EDUCATION DEPARTMENT

VILLUPURAM DISTRICT

MATHEMATICS

10

Question Bank &
Govt. Qn. Paper - Sep. 2020

2022-23

BEST WISHES
Mrs. K.Krishnapriya, B.Sc., M.A., B.Ed.,
Chief Educational Officer, Villupuram District.

At the state level, we secured the 11th place in the Government Public Examination through our hard work and deserve appreciation.

தன்னம்பிக்கை + விடாமுயற்சி + கடின உழைப்பு = வெற்றி
"The Struggle you're in Today will definitely develop the strength you need for Tomorrow.

A. SIVAMOORTHY, Government High School, Perumbakkam, **Villupuram District.**
Mobile: 9750827997

MESSAGE TO TEACHERS

First and Foremost I would like to express by hearty gratitude to all the teachers who are taking much effort to attain the outstanding performance in Tenth Public Examination last year.

Congratulations to all the teachers who are taking atmost care to improve the level of gifted students as well as the slow learners with colourful marks.

Still we are in a position to enhance the percentage of X Standard result in Villupuram District in the State Level.

"In my point of view a dedicated and service - minded teacher is blessed by the God Over"

Hence it is my appeal to all the Tenth handing teachers to devote more time for the welfare and upliftment of the poor, the destitute, the down trodden and the rural pupils fruitfully.

With Best Wishes

Mrs.K.Krishnapriya, B.Sc., M.A., B.Ed.,

Chief Educational Officer, Villupuram District.

Preface

This material has been prepared in accordance with The TamilNadu Government State Board Syllabus. I am very happy to inform you that by practicing all the problems in this material thoroughly will definitely make the students to score more than 90 percentage of marks in Mathematics in the Public Examination. I am in a position to express my hearty gratitude to our respected CEO Madam and DEO Sir for having encouraged my serious attempt to prepare this material for the welfare of the students. Constructive criticisms and valuable suggestions are always welcome.

A. SIVAMOORTHY,
Government High School,
Perumbakkam,
Villupuram District.

Approximate marks will be obtained by studying both State 1 and Stage 2.

Chapter	Title	Stage - 1 Marks				Stage - 2 Marks			
		1	2	5	8	1	2	5	8
1	Relations and Functions	2	1	1	-	2	2	2	-
2	Numbers and Sequences	2	1	1	-	2	1	1	-
3	Algebra	2	1	1	1	2	1	2	1
4	Geometry	2	1	1	1	2	1	1	1
5	Coordinate Geometry	2	1	1	-	2	1	1	-
6	Trigonometry	1	1	-	-	1	1	1	-
7	Mensuration	2	1	1	-	2	1	1	-
8	Statistics and Probability	1	2	1	-	1	2	1	2
	Total Questions	14(12)	14(9)	14(7)	2(2)	14(12)	14(10)	14(10)	2(2)
	Total Marks	12	18	35	16	12	20	50	16

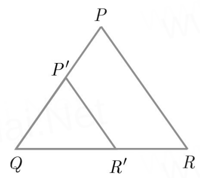
STAGE - I

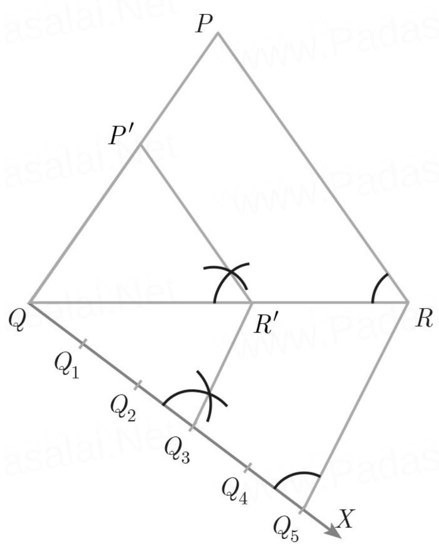
4 GEOMETRY

1. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)

Solution:

Rough diagram





Given a triangle PQR we are required to construct another triangle whose sides are 3/5 of the corresponding sides of the triangle PQR

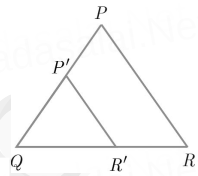
Steps of Construction:

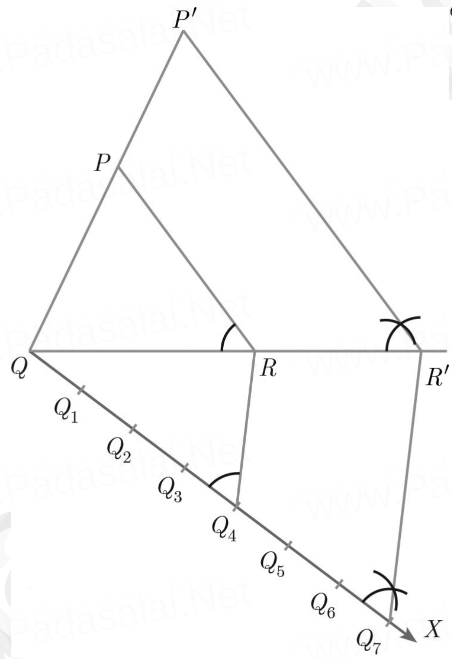
1. Construct a Δ PQR with any measurement
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P
3. Locate 5(the greater of 3 and 5 in $\frac{3}{5}$) points. Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$.
4. Join Q_5R and draw a line through Q_3(the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q5R to intersect QR at R.
5. Draw a line through R' parallel to the line RP to intersect QP at P'. Then, Δ P'QR' is the required triangle each of whose sides is three-fifths of the corresponding sides of ΔPQR.

2. Cons Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)

Solution:

Rough diagram



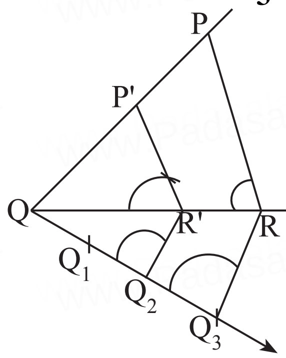


Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{7}{4}$ of the corresponding sides of the triangle PQR

Steps of Construction:

1. Construct a Δ PQR with any measurement
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P
3. Locate 7 (the greater of 7 and 4 in $\frac{7}{4}$). Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$.
4. Join Q_4 (the 4^{th} point, 4 being smaller of 4 and 7 in $\frac{7}{4}$) to R and draw a line through Q7 parallel to Q_4R, intersecting the extended line segment QR at R'.
5. Draw a line through R' parallel to the line RP to intersect QP at P'. Then, Δ P'QR' is the required triangle each of whose sides is seven - fourths of the corresponding sides of ΔPQR.

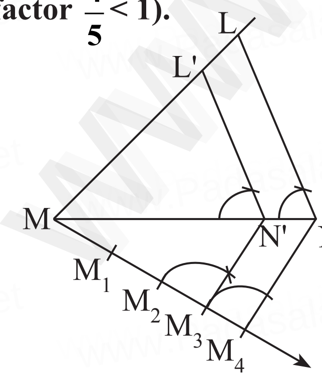
3. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3} < 1$).



Steps of Construction

1. Draw a Triangle PQR with any measurement
2. Draw any ray QX making an acute angle with QR on the side opposite to the vertex P.
3. Locate 3 (the greater of 2 and 3 in $\frac{2}{3}$) points. Q_1, Q_2, Q_3 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3$.
4. Join Q_3, R and draw a line through Q_2 (the second point, 2 being smaller of 2 and 3 in $\frac{2}{3}$) parallel to Q_3 R to intersect QR at R'.
5. Draw line through R^' parallel to the line RP to intersect QP at P'.
6. The ΔP'QR' is the required triangle each of the whose sides is $\frac{2}{3}$ of the corresponding sides of ΔPQR.

4. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5} < 1$).

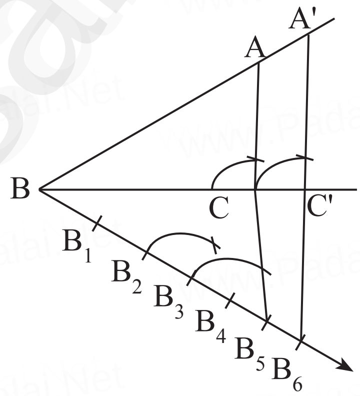


Steps of Construction

1. Draw a Triangle LMN with any measurement
2. Draw any ray making an acute angle to the vertex L.
3. Locate 5 points (the greater of 4 and 5 in —). M_1, M_2, M_3, M_4, M_5 and MX so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$.
4. Join M_5N and draw a line parallel to M_5N through M_4 (the fourth point, 4 being smaller of 4 and 5 in $\frac{4}{5}$) to intersect in MN at N'.
5. Draw line through N' paralel to the line NL to intersect ML at L'

   The ΔL'MN' is the required triangle each of the whose sides is $\frac{4}{5}$ of the corresponding sides of ΔLMN.

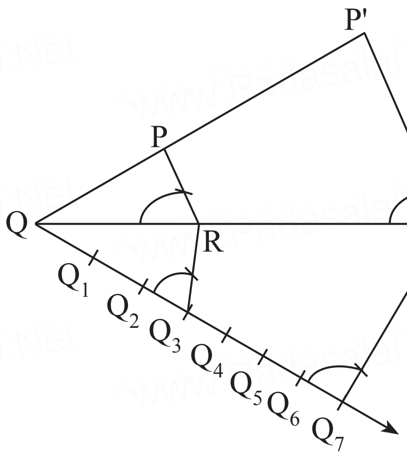
5. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$). SEP-20



Steps of Construction

1. Draw a Triangle ABC with any measurement
2. Draw any ray BX making an acute angle with BC on the opposite side to the vertex A.
3. Locate 6(the greater of 6 and 5 in 6/5) points in BX. B_1, B_2, B_3, B_4, B_5, B_6 so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6$.
4. Join B_3, R(the fifth points, 5 being smaller of 5 and 6 in $\frac{6}{5}$) to C and draw a line through B6 parallel to B5C intersecting the extended line segment BC at C'.
5. Draw a line through C' parallel to CA intersecting the extended line segment BA at A'
6. The ΔA'BC' is the required triangle each of the whose sides is $\frac{6}{5}$ of the corresponding sides of ΔABC.

6. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3} > 1$).



Steps of Construction

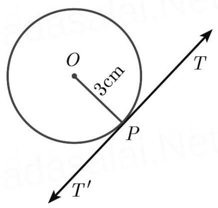
1. Draw a Triangle PQR with any measurement
2. Draw any ray QX making an acute angle with QR on the opposite side to the vertex P.
3. Locate 7 points(the greater of 7 and 3 in $\frac{7}{3}$) points. Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 and Q_7 so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 = Q_6Q_7$
4. Join Q_3R and draw a line segment through Q_7 parallel to Q_3R to intersecting the extended line segment QR at R'.
5. Draw line segment through R' parallel to the PR to intersecting the extended line segment QP at P'
6. The ΔP'QR' is the required triangle each of the whose sides is $\frac{7}{3}$ of the corresponding sides of ΔPQR.

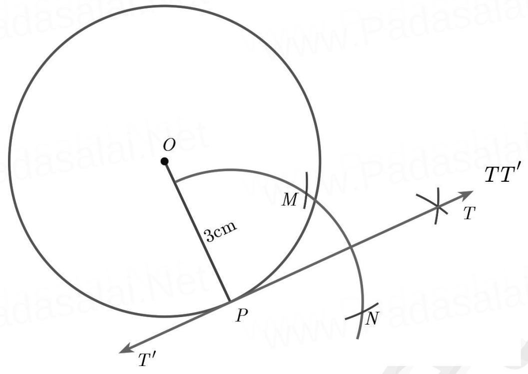
7. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution:

Given, radius r = 3 cm

Rough diagram





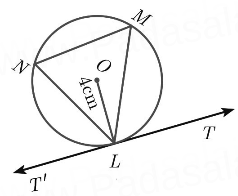
Construction

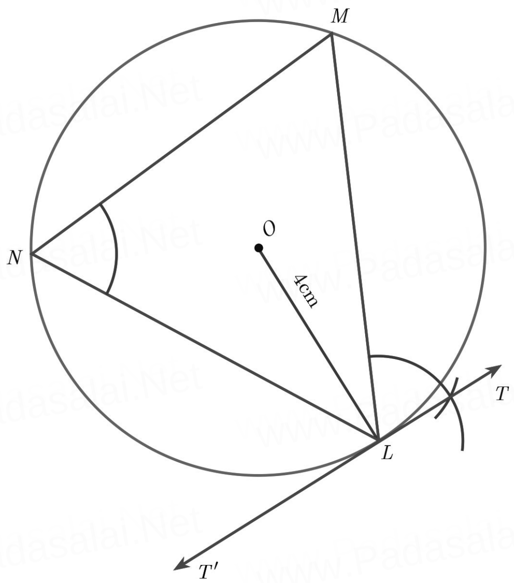
1. Draw a circle with centre at O of radius 3 cm.
2. Take a point P on the circle. Join OP.
3. Draw perpendicular time TT' to OP which passes through P.
4. TT' is the required tangent.

8. Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution:

Rough diagram





Construction

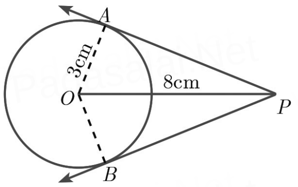
1. With O as the centre, draw a circle of radius 4cm.
2. Take a point L on the circle. Through L draw any chord LM.
3. Take a point N distinct from L and M on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

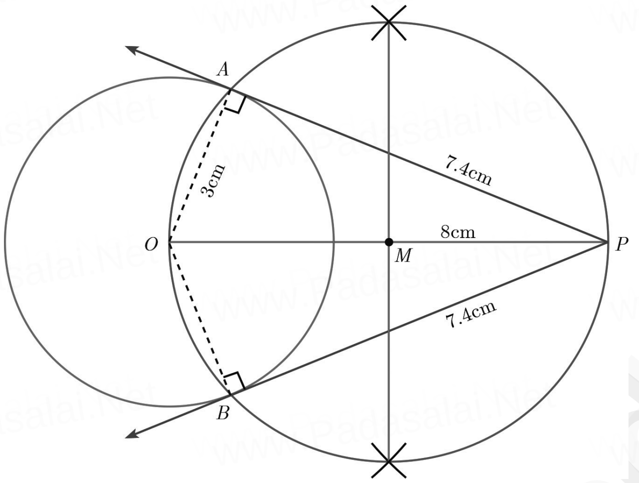
4. Through L draw a tangent TT' such that ∠TLM = ∠MNL .
5. TT' is the required tangent .

9. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

Solution:

Rough diagram





Construction:

1. With centre at O, draw a circle of radius 3 cm.
2. Draw a line OP of length 8 cm.
3. Draw a perpendicular bisector of OP, which curs OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4 cm,

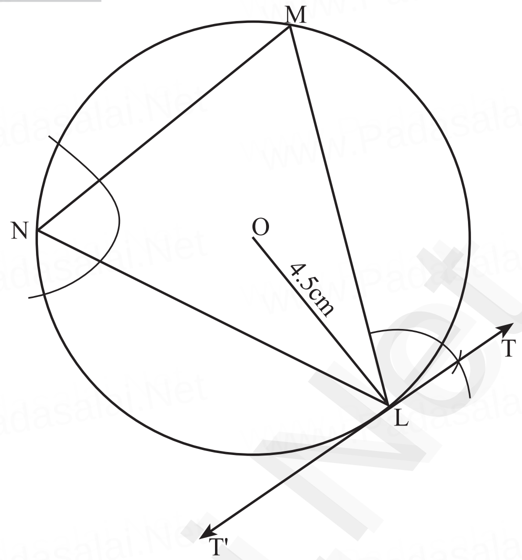
   Verification: In the right angle triangle OAP.

   $PA^2 - OA^2 = 64 - 9 = 55$

   $PA = \sqrt{55} = 7.4$ cm

10. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

Solution:

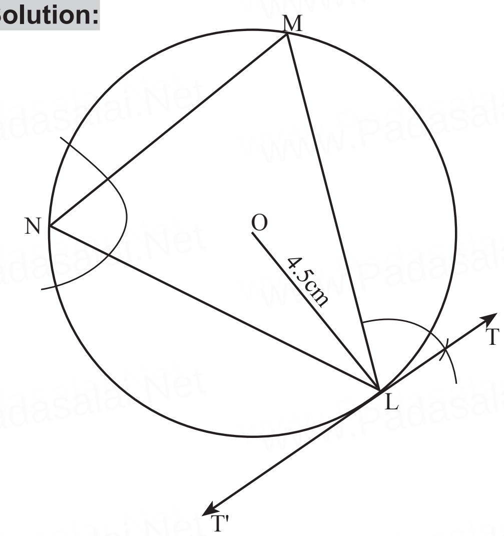


Construction:

1. Draw a circle with centre at P of radius 4.5 cm.
2. Take a point L on the circle. Though L draw any chord LM.
3. Take a point M distinct from L and N on the circle, So thatL, M and N are in anticlockwise direction. Join LN and NM.
4. Through L draw a tangent TT' such that ∠TLM = ∠MNL.
5. TT' is the required tangent.

11. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

Solution:



Construction:

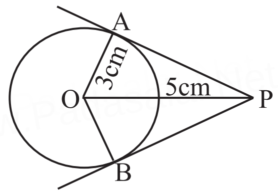
1. Draw a circle with centre at P of radius 4.5 cm.
2. Take a point L on the circle. Though L draw any chord LM.

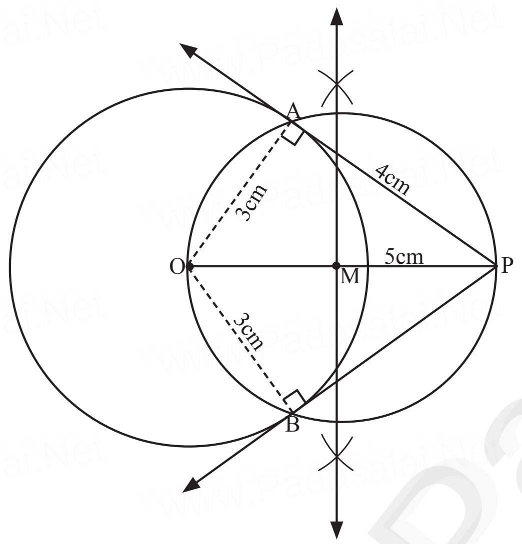
3. Take a point M distinct from L and N on the circle, So thatL, M and N are in anticlockwise direction. Join LN and NM.
4. Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.
5. TT' is the required tangent.

12. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents. SEP-20

Solution:

Rough Diagram





Construction:

1. With centre at O, draw a circle of radius 5cm.
2. Draw a line OP = 10 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 8.7 cm.

Proof:

In ΔOPA

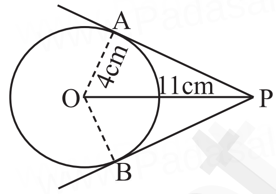
$PA^2 = OP^2 - OA^2$

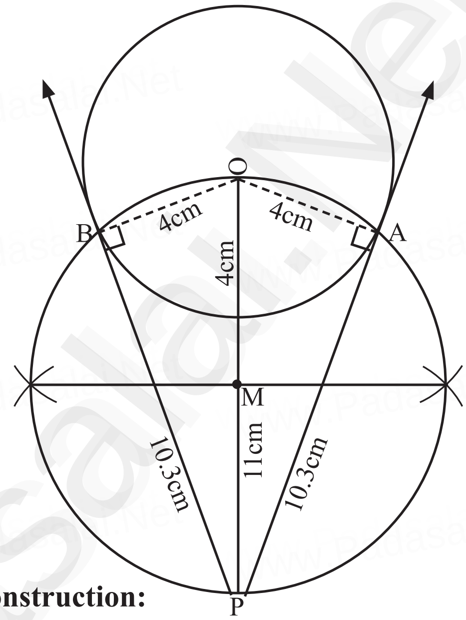
$= 10^2 - 5^2 = 100 - 25 = 75$

$PA = \sqrt{75} = 8.6$ cm (approx)

13. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

Rough Diagram





Construction:

1. With centre at O, draw a circle of radius 4cm.
2. Draw a line OP = 11 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 10.2 cm.

