o^2 = \varepsilon_r U_o$$

Since $\varepsilon_r > 1$ we have $U > U_o$

20. Derive the expression for resultant capacitance when capacitors are connected in series and in parallel (Mar 20)

Capacitance in series



- ✓ C_1, C_2 and C_3 connected in series .C_S is effective capacitance
- ✓ Charge in each capacitor is same
- ✓ Voltage across each capacitor is different

$V = V_1 + V_2 + V_3$

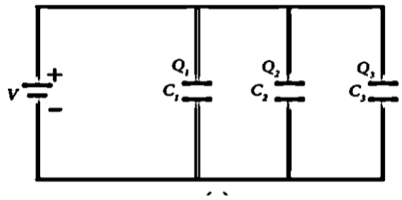
$V = \frac{q}{C_S} \quad V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3}$

$$\frac{q}{C_S} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The inverse of the equivalent capacitance C_S of three capacitors connected in series is equal to the sum of the inverses of each capacitance

Capacitance in parallel



- ✓ C_1, C_2 and C_3 connected in Parallel .C_P is effective capacitance
- ✓ Potential in each capacitor is same
- ✓ Charge in each capacitor is different

$q_1 = C_1 V \,, q_2 = C_2 V \,, q_3 = C_3 V$

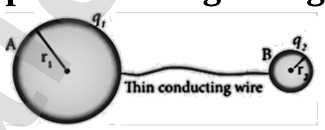
$$q = C_P V$$

$C_P V = C_1 V + C_2 V + C_3 V$

$C_P = C_1 + C_2 + C_3$

The equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.

21.Explain in detail how charges are distributed in a conductor & the principle behind lightning conductor



Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire The distance between the spheres is much greater than the radii of either spheres.

If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium.

Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that $Q = q_1 + q_2$. The charges are distributed only on the surface and there is no net charge inside the conductor.

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1}$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_2}$$

The surface of the conductor is an equipotential. Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$V_A = V_B$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

Let us take the charge density on the surface of sphere A is σ1 and charge density on the surface of sphere B is σ_2. This implies that $q_1 = 4\pi r_1^2\, \sigma_1$ and $q_2 = 4\pi r_2^2\, \sigma_2$

σr = constant

Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa

Lightning conductor is a device used to protect tall buildings from lightning strikes. It works on the principle of action at points or corona discharge. The leakage of charges from sharp pointed conductor is called corona discharge

22.Explain in detail the construction and working of a van de Graff generator

It is a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V).

Principle

Electrostatic induction and action at points

Construction

1) A large hollow spherical conductor is fixed on the insulating stand .
2) Pulley B is mounted at the center of the hollow sphere
3) Pulley C is fixed at the bottom.
4) A belt made up of insulating materials like silk or rubber runs over both pulleys.
5) The pulley C is driven continuously by the electric motor.
6) Two comb shaped metallic conductors E and D are fixed near the pulleys.
7) The comb D is maintained at a positive potential of 10^4 V by a power supply.
8) The upper comb E is connected to the inner side of the hollow metal sphere.

Working

(i) Action of points

1) Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D.
2) The positive charges stick to the belt and move up.

(ii) Electrostatic induction

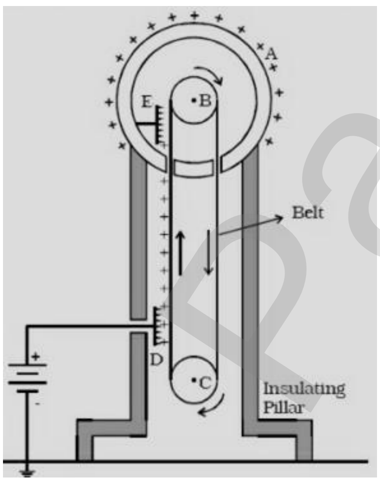
1) When the positive charges reach the comb E, a large amount of negative

and positive charges are induced on either side of comb E due to electrostatic induction.

2) As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere.
3) At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.
4) When the belt descends, it has almost no net charge.
5) This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value.

Leakage and prevention

1) We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air.
2) The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.



Uses

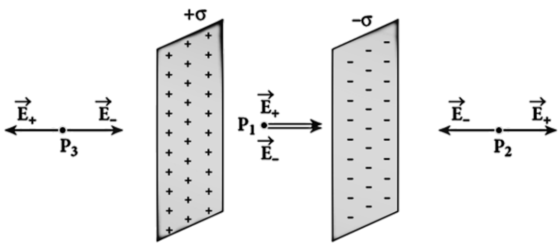
The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

Book Inside

1.Explain the properties of electric field lines

- ✓ The electric field lines start from a positive charge and end at negative charges or at infinity.
- ✓ The electric field vector at a point in space is tangential to the electric field line at that point.
- ✓ The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude.
- ✓ No two electric field lines intersect each other.
- ✓ The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges.

