XII Physics

English Medium

Study material

Important derivations steps

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Electric field (E)

charge → Two charge Interaction

Point space

formula

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

1m

[Electric field is discovered by Miheal Faraday.]

ε_0 → Permittivity of free space

$$\boxed{\varepsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}}$$

q → charge

r → distance

Vector form : $(\rightarrow)$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$\wedge$ → unit vector

unit :

NC^{-1} (or) N/C

Quantity :

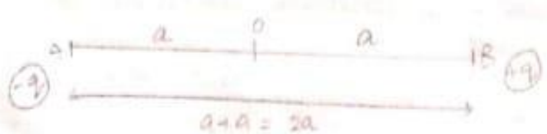
Vector Quantity.

Electric dipole

Pg no 20

2m

two → -q, +q



Statement [Two equal and opposite charge separated by a small distance constitute an electric dipole]

Ex. :

CO (carbon - mono - oxide)
H_2O (water)
NH_3 (Ammonia)
HCl (Hydrochloride acid)

formula :

$$\boxed{P = q \times 2a}$$

$P \to$ Electric dipole moment.
$q \to$ charge.
$2a \to$ small distance

Unit.

$C_m \to$ Coulomb metre

Quantity :

Vector Quantity.

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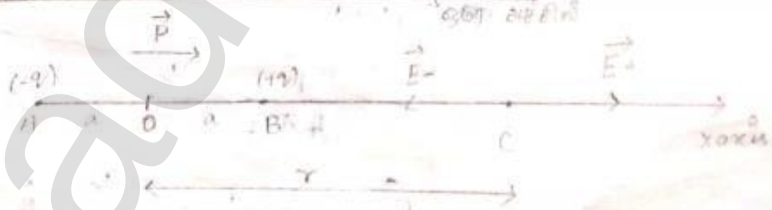
5m
Book back 4.

i) **Electric field due to an electric dipole at points on the axial line.**



AB two point
$-q \to$ negative charge
$+q \to$ positive charge
$O \to$ centre of the point

$\boxed{OC = r}$

$a \rightarrow$ distance

$2a \rightarrow$ small distance

$r \rightarrow$ distance

$OA = a$

$OB = a$

$AB = 2a$

$E \rightarrow$ total Electric field

$P \rightarrow$ electric dipole moment

$$\boxed{P = q \times 2a}$$

B point $\rightarrow$ (positive charge) $(+q)$

Electric field.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$\boxed{\vec{E}_+ = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-a)^2} \hat{P}} \quad \text{— (1)}$$

A point $\rightarrow$ (negative charge) $(-q)$

Electric field

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$\boxed{\vec{E}_- = -\frac{1}{4\pi\varepsilon_0} \frac{q}{(r+a)^2} \hat{P}} \quad \text{— (2)}$$

Total Electric field

$$\boxed{\vec{E}_{total} = \vec{E}_{+} + \vec{E}_{-}}$$

$$\vec{E}_{total} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p}\right) + \left(-\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}\right)$$

$$\vec{E}_{total} = \left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{total} = \left[\frac{q}{4\pi\varepsilon_0}\frac{1}{(r-a)^2} - \frac{q}{4\pi\varepsilon_0}\frac{1}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2(r+a)^2}\right]\hat{p}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$
$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r^2+a^2+2ra) - (r^2+a^2-2ra)}{(r^2-a^2)^2}\right]\hat{p}$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cancel{r^2}+a^2+2ra-\cancel{r^2}-a^2+2ra}{(r^2-a^2)^2}\right]\hat{p}$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{p}$$

$\boxed{r >> a}$ neglected (a)

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{(r^2)^2}\right]\hat{p}$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4\gamma a}{\gamma^4}\right]\hat{p}$$

$$\vec{E}_{total} = \frac{q}{4\pi\varepsilon_0}\frac{4a}{\gamma^3}\hat{p}$$

$$\vec{E}_{total} = \frac{1}{4\pi\varepsilon_0}\frac{q \times 4a}{\gamma^3}\hat{p}$$