Verification:

In ΔOPA $AP^2 = OP^2 - OA^2$

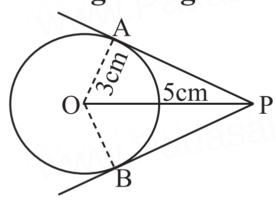
$= 11^2 - 4^2 = 121 - 16 = 105$

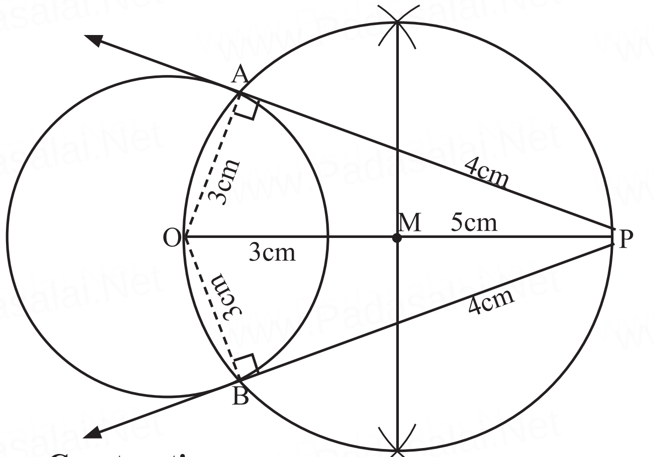
$AP = \sqrt{105} = 10.2$ cm

14. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents. SEP-21

Solution:

Rough Diagram





Construction:

1. With centre at O, draw a circle of radius 3cm.
2. Draw a line OP = 5cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 4 cm.

Verification

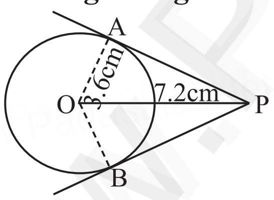
In ΔOPA $AP^2 = OP^2 - OA^2$

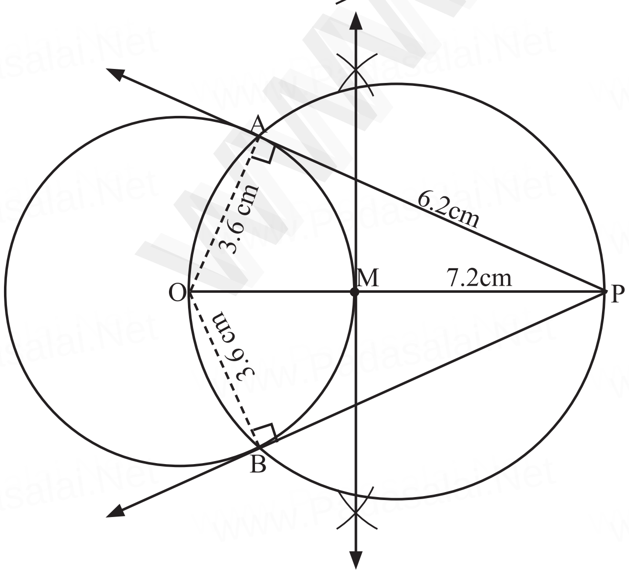
$= 5^2 - 3^2 = 25 - 9 = 16$

$AP = \sqrt{16} = 4$ cm

15. Draw a tangent to the circle from the point P having radius 3.6cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

Solution: **Rough Diagram**





Construction:

1. With centre at O, draw a circle of radius 3.6 cm.
2. Draw a line OP = 7.2 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 6.2 cm.

Verification:

In ΔOPA, $PA^2 = OP^2 - OA^2$

$= 7.2^2 - 3.6^2$

$= 51.84 - 12.96$

$= 38.88$

$PA = \sqrt{38.88} = 6.2$ cm (approx)

For Practice

1. Construct a ΔPQR in which PQ = 8 cm, R = 60° and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.
2. Construct a triangle ΔPQR such that QR = 5 cm, $\angle P = 30°$ and the altitude from P to QR is of length 4.2 cm.
3. Draw a triangle ABC of base BC = 8 cm, A = 60° and the bisector of $\angle A$ meets BC at D such that BD = 6 cm.
4. Construct a ΔPQR in which QR = 5 cm, P = 40° and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.
5. Construct a ΔPQR such that QR = 6.5 cm, P = 60° and the altitude from P to QR is of length 4.5 cm.
6. Construct a ΔABC such that AB = 5.5 cm, C = 25° and the altitude from C to AB is 4 cm.
7. Draw a triangle ABC of base BC = 5.6 cm, A = 40° and the bisector of $\angle A$ meets BC at D such that CD = 4 cm.
8. Draw ΔPQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.

***

GRAPH

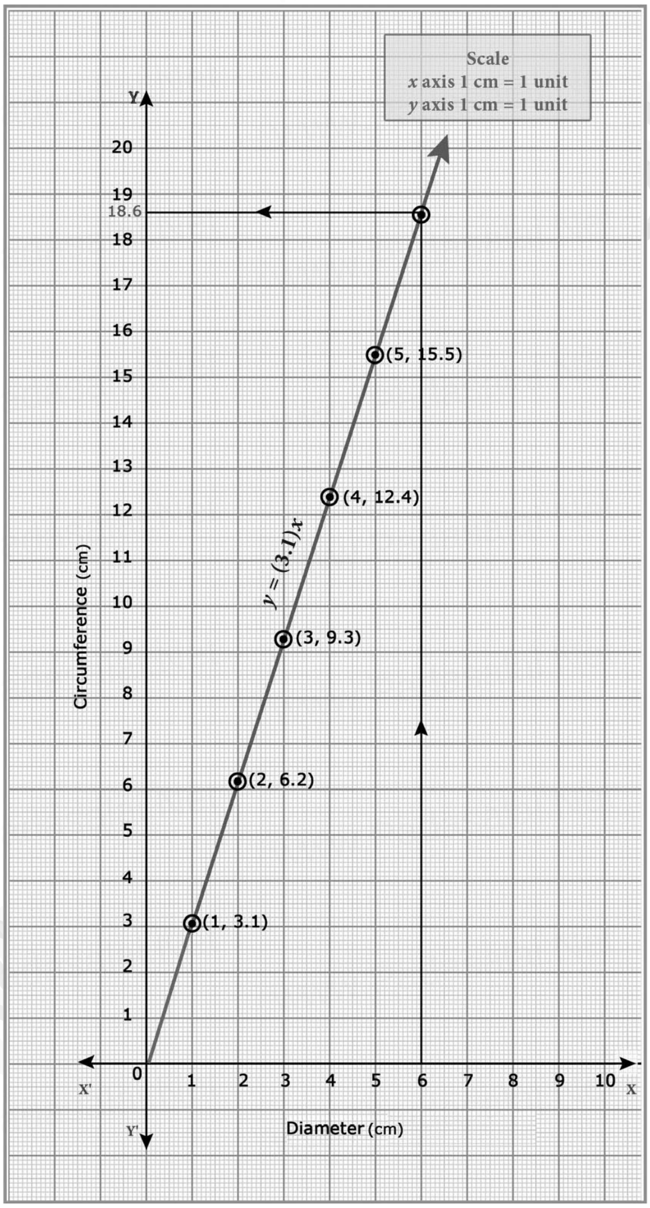
1. Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

Solution:

I. Table (Given)

Diameter(x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5



II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

Let y = kx, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{y}{x} = \frac{3.1}{1} = \frac{6.2}{2} = \frac{9.3}{3} = \frac{12.4}{4} = ... = 3.1$$

$\therefore y = 3.1x$

III. **Points**

(1, 3.1) (2, 6.2) (3, 9.3), (4, 12.4) and (5, 15.5)

IV. **Solution:**

From the graph, when diameter is 6 cm, its circumference is 18.6 cm.

Verify:

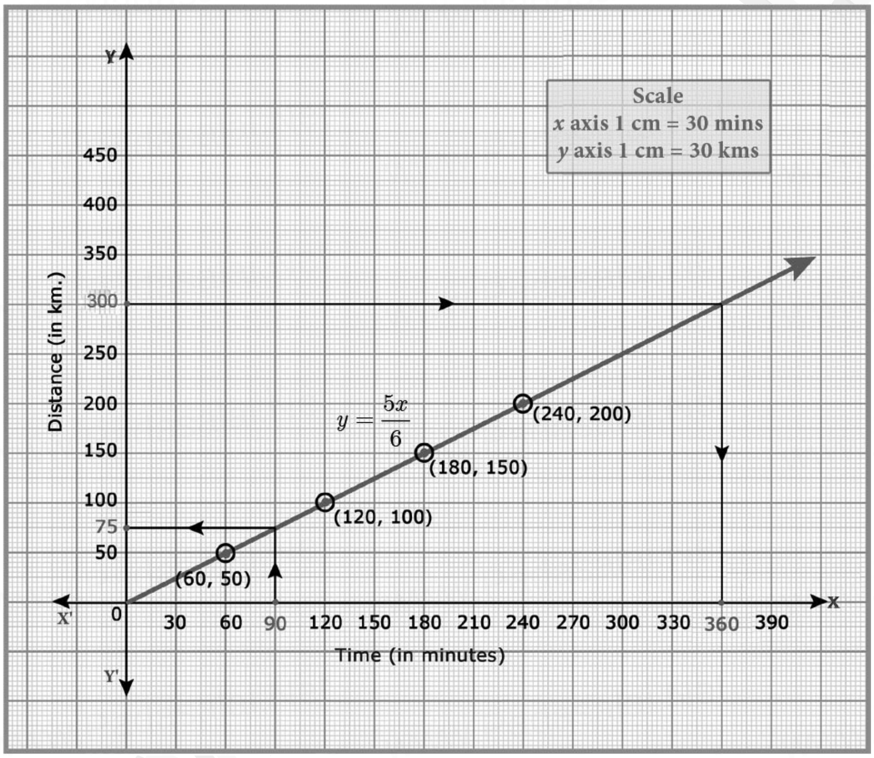
When $x = 6$, y = (3.1) × 6 = 18.6

2. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find (i) the constant of variation (ii) how far will it travel in 1½ hr (iii) the time required to cover a distance of 300 km from the graph.

Solution:

I. Table:

Time taken x (in minutes)	60	120	180	240	300	360
Distance y (in km)	50	100	150	200	250	300



II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

$$\frac{y}{x} = \frac{50}{60} = \frac{100}{120} = \frac{5}{6}$$

∴ Equation $y = \frac{5}{6}x$

III. Points:

(60, 50), (120, 100), (180, 150), (240, 200), (300, 250)

IV. Solution:

(i) the constant of variation $k = \frac{y}{x} = \frac{5}{6}$

(ii) from the graph, the bus will travel 75 km in 90 mins

(verify : $y = \frac{5}{6} \times 90 = \frac{450}{6} = 75$)

(iii) from the graph, the time required to cover a distance of 300 km is 360 minutes.

3. **A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.**

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

(i) Graph the above data and identify the type of variation.

(ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers?

(iii) If the work has to be completed by 30 days, how many workers are required?

Solution:

I. Table (Given)

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

II. Variation:

When 'x' increases, 'y' also decreases. Hence, inverse variation.

i.e. $xy = k$

$xy = 40 \times 150 = 50 \times 120 = \ldots.. 6000$ (k)

$\therefore$ Required Equation $xy = 6000$

III. Points

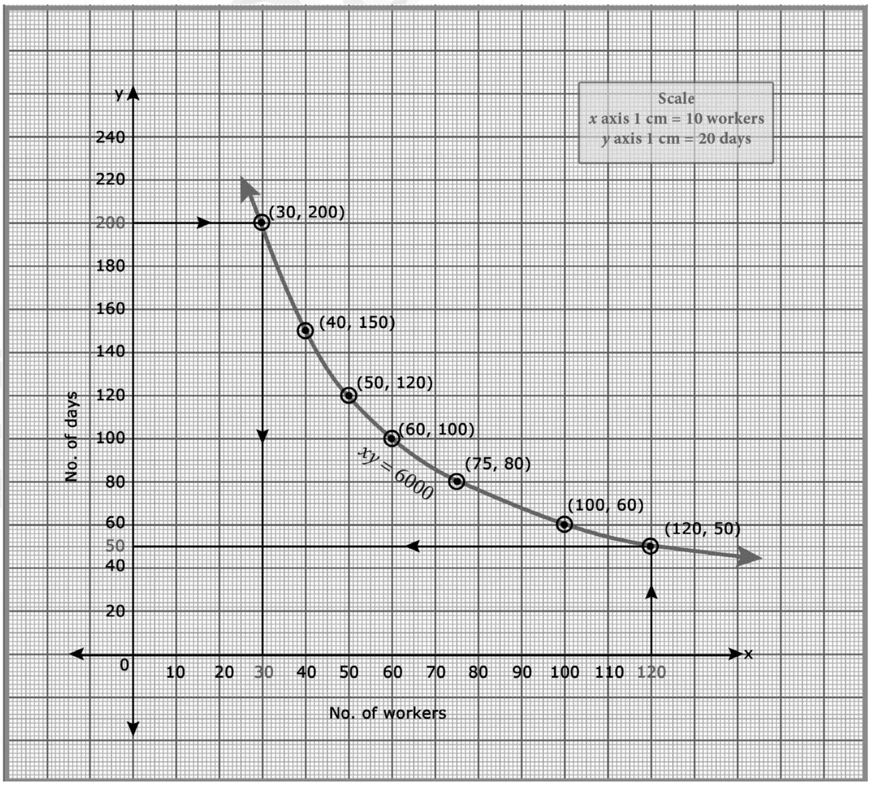
(40, 150) (50, 120) (60, 100), (75, 80)

IV. Solution:

(i) Inverse Variation

(ii) When $x = 120 \Rightarrow 120 \times y = 6000 \Rightarrow y = \frac{6000}{120} = 50$. Also from the graph, the No. of days required to complete the work if the company decides to opt for 120 workers is 50 days.

(iii) When $y = 200 \Rightarrow x \times 200 = 6000 \Rightarrow x = \frac{6000}{200} = 30$. Also from the graph, the No. of workers required to complete in 200 days is 30.



4. **Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Ponmozhi, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hours respectively. Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.**

Solution:

I. Table:

Speed x(km / hr)	12	6	4	3	2
Time y (hours)	1	2	3	4	6

II. Variation:

From the table, we observe that as x decreases, y increases. Hence, the type is inverse variation.

Let $y = \frac{k}{x}$

$\Rightarrow xy = k$, $k > 0$ is called the constant of variation.

From the table $k = 12 \times 1 = 6 \times 2 = \dots = 2 \times 6 = 12$ (k)

Therefore, $xy = 12$.

III. Points:

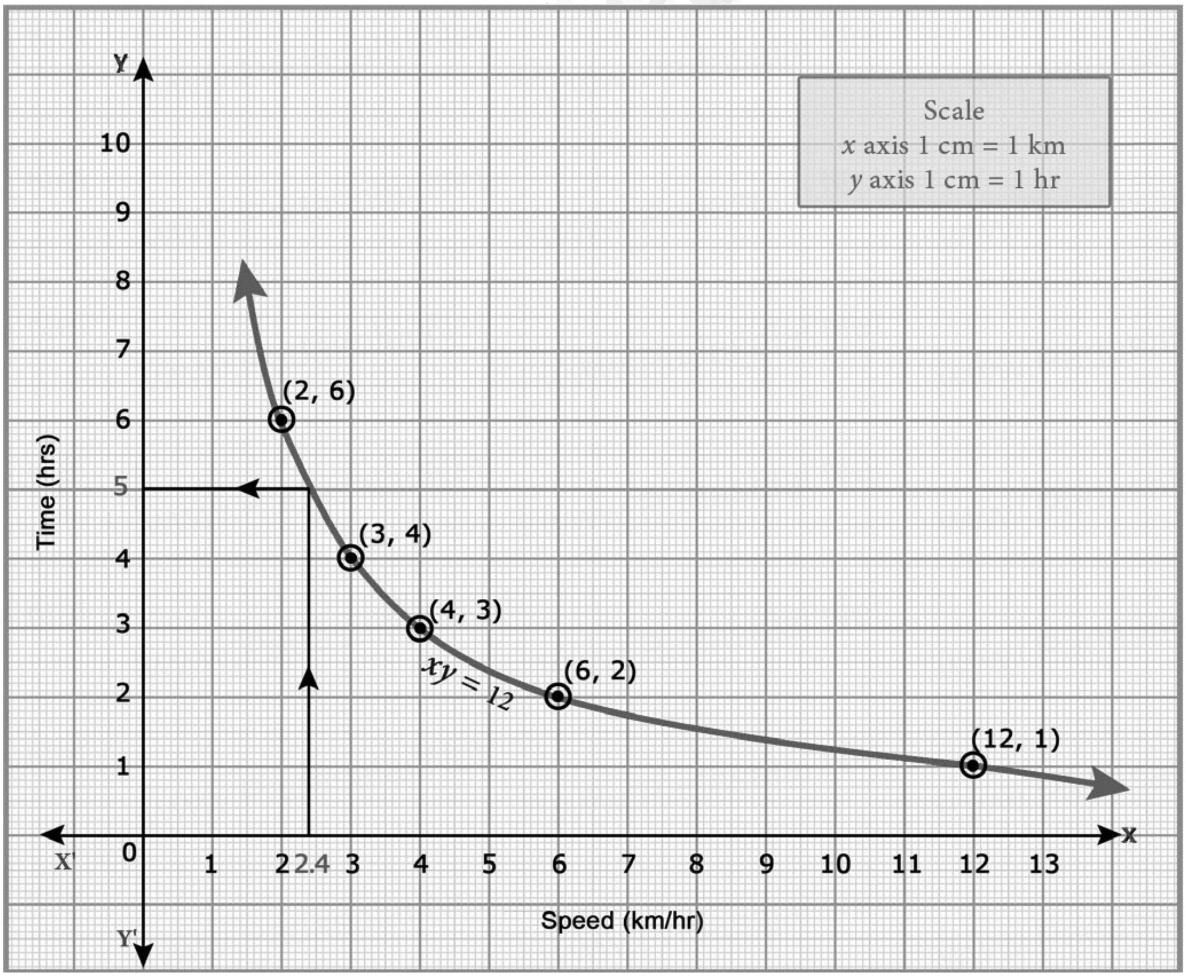
(12, 1), (6, 2), (4, 3), (3, 4), (2, 6)

IV. Solution:

When $x = 2.4 \Rightarrow 2.4 \times y = 12$.

$$y = \frac{12}{2.4} = 5$$

Also, from the graph, the time taken to Kaushik with his speed of 2.4 km / hr is 5 hours .



5. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find

(i) the marked price when a customer gets a discount of ₹ 3250 (from graph)

(ii) the discount when the marked price is ₹ 2500

Solution:

I. Table (Given)

Marked Price ₹ (x)	1000	2000	3000	4000	5000	6000
Discounted Price ₹ (y)	500	1000	1500	2000	2500	3000

II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

Let y = kx, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{y}{x} = \frac{500}{1000} = \frac{1000}{2000} = \ldots = \frac{1}{2}$$

∴ Required Equation is $y = \frac{1}{2}x$

III. Points:

(1000, 500), (2000, 1000), (3000, 1500), (4000, 2000), (5000, 2500), (6000, 3000)

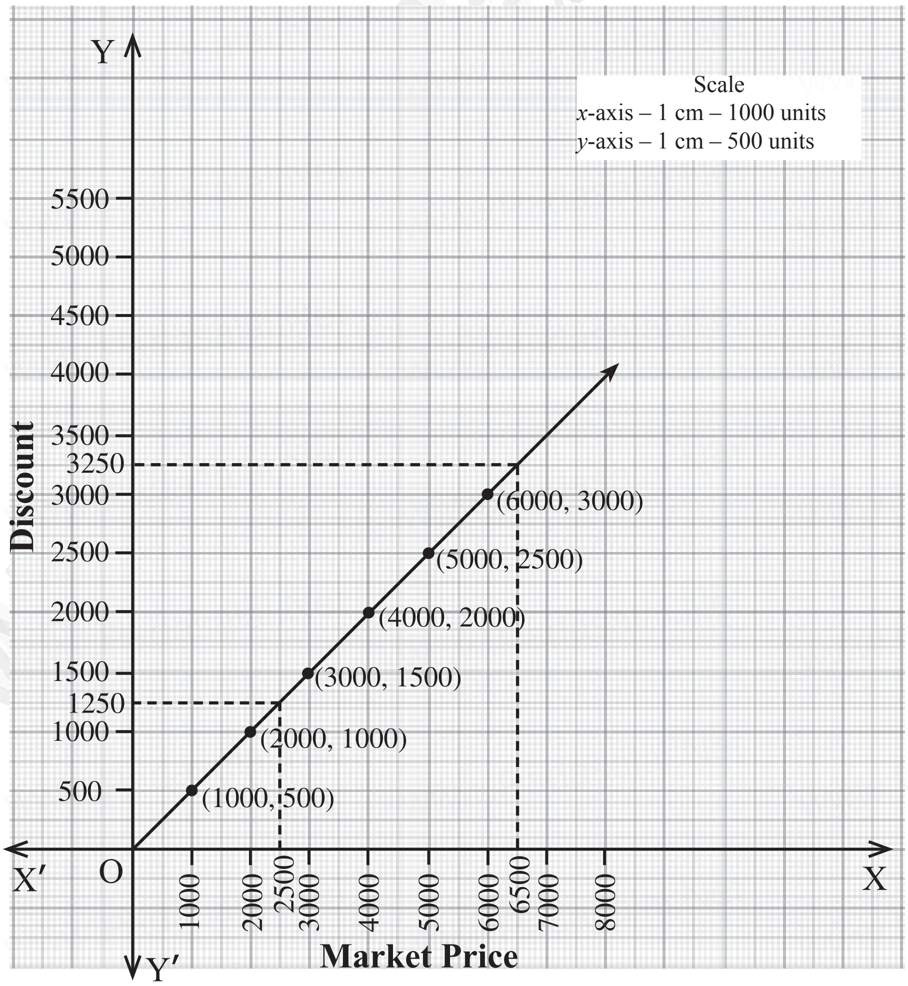
IV. Solution:

(i) From the graph, when a discount price is ₹ 3250, the marked price is ₹ 6500

(ii) From the graph, when the marked price is ₹ 2500, the discounted price is ₹ 1250

Verify:

When $x = 6$, y = (3.1) × 6 = 18.6



6. Draw the graph of $xy = 24$, x, $y > 0$. Using the graph find,

(i) y when $x = 3$ and (ii) x when $y = 6$.

Solution:

I. Table: (Given)

x	1	2	3	4	6	12	24
y	24	12	8	6	4	2	1

II. Variation:

When 'x' increases, 'y' also decreases. Hence, inverse variation.

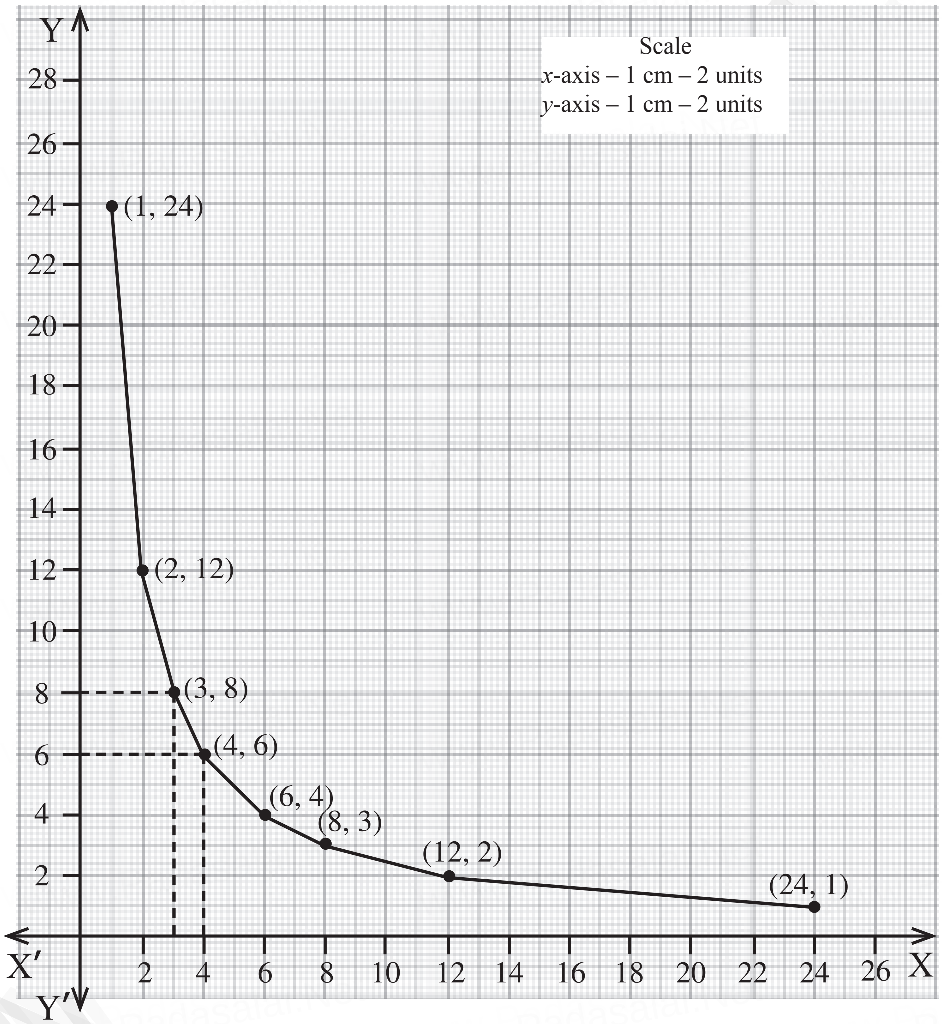
i.e. $xy = k$

$xy = 1 \times 24 = 2 \times 12 = \ldots..12 \times 2 = 24$

$\therefore$ Required Equation $xy = 24$

III. Points:

(1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (12, 2)



IV. Solution:

(i) $x = 3 \Rightarrow 3 \times y = 24$

$$y = \frac{24}{3} = 8 \qquad y = 8$$

(ii) $y = 6 \Rightarrow x \times 6 = 24$

$$x = \frac{24}{6} = 4 \qquad x = 4$$

Also, Verified in the Graph.

7. **Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also (i) find y when $x = 9$ (ii) find x when y = 7.5.**

Solution:

I. Table: (Given)

x	2	4	6	8	10
y	1	2	3	4	5

II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

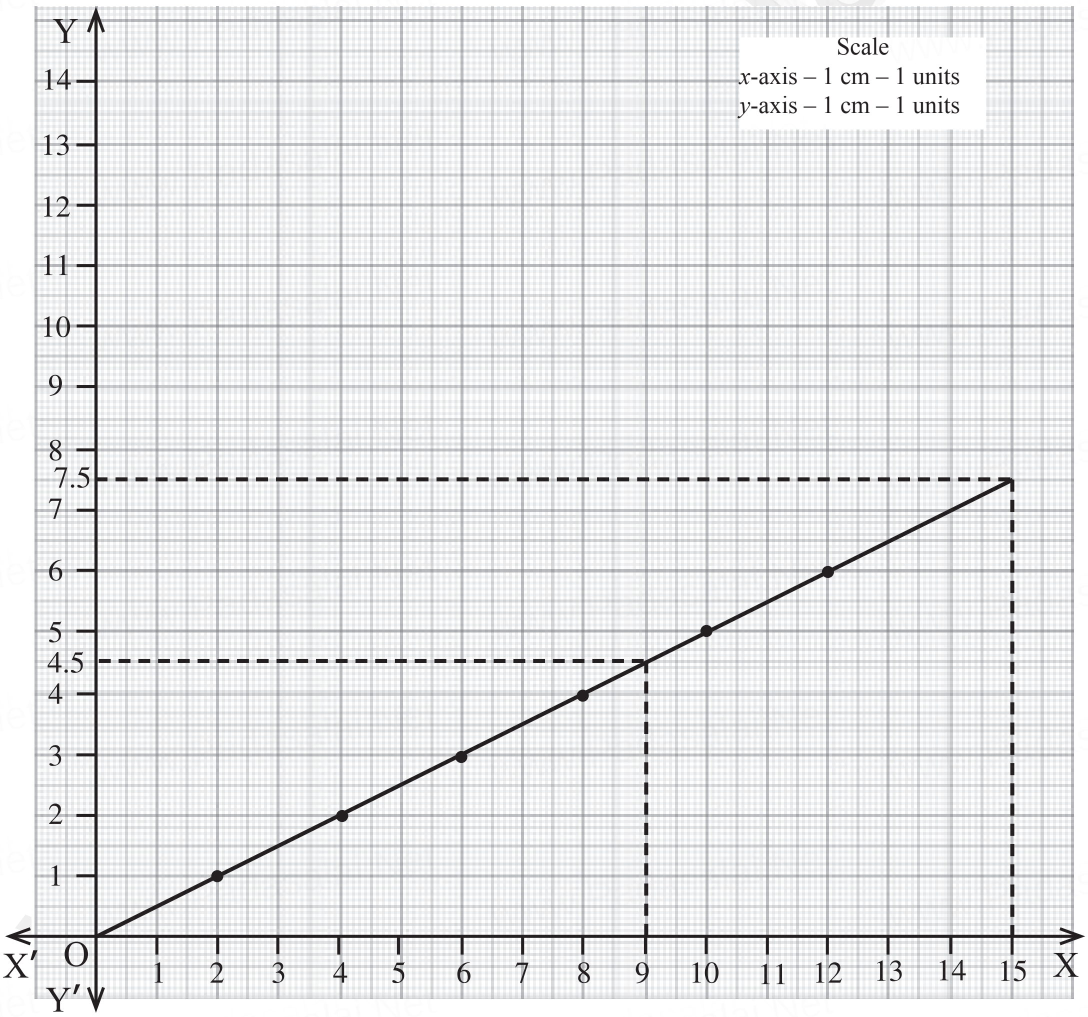
Let y = kx, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{y}{x} = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \ldots = \frac{1}{2}$$

$\therefore$ Required Equation is $y = \frac{1}{2}x$

III. Points:

(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)



IV. Solution:

From the graph, when $x = 9$, y = 4.5

From the graph, when y = 7.5, $x = 15$

8. The following table shows the data about the number of pipes and the time taken to till the same tank.

No. of pipes (x)	**2**	**3**	**6**	**9**
Time Taken (in min) (y)	**45**	**30**	**15**	**10**

Draw the graph for the above data and hence

(i) find the time taken to fill the tank when five pipes are used

(ii) Find the number of pipes when the time is 9 minutes.

Solution:

I. Table (Given):

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

II. Variation:

When 'x' increases, 'y' also decreases. Hence, inverse variation.

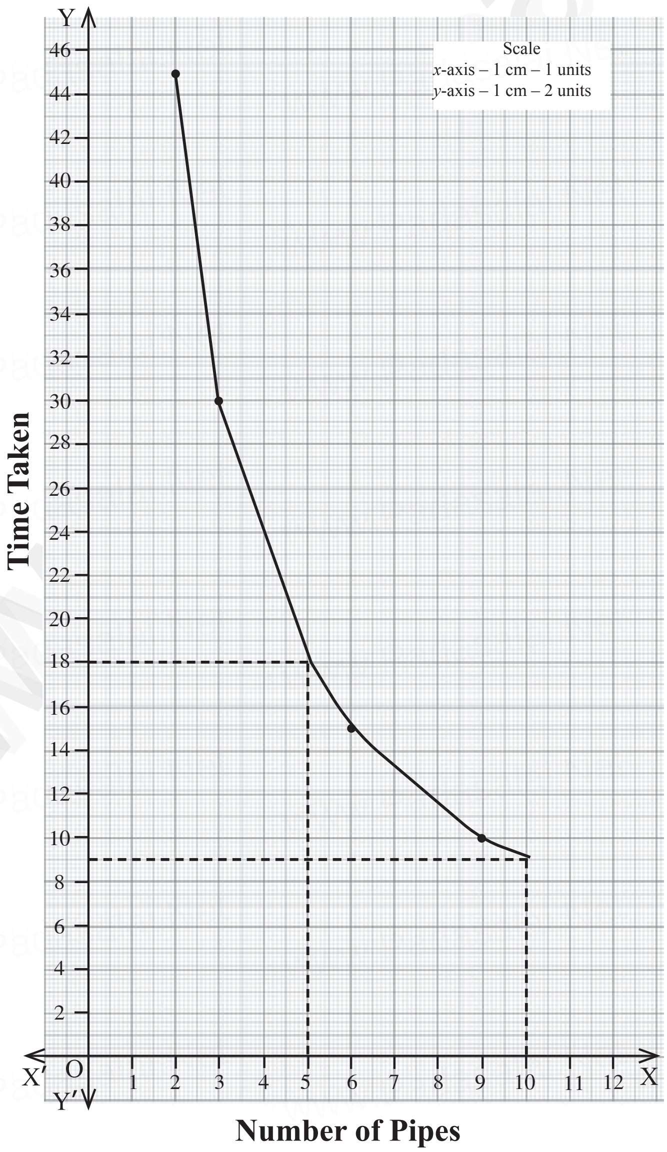
i.e. $xy = k$

$xy = 2 \times 45 = 3 \times 30 = \ldots..6\times15 = 9\times10 = 90$

$\therefore$ Required Equation $xy = 90$

III. Points:

(2, 45), (3, 30), (6, 15), (9, 10)



IV. Solution:

$x = 5 \Rightarrow 5 \times y = 90$

$y = \frac{90}{5} = 18$ (Verified with Graph)

Hence, the time taken to fill the tank when five pipes are used is 18.

$y = 9 \Rightarrow x \times 9 = 90$

$x = \frac{90}{9} = 10$ (Verified with Graph)

Hence, the No. of pipes when the time 9 minutes is 10

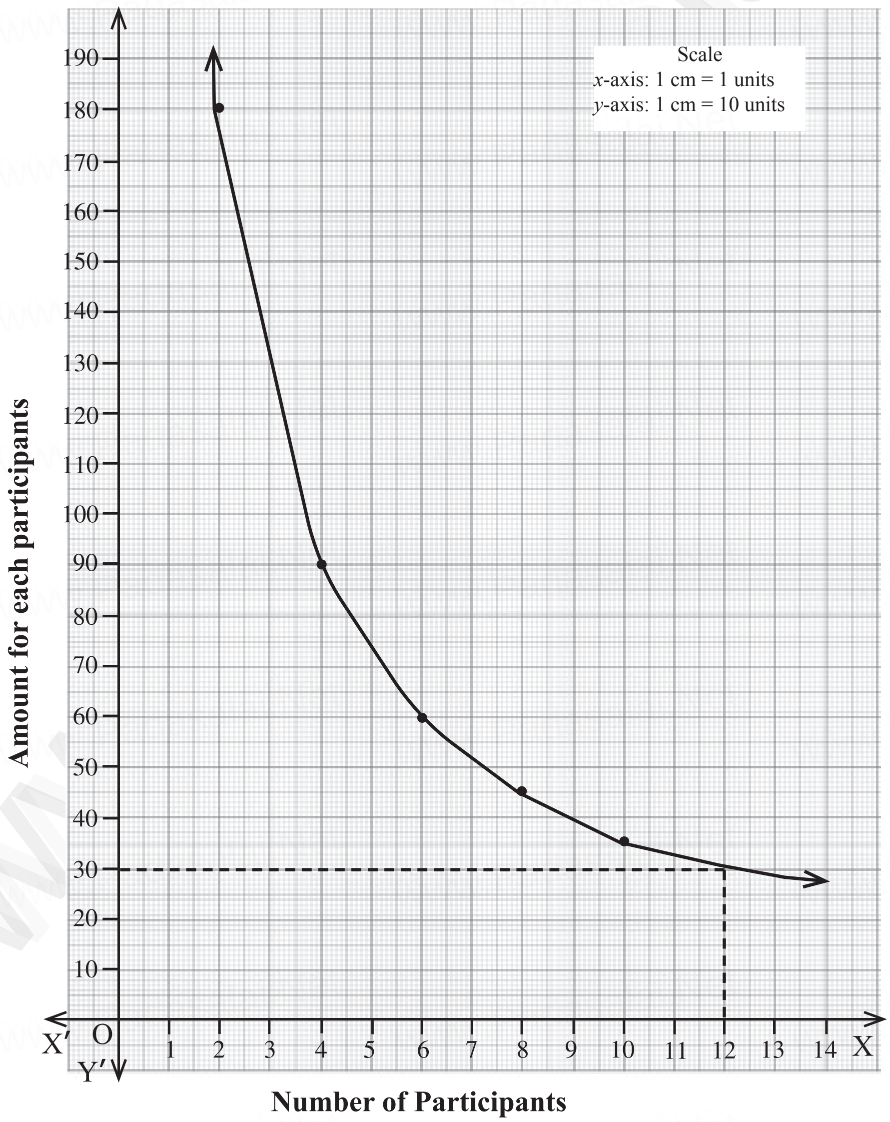
9. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

Solution:



I. Table(Given)

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

II. Variation:

When 'x' increases, 'y' also decreases. Hence, inverse variation.

i.e. $xy = k$

$xy = 2 \times 180 = 4 \times 90 = \ldots..10 \times 36 = 360 = k$

$\therefore$ Required Equation $xy = 360$

III. Points:

(2, 180), (4, 90), (6, 60), (8, 45), (10, 36)

IV. Solution:

Constant of Variation: k = 360

When $x = 12 \Rightarrow xy = 360 \Rightarrow 12y = 360$

$$y = \frac{360}{12} = 30 \quad \text{(Verified with Graph)}$$

Hence, When the number of participants are 12, then each participant will get ₹30

10. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	**4**	**8**	**12**	**24**
Amount ₹ (y)	**60**	**120**	**180**	**360**

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr; (ii) find the parking duration when the amount paid is ₹150.

Solution:

I. Table (Given):

Time (in hours) (x)	4	8	12	24
Amount ₹ (y)	60	120	180	360

II. Variation:

When 'x' increases, 'y' also increases. Thus, the variation is a direct variation.