2.Derive the expression for electric field due two long parallel charged infinite sheets



- ✓ Consider two infinitely large charged plane sheets with equal and opposite charge densities +σ and -σ which are placed parallel to each other.

- ✓ The electric field between the plates and outside the plates is found using Gauss law.
- ✓ The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_0}$

Outside the plates

- ✓ At the points P_2 and P_3, the electric field due to both plates are equal in magnitude and opposite in direction.
- ✓ As a result, electric field at a point outside the plates is zero.

Inside the plates

- ✓ Inside the plate electric fields are in same direction i.e., towards the right, the total electric field at a point P_1

$$E_{inside} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

- ✓ The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plate.

3.Explain the working of microwave oven

- ✓ Microwave oven works on the principle of torque acting on an electric dipole.
- ✓ The food we consume has water molecules which are permanent electric dipoles.
- ✓ Oven produces microwaves that are oscillating electromagnetic fields and produce torque on the water molecules.
- ✓ Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy.
- ✓ Thus, heat generated is used to heat the food.

Numerical Problems Solved

1. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

Given:

$q = 50\ nc = 50 \times 10^{-9} c \quad e^- = 1.6 \times 10^{-19} C$

To find n

Solution:

$$n = \frac{q}{e} = \frac{50 \times 10^{-9}}{1.6 \times 10^{-19}}$$

$$n = \frac{50 \times 10^{10}}{1.6} = \frac{25 \times 10^{10}}{8 \times 10^{-1}}$$

$$\boldsymbol{n = 3.125 \times 10^{11}\ electrons}$$

2. The total number of electrons in the human body is typically in the order of 10^{28}. Suppose, due to some reason, you and your friend lost 1% of this number of electrons. Calculate the electrostatic force between you and your friend separated at a distance of 1m. Compare this with your weight. Assume mass of each person is 60 kg and use point charge approximation.

Given:

$n = 10^{28}, r = 1m, m = 60kg \quad, n' = 1\%\ 10^{28} = \frac{1}{100} \times 10^{28} = 10^{26}$

$F_e = ? \quad , \quad w = ?$

Solution:

(i) $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q\,q'}{r^2}$

$q = ne = 10^{26} \times 1.6 \times 10^{-19}$

$= 1.6 \times 10^{7} c$

$q' = n'e = 10^{26} \times 1.6 \times 10^{-19}$

$= 1.6 \times 10^{7} c$

$$Fe = \frac{9 \times 10^9 \times 1.6 \times 10^7 \times 1.6 \times 10^7}{1^2}$$

$Fe = 9 \times 2.56 \times 10^{23}$

$Fe = 23.04 \times 10^{23}\ N$

$\therefore\ F_e = \mathbf{23 \times 10^{23} N}$

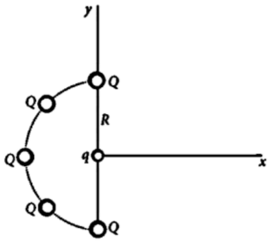
(ii) Weight, $w = mg = 60 \times 9.8$

$w = 588N$

$$\frac{F_e}{w} = \frac{23 \times 10^{21}}{588}$$

$$\therefore \frac{\boldsymbol{F_e}}{\boldsymbol{w}} = \mathbf{3.9 \times 10^{21}}$$

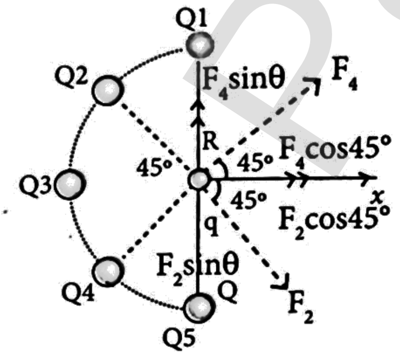
3. **Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the center of the circle of radius R. Calculate the electrostatic force experienced by the charge q.**



Force experienced by q =?

Solution:

$Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = Q$



- Forces acting on q due to Q_1 and Q_5 are equal and opposite.
- Hence $\vec{F}_1$ & $\vec{F}_5$ get cancelled.
- Net force is zero
- Forces acting on q due to Q_2 $(\vec{F}_2)$ and Q_4 $(i,e.\vec{F}_4)$ is resolved into two two components
- $F_2 \sin\theta$ and $F_4 \sin\theta$ are equal in magnitude but opposite in direction, get cancelled.
- $F_2 \cos\theta$ and $F_4 \cos\theta$ are horizontal components acts in same direction, gets added.