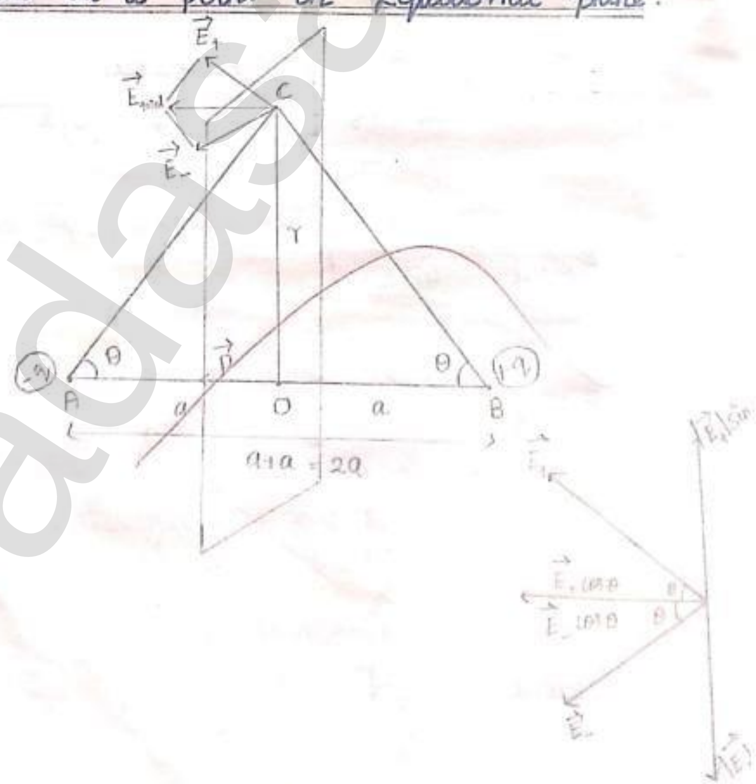
Electric dipole moment.

$$P = q \times 2a$$

$$\boxed{2\vec{p} = q \times 4a.\hat{p}}$$

$$\boxed{\vec{E}_{total} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{\gamma^3}}$$

5m. ii) Electric field due to an Electric dipole at a point on Equatorial plane.



$\sin\theta$ → Opposite direction.
Vertical component.
Cancel each other

$\cos\theta$ → Same direction
Horizontal component.

AB → two point.
$-q$ → negative charge.
$+q$ → positive charge.
$2a$ → Small distance.

$AB = 2a$
$OA = a$
$OB = a$
$OC = r$

r → distance.
E → total electric field.
P → electric dipole moment

$$\boxed{P = q \times 2a}$$

B point ($+q$ charge)

$$\boxed{|\vec{E}_+| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}} \quad \text{--- (1)}$$

A point ($-q$ charge)

$$\boxed{|\vec{E}_-| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}} \quad \text{(2)}$$

Total electric field.

$$\vec{E}_{total} = -|\vec{E}_+|\cos\theta\,\hat{p} - |\vec{E}_-|\cos\theta\,\hat{p}$$

$$\vec{E}_{total} = -2\vec{E}\cos\theta\,\hat{p}$$

$$\vec{E}_{total} = -2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \times \cos\theta \cdot \hat{p}$$

$$\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{(r^2+a^2)} \times \cos\theta \cdot \hat{p}$$

$$\boxed{\cos\theta = \frac{a}{\sqrt{r^2+a^2}}}$$

$$\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{(r^2+a^2)} \times \frac{a}{\sqrt{r^2+a^2}} \cdot \hat{p}$$

$$\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{2qa}{(r^2+a^2)\sqrt{r^2+a^2}} \cdot \hat{p}$$

$$\boxed{\hat{p} \cdot q \times 2a = \vec{p}}$$

$$\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)(r^2+a^2)^{1/2}}$$

$$\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)^{3/2}}$$

$$\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2)^{3/2}}$$

$\boxed{r >> a}$

$a \rightarrow$ neglected

$$\boxed{\vec{E}_{total} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}}$$

Torque $\cdot (\tau) \rightarrow$ Greek letter
(திருப்புவிசை)

$$\tau = PE\sin\theta \rightarrow \text{angle}$$

τ: torque

P: Electric dipole $P = q \times 2a$

E: Electric field

Vector form :

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Conditions :

$$\tau = PE \sin \theta$$

i) maximum range. (பெரும)

$$\boxed{\theta = 90^\circ}$$

$$\tau = PE \sin(90^\circ) \qquad \sin 90^\circ = 1$$

$$\tau = PE(1)$$

$$\boxed{\tau = PE}$$

ii) minium range (சிறுமம்)

$$\boxed{\theta = 0^\circ}$$

$$\tau = PE \sin(0^\circ) \qquad \sin 0^\circ = 0$$

$$\tau = PE(0)$$

$$\boxed{\tau = PE}$$

Note :