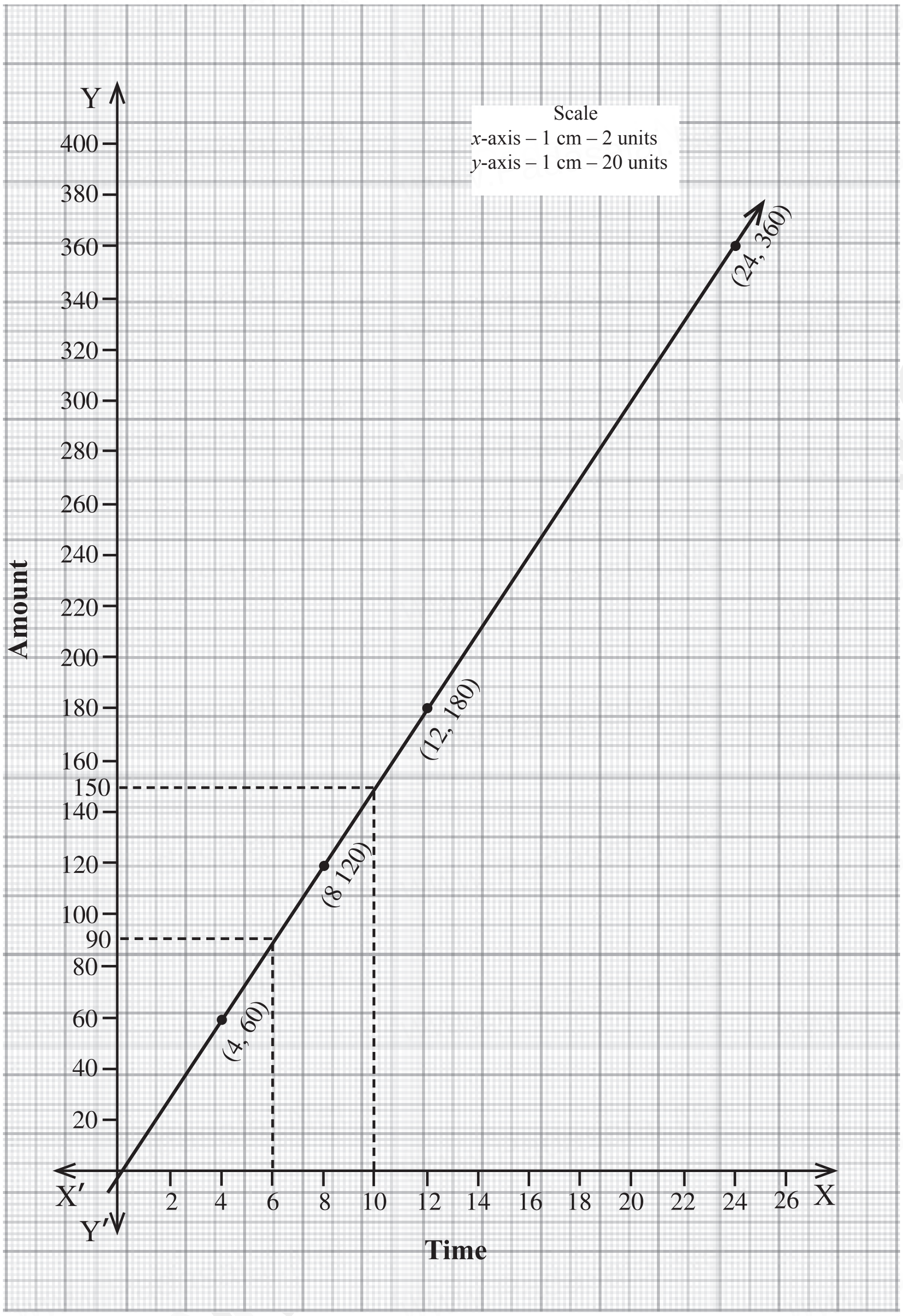
Let $y = kx$, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{y}{x} = \frac{60}{4} = \frac{120}{8} = \ldots. \frac{180}{12} = \frac{360}{24} = 15 = k$$

$\therefore$ Required Equation is $y = 15x$

III. Points:

(4, 60), (8, 120), (12, 180), (24, 360)



IV. Solution:

From the graph, when parking time is 6 hours, then the amount to be paid is ₹ 90.

From the graph, when them amount paid is 1510, then the parking duration is 10 hours.

11. Discuss the nature of solutions of the following quadratic equations. SEP-20 SEP-21

(i) $x^2 + x - 12 = 0$ **(ii)** $x^2 - 8x + 16 = 0$ **(iii)** $x^2 + 2x + 5 = 0$

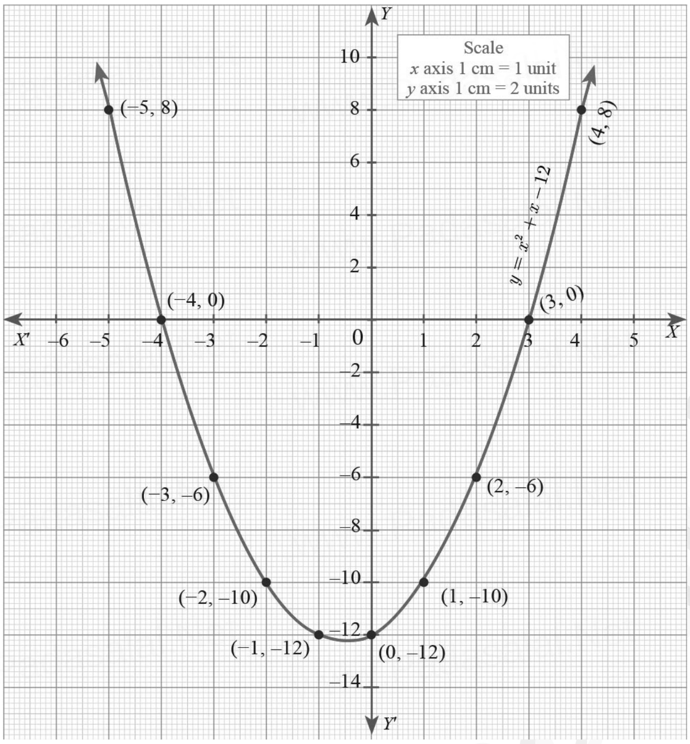
Solution:

i) Table:

x	–5	–4	–3	–2	–1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
x	–5	–4	–3	–2	–1	0	1	2	3	4
–12	–12	–12	–12	–12	–12	–12	–12	–12	–12	–12
y	8	0	–6	–10	–12	–12	–10	-6	0	8

Points: (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1, 12), (2, 6), (3, 2), (4, 0)

Points of Parabola intersect *x* axis: (–4, 0) and (3, 0). *x*– coordinates –4 and 3



Nature of Solution:

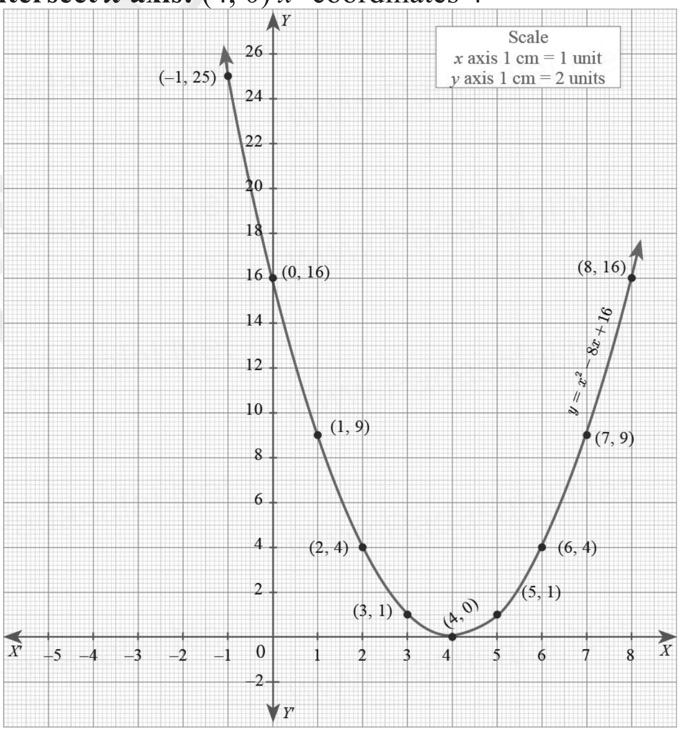
There are two points of intersection with the X axis, the quadratic equation has real and unequal roots.

ii) Table:

x	–1	0	1	2	3	4	5	6	7	8
x^2	1	0	1	4	9	16	25	36	49	64
$-8x$	8	0	–8	–16	–24	–32	–40	–48	–56	–64
+7	7	7	7	7	7	7	7	7	7	7
y	25	16	9	4	1	0	1	4	9	16

Points: (–1, 25), (0, 16), (1, 9), (2, 4), (3, 1), (4, 0), (5, 1), (6, 4), (7, 9), (8, 16)

Points of Parabola intersect *x* axis: (4, 0) *x*- coordinates 4



Nature of Solution:

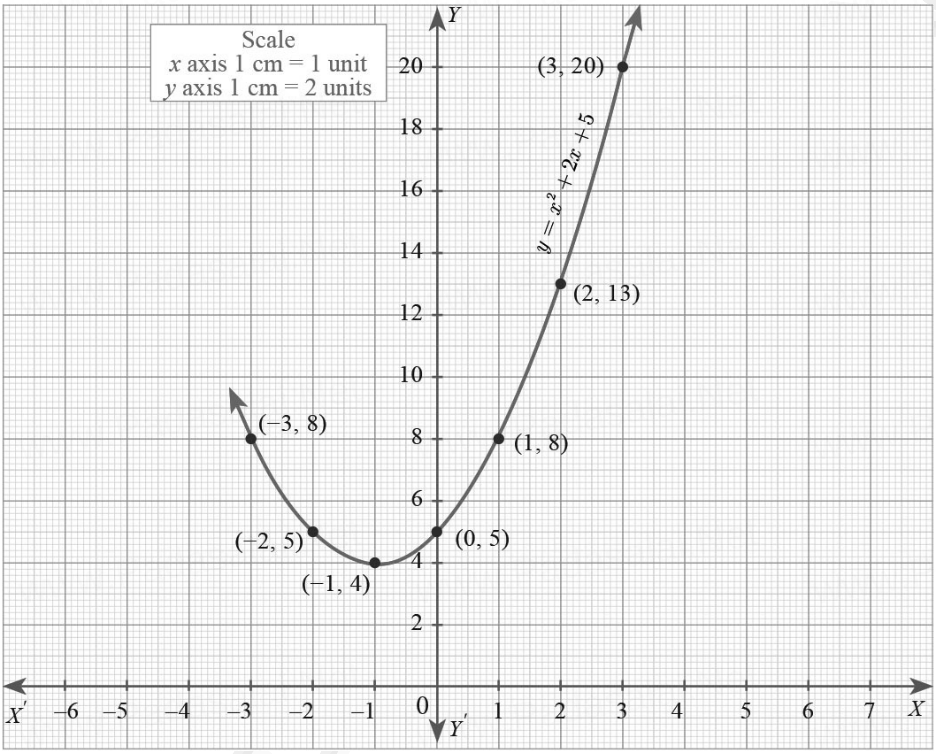
The quadratic equation has real and equal roots.

iii) Table:

x	–3	–2	–1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$2x$	–6	–4	–2	0	2	4	6	8
5	5	5	5	5	5	5	5	5
y	8	5	4	5	8	13	20	29

Points: (–3, 8), (–2, 5), (–1, 4), (0, 5), (1, 8), (2, 13), (3, 20), (4, 29)

Points of Parabola intersect *x* axis: the parabola doesn't intersect or touch the X axis.



Nature of Solution:

There is no real root for the given quadratic equation.

12. Graph the following quadratic equations and state their nature of solutions.

(i) $x^2 - 9x + 20 = 0$ (ii) $x^2 - 4x + 4 = 0$ (iii) $x^2 + x + 7 = 0$

(iv) $x^2 - 9 = 0$ (v) $x^2 - 6x + 9 = 0$ (vi) $(2x - 3)(x + 2) = 0$

Solution:

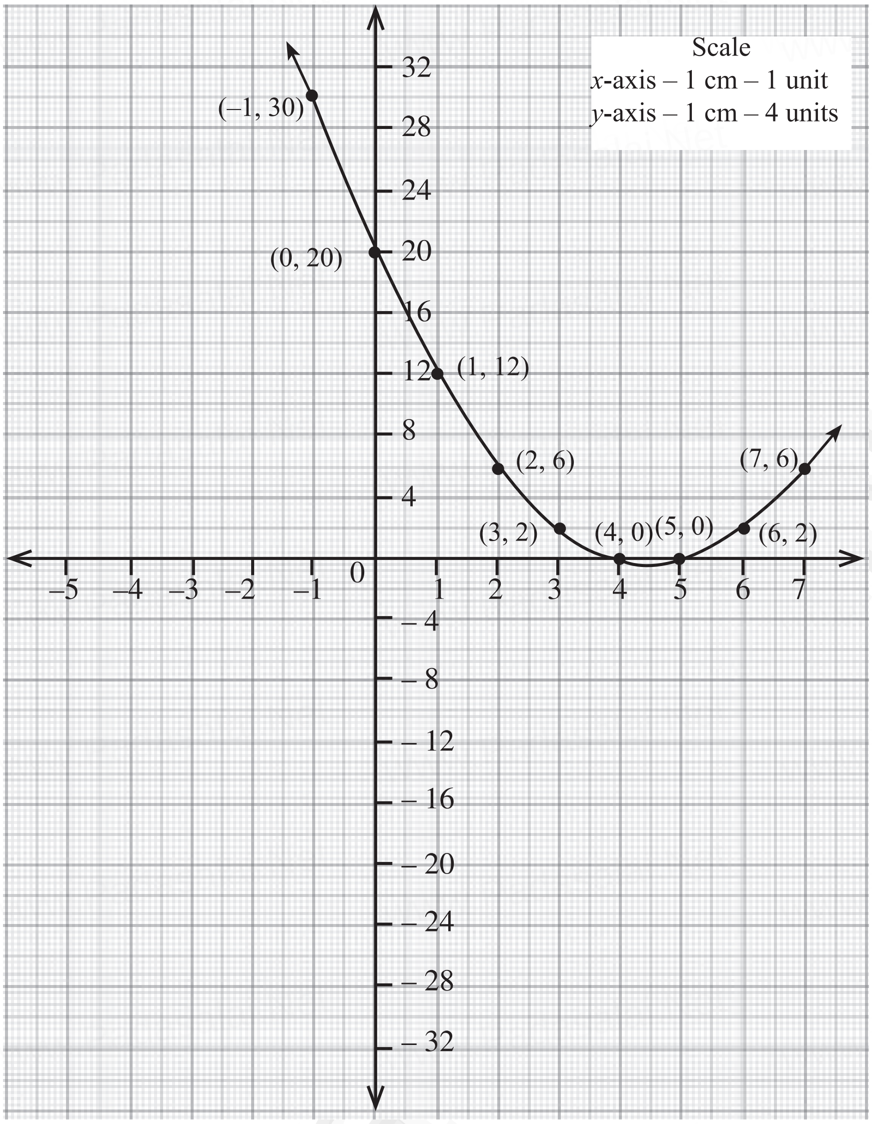
i. $x^2 - 9x + 20 = 0$

Table:

x	–4	–3	–2	–1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$-9x$	36	27	18	9	0	–9	–18	–27	–36	–45
+20	20	20	20	20	20	20	20	20	20	20
y	72	56	42	30	20	12	6	2	0	0

Points: (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1,12), (2, 6), (3,2), (4, 0)

Point of intersection of Parabola at *x* axis: (4, 0) and (5, 0). X- Coordinates are 4 and 5



Solution:

Since there are two points of intersection with the x axis, the quadratic equation $x^2 - 9x + 20 = 0$ has real and unequal roots

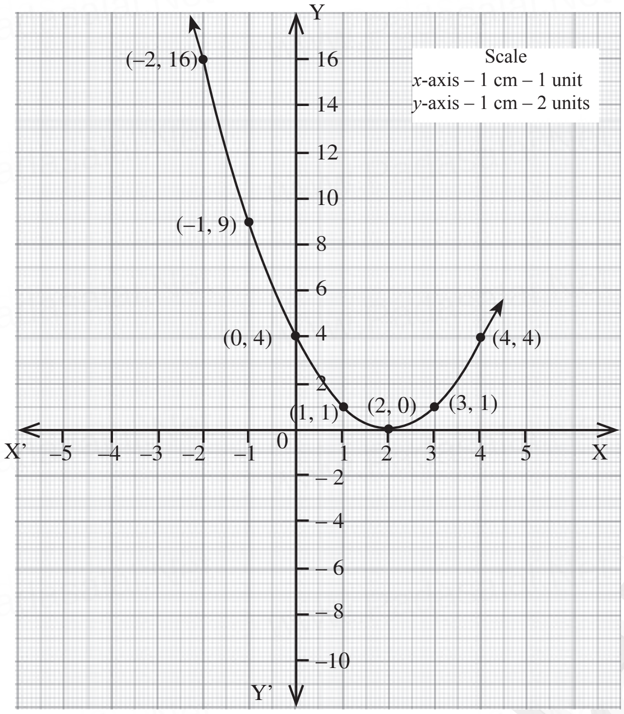
(ii) $x^2 - 4x + 4 = 0$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	–4	–8	–12	–16
+4	4	4	4	4	4	4	4	4	4
y	36	25	16	9	4	1	0	1	4

Points:

(–4, 36), (–3, 25), (–2, 16), (–1, 9), (0, 4), (1, 1), (2, 0), (3, 1), (4, 4)



Solution:

Here, the curve meets the x - axis at (2, 0).

∴ The equation 2 equal roots. ∴ The x - co ordinates of the points is $x = 2$.

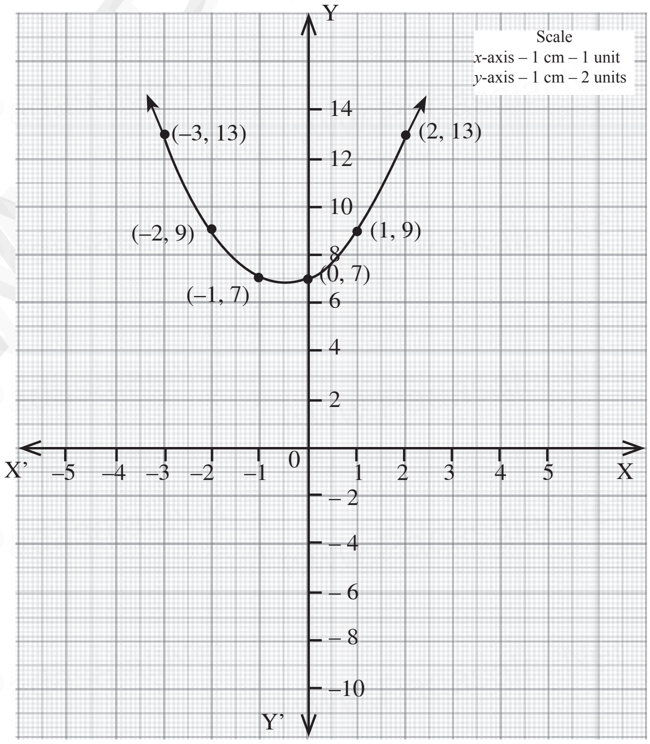
∴ Solution = {2, 2}

(iii) $x^2 + x + 7 = 0$.

Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$+x$	–4	–3	–2	–1	0	1	2	3	4
+7	7	7	7	7	7	7	7	7	7
y	19	13	9	7	7	9	13	19	27

Points (–4, 19), (–3, 13), (–2, 9), (–1, 7), (0, 7), (1, 9), (2, 13), (3, 9), (4, 27)



Solution:

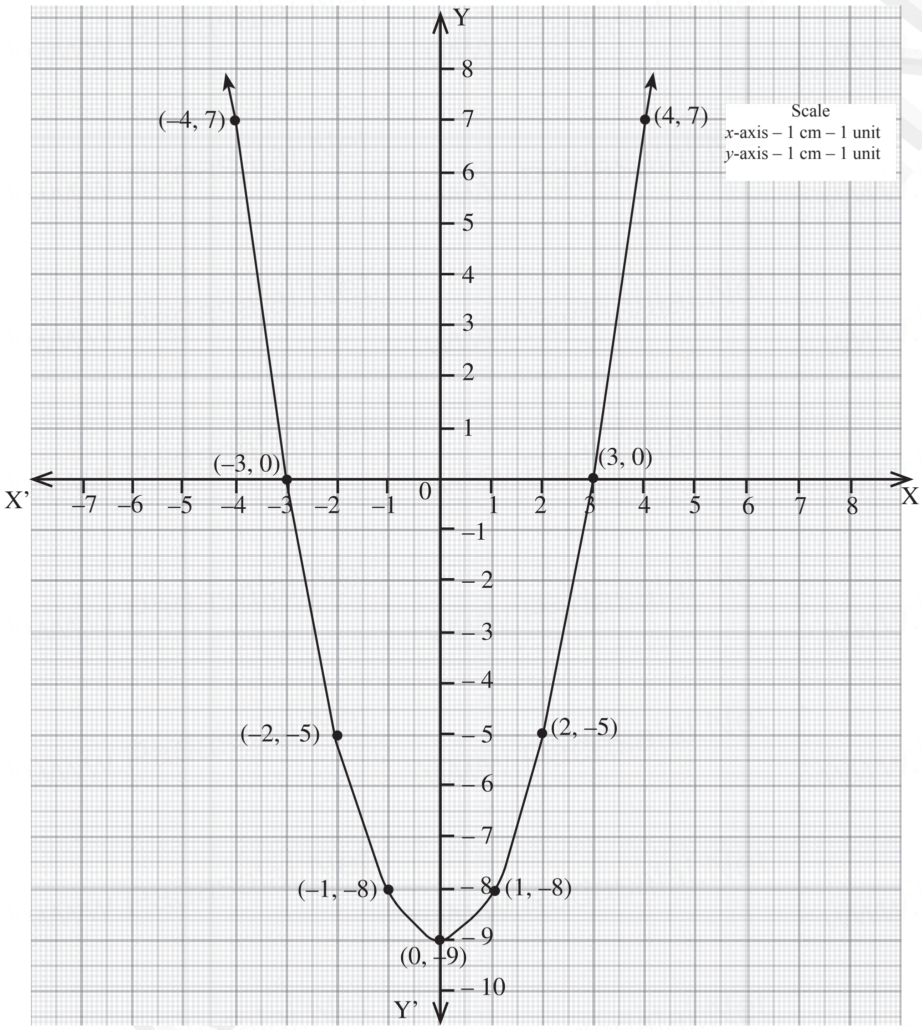
Here the curve does not meet the x – axis and the curve has no real roots

(iv) $x^2 - 9 = 0$.

Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–9	–9	–9	–9	–9	–9	–9	–9	–9	–9
y	7	0	–5	–8	–9	–8	–5	0	7

Points: (–4, 7), (–3, 0), (–2, –5), (–1, –8), (0, –9), (1, –8), (2, –5), (3, 0), (4, 7)



Solution:

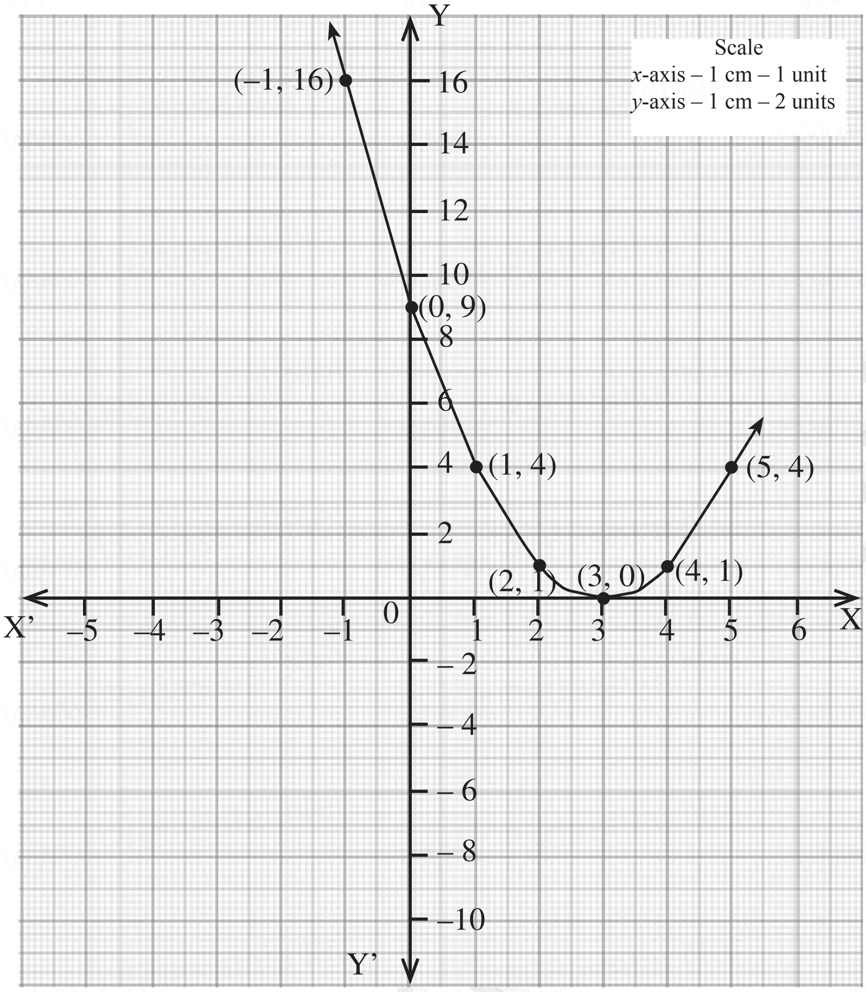
Here the curve meets the x – axis at 2 points (–3, 0), (3, 0). ∴ The equation has real and unequal roots.
∴ The x-coordinates are 3 , –3 will be the solution. ∴ Solution = {–3, 3}and the curve has no real roots

(v) $x^2 - 6x + 9 = 0$.

Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-6x$	24	18	12	6	0	–6	–12	–18	–24
+9	9	9	9	9	9	9	9	9	9
y	49	36	25	16	9	4	1	0	1

Points (–4, 49), (–3, 36), (–2, 25), (–1, 16), (0, 9), (1, 4), (2, 1), (3, 0), (4, 1)



Solution

Here the curve meets the x-axis at only one point (3, 0). and the equation has real and equal roots.

∴ The x-coordinates are 3 will be the solution. ∴ Solution = {3, 3}

(vi) (2*x* – 3) (*x* + 2) = 0.

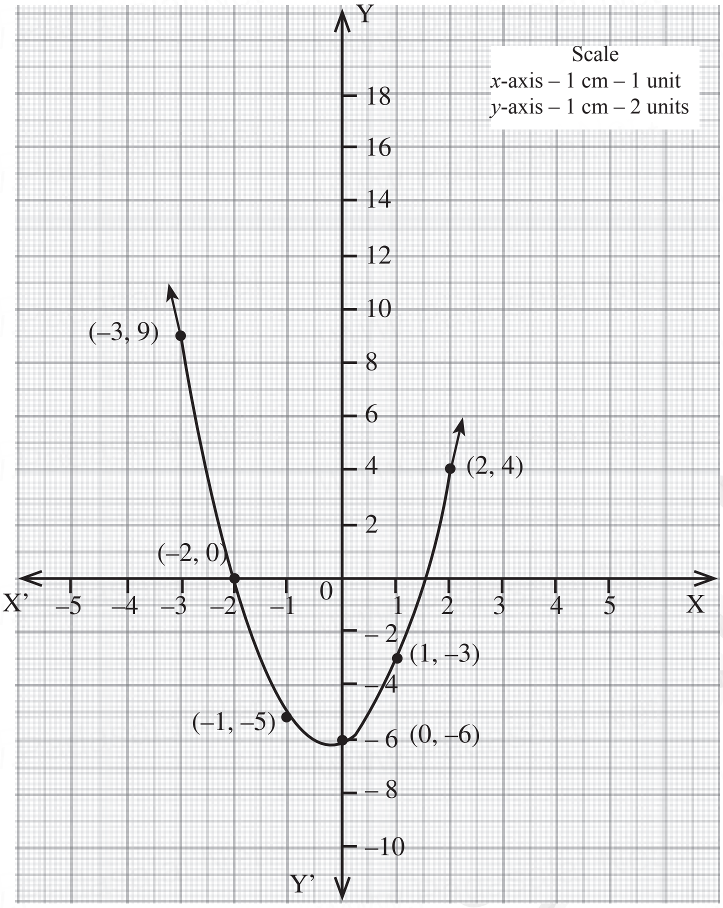
$$y = (2x - 3)(x + 2)$$
$$= 2x^2 + 4x - 3x - 6$$
$$= 2x^2 + x - 6$$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$+x$	–4	–3	–2	–1	0	1	2	3	4
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	22	9	0	–5	–6	–3	4	15	30

Points:

(–4, 22), (–3, 9), (–2, 0), (–1,–5), (0,–6), (1, –3), (2, 4), (3, 15), (4, 30)



Solution:

Here the curve meets the x-axis at 2 points (–2, 0), (1.5, 0).

∴ The equation has real and unequal roots.

∴ The x-coordinates are –2, 1.5 will be the solution.

∴ Solution = {–2, 1.5}

13. Draw the graph of y = $x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

Solution:

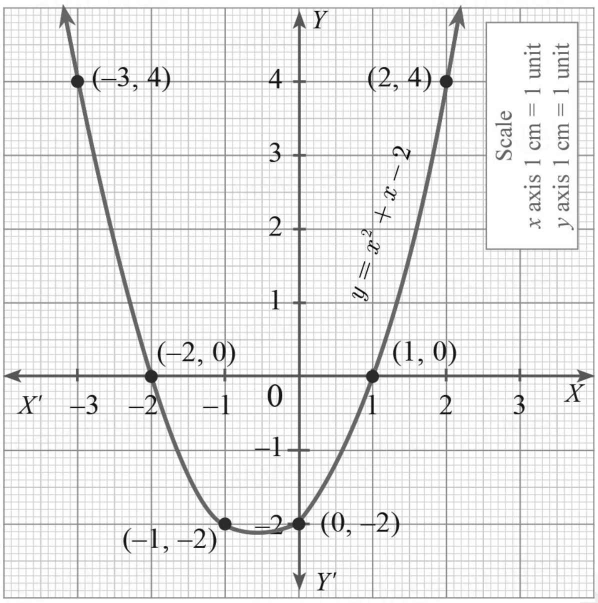
Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+x	–4	–3	–2	–1	0	1	2	3	4
–2	–2	–2	–2	–2	–2	–2	–2	–2	–2
y	10	4	0	–2	–2	0	4	10	18

Points: (–4, 10), (–3, 4), (–2, 0), (–1, –2), (0, –2), (1, 0), (2, 4), (3, 10), (4, 18)

Subtraction

$$\begin{array}{rcl} y & = & x^2 + x - 2 \\ 0 & = & x^2 + x - 2 \\ (-) & & (-)\ (-)\ (-) \\ \hline y & = & 0 \end{array}$$



Solution: –2 and 1

14. Draw the graph of y = $x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$ **SEP-21**

Solution:

Table:

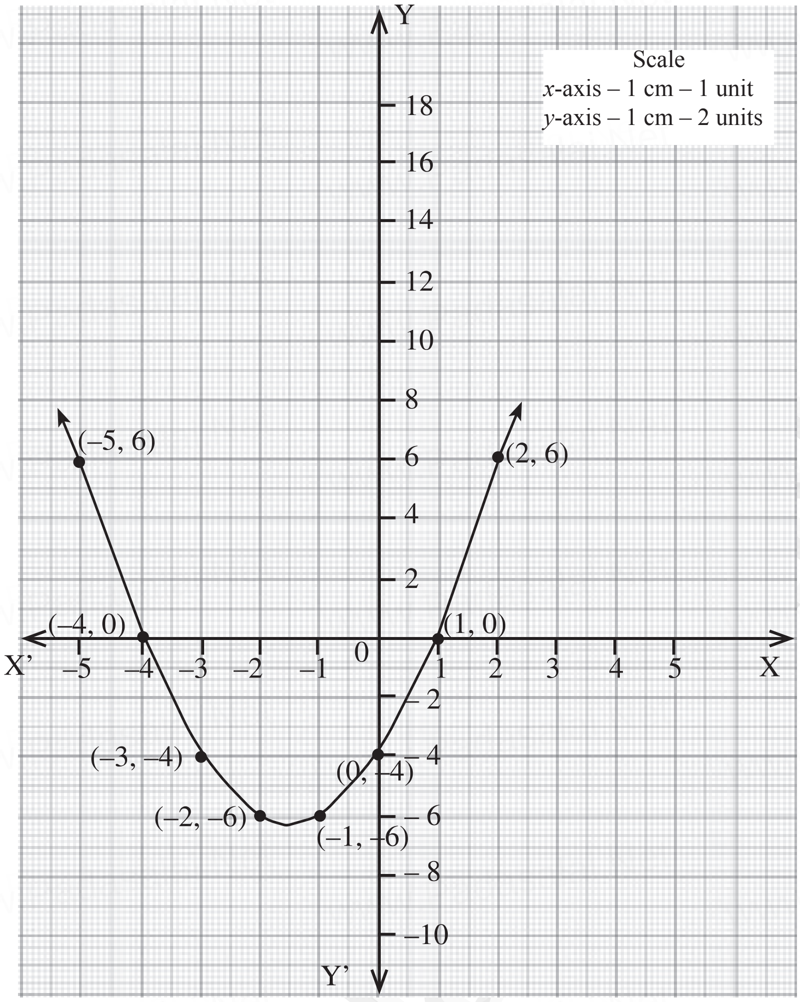
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$3x$	–12	–9	–6	-3	0	3	6	9	12
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
y	0	-4	-6	-6	-4	0	6	14	24

Points:

(–4, 0), (–3, –4), (–2, –6), (–1, –6), (0, –4), (1, 0), (2, 6), (3, 14), (4, 24)

Subtraction

$$\begin{array}{rcl} y & = & x^2 + 3x - 4 \\ 0 & = & x^2 + 3x - 4 \\ (-) & & (-)\ (-)\ (+) \\ \hline y & = & 0 \end{array}$$



Solution:

The curve meets x–axis at (–4, 0), (1, 0) and

the co-ordinates of the points $x = –4$, $x = 1$ will be the solution of $x^2 + 3x – 4 = 0$

$\therefore$ Solution = {–4, 1}

15. Draw the graph of $y = x^2 – 5x – 6$ and hence solve $x^2 – 5x – 14 = 0$

Solution:

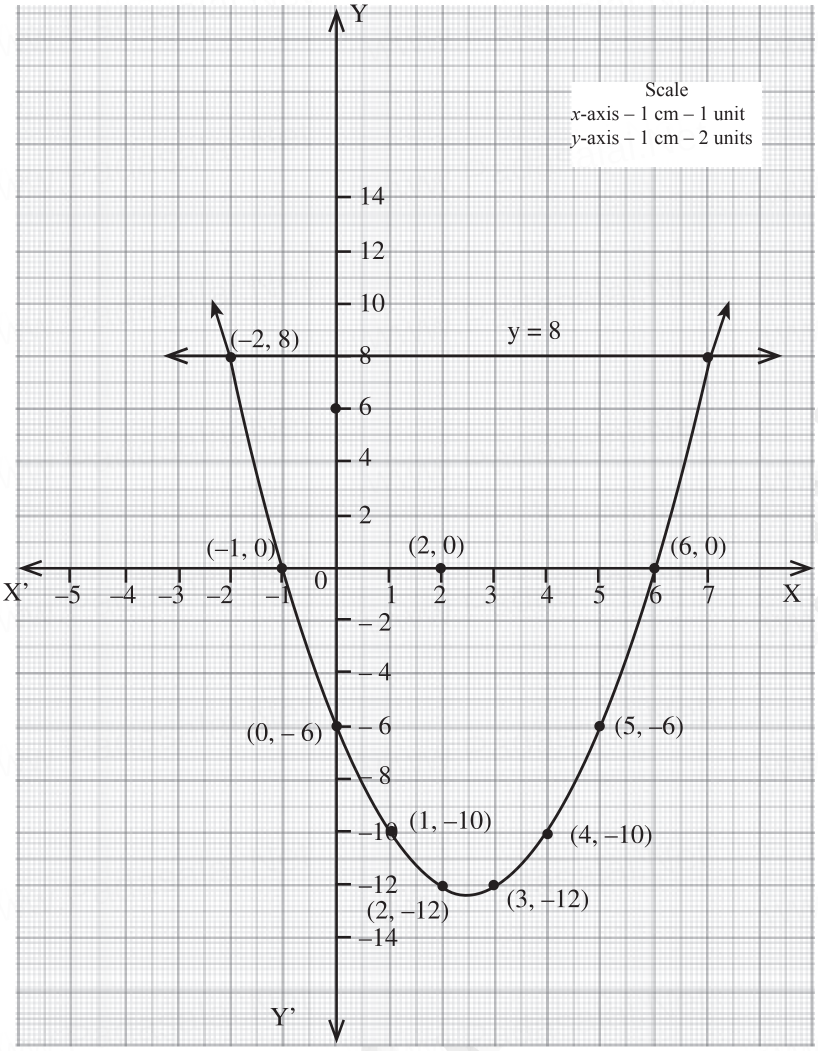
Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$–5x$	20	15	10	5	0	–5	–10	–15	–20
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	30	18	8	0	–6	–10	–12	–12	–10

Points (–4, 30), (–3, 4), (–2, 8), (–1, 0), (0, –6), (1, –10), (2, –12), (3, –12), (4, 10)

Subtraction

$$\begin{array}{rcl} y & = & x^2 – 5x – 6 \\ 0 & = & x^2 – 5x – 14 \\ (–) & & (–)\ (+)\ (+) \\ \hline y & = & 8 \end{array}$$



Solution

The x co-ordinates of the points $x = -2$, $x = 7$ will be the solution $x^2 - 5x - 14 = 0$

$\therefore$ Solution = $\{-2, 7\}$

For Practice

1. Draw the graph of y = $2x^2$ and hence solve $2x^2 - x - 6 = 0$
2. Draw the graph of y = $x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$
3. Draw the graph of y = $x^2 - 4x + 3$ and use it to solve $x^2 - 6x + 9 = 0$
4. Draw the graph of y = $x^2 - 4$ and hence solve $x^2 - x - 12 = 0$
5. Draw the graph of y = $x^2 + x$ and hence solve $x^2 + 1 = 0$
6. Draw the graph of y = $x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$
7. Draw the graph of y = $2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$ SEP-20
8. Draw the graph of y = $(x - 1)(x + 3)$ and hence solve $x^2 - x - 6 = 0$

***

1 Relations and Functions

Exercise 1.6

Multiple choice Questions

1. **If n(A×B) = 6 and A = {1, 3} then n(B) is** SEP-21
   A) 1 B) 2
   C) 3 D) 6

2. **A = {a, b, p}, B = {2, 3}, C = {p, q, r, s} then n[(A×C)×B] is**
   A) 8 B) 20 C) 12 D) 16

3. **If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8} then state which of the following statement is true.** SEP-20
   A) (A×C) ⊂ (B×D) B) (B×D) ⊂ (A×C)
   C) (A×B) ⊂ (A×D) D) (D×A) ⊂ (B×A)

4. **If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is**
   A) 3 B) 2 C) 4 D) 8

5. **The range of the relation R = {(x, x^2) | x is a prime number less than 13} is**
   A) {2,3,5,7} B) {2,3,5,7,11}
   C) {4,9,25,49,121} D) {1,4,9,25,49,121}

6. **If the ordered pairs (a +2, 4) and (5, 2a +b) are equal then (a, b) is** MAY-22
   A) (2, –2) B) (5, 1) C) (2, 3) D) (3, –2)

7. **Let n(A) = m and n(B) = n then the total number of non-empty relations that can be defined from A to B is**
   A) m^n B) n^m C) $2^{mn} - 1$ D) 2^{mn}

8. **If {(a, 8), (6, b)} represents an identity function, then the value of a and b are respectively**
   A) (8, 6) B) (8, 8)
   C) (6, 8) D) (6, 6)

9. **Let A = {1, 2, 3, 4} and B = {4, 8, 9, 10}. A function f : A → B given by f = {(1, 4), (2, 8), (3, 9), (4, 10)} is a**
   A) Many-one function
   B) Identity function
   C) One-to-one function
   D) Into function

10. **If f(x) = $2x^2$ and g(x) = $\frac{1}{3x}$, then fog is**
    A) $\frac{3}{2x^2}$ B) $\frac{2}{3x^2}$
    C) $\frac{2}{9x^2}$ D) $\frac{1}{6x^2}$