Force acting on q due to Q_3 is $\vec{F}_3$.

Total force acting on q is

$\vec{F} = \vec{F}_3 + F_2 \cos\theta\,\hat{\imath} + F_4 \cos\theta\,\hat{\imath}$

$(\theta = 45°)$

$$\vec{F} = \frac{kqQ}{R^2}\hat{\imath} + \frac{kqQ}{R^2}\cos 45°\hat{\imath} + \frac{kqQ}{R^2}\cos 45°\hat{\imath}$$

$$= \frac{kqQ}{R^2}\hat{\imath}\left[1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{kqQ}{R^2}\hat{\imath}\left[1 + \frac{2}{\sqrt{2}}\right]$$

$$= \frac{kqQ}{R^2}\hat{\imath}\left[1 + \frac{\sqrt{2}\sqrt{2}}{\sqrt{2}}\right]$$

$$\vec{\boldsymbol{F}} = \frac{\mathbf{1}}{\mathbf{4\pi\varepsilon_0}}\frac{\boldsymbol{qQ}}{\boldsymbol{R^2}}[\mathbf{1} + \sqrt{\mathbf{2}}]\hat{\boldsymbol{\imath}}\ \boldsymbol{N} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

4. Suppose a charge +q on Earth's surface and another +q charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change?

Given:

$M_E = 5.9 \times 10^{24} kg$,

$M_M = 7.9 \times 10^{22} kg$

(i) $q = ?$ (ii) $If\ r = \frac{r}{2}, q = ?$

Solution:

(i) $\boldsymbol{F_e = F_g}$

$$\frac{1}{4\pi\varepsilon_0}\frac{q.q}{r^2} = \frac{GM_E M_M}{r^2}$$

$$9 \times 10^9 \times q^2 = 6.67 \times 10^{-11} \times 5.9 \times 10^{24} \times 7.9 \times 10^{22}$$

$$q^2 = \frac{6.67 \times 5.9 \times 7.9 \times 10^{35}}{9 \times 10^9}$$

$$q = \sqrt{\frac{6.67 \times 5.9 \times 7.9 \times 10^{26}}{9}}$$

$$q = 10^{13}\sqrt{\frac{6.67 \times 5.9 \times 7.9}{9}}$$

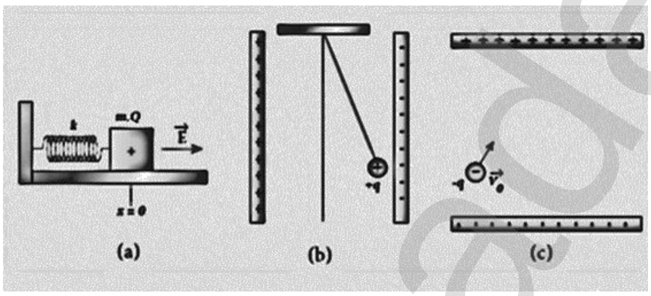
$\therefore \boldsymbol{q = 5.87 \times 10^{13} C.}$

(ii) $\boldsymbol{kq^2 = GM_E M_M}$

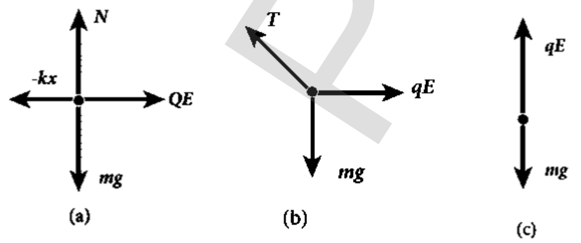
If $r = \frac{r}{2}$. The charge 'q' will not change

There is no r term

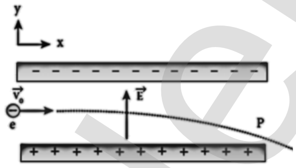
5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).



Solution



6. Consider an electron travelling with a speed v_o and entering into a uniform electric field $\vec{E}$ which is perpendicular to $\vec{v}_o$ as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.



Solution:

(i) $F = ma, a = \frac{F}{m}$

$\therefore$ force experienced by $e^-(F) = eE$

$\therefore$ acceleration , $\vec{a} = \frac{eE}{m}(-\hat{j}) = -\frac{eE}{m}\hat{j}$

(ii) velocity $(\vec{v})$:

$$\vec{v} = \vec{u} + \vec{a}t, \vec{u} = v_0\hat{\imath}, \vec{a} = -\frac{eE}{m}\hat{j}$$

$$\vec{v} = v_0\hat{\imath} - \frac{eE}{m}t\,\hat{j}$$

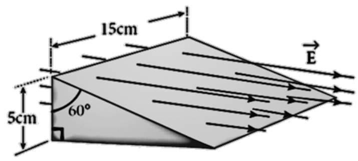
(iii) Position vector $(\vec{r})$:

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r} = v_0\hat{\imath} + \frac{1}{2}\left(-\frac{eE}{m}\hat{j}\right)t^2$$

$$\vec{r} = v_0\hat{\imath} - \frac{1}{2}\frac{eE}{m}t^2\hat{j}$$

7. A closed triangular box is kept in an electric field of magnitude $E = 2 \times 10^3\ NC^{-1}$ as shown in the figure.



Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) entire surface.

Given

$\boldsymbol{E = 2 \times 10^3\ NC^{-1}}$

$\boldsymbol{\phi_E = ?}$

Solution

a) Vertical rectangle surface

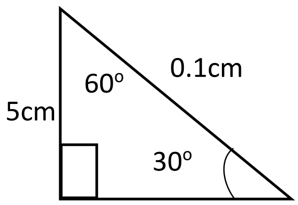
$\phi_E = EA\cos\theta, \quad A = 15\ cm \times 5\ cm$

$A = 75 \times 10^{-4} m^2$

$\phi_E = 2 \times 10^3 \times 75 \times 10^{-4} \times \cos 180°$

$= 150 \times 10^{-1}$

$\phi_E = -15\ N\ m^2\ C^{-1}$



b) Slanted surface :

$\phi_E = EA\cos\theta$

Area of the slanted surface,

$A = 15 \times 10^2 \times 10 \times 10^{-2} m^2$

$A = 150 \times 10^{-4}\ m^2$

$\phi_E = 2 \times 10^3 \times 150 \times 10^{-4} \times \cos 60°$

$\phi_E = 300 \times 10^{-1} \times \frac{1}{2}$

$\boldsymbol{\phi_E = 15\ N\ m^2\ C^{-1}}$

Horizontal surface

$\theta = 90°, \cos 90° = 0$

$\phi_E = 0$

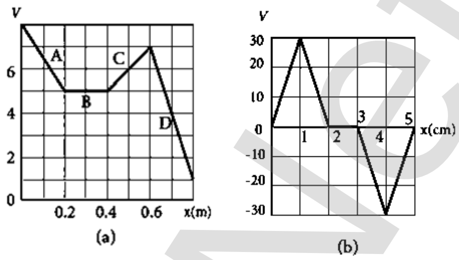
c) Entire surface:

$\phi_{Total} = \phi_{V.S} + \phi_{S.S} + \phi_{H.S}$

$= 15 - 15 + 0 = 0$

$\therefore$ **Net flux is zero**

8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).



Given:

$E_x = ?$

Solution

For figure (a)

(i) Region A:

$$E_x = -\frac{dV}{dx}$$

From 0 to 0.2 m slope is $-ve$ (negative)

$$\therefore E_x = -\left(-\frac{dV}{dx}\right) = \frac{dV}{dx} = \frac{3}{0.2}$$

$$= 15\ Vm^{-1}$$

(ii) Region B:

From 0.2m to 0.4m, potential is constant

$V = 5v,$

$$E_x = -\frac{dV}{dx} = 0 \qquad (V = constant)$$

(iii) Region C:

From 0.4 m to 0.6 m , slope is $+ve$

$$E_x = -\frac{dV}{dx} = -\frac{2}{0.2} = -10Vm^{-1}$$

(iv) Region D:

From 0.6 m to 0.8 m ,slope is $-ve$

$$E_x = -\left(-\frac{dV}{dx}\right) = \frac{dV}{dx} = \frac{6}{0.2} = 30\ Vm^{-1}$$

For figure (b):

(i) From 0 to 1 m , slope is $+ve$ (positive)

$$E_x = -\frac{dv}{dx} = -\frac{30}{1}$$

$$E_x = -30\ Vcm^{-1}$$

(ii) From 1 to 2 cm , slope is $-ve$

$$E_x = -\left(-\frac{dV}{dx}\right) = \frac{dV}{dx} = \frac{30}{1} = 30\ Vcm^{-1}$$

(iii) From 2 to 3 cm , $V = 0$

$$E_x = -\frac{dV}{dx} = 0$$

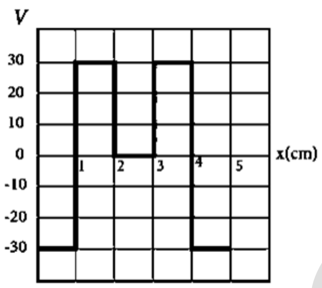
(iv) From 3 to 4cm, slope is $-ve$ (negative)

$$E_x = -\left(-\frac{dV}{dx}\right) = \frac{dV}{dx} = \frac{30}{1} = 30\ Vcm^{-1}$$

(v) From 4 to 5 cm, slope is $+ve$ (positive)

$$E_x = -\frac{dV}{dx} = -\frac{30}{1} = -30\ Vcm^{-1}$$

Electric field as a function of 'x' diagram for 'b' is



(b)

9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure.