$\cos 0^\circ = 1$

$\cos 90^\circ = 0$

$\cos 180^\circ = -1$

$\cos(180^\circ - \theta) = -\cos\theta$

Pg no: Example 1.11

Soln

HCl gas

Uniform Electric field $E = 3 \times 10^4 NC^{-1}$

Dipole moment $P = 3.4 \times 10^{-30}$ Cm maximum torque = ?

torque

$$\tau = PE\sin\theta$$

maximum $(\theta = 90^\circ)$

$$\tau = PE\sin(90^\circ)$$

$$\tau = PE(1)$$

$$\boxed{\tau = PE}$$

$$\tau = 3.4\times10^{-30}\ Cm \times 3\times10^{4}\ NC^{-1}$$

$$\tau = 3.4\times3\times10^{-30}\times10^{4}\ Nm$$

$$\frac{34\times3}{10.2}$$

$$\boxed{\tau_{max} = 10.2\times10^{-26}\ Nm}$$

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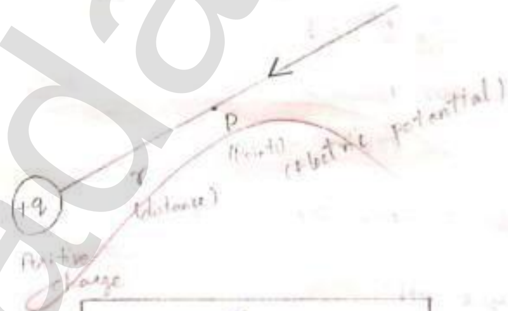
5m 6. Electric potential due to an point change. Unit : Volt.



$$\boxed{V = \int_{\infty}^{r} (-\vec{E})\cdot d\vec{r}}$$

$\infty \rightarrow$ Infinity

$r \rightarrow$ Distance

Electric potential

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cdot\hat{r}}$$

$$V = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\vec{dr}\cdot\hat{r}$$

$$V = -\int_{\infty}^{r}\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\vec{dr}\cdot\hat{r}$$

$$V = -\int_{\infty}^{r}\frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\,\vec{dr}\cdot\hat{r}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{r^2}\,\vec{dr}\cdot\hat{r}$$

$$\boxed{\vec{dr} = dr\,\hat{r}}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{r^2}\,dr\,\hat{r}\cdot\hat{r}$$

$$\boxed{\hat{r}\cdot\hat{r} = 1}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{r^2}\,dr$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} r^{-2}\,dr \quad \int x^n dx = \frac{x^{n+1}}{n+1}+C$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[\frac{r^{-2+1}}{-2+1}\right]_{\infty}^{r}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[\frac{r^{-1}}{-1}\right]_{\infty}^{r}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[-(r^{-1})\right]_{\infty}^{r}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\boxed{\frac{1}{\infty} = 0}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - 0\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

3m

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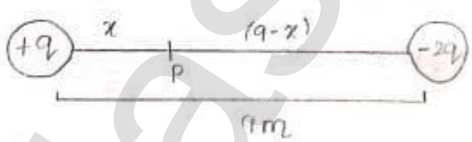
Example 1.13

Soln:



Electric potential:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0$$

$$\frac{q}{x} - \frac{2q}{(9-x)} = 0$$

$$\frac{q}{x} = \frac{2q}{(9-x)}$$

$$\frac{1}{x} = \frac{2}{9-x}$$

$$9-x = 2x$$

$$9 = 2x + x$$

$$9 = 3x$$

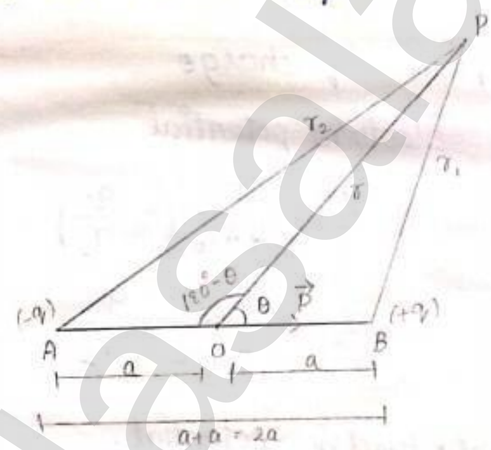
$$\boxed{x = 3m}$$

$x = 3m$

$(9-x) = 9-3 = 6\ m$

(X) 5m.
Pg.no: 30

Electrostatic potential at a point due to an electric dipole.