11. **If f : A → B is a bijective function and if n(B) = 7 , then n(A) is equal to**
    A) 7 B) 49 C) 1 D) 14

12. **Let f and g be two functions given by**
    f = {(0, 1), (2, 0), (3, −4), (4, 2), (5, 7)}
    g = {(0, 2), (1, 0), (2, 4), (−4, 2), (7, 0)} then the range of fog is
    A) {0, 2, 3, 4, 5} B) {–4, 1, 0, 2, 7}
    C) {1, 2, 3, 4, 5} D) {0, 1, 2}

13. **Let $f(x) = 1 + x^2$ then**
    A) f (xy) = f(x) . f (y)
    B) $f(xy)^3 \geq f(x) . f(y)$
    C) f (xy) ≤ f (x) . f (y)
    D) None of these

14. **If g = {(1, 1), (2, 3), (3, 5), (4, 7)} is a function given by g(x) = $\alpha x + \beta$ then the values of a and b are**
    A) (–1, 2) B) (2, –1) C) (–1, –2) D) (1, 2)

15. **$f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is**
    A) linear B) cubic
    C) reciprocal D) quadratic

2 Marks STAGE 1

1. **If A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.** SEP-20

   Solution:

   A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then
   A = {Set of all first coordinates of elements of A × B} ∴ A = {3, 5}
   B = {Set of all second coordinates of elements of A × B} ∴ B = {2, 4}
   Thus A = {3, 5} and B = {2, 4}

2. **Find A × B, A × A and B × A**
   i) A={2, −2, 3} and B = {1,−4}
   ii) A = B = {p, q}
   iii) A = {m, n} ; B = f

Solution:

i. A × B = {2, –2, 3} × {1, –4}
= {(2, 1), (2, –4), (–2, 1), (–2, –4), (3, 1), (3, –4)}

A × A = {2, –2, 3} × {2, –2, 3}
= {(2, 2), (2, –2), (2, 3), (–2, 2), (–2, –2), (–2, 3), (3, 2), (3, –2), (3, 3)}

B × A = {1, –4} ×{2, –2, 3}
= {(1, 2), (1, –2), (1, 3), (–4, 2), (–4, –2), (–4, 3)}

ii. Given A = B = {p, q}

A × B = {p, q} × {p, q}
= {(p, p), (p, q), (q, p),(q ,q)}

A × A = {p, q} × p, q}
= {(p, p), (p, q), (q, p),(q ,q)}

B × A = {p, q} × {p, q}
= {(p, p), (p, q), (q, p),(q ,q)}

iii. A = {m, n}, B = φ

A × B = {(m, n) × { } = { }

A × A = {(m, n) } × {m, n}
= {(m, m), (m, n), (n, m), (n,n)}

B × A = { } × {m, n} = { }

3. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A × B and B × A. MAY-22

Solution:

A = {1, 2, 3} B = {2, 3, 5, 7}

A × B = {1, 2, 3} × {2, 3, 5, 7}
= {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {2, 3, 5, 7} × {1, 2, 3}
= {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

4. If B × A = {(−2, 3), (−2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.

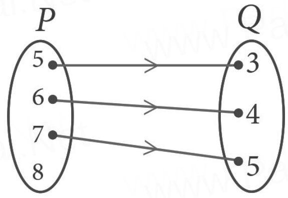
Solution:

A = {Set of all second coordinates of elements of B × A} ∴ A = {3, 4}

B = {Set of all first coordinates of elements of B × A} ∴ B = {–2, 0, 3}

Thus, A = {3, 4} B = {–2, 0, 3}

5. The arrow diagram shows a relationship between the sets P and Q. Write the relation in



(i) Set builder form (ii) Roster form

(iii) What is the domain and range of R.

Solution:

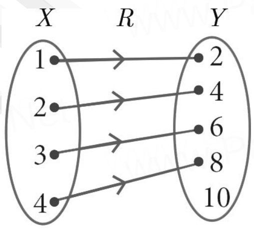
i. Set builder form of R = {(x, y) | y = x – 2, x ∈ P, y ∈ Q}

ii. Roster form R = {(5, 3), (6, 4), (7, 5)}

iii. Domain of R = {5, 6, 7} and range of R = {3, 4, 5}

6. Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and R = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that R is a function and find its domain, co-domain and range?

Solution:



Pictorial representation of R is given diagram, From the diagram, we see that for each x ∈ X, there exists only one y ∈ Y.

Thus all elements in X have only one image in Y.

Therefor R is a function.

Domain X = {1, 2, 3, 4}

Co-domain Y = { 2, 4, 6, 8, 10}

Range of f ={2, 4, 6, 8}

7. Let A = {1, 2, 3, 4, ..., 45} and R be the relation defined as "is square of a number" on A. Write R as a subset of A×A. Also, find the domain and range of R. SEP-21

Solution:

A = {1, 2, 3, …., 45}

R = {(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)}

R ⊂ (A × A)

∴ Domain of R = {1, 2, 3, 4, 5, 6}

Range of R = {1, 4, 9, 16, 25, 36}

8. A Relation R is given by the set {(x, y) /y = x + 3, x ∈ {0, 1, 2, 3, 4, 5}}. Determine its domain and range.

Solution:

$x = \{0, 1, 2, 3, 4, 5\}$

$f(x) = y = x + 3$

$f(0) = 3;$ $f(1) = 4;$ $f(2) = 5;$

$f(3) = 6;$ $f(4) = 7;$ $f(5) = 8$

$\therefore R = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$

Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {3, 4, 5, 6, 7, 8}

9. Given the function f: $x \to x^2 - 5x + 6$, evaluate (i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x-1)$

Solution:

Given: f: $x \to x^2 - 5x + 6$

$\Rightarrow f(x) = x^2 - 5x + 6$

i. $f(-1) = (-1)^2 - 5(-1) + 6$
$= 1 + 5 + 6$
$= 12$

ii. $f(2a) = (2a)^2 - 5(2a) + 6$
$= 4a^2 - 10a + 6$

iii. $f(2) = (2)^2 - 5(2) + 6$
$= 4 - 10 + 6$
$= 0$

iv. $f(x-1) = (x-1)^2 - 5(x-1) + 6$
$= x^2 - 2x + 1 - 5x + 5 + 6$
$= x^2 - 7x + 12$

10. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution:

$f(x) = 3 - 2x$

$f(x^2) = [f(x)]^2$

$3 - 2x^2 = [3 - 2x]^2$

$\Rightarrow$ $3 - 2x^2 = 9 + 4x^2 - 12x$

$3 - 2x^2 - 9 - 4x^2 + 12x = 0$

$\Rightarrow$ $-6x^2 + 12x - 6 = 0 \div -6$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$ $x = 1, 1$

11. Let A = {1, 2, 3, 4} and B = N. Let f : A→ B be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function.

Solution:

A = {1, 2, 3, 4}, B = N

$f : A \to B$, $f(x) = x^3$

$f(1) = (1)^3 = 1;$ $f(2) = (2)^3 = 8;$

$f(3) = (3)^3 = 27;$ $f(4) = (4)^3 = 64$

i) Range of f={1, 8, 27, 64}

ii) It is one-one and into function.

5 Marks

STAGE 1

1. If A = {1, 3, 5} and B = {2, 3} then
(i) find A×B and B×A.
(ii) Is A×B = B×A? If not why?
(iii) Show that n(A×B) = n(B×A) = n(A)× n(B)

SEP-21

Solution:

Given that A = {1, 3, 5} and B = {2, 3}

i. $A \times B = \{1, 3, 5\} \times \{2 \times 3\}$
$= \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$ ....(1)

$B \times A = \{2 \times 3\} \times \{1, 3, 5\}$
$= \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$ ....(2)

ii. From (1) and (2) we conclude that $A \times B \neq B \times A$ as $(1, 2) \neq (2, 1)$ and $(1, 3) \neq (3, 1)$ etc

iii. n(A) = 3; n(B) = 2

From (1) and (2) we observe that,

$n(A \times B) = n(B \times A) = 6;$

We see that, $n(A) \times n(B) = 3 \times 2 = 6$

Thus, $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

2. Let A = $\{x \in N \mid 1 < x < 4\}$, B = $\{x \in W \mid 0 \le x < 2\}$ and C = $\{x \in N \mid x < 3\}$. Then verify that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution:

Given $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$,
$B = \{x \in W \mid 0 \le x < 2\} = \{0, 1\}$,
$C = \{x \in N \mid x < 3\} = \{1, 2\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{0,1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C)$
$= \{2, 3\} \times \{0, 1, 2\}$
$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$
$= (2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$
$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C)$
$= \{(2,0), (2,1), (3, 0), (3,1)\} \cup \{(2,1), (2, 2), (3, 1), (3, 2)\}$
$= \{(2, 0), (2, 1), (2, 2), (3,0), (3, 1), (3, 2)\}$ ..... (2)

From (1) = (2).

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\}$

$= \{(2, 1), (3, 1)\}$ ..... (1)

$A \times B = \{2, 3\} \times \{0, 1\}$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\}$

$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C)$

$= \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 1), (3, 1)\}$ ..... (2)

(1) = (2)

$\therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence it is Verified

3. If A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}, Show that A × A = (B × B) ∩ (C × C).

Solution:

Given $A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$

LHS:

$A \times A = \{5, 6\} \times \{5, 6\}$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (1)

RHS = $(B \times B) \cap (C \times C)$.

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$= \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$= \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore (B \times B) \cap (C \times C)$

$= \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ..... (2)

∴ From (1) and (2). LHS = RHS

4. Given A = {1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if (A∩C) × (B∩D) = (A×B) ∩ (C × D) is true?

Solution:

$A \cap C = \{1, 2, 3\} \cap \{3, 4\}$

$A \cap C = \{3\}$,

$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D)$

$= \{3\} \times \{3, 5\} = \{(3, 3), (3, 5)\}$ ..... (1)

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1, 2), (1, 3), (1,5), (2, 2), (2, 3), (2, 5),(3, 2), (3, 3), (3, 5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$

$= \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ..... (2)

(1), (2) are equal.

$\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

Hence it is verified.

5. Let A = {$x \in W \mid x < 2$}, B = {$x \in N \mid 1 < x \leq 4$} and C = {3, 5}. Verify that
(i) A×(B ∪ C) = (A×B)∪(A×C)
(ii) A×(B ∩ C)=(A×B) ∩ (A×C) SEP-21
(iii) (A ∪ B) × C = (A × C) ∪ (B × C)

Solution:

Given:

$A = \{x \in W / x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N / 1 < x \leq 4\}$

$\Rightarrow B = \{2, 3, 4\}; C = \{3, 5\}$

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$B \cup C = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$

$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cup (A \times C)$

$= \{(0, 2), (0, 3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ ..... (2)

∴(1) = (2) Hence Verified.

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$B \cap C = \{2, 3, 4\} \cap \{3, 5\} = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ..... (1)

$A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0,2), (0,3), (0,4), (1,2), (1,3), (1,4)\}$

$A \times C = \{0,1\} \times \{3, 5\}$

$= \{(0,3), (0,5), (1,3), (1,5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ..... (2)

∴ (1) = (2). Hence Proved.

iii. $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$A \cup B = \{0, 1\} \cup \{2, 3, 4\}$

$= \{0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$

= {(0,3), (0,5), (1,3), (1,5), (2,3), (2,5), (3,3), (3,5) , (4,3), (4,5)} ..... (1)

A × C = {0, 1} × {3, 5}

= {(0,3), (0,5), (1,3), (1,5)}

B × C = {2, 3, 4} × {3, 5}

= {(2,3), (2,5), (3,3), (3,5), (4,3), (4,5)}

∴ (A × C) ∪ (B × C)

= {(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)} ..... (2)

∴ From (1) and (2) LHS = RHS.

6. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) (A ∩ B) × C = (A × C) ∩ (B × C) SEP-20

(ii) A × (B − C) = (A × B) − (A × C) MAY-22

Solution:

Given A = {1, 2, 3, 4, 5, 6, 7}

B = {2, 3, 5, 7} C = {2}

To verify (A ∩ B) × C = (A × C) ∩ (B × C)

A ∩ B = {1, 2, 3, 4, 5, 6, 7} ∩ {2, 3, 5, 7}

= {2, 3, 5, 7}

(A ∩ B) × C = {2, 3, 5, 7} × {2}

∴ (A ∩ B) × C = {(2,2), (3,2), (5,2), (7,2)} .....(1)

A × C = {1, 2, 3, 4, 5, 6, 7} × {2}

= {(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)}

B × C = {2, 3, 5, 7} × {2}

= {(2,2), (3,2), (5,2), (7,2)}

(A × C) ∩ (B × C)

= {(2, 2), (3, 2), (5, 2), (7, 2)} ..... (2)

∴ From (1) and (2), LHS = RHS

ii. To verify A × (B – C)=(A × B) – (A × C)

B – C = {2, 3, 5, 7} – {2} = {3, 5, 7}

A×(B – C) = {1, 2, 3, 4, 5, 6, 7}×{3, 5, 7}

= {(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3, 7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)} ..... (1)

A × B = {1, 2, 3, 4, 5, 6, 7} × {2, 3, 5, 7}

= {(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7), (5,2), (5,3), (5,5), (5,7), (6,2), (6,3), (6,5),(6,7), (7,2), (7,3), (7,5), (7,7)}

A × C = {1, 2, 3, 4, 5, 6, 7} × {2}

= {(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)}

(A × B) – (A × C)

= {(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)} ..... (2)

(1), (2) are equal.

∴ A × (B – C) = (A × B) – (A × C).

Hence it is verified.

7. Let A = {3, 4, 7, 8} and B = {1, 7, 10}. Which of the following sets are relations from A to B?

(i) R_1 = {(3, 7), (4, 7), (7, 10), (8, 1)}

(ii) R_2 = {(3, 1), (4, 12)}

(iii) R_3 = {(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)}

Solution:

A × B = {(3,1), (3,7), (3,10), (4,1), (4,7), (4,10), (7,1), (7,7), (7,10), (8,1), (8,7), (8,10)}

i. We note that, $R_1 \subseteq A \times B$.

Thus R_1 is a relation from A and B.

ii. Here (4, 12) ∈ R_2 , but (4, 12) ∉ A × B.

So R_2 is not a relation from A to B.

iii. Here (7, 8) ∈ R_3 , but (7, 8) ∉ A × B.

So R_3 is not a relation from A to B.

8. Let A = {1, 2, 3, 7} and B = {3, 0, –1, 7}, which of the following are relation from A to B ?

(i) R_1 = {(2, 1), (7, 1)} (ii) R_2 = {(–1, 1)}

(iii) R_3 = {(2, –1), (7, 7), (1, 3)}

(iv) R_4 = {(7, –1), (0, 3), (3, 3), (0, 7)}

Solution:

Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

∴ A × B

= {1, 2, 3, 7} × {3, 0, –1, 7}

= {(1, 3),(1, 0), (1, –1), (1, 7), (2, 3), (2, 0), (2, –1), (2, 7), (3, 3), (3, 0), (3, –1), (3, 7), (7, 3), (7, 0), (7, –1), (7, 7)}

i. R_1 = {(2, 1), (7, 1)} , (2, 1) ∈ R_1

but (2, 1) ∉ A × B

∴ R_1 is not a relation from A to B.

ii. R_2 = {(–1, 1)}, (–1, 1) ∈ R_2

but (–1, 1) ∉ A × B

∴ R_2 is not a relation from A to B.

iii. $R_3 = \{(2, -1), (7, 7), (1, 3)\}$

We note that $R_3 \subseteq A \times B$

$\therefore R_3$ is a relation.

iv. $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$, (0, 3), (0, 7) $\in R_4$ but not in A × B.

$\therefore R_4$ is not a relation from A to B.

9. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

(ii) $\{(x, y) | y = x + 3,$ x, y are natural numbers < 10}

Solution:

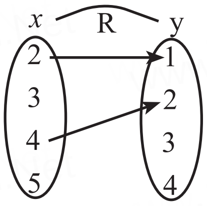
i. $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

$x = 2y$

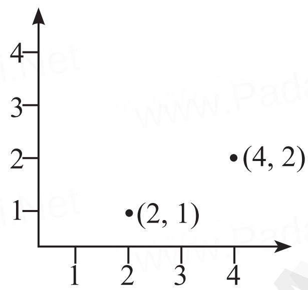
$f(x) = \frac{x}{2}$; $f(2) = \frac{2}{2} = 1$; $f(3) = -$;

$f(4) = \frac{4}{2} = 2$; $f(5) = \frac{5}{2}$

a) An Arrow diagram



b) Graph



c) Roster Form

{(2, 1), (4, 2)}

ii. **$\{(x, y) | y = x + 3,$ x, y are natural numbers < 10}**

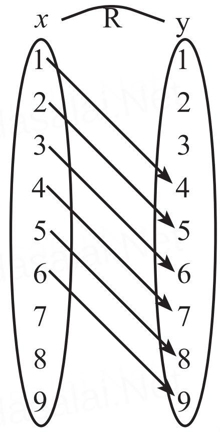
Solution:

$f(x) = x + 3$;

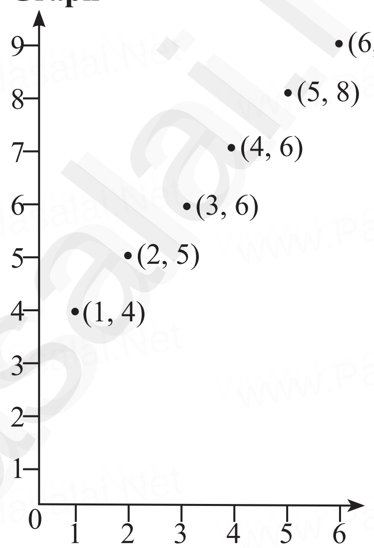
$f(1) = 4$; $f(2) = 5$; $f(3) = 6$;

$f(4) = 7$; $f(5) = 8$; $f(6) = 9$

a) An Arrow diagram



b) Graph



c) Roster Form

(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

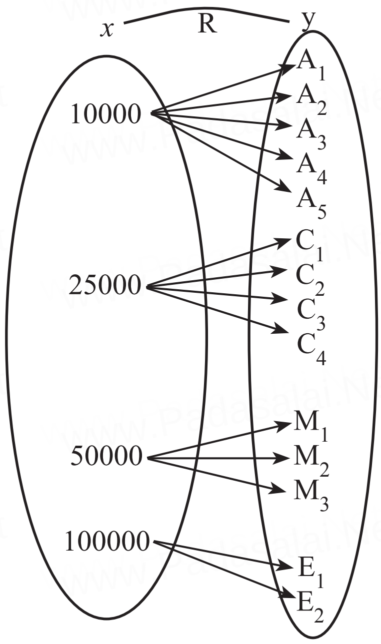
10. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.

Solution:

a) Ordered Pair:

$\{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5)$ $(25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2), (50000, C_3)$ $(100000, E_1), (100000, E_2)\}$

b) Arrow Diagram:



11. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A → B be a function given by f(x) = 3x − 1. Represent this function

(i) by arrow diagram

(ii) in a table form

(iii) as a set of ordered pairs

(iv) in a graphical form **SEP-20**

Solution:

A = {1, 2, 3, 4}, B = {2, 5, 8, 11, 14}

f(x) = 3x – 1

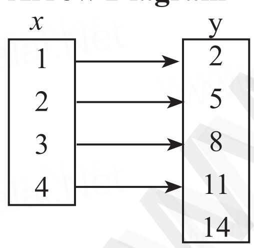
f(1) = 3(1) – 1 = 3 – 1 = 2;

f(2) = 3(2) – 1 = 6 – 1 = 5 f(3) = 3(3) – 1 = 9 – 1 = 8;

f(4) = 3(4) – 1 = 12 – 1 = 11.

R = {(1, 2), (2, 5), (3, 8), (4, 11)}

i) Arrow Diagram



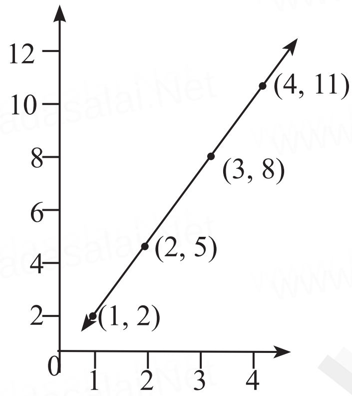
ii) Table

x	1	2	3	4
y	2	5	8	11

iii) Set of Ordered pairs

{(1, 2), (2, 5), (3, 8), (4, 11)}

iv) Graphical Form



12. Let f be a function f : N → N be defined by $f(x) = 3x + 2, x \in N$

(i) Find the images of 1, 2, 3

(ii) Find the pre-images of 29, 53

(iii) Identify the type of function

Solution:

The function f : N→ N be defined by

f(x) = 3x + 2

i. If x = 1, f(1) = 3(1) + 2 = 5

If x = 2, f(2) = 3(2) + 2 = 8; If x = 3, f(3) = 3(3) + 2 = 11

The images of 1, 2, 3 are 5, 8, 11 respectively.

ii. If x is the pre-image of 29, then f(x) = 29. Hence 3x + 2 = 29 ; 3x = 27 ⇒ x = 9.

Similarly, if x is the pre-image of 53 then f(x) = 53. Hence 3x + 2 = 53

3x = 53 – 2 ⇒ 3x = 51 ⇒ x = 17.

Thus the pre-image of 29 and 53 are 9 and 17 respectively.

iii. Since different elements of N have different images in the co-domain, the function f is one-one function. The co-domain of f is N. But the range of f = {5, 8, 11, 14, 17,….}is a proper subset of N . Therefore f is not an onto function. That is, f is an into function. Thus f is one-one and into functions.

13. Let f : A → B be a function defined by $f(x) = \frac{x}{2} - 1$ where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9} . Represent f by

i) set of ordered pairs ii) a table

iii) an arrow diagram iv) a graph

Solution:

Given $f(x) = \frac{x}{2} - 1$

$x = 2 \Rightarrow f(2) = 1 - 1 = 0$

$x = 4 \Rightarrow f(4) = 2 - 1 = 1$

$x = 6 \Rightarrow f(6) = 3 - 1 = 2$

$x = 10 \Rightarrow f(10) = 5 - 1 = 4$

$x = 12 \Rightarrow f(12) = 6 - 1 = 5$

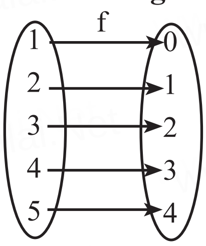
i) Set of Ordered Pairs:

f = {(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

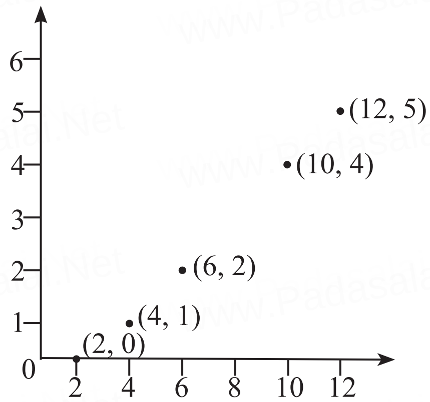
ii) Table

x	2	4	6	10	12
f(x)	0	1	2	4	5

iii) Arrow Diagram



iv) Graph



14. Represent the function

f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} through

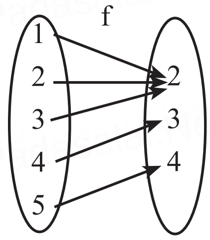
(i) an arrow diagram

(ii) a table form

(iii) a graph

Solution:

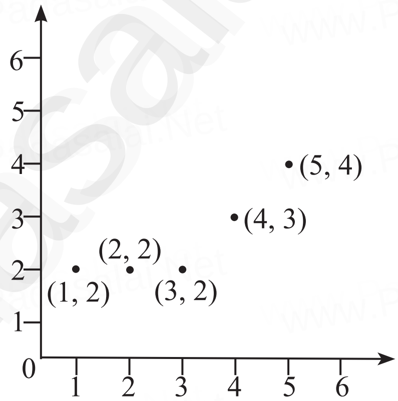
i) Arrow Diagram



ii) Table Form:

x	1	2	3	4	5
f(x)	2	2	2	3	4

iii) Graph



FORMULAE AND DEFINITIONS

1.	**Cartesian Product**	If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$, $b \in B$ is called the Cartesian Product of A and B and is denoted by A×B. Thus $A \times B = \{(a, b) \mid a \in A, b \in B\}$
2.	**Relations**	Let A and B be any two non empty sets. A relation R from A to B is a subset of A×B satisfying some specified conditions. That is $R \subseteq A \times B$.
3.	**Null Relation**	A relation which contains no elements is called a 'Null reation.'
4.	If $n(A) = p$ and $n(B) = q$, then $\boldsymbol{n(A \times B) = pq}$	
5.	If $n(A) = p$, $n(B) = q$, then the **total number of relation** that exist between A and B is $\mathbf{2^{pq}}$.	
6.	f $n(A) = p$, $n(B) = q$, then the **total number of non-empty relation** that exist between A and B is $\mathbf{2^{pq} - 1}$.	
7.	Kinds of functions	(i). one-one (ii) many to one (iii) onto (iv) into (v) constant (vi) identity
8.	Onto function	Range = Co do main

***

2 Numbers and Sequences

Exercise 2.10

Multiple choice Questions

1. **Euclid's division lemma states that for positive integers a and b, there exist unique integers q and r such that a = bq +r, where r must satisfy.**
   A) $1 < r < b$ B) $0 > r > b$
   C) $0 \le r < b$ D) $0 < r \le b$

2. **Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are** SEP-20
   A) 0, 1, 8 B) 1, 4, 8
   C) 0, 1, 3 D) 1, 3, 5

3. **If the HCF of 65 and 117 is expressible in the form of 65m -117 , then the value of m is**
   A) 4 B) 2 MAY-22
   C) 1 D) 3

4. **The sum of the exponents of the prime factors in the prime factorization of 1729 is**
   A) 1 B) 2 SEP-21
   C) 3 D) 4

5. **The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is**
   A) 2025 B) 5220
   C) 5025 D) 2520

6. **74k ≡ _____ (mod 100)**
   A) 1 B) 2
   C) 3 D) 4

7. **Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is** SEP-21
   A) 3 B) 5
   C) 8 D) 11

8. **The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.**
   A) 4551 B) 10091
   C) 7881 D) 13531

9. **If 6 times of 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P. is**
   A) 0 B) 6
   C) 7 D) 13

10. **An A.P. consists of 31 terms. If its 16th term is m, then the sum of all the terms of this A.P. is**
    A) 16 m B) 62 m
    C) 31 m D) $\frac{31}{2}$ m

11. **In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?**
    A) 6 B) 7
    C) 8 D) 9 **Ans: C) 8**

12. **If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + ... + 2^0$ which of the following is true?** SEP-20
    A) B is 264 more than A
    B) A and B are equal
    C) B is larger than A by 1
    D) A is larger than B by 1

13. **The next term of the sequence 3**
    A) 124 B) 127
    C) 23 D) 181

14. **If the sequence $t_1, t_2, t_3, ...$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, ...$ is**
    A) a Geometric Progression
    B) an Arithmetic Progression
    C) neither an Arithmetic Progression nor a Geometric Progression
    D) a constant sequence

15. **The value of $(1^3 + 2^3 + 3^3 + ... + 15^3) - (1 + 2 + 3 + ... + 15)$ is**
    A) 14400 B) 14200
    C) 14280 D) 14520

2 Marks — STAGE 1

1. **'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.**

Solution:

$800 = a^b \times b^a$

2	800
2	400
2	200
2	100
2	50
5	25
	5

$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^5 \times 5^2$

$\therefore a = 2, b = 5$ (or) $a = 5, b = 2$

2. **Find the HCF of 252525 and 363636.**

Solution:

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
13	481
37	37
	1

5	252525
5	50505
3	10101
7	3367
13	481
37	37
	1

$252525 = 3 \times 5^2 \times 7 \times 13 \times 37$

$363636 = 2^3 \times 3^3 \times 7 \times 13 \times 37$

H.C.F of 252525 and 363636

$= 3 \times 7 \times 13 \times 37$

$= 10101.$

3. **If $13824 = 2^a \times 3^b$ then find a and b.** MAY-22

Solution:

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$\Rightarrow 13824 = 2^9 \times 3^3$

$\therefore a = 9, b = 3$

4. **Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.**

Solution:

2	408
2	204
2	102
3	51
	17

2	170
5	85
	17

$408 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

H.C.F. of 408 & 170 $= 2 \times 17 = 34$

L.C.M. of 408 & 170 $= 2^3 \times 3 \times 5 \times 17$

$= 2040$

5. **The general term of a sequence is defined as**

$$a_n = \begin{cases} n(n+3);\ n \in N \text{ is odd} \\ n^2+1\ ;\ n \in N \text{ is even} \end{cases}$$

Find the eleventh and eighteenth terms.

Solution:

To find a_{11}, since 11 is odd,

we put n = 11 in

$a_n = n(n+3)$

Thus,

the eleventh term $a_{11} = 11(11+3) = 154$.

To find a_{18}, since 18 is even,

we put n = 18 in

$a_n = n^2 + 1$

Thus, the eighteenth term $a_{18} = 18^2 + 1 = 325$.

6. **Find the indicated terms of the sequences whose n^{th} terms are given by**

(i) $a_n = \dfrac{5n}{n+2}$; a_6 and a_{13}

(ii) $a_n = -(n^2-4)$; a_4 and a_{11}

Solution:

i. $a_n = \dfrac{5n}{n+2}$

$a_6 = \dfrac{30}{8} = \dfrac{15}{4}$; $a_{13} = \dfrac{65}{15} = \dfrac{13}{3}$

ii. $a_n = -(n^2-4)$

$a_4 = -(16-4) = -12$;

$a_{11} = -(121-4) = -117$

7. Find a_8 and a_{15} whose nth term is

$$a_n = \begin{cases} \dfrac{n^2-1}{n+3} & ; n \text{ is even}, \quad n \in N \\ \dfrac{n^2}{2n+1} & ; n \text{ is odd}, \quad n \in N \end{cases}$$

Solution:

To find a_8 here n is even, so $a_n = \dfrac{n^2-1}{n+3}$

$$a_8 = \frac{64-1}{11} = \frac{63}{11}$$

To find a_{15}, here n is odd, so $a_n = \dfrac{n^2}{2n+1}$

$$a_{15} = \frac{(15)^2}{30+1} = \frac{225}{31}$$

8. Find the 19th term of an A.P. –11, –15, –19, ...

Solution:

General Form of an A.P. is $t_n = a + (n-1)d$

$a = -11$; $d = -15+11 = -4$; $n = 19$

$t_{19} = -11 + 18(-4)$

$= -11 - 72$

$t_{19} = -83$

9. Which term of an A.P. 16, 11, 6, 1,... is –54 ? **MAY-22**

Solution:

$$n = \left(\frac{l-a}{d}\right) + 1$$

$a = 16$; $d = 11 - 16 = -5$; $l = -54$

$$n = \frac{-54-16}{-5} + 1 = \frac{-70}{-5} + 1$$

$n = 14 + 1$

$n = 15$

10. Find the middle term(s) of an A.P. 9, 15, 21, 27, …, 183.

Solution:

$a = 9$, $d = 6$, $l = 183$

$$n = \left(\frac{l-a}{d}\right) + 1$$

$$= \frac{183-9}{6} + 1 = \frac{174}{6} + 1 = 29 + 1 = 30$$

∴ 15 and 16 are the middle terms.

$t_n = a + (n-1)d$

$\therefore t_{15} = a + 14d \qquad t_{16} = a + 15d$

$= 9 + 14(6) \qquad = 9 + 15(6)$

$= 9 + 84 \qquad = 9 + 90$

$= 93 \qquad = 99$

∴ 93, 99 are the middle terms of A.P.

11. If 3 + k, 18 – k, 5k + 1 are in A.P. then find k. **SEP-21**

Solution:

3 + k, 18 – k, 5k + 1 is a A.P

$$t_2 - t_1 = t_3 - t_2$$

$$(18-k) - (3+k) = (5k+1) - (18-k)$$

$$15 - 2k = 6k - 17$$

$$-2k - 6k = -17 - 15$$

$$-8k = -32$$

$$k = 4$$

12. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution:

First Term, a = 20

Common Difference, d = 2

∴ Number of seats in the last row

$= t_n = a + (n-1)d$

$t_{30} = a + 29d = 20 + 29(2) = 20 + 58 = 78$

13. Write an A.P. whose first term is 20 and common difference is 8.

Solution:

First Term, a = 20;

Common Difference, d = 8

Arithmetic Progression is a, a+d, a+3d, …

In this case,

we get 20, 20 + 8, 20 + 2(8), 20 + 3(8), ...

So, the required A.P. is 20, 28, 36, 44, ...

14. Find the number of terms in the A.P. 3, 6, 9, 12, …, 111. **SEP-21**

Solution:

First term a = 3,

Common difference d = 6 – 3 = 3,

Last term, $l = 111$

We know that, $n = \left(\dfrac{l-a}{d}\right) + 1$

$$n = \left(\frac{111-3}{3}\right) + 1 = 37$$

Thus the A.P. contains 37 terms.

15. Write the first three terms of the G.P. whose first term and the common ratio are given below.

(i) a = 6, r = 3 **(ii) a = $\sqrt{2}$, r = $\sqrt{2}$**

(iii) a = 1000, r = $\frac{2}{5}$

Solution:

i. General Form of an G.P. $\Rightarrow$ a, ar, ar^2, …..

a = 6, r = 3 G.P. $\Rightarrow$ 6, 6(3), $6(3)^2$…

$\Rightarrow$ 6, 18, 54, ….

ii. G.P. $\Rightarrow$ a, ar, ar^2, …

$a = \sqrt{2}$, $r = \sqrt{2}$

G.P. $\Rightarrow \sqrt{2}, \sqrt{2}\ \sqrt{2}, \sqrt{2}\left(\sqrt{2}\right)^2$

$\Rightarrow \sqrt{2}, 2, 2\sqrt{2}$

iii. G.P. $\Rightarrow$ a, ar, ar^2, …..

$a = 1000,\ r = \frac{2}{5}$

G.P. $\Rightarrow 1000, 1000 \times \frac{2}{5}, 1000 \times \left(\frac{2}{5}\right)^2 \ldots$

G.P. $\Rightarrow$ 1000, 400, 160,….

16. In a G.P. 729, 243, 81, … find t_7.

Solution:

$t_n = ar^{n-1}$

$a = 729, r = \frac{243}{729} = \frac{1}{3}, n = 7$

$t_7 = 729 \times \left(\frac{1}{3}\right)^{7-1}$

$t_7 = 729 \times \left(\frac{1}{3}\right)^6$

$t_7 = 729 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

17. Find x so that x + 6, x + 12 and x + 15 are consecutive terms of a Geometric Progression.

Solution:

Given $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a G.P.

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$$(x+12)^2 = (x+6)(x+15)$$

$$x^2 + 24x + 144 = x^2 + 21x + 90$$

$$24x - 21x = 90 - 144$$

$$3x = -54$$

$$x = -\frac{54}{3} = -18$$

18. Find the number of terms in the following G.P.

(i) 4, 8, 16, …, 8192?

(ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$

Solution:

i. G.P. $\Rightarrow$ 4, 8, 16, …, 8192.

Here a = 4, r = 2, t_n = 8192

$ar^{n-1} = t_n \Rightarrow 4(2)^{n-1} = 8192;$

$2^{n-1} = \frac{8192}{4} = 2048$

$2^{n-1} = 2^{11}$; n – 1 = 11

$\Rightarrow$ n = 12

ii. G.P. $\Rightarrow \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$.

Here $a = \frac{1}{3}, r = \frac{1}{3}, t_n = \frac{1}{2187}$

$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187}$

$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{2187} \times 3$

$\left(\frac{1}{3}\right)^{n-1} = \frac{1}{729} = \left(\frac{1}{3}\right)^6$;

n – 1 = 6 $\Rightarrow$ n = 7

19. In a G.P. the 9^{th} term is 32805 and 6^{th} term is 1215. Find the 12^{th} term.

Solution:

From the given

$t_9 = 32805 \Rightarrow ar^8 = 32805$ ..... (1)

$t_6 = 1215 \Rightarrow ar^5 = 1215$ ..... (2)

(1) ÷ (2) $\Rightarrow r^3 = 27 \quad \Rightarrow r = 3$

(2) $\Rightarrow a(3)^5 = 1215 \Rightarrow a = 5$

To find t_{12},

$t_n = ar^{n-1}$

$t_{12} = (5)(3)^{11}$

20. Find the first term of a G.P. in which S_6 = 4095 and r = 4.

Solution:

Common ratio, = 4 > 1,

Sum of first 6 terms S_6 = 4095

Hence, $S_n = \frac{a\left(r^n - 1\right)}{r-1} = 4095$

r = 4, $\frac{a\left(4^6 - 1\right)}{4-1} = 4095$

$\Rightarrow a \times \frac{4095}{3} = 4095$

First term, a = 3.

21. Find the value of

1 + 2 + 3 + ... + 50

Solution:

$1 + 2 + 3 + \ldots + 50$

Using $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

$1 + 2 + 3 + \ldots + 50 = \frac{50 \times (50+1)}{2} = 1275$

22. Find the sum of the following series

1 + 2 + 3 + ... + 60

Solution:

$1 + 2 + 3 + \ldots + 60 = \frac{n(n+1)}{2}$

$= \frac{60 \times 61}{2}$

$= 30 \times 61 = 1830$

23. Find the sum of

(i) 1 + 3 + 5 + ... to 40 terms

(ii) 2 + 4 + 6 + ... 80

(iii) 1 + 3 + 5 + ... + 55

Solution:

i. $1 + 3 + 5 + \ldots + n \text{ terms} = n^2$

$1 + 3 + 5 + \ldots + 40 \text{ terms} = (40)^2 = 1640$

ii. $2 + 4 + 6 + \ldots + 80$

$= 2\,[1 + 2 + 3 + \ldots + 40]$

$= 2\left[\frac{n(n+1)}{2}\right] = 40 \times 41 = 1640$

iii. $1 + 3 + 5 + \ldots + 55$

Here the number of terms is not given.

Now, we have to find the number of terms using the formula.

$n = \frac{(55-1)}{2} + 1 = 28$

Therefore,

$1 + 3 + 5 + \ldots + 55 = (28)^2 = 784$

24. Find the sum of

(i) $1^2 + 2^2 + \ldots + 19^2$

(ii) $5^2 + 10^2 + 15^2 + \ldots + 105^2$

Solution:

i. $1^2 + 2^2 + \ldots + 19^2$

$= \frac{n(n+1)(2n+1)}{6}$

$= \frac{19 \times (19+1)(2 \times 19+1)}{6}$

$= \frac{19 \times 20 \times 39}{6} = 2170$

ii. $5^2 + 10^2 + 15^2 + \ldots + 105^2$

$= 5^2(1^2 + 2^2 + 3^2 + \ldots + 21^2)$

$= 25 \times \frac{21 \times (21+1) \times (2 \times 21+1)}{6}$

$= 25 \times \frac{21 \times 22 \times 43}{6} = 82775$

25. Find the sum of $1^3 + 2^3 + 3^3 + \ldots + 16^3$

Solution:

$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$1^3 + 2^3 + 3^3 + \ldots + 16^3 = \left[\frac{16 \times 17}{2}\right]^2$

$= [136]^2 = 18496$

26. If 1 + 2 + 3 + ... + n = 666 then find n.

Solution:

$1 + 2 + 3 + \ldots\ldots + n = 666$

$\frac{n(n+1)}{2} = 666$

$n^2 + n = 1332$

$n^2 + n - 1332 = 0$

$(n - 36)(n + 37) = 0$

$n = -37$ or $n = 36$

But $n \neq -37$ (Since n is a natural number)

Hence n = 36.

27. If 1 + 2 + 3 + ... + k = 325 , then find $1^3 + 2^3 + 3^3 + \ldots + k^3$.

Solution:

$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} = 325$

$1^3 + 2^3 + 3^3 + \ldots + k^3$

$= \left[\frac{k(k+1)}{2}\right]^2 = (325)^2 = 105625$

28. If $1^3 + 2^3 + 3^3 + \ldots + k^3 = 44100$ then find 1 + 2 + 3 + ... + k.

Solution:

$1^3 + 2^3 + 3^3 + \ldots + k^3 = 44100 = \left[\frac{k(k+1)}{2}\right]^2$

$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} = 210$

29. How many terms of the series $1^3 + 2^3 + 3^3 + \ldots$ should be taken to get the sum 14400?

Solution:

$1^3 + 2^3 + 3^3 + \ldots + k^3 = \left[\frac{k(k+1)}{2}\right]^2 = 14400$

$\Rightarrow \frac{k(k+1)}{2} = \sqrt{14400} = 120$

$k(k + 1) = 240$

$k^2 + k - 240 = 0$

$(k - 15)(k + 16) = 0$

$k = +15$ or $k = -16$

k can't be negative

$\therefore k = 15$

5 Marks

1. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4

Solution:

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

$11340 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore P_1 = 2,\ P_2 = 3,\ P_3 = 5,\ P_4 = 7$

$x_1 = 3,\ x_2 = 4,\ x_3 = 2,\ x_4 = 1$

2. If $a_1 = 1$, $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$ $n \geq 3, n \in N$, then find the first six terms of the sequence.

Solution:

Given $a_1 = a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$

$a_3 = 2a_2 + a_1 = 2(1) + 1 = 3;$

$a_4 = 2a_3 + a_2 = 2(3) + 1 = 7$

$a_5 = 2a_4 + a_3 = 2(7) + 3 = 17;$

$a_6 = 2a_5 + a_4 = 2(17) + 7 = 41$

3. Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Solution:

A.P. $\Rightarrow x$, 10, y, 24, z

That is $y = \frac{10+24}{2} = \frac{34}{2} = 17$

$\therefore$ A.P = x, 10, 17, 24, z

Here we know that d = 17 – 10 = 7

$\therefore x = 10 - 7 = 3$

$z = 24 + 7 = 31$

$\therefore x = 3, y = 17, z = 31.$

4. Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution:

$S_n = 5 + 55 + 555 + \ldots + n$ terms

$= 5\,[1 + 11 + 111 + \ldots + n \text{ terms}]$

$= \frac{5}{9}\,[9 + 99 + 999 + \ldots + n \text{ terms}]$

$= \frac{5}{9}\,[10 - 1 + 100 - 1 + 1000 - 1 + \ldots n \text{ terms}]$

$= \frac{5}{9}\,[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$

$= \frac{5}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

$= \frac{50}{81}\left[(10^n - 1) - \frac{5}{9}n\right]$

5. Find the sum to n terms of the series
(i) 0.4 + 0.44 + 0.444 + ... to n terms
(ii) 3 + 33 + 333 + ... to n terms

Solution:

i. 0.4 + 0.44 + 0.444 + ... n terms

$= \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ldots n\,terms$

$= 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n\,terms\right]$

$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots n\,terms\right]$

$= \frac{4}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots n\,terms\right]$

$= \frac{4}{9}\left[(1+1+1+\ldots n \text{ terms}) - \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n\,terms\right)\right]$

$= \frac{4}{9}\left[n - \frac{1}{10}\left[\frac{1 - \left(\frac{1}{10}\right)^n}{1 - \frac{1}{10}}\right]\right] = \frac{4}{9}\left[n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right]$

ii. 3 + 33 + 333 + ... n

$= 3(1 + 11 + 111 + \ldots + n \text{ terms})$

$= \frac{3}{9}\,(9 + 99 + 999 + \ldots + n \text{ terms})$

$= \frac{3}{9}\,((10-1) + (100-1) + (1000-1) + \ldots + n \text{ terms})$

$= \frac{3}{9}$ (10 + 100 + 1000 + ... + n terms) – (1 + 11 + 111 + ….. + n terms)

$$= \frac{3}{9}\left(10\left(\frac{10^n - 1}{9}\right) - n\right)$$

$$= \frac{30}{81}(10n - 1) - \frac{3n}{9}$$

6. Find the sum of the Geometric series 3 + 6 + 12 + ... + 1536

Solution:

3 + 6 + 12 + ... + 1536

a = 3, r = 2

$t_n = 1536$

$ar^{n-1} = 1536$

$3(2)^{n-1} = 1536$

$3(2)^{n-1} = 3(2)^9$

$2^{n-1} = 2^9$

n–1 = 9

∴ n = 10

To find S_n,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$$

= 3 (1023) = 3069

7. Find the value of 16 + 17 + 18 + ... + 75

Solution:

16 + 17 + 18 + … + 75

= (1 + 2 + 3 + ….. + 75) – (1 + 2 + 3 + … + 15)

$$= \frac{75(75+1)}{2} - \frac{15(15+1)}{2}$$

= 2850 – 120

= 2730

8. Find the sum of $9^3 + 10^3 + ... + 21^3$

Solution:

$9^3 + 10^3 + ... + 21^3$

$= (1^3 + 2^3 + 3^3 ... + 21^3) - (1^3 + 2^3 + 3^3 ... + 8^3)$

$$= \left[\frac{21\times(21+1)}{2}\right]^2 - \left[\frac{8\times(8+1)}{2}\right]^2$$

$= (231)^2 - (36)^2$

= 52065

9. Find the sum of the following series

(i) $6^2 + 7^2 + 8^2 + ... + 21^2$

(ii) $10^3 + 11^3 + 12^3 + ... + 20^3$

Solution:

i. $6^2 + 7^2 + 8^2 + ... + 21^2$

$= (1^2 + 2^2 + 3^2 ... + 21^2) - (1^2 + 2^2 + 3^2 + ... + 5^2)$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{21\times(21+1)(42+1)}{6} - \frac{5\times(5+1)(10+1)}{6}$$

$$= \frac{21\times22\times43}{6} - \frac{5\times6\times11}{6}$$

= 3311 – 55 = 3256

ii. $10^3 + 11^3 + 12^3 + ... + 20^3$

$= 1^3 + 2^3 + 3^3 + ... + 20^3 - 1^3 + 2^3 + 3^3 + ... + 9^3$

$$= \left[\frac{n(n+1)}{2}\right]^2 - \left[\frac{n(n+1)}{2}\right]^2$$

$$= \left[\frac{20\times21}{6}\right]^2 - \left[\frac{9\times10}{3}\right]^2$$

$= [210]^2 - (45)^2$

= 44100 – 2025 = 42075

10. The sum of the cubes of the first *n* natural numbers is 2025, then find the value of *n*.

Solution:

$1^2 + 2^2 + 3^2 + ... + n^2 = 285$

$$\frac{n(n+1)(2n+1)}{2\times3} = 285$$

$$\frac{n(n+1)(2n+1)}{6} = 285$$

n(n+1) (2n+1) = 285 × 6 ..... (1)

$1^3 + 2^3 + 3^3 + ... + n^3 = 2025$

$$\left[\frac{n(n+1)}{2}\right]^2 = 2025$$

$$\frac{n(n+1)}{2} = \sqrt{2025} = 45$$

n(n+1) = 45 × 2 ..... (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{n(n+1)(2n+1)}{n(n+1)} = \frac{258\times6}{45\times2}$$

2n + 1 = 19

2n = 19 – 1

⇒ 2n = 18

∴ n = 9

11. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, …, 24 cm. How much area can be decorated with these colour papers?

Solution:

The Required Area

$= 10^2 + 11^2 + 12^2 + ... + 24^2$

Area $= (1^2 + 2^2 + 3^2 + ... + 24^2) - (1^2 + 2^2 + ... + 9^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{24\times25\times49}{6} - \frac{9\times10\times19}{6}$

$= 4900 - 285 = 4615$ cm^2

Therefore Rekha has 4615 cm^2 colour paper. She can decorate 4615cm^2 area with these colour papers.

12. Find the sum of $15^2 + 16^2 + 17^2 + ... + 28^2$

$15^2 + 16^2 + 17^2 + ... + 28^2$

$= (1^2 + 2^2 + 3^2 ... + 28^2) - (1^2 + 2^2 + 3^2 ... + 14^2)$

$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$= \frac{28\times29\times57}{2\times3} - \frac{14\times15\times29}{2\times3}$

$= 14 \times 29 \times 19 - 7 \times 5 \times 29$

$= 7714 - 1015 = 6699$

FORMULAE AND DEFINITIONS

1.	**Euclid's Divisions Lemma**	Let a and b ($a > b$) be any two positive integers. Then there exist unique integers q and r such that $\boldsymbol{a = bq+r,\ 0 \le r < b}$
2.	**Fundamental theorem of Arithmetic**	Every composite number can be written uniquely as the product of power of primes is called Fundamental Theorem of Arithmetic.
3.	General Form of an A.P	$a, a + d, a + 2d, a + 3d, ...$
4.	Common Difference of an A.P	$d = t_2 - t_1$
5.	General term or n^{th} term of an A.P	$t_n = a + (n - 1) d$
6.	Number of terms of an A.P	$n = \left(\frac{l-a}{d}\right) + 1$
7.	Three consecutive terms of an A.P	$a - d, a, a + d$
8.	Four consecutive terms of an A.P	$a - 3d, a - d, a + d, a + 3d$
9.	Condition for three numbers (a, b, c) to be in A.P	$2b = a + c$

10.	In an A.P first term a and last term l are given then the nth last term	$S_n = \frac{n}{2}[2a+(n-1)d]$ (OR) $S_n = \frac{n}{2}[a+l]$
11.	General form of an G.P	$a, ar, ar^2, ... ar^{n-1}$ where a = first term; r - common ratio
12.	Common ratio	$r = \frac{t_2}{t_1}$
13.	nth term of an G.P.	$t_n = ar^{n-1}$
14.	S consecutive terms of an G.P.	$\frac{a}{r}, a, ar$
15.	If a, b, c are in G.P then the condition	$b^2 = ac$
16.	Sum of first n terms of an G.P.	$S_n = \frac{a(r^n - 1)}{r-1},\ r \ne 1$
17.	Sum to infinite terms of an G.P.	$S_\infty = \frac{a}{1-r}\quad -1<r<1$

***

3 Algebra

Exercise 3.20

Multiple choice Questions

1. **A system of three linear equations in three variables is inconsistent if their planes**
   A) intersect only at a point
   B) intersect in a line
   C) coincides with each other
   D) do not intersect

2. **The solution of the system $x + y - 3z = -6$, $-7y + 7z = 7$, $3z = 9$ is**
   A) $x = 1, y = 2, z = 3$
   B) $x = -1, y = 2, z = 3$
   C) $x = -1, y = -2, z = 3$
   D) $x = 1, y = -2, z = 3$

3. **If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is** MAY-22
   A) 3 B) 5 C) 6 D) 8

4. **$\dfrac{3y-3}{y} \div \dfrac{7y-7}{3y^2}$ is**
   A) $\dfrac{9y}{7}$ B) $\dfrac{9y^3}{(21y-21)}$
   C) $\dfrac{21y^2-42y+21}{3y^3}$ D) $\dfrac{7(y^2-2y+1)}{y^2}$

5. **$y^2 + \dfrac{1}{y^2}$ is not equal to**
   A) $\dfrac{y^4+1}{y^2}$ B) $\left(y+\dfrac{1}{y}\right)^2$
   C) $\left(y-\dfrac{1}{y}\right)^2+2$ D) $\left(y+\dfrac{1}{y}\right)^2-2$

6. **$\dfrac{x}{x^2-25} - \dfrac{8}{x^2+6x+5}$ gives**
   A) $\dfrac{x^2-7x+40}{(x-5)(x+5)}$ B) $\dfrac{x^2+7x+40}{(x-5)(x+5)(x+1)}$
   C) $\dfrac{x^2-7x+40}{(x^2-25)(x+1)}$ D) $\dfrac{x^2+10}{(x^2-25)(x+1)}$

7. **The square root of is $\dfrac{256x^8y^4z^{10}}{25x^6y^6z^6}$ equal to** SEP-21
   A) $\dfrac{16}{5}\left|\dfrac{x^2z^4}{y^2}\right|$ B) $16\left|\dfrac{y^2}{x^2z^4}\right|$
   C) $\dfrac{16}{5}\left|\dfrac{y}{xz^2}\right|$ D) $\dfrac{16}{5}\left|\dfrac{xz^2}{y}\right|$

8. **Which of the following should be added to make $x^4 + 64$ a perfect square** MAY-22
   A) $4x^2$ B) $16x^2$ C) $8x^2$ D) $-8x^2$

9. **The solution of $(2x - 1)^2 = 9$ is equal to**
   A) –1 B) 2
   C) –1, 2 D) None of these

10. **The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are**
    A) 100, 120 B) 10, 12
    C) –120, 100 D) 12, 10

11. **If the roots of the equation $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in ______.**
    A) A.P B) G.P
    C) Both A.P and G.P D) None of these

12. **Graph of a linear equation is a ________** SEP-21
    A) straight line B) circle
    C) parabola D) hyperbola

13. **The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is** MAY-22
    A) 0 B) 1 C) 0 or 1 D) 2

14. **For the given matrix**
    $$A = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$$ **the order of the matrix A^T is**
    A) 2×3 B) 3×2 C) 3×4 D) 4×3

15. **If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have**
    A) 3 B) 4 C) 2 D) 5

16. **If number of columns and rows are not equal in a matrix then it is said to be a**
    A) diagonal matrix
    B) rectangular matrix
    C) square matrix
    D) identity matrix

17. **Transpose of a column matrix is** SEP-20
    A) unit matrix B) diagonal matrix
    C) column matrix D) row matrix

18. Find the matrix X if 2X + $\begin{pmatrix}1 & 3\\5 & 7\end{pmatrix} = \begin{pmatrix}5 & 7\\9 & 5\end{pmatrix}$

A) $\begin{pmatrix}-2 & -2\\2 & -1\end{pmatrix}$ B) $\begin{pmatrix}2 & 2\\2 & -1\end{pmatrix}$

C) $\begin{pmatrix}1 & 2\\2 & 2\end{pmatrix}$ D) $\begin{pmatrix}2 & 1\\2 & 2\end{pmatrix}$

19. Which of the following can be calculated from the given matrices

$A = \begin{pmatrix}1 & 2\\3 & 4\\5 & 6\end{pmatrix}$, $B = \begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{pmatrix}$

(i) A^2 (ii) B^2 (iii) AB (iv) BA

A) (i) and (ii) only
B) (ii) and (iii) only
C) (ii) and (iv) only
D) all of these

20. If $A = \begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix}$, $B = \begin{pmatrix}1 & 0\\2 & -1\\0 & 2\end{pmatrix}$ and $C = \begin{pmatrix}0 & 1\\-2 & 5\end{pmatrix}$.

Which of the following statements are correct?

(i) AB+C = $\begin{pmatrix}5 & 5\\5 & 5\end{pmatrix}$ (ii) BC = $\begin{pmatrix}0 & 1\\2 & -3\\-4 & 10\end{pmatrix}$

(iii) BA+C = $\begin{pmatrix}2 & 5\\3 & 0\end{pmatrix}$ (iv) (AB)C = $\begin{pmatrix}-8 & 20\\-8 & 13\end{pmatrix}$

A) (i) and (ii) only B) (ii) and (iii) only
C) (iii) and (iv) only D) all of these

2 Marks STAGE 1

1. Find the LCM of the given polynomials

(i) $4x^2y, 8x^3y^2$
(ii) $9a^3b^2, 12a^2b^2c$
(iii) $16m, 12m^2n^2, 8n^2$
(iv) $p^2 - 3p + 2, p^2 - 4$
(v) $2x^2 - 5x - 3, 4x^2 - 36$
(vi) $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

Solution:

i. $4x^2y, 8x^3y^2$

$4x^2y = 2^2x^2y$

$8x^3y^2 = 2^3x^3y^2$

$\therefore$ LCM$(4x^2y, 8x^3y^2) = 2^3x^3y^2 = 8x^3y^2$

ii. $9a^3b^2, 12a^2b^2c$

$9a^3b^2 = (1)(3)^2 a^3b^2$

$12a^2b^2c = 2^2 \times 3 \times a^2 \times b^2 \times c$

$\therefore$ LCM$(9a^3b^2, 12a^2b^2c)$

$= (1) \times 2^2 \times 3^2 \times a^3 \times b^2 \times c = 36a^3b^2c$

iii. $16m, 12m^2n^2, 8n^2$

$16m = 2^4 \times m$

$12m^2n^2 = 2^2 \times 3 \times m^2 \times n^2$

$8n^2 = 2^3 \times n^2$

$\therefore$ LCM$(16m, 12m^2n^2, 8n^2)$

$= 2^4 \times 3 \times m^2 \times n^2 = 48m^2n^2$

iv. $p^2 - 3p + 2, p^2 - 4$

$p^2 - 3p + 2 = (p - 1)(p - 2)$

$p^2 - 4 = (p + 2)(p - 2)$

$\therefore$ LCM$(p^2 - 3p + 2, p^2 - 4)$

$= (p - 1)(p + 2)(p - 2)$

v. $2x^2 - 5x - 3, 4x^2 - 36$

$2x^2 - 5x - 3 = (x - 3)(2x + 1)$

$4x^2 - 36 = 4(x + 3)(x - 3)$

$\therefore$ LCM$(2x^2 - 5x - 3, 4x^2 - 36)$

$= 4(x - 3)(x + 3)(2x + 1)$

vi. $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

$(2x^2 - 3xy)^2 = x^2(2x - 3y)^2$

$(4x - 6y)^3 = 2^3(2x - 3y)^3$

$8x^3 - 27y^3 = (2x)^3 - (3y)^3$

$= (2x - 3y)(4x^2 + 6xy + 9y^2)$

$\therefore$ LCM$((2x^2 - 3xy)^2, (4x - 6y)^3, (8x^3 - 27y^3))$

$= 2^3 \times x^2 \times (2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

$= 8x^2(2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

2. Simplify:

i) $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4}$

ii) $\dfrac{p^2 - 10p + 21}{p - 7} \times \dfrac{p^2 + p - 12}{(p - 3)^2}$

iii) $\dfrac{5t^3}{4t - 8} \times \dfrac{6t - 12}{10t}$

Solution:

i. $\dfrac{4x^2y}{2z^2} \times \dfrac{6xz^3}{20y^4} = \dfrac{3x^3z}{5y^3}$

ii. $\dfrac{p^2 - 10p + 21}{p - 7} \times \dfrac{p^2 + p - 12}{(p - 3)^2}$

$= \dfrac{(p - 7)(p - 3)}{(p - 7)} = \dfrac{(p + 4)(p - 3)}{(p - 3)^2} = (p + 4)$

iii. $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$

$= \frac{5t^3}{4(t-2)} \times \frac{6(t-2)}{10t} = \frac{3t^2}{4}$

3. Simplify: $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

Solution:

$\frac{x^3}{x-y} + \frac{y^3}{y-x} = \frac{x^3-y^3}{x-y}$

$= \frac{(x^2+xy+y^2)(x-y)}{(x-y)}$

$= x^2 + xy + y^2$

4. Find the excluded values of the following expressions (if any). **MAY-22**

i) $\frac{x+10}{8x}$ ii) $\frac{7p+2}{8p^2+13p+5}$

Solution:

i. The expression $\frac{x+10}{8x}$ is undefined when