To create the spark, an electric field of magnitude $3 \times 10^6\ Vm^{-1}$ is required.

(a) What potential difference must be applied to produce the spark?

(b) If the gap is increased, does the potential difference increase, decrease or remains the same?

(c) find the potential difference if the gap is 1 mm.

Given:

$E = 3 \times 10^6\ Vm^{-1}$,

$d = 0.6mm = 6 \times 10^{-4} m$, $V = ?$

Solution:

a) Potential difference

$$V = E \times d$$

$$V = 3 \times 10^6 \times 6 \times 10^{-4}$$

$$= 18 \times 10^2\ V$$

$$\boldsymbol{V = 1800\ V}$$

b) $V \propto d$

If gap increases, then potential also will increase.

c) If $d = 1mm$,

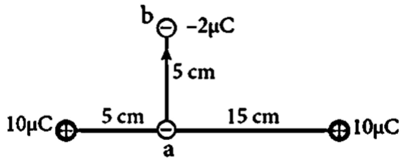
$$V = E \times d$$

$$V = 3 \times 10^6 \times 1 \times 10^{-3}$$

$$V = 3 \times 10^3\ V$$

$$\boldsymbol{V = 3000\ V}$$

10. A point charge of +10 μC is placed at a distance of 20 cm from another identical point charge of +10 μC. A point charge of -2 μC is moved from point a to b as shown in the figure. Calculate the change in potential energy of the system? Interpret your result.



Given:

$q_1 = 10\mu c,\ q_2 = -2\mu c$,

$r_1 = 5\ cm,\ r_2 = 15\ cm$

Solution

$r_1' = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}\ cm$

$r_2' = \sqrt{15^2 + 5^2} = \sqrt{250} = \sqrt{25 \times 10}$

$= 5\sqrt{10}\ cm$

change in $P.E \quad \Delta U = ?$

(i) Initial potential energy (u_i) when $-2\mu c$ charge at 'a'

$$U_i = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{r_2}$$

$$= \frac{q_1 q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} + \frac{1}{r_2}\right] \qquad [q_1 = q_2]$$

$$= 9 \times 10^9 \times (-2 \times 10^{-6}) \times 10 \times 10^{-6} \times \left[\frac{1}{5} + \frac{1}{15}\right] \times \frac{1}{10^{-2}}$$

$$= -18 \times 10^{-2} \times \frac{4}{15} \times \frac{1}{10^{-2}}$$

$$= -\frac{24}{5}$$

$U_i = -4.85$

(ii) Final potential energy (u_f), when a point charge $(-2\mu c)$ moved to 'b'

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{r_2}$$

$$= \frac{q_1 q}{4\pi\varepsilon_0}\left[\frac{1}{r_1'} + \frac{1}{r_2'}\right] \qquad (q_1 = q_2)$$

$$= 9 \times 10^9 \times (-2 \times 10^{-6}) \times 10 \times 10^{-8} \times \left[\frac{1}{5\sqrt{2}} + \frac{1}{5\sqrt{10}}\right]$$

$$= \frac{-18 \times 10^{-2}}{5 \times 10^{-2}} \times \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{10}}\right]$$

$$= -3.6 \times \left[\frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{\sqrt{10}}{\sqrt{10} \times \sqrt{10}}\right]$$

$$= -3.6 \times \left[\frac{1.414}{2} + \frac{3.16}{10}\right]$$

$$= -3.6 \times [0.707 + 0.316]$$

$$= -3.6 \times 1.023$$

$$\boldsymbol{U_f = -3.683\ J}$$

∴ Change in potential energy

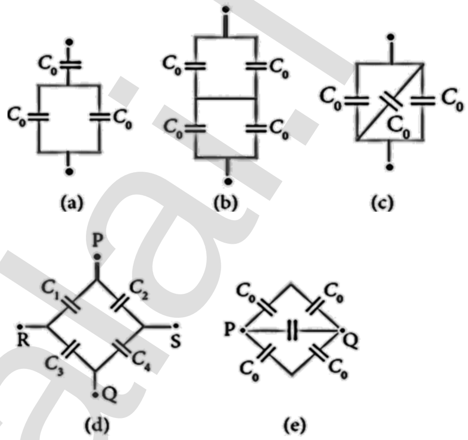
$$\boldsymbol{\Delta U = U_f - U_i}$$

$$= -3.683 - (-4.8)$$

$$= 1.117\ J$$

$$\boldsymbol{\Delta U = 1.12\ J}$$

11. Calculate the resultant capacitances for each of the following combinations of capacitors.



Solution:

(i) The equivalent capacitance of capacitors connected in series ,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} ..$$

The equivalent capacitance in capacitors connected in parallel,

$$\boldsymbol{C_p = C_1 + C_2 + C_3 + \cdots}$$

Figure (a):

c_0 *and* c_0 in parallel,

$C_p = c_0 + c_0 = 2c_0$

c_0 & $2c_0$ in series,

$$\frac{1}{C_s} = \frac{1}{c_0} + \frac{1}{2c_0} = \frac{2+1}{2c_0} = \frac{3}{2c_0}$$

$$\therefore \boldsymbol{C_s = \frac{2c_0}{3}}$$

Figure (b):

c_0 *and* c_0 in parallel,

$C_p = c_0 + c_0 = 2c_0$

Now,

$2c_0$ *and* $2c_0$ are in series,

$$\frac{1}{C_s} = \frac{1}{2c_0} + \frac{1}{2c_0} = \frac{2}{2c_0} = \frac{1}{c_0}$$

$$\therefore \boldsymbol{C_s = c_0}$$

Figure (c):

Now, c_0, c_0 *and* c_0 in parallel,

$C_p = c_0 + c_0 + c_0$

$\therefore \boldsymbol{C_p = 3c_0}$

Figure (d):

Equivalent capacitance between P and Q:

Now c_1 *and* c_3 in series

$$\frac{1}{C_{s_1}} = \frac{1}{c_1} + \frac{1}{c_3} = \frac{c_3 + c_1}{c_1 c_3} \;;\; C_{s_1} = \frac{c_1 c_3}{c_1 + c_3}$$

c_2 *and* c_4 in series

$$\frac{1}{C_{s_2}} = \frac{1}{c_2} + \frac{1}{c_4} = \frac{c_4 + c_2}{c_2 c_4} \;;\; C_{s_2} = \frac{c_2 c_4}{c_2 + c_4}$$

Now, C_{s_1} *and* C_{s_2} in parallel,

$$C_{PQ} = C_{s_1} + C_{s_2}$$

$$= \frac{c_1 c_3}{c_1 + c_3} + \frac{c_2 c_4}{c_2 + c_4}$$

$$\boldsymbol{C_{PQ} = \frac{c_1 c_3 (c_2 + c_4) + c_2 c_4 (c_1 + c_3)}{(c_1 + c_3)(c_2 + c_4)}}$$

Equivalent capacitance between R and S:

Now c_1 *and* c_2 in series

$$\frac{1}{C_{s_1}} = \frac{1}{c_1} + \frac{1}{c_2} = \frac{c_2 + c_1}{c_1 c_2} \;;\; C_{s_1} = \frac{c_1 c_2}{c_1 + c_2}$$

c_3 *and* c_4 in series

$$\frac{1}{C_{s_2}} = \frac{1}{c_3} + \frac{1}{c_4} = \frac{c_4 + c_3}{c_3 c_4} \;;\; C_{s_2} = \frac{c_3 c_4}{c_3 + c_4}$$

Now, C_{s_1} *and* C_{s_2} in parallel,

$$C_{RS} = C_{s_1} + C_{s_2}$$

$$= \frac{c_1 c_2}{c_1 + c_2} + \frac{c_3 c_4}{c_3 + c_4}$$

$$C_{RS} = \frac{c_1 c_2 (c_3 + c_4) + c_3 c_4 (c_1 + c_2)}{(c_1 + c_2)(c_3 + c_4)}$$

$$\boldsymbol{C_{RS} = \frac{c_1 c_2 c_3 + c_1 c_2 c_4 + c_1 c_3 c_4 + c_2 c_3 c_4}{(c_1 + c_2)(c_3 + c_4)}}$$

$$\boldsymbol{C_{PQ} = \frac{c_1 c_2 c_3 + c_1 c_3 c_4 + c_1 c_2 c_4 + c_2 c_3 c_4}{(c_1 + c_3)(c_2 + c_4)}}$$

Figure (e):

Now, c_0 *and* c_0 in series,

$$\frac{1}{C_{s_1}} = \frac{1}{c_0} + \frac{1}{c_0} = \frac{2}{c_0}; \qquad C_{s_1} = \frac{c_0}{2}$$

$$\frac{1}{C_{s_2}} = \frac{1}{c_0} + \frac{1}{c_0} = \frac{2}{c_0}; \qquad C_{s_2} = \frac{c_0}{2}$$