AB → two point
-q → negative charge
+q → positive charge
r → distance
2a → small distance
a → distance
θ → angle
P → Electric dipole moment

OP = r
OA = a
OB = a
AB = 2a

$\boxed{P = q \times 2a}$

$O \to$ centre of a point

$BP = r_1$

$AP = r_2$

Electric Potential

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

B point +q charge

Electric potential

$$\boxed{V_1 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_1}} \quad (1)$$

A point -q charge

Electric potential

$$V_2 = \frac{1}{4\pi\varepsilon_0} \left(-\frac{q}{r_2}\right)$$

$$\boxed{V_2 = -\frac{1}{4\pi\varepsilon_0} \frac{q}{r_2}} \quad (2)$$

Total Electric potential.

$$\boxed{V = V_1 + V_2}$$

$$V = \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r_1}\right) + \left(-\frac{1}{4\pi\varepsilon_0} \frac{q}{r_2}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_1} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_2}$$

$$V = \frac{q}{4\pi\varepsilon_0} \frac{1}{r_1} - \frac{q}{4\pi\varepsilon_0} \frac{1}{r_2}$$

$$\boxed{V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \quad -③$$

Find $\frac{1}{r_1}$ in Cosine law

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

Rearrange

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2ra}{r^2}\cos\theta\right)$$

$$r_1^2 = r^2\left(1 + \underbrace{\frac{a^2}{r^2}}_{\text{neglected}} - \frac{2a}{r}\cos\theta\right)$$

$$r_1^2 = r^2\left(1 - \frac{2a}{r}\cos\theta\right)$$

$$r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{1/2}$$

Inverse

$$\frac{1}{r_1} = \frac{1}{r}\frac{1}{\left(1 - \frac{2a}{r}\cos\theta\right)^{1/2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Binomical theorem

$$\boxed{\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)} \quad -④$$

Find $\frac{1}{r_2}$ in Cosine law

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$r_2^2 = r^2 + a^2 - 2ra\cos(180° - \theta)$$

$$r_2^2 = r^2 + a^2 - 2ra(-\cos\theta)$$

$$r_2^2 = r^2 + a^2 + 2ra\cos\theta$$

Rearrange.

$$r_2^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2ra}{r^2}\cos\theta\right)$$

$$r_2^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$$

$$r_2^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$$

$$r_2 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{1/2}$$

Inverse

$$\frac{1}{r_2} = \frac{1}{r}\frac{1}{\left(1 + \frac{2a}{r}\cos\theta\right)^{1/2}}$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Binomical theorem.

$$\boxed{\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)} \quad - (5)$$

from (3) equation

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\boxed{\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)}$$

$$\boxed{\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) - \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\left(1 + \frac{a}{r}\cos\theta\right) - \left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[1+\frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[\frac{2a}{r}\cos\theta\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q\times 2a\cos\theta}{r^2}$$ $\boxed{q\times 2a = P}$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}}$$ (or) $\boxed{P\cdot\cos\theta = \vec{P}\cdot\hat{r}}$

$$\boxed{\vec{V} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}}{r^2}\cdot\hat{r}}$$

Special case :

i) +q charge (axis line) $\boxed{\theta = 0^\circ}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos(0^\circ)}{r^2}$$

$\boxed{\cos 0^\circ = 1}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P(1)}{r^2}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^2}}$$

ii) Axis line (-q) charge $\boxed{\theta = 180^\circ}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos(180^\circ)}{r^2}$$

$\boxed{\cos(180^\circ) = -1}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P(-1)}{r^2}$$

$$\boxed{V = -\frac{1}{4\pi\varepsilon_0}\frac{P}{r^2}}$$

iii) Equatorial line $\boxed{\theta = 90^\circ}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos(90^\circ)}{r^2}$$

$\boxed{\cos(90^\circ) = 0}$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{P(0)}{r^2}$$

$$\boxed{V = 0}$$