Now, C_{s_1}, c_0 *and* C_{s_2} in parallel

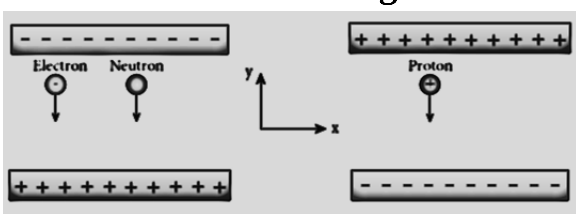
$$C_p = C_{s_1} + c_0 + C_{s_2}$$

$$= \frac{c_0}{2} + c_0 + \frac{c_0}{2}$$

$$C_p = \frac{c_0 + 2c_0 + c_0}{2} = \frac{4c_0}{2}$$

$$\boldsymbol{C_p = 2c_0}$$

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5 V and separation distance h = 1 mm as shown in the figure.



(a) Calculate the time of flight for both electron and proton

(b) Suppose if a neutron is allowed to fall, what is the time of flight?

(c) Among the three, which one will reach the bottom first?

Given:

$m_P = 1.6 \times 10^{-27} kg,$

$m_e = 9.1 \times 10^{-31} kg\,,\quad g = 10\,ms^{-2}$

Solution:

$h = 1mm = 1 \times 10^{-3} m; \qquad V = 5\,volt$

$E = \frac{V}{h} = 5 \times 10^3 NC^{-1}$

(i) Time of flight (t_f) for electron,

$$s = ut + \frac{1}{2}at^2.$$

$$s = h, u = 0, a = \frac{F}{m} = \frac{eE}{m}, t = t_e$$

$$h = 0 + \frac{1}{2}\left(\frac{eE}{m_e}\right)t_e^2 = \frac{1}{2}\left(\frac{eE}{m_e}\right)t_e^2$$

$$t_e = \sqrt{\frac{2m_e h}{eE}}$$

$$= \sqrt{\frac{2 \times 9.1 \times 10^{-31} \times 1 \times 10^{-3}}{1.6 \times 10^{-19} \times 5 \times 10^3}}$$

$$t_e = \sqrt{\frac{18.2 \times 10^{-18}}{8}} = \frac{10^{-9}}{2}\sqrt{9.1}$$

$$t_e = \frac{3 \times 10^{-9}}{2}$$

$$\boldsymbol{t_e = 1.5\,ns}$$

(ii) Time of flight for proton (t_p):

$$s = ut + \frac{1}{2}at^2\,;$$

$$s = h, u = 0, t = t_p, a = \frac{Ee}{m_P}$$

$$h = 0 + \frac{1}{2}\left(\frac{eE}{m_P}\right)t_p^2$$

$$t_p = \sqrt{\frac{2m_p h}{eE}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-27} \times 1 \times 10^{-3}}{1.6 \times 10^{-19} \times 5 \times 10^3}}$$

$$t_p = \sqrt{0.4 \times 10^{-14}} = 10^{-7}\sqrt{0.4}$$

$$t_p = 6.3 \times 10^{-7} \times 10^{-1} = 6.3 \times 10^{-8} s$$

$$\boldsymbol{t_p = 63\,n\,s}$$

(iii) Time of flight for neutron (t_n):

$$s = ut + \frac{1}{2}at^2\,;$$

$$s = h, u = 0, t = t_n, a = g$$

$$t_n = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1 \times 10^{-3}}{10}}$$

$$t_n = 10^{-2}\sqrt{2} = 1.414 \times 10^{-2}\,s$$

$$= 14.14 \times 10^{-3} s$$

$$t_n = 14.14\,ms$$

$$\therefore \boldsymbol{t_e < t_p < t_n}$$

Therefore, electrons will reach the bottom first.

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air ($3 \times 10^6 Vm^{-1}$), lightning will occur.

(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Given:

$E = 3 \times 10^6 V m^{-1}$;

$d = 1000 m, \ q = 25\ C, V = ?, \ U = ?$

Solution:

(i) Electric potential difference

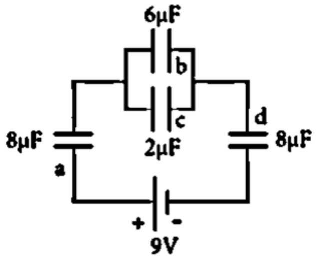
$V = E \times d$

$V = 3 \times 10^6 \times 1000 = 3 \times 10^9 V$

(ii) Electrostatic potential energy (U)

$U = V \times q = 3 \times 10^9 \times 25 = 75 \times 10^9 \ \text{J}$

14. For the given capacitor configuration
(a) Find the charges on each capacitor
(b) potential difference across them
(c) energy stored in each capacitor



Solution:

$V = 9\ V\ , C_a = 8\mu F, \ C_b = 6\mu F,$

$C_c = 2\mu F, \quad C_d = 8\mu F$

$C_b \ and \ C_c$ in parallel,

$C_{bc} = C_b + C_c = 6 + 2 = 8\mu F$

Potential difference in each capacitor:

$V_a = \frac{V}{3} = \frac{9}{3} = 3\ volt$

$V_b = \frac{V}{3} = \frac{9}{3} = 3\ volt$

$V_c = \frac{V}{3} = \frac{9}{3} = 3\ volt$

$V_d = \frac{V}{3} = \frac{9}{3} = 3\ volt$

Charge on each capacitor

$q_a = C_a V_a \quad = 8 \times 10^{-6} \times 3$

$\qquad = 24 \times 10^{-6}\ c$

$q_a = 24\ \mu c$

$q_b = C_b V_b = 6 \times 10^{-6} \times 3$

$\qquad = 18 \times 10^{-6} c$

$q_b = 18 \mu c$

$q_c = C_c V_c = 2 \times 10^{-6} \times 3$

$\qquad = 6 \times 10^{-6} c,$

$q_c = 6 \mu c$

$q_d = C_d V_d \quad = 8 \times 10^{-6} \times 3$

$\qquad = 24 \times 10^{-6} c,$

$q_d = 24 \mu c$

Energy stored in each capacitor

$U_a = \frac{1}{2} C_a V_a^2 = \frac{1}{2} \times 8 \times 10^{-6} \times (3)^2$

$\qquad = 36 \times 10^{-6} J$

$\therefore U_a = 36\ \mu J$

$U_b = \frac{1}{2} C_b V_b^2 = \frac{1}{2} \times 6 \times 10^{-6} \times (3)^2$

$\qquad = 27 \times 10^{-6} J$

$\therefore U_b = \ 27 \mu J$

$U_c = \frac{1}{2} C_c V_c^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (3)^2$

$\qquad = 9 \times 10^{-6} J$

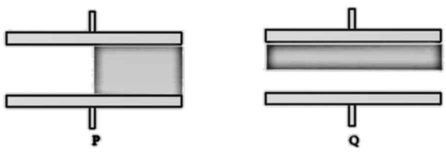
$\therefore U_c = \ 9 \mu J$

$U_d = \frac{1}{2} C_d V_d^2 = \frac{1}{2} \times 8 \times 10^{-6} \times (3)^2$

$\qquad = 36 \times 10^{-6} J$

$\therefore U_d = 36\ \mu J$

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ε_r as shown in the figure. Calculate the capacitance of capacitors P and Q.



Capacitor 'P'

capacitance of a parallel plate capacitor,

$C = \frac{\varepsilon_0 A}{d}$

with dielectric, $C = \frac{\varepsilon_r \varepsilon_0 A}{d}$

In capacitor 'P': C_1 *and* C_2 parallel ,

$C_p = C_1 + C_2$

$$C_1 = \frac{\varepsilon_0 A}{d}; \quad here\ A = \frac{A}{2}$$

$$C_1 = \frac{\varepsilon_0 \left(\frac{A}{2}\right)}{d} = \frac{\varepsilon_0 A}{2d}$$

$$C_2 = \frac{\varepsilon_r \varepsilon_0 \left(\frac{A}{2}\right)}{d} = \frac{\varepsilon_r \varepsilon_0 A}{2d}$$

$$C_p = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_r \varepsilon_0 A}{2d}$$

$$\boldsymbol{C_p = \frac{\varepsilon_0 A}{2d}(1 + \varepsilon_r)}$$

In capacitor 'Q':

Now, $A = A, d = \frac{d}{2}$

C_1 *and* C_2 are in series,

$$\frac{1}{C_Q} = \frac{1}{C_1} + \frac{1}{C_2};$$

$$C_1 = \frac{\varepsilon_r \varepsilon_0 A}{\frac{d}{2}}; \quad C_1 = \frac{2\varepsilon_r \varepsilon_0 A}{d} \quad ,$$

$$C_2 = \frac{\varepsilon_0 A}{\frac{d}{2}}; \quad C_2 = \frac{2\varepsilon_0 A}{d}$$

$$\frac{1}{C_Q} = \frac{d}{2\varepsilon_r \varepsilon_0 A} + \frac{d}{2\varepsilon_0 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left[\frac{1}{\varepsilon_r} + 1\right]$$

$$\frac{1}{C_Q} = \frac{d}{2\varepsilon_0 A}\left[\frac{1 + \varepsilon_r}{\varepsilon_r}\right]$$

$$\boldsymbol{C_Q = \frac{2\varepsilon_0 A}{d}\left[\frac{\varepsilon_r}{1 + \varepsilon_r}\right]}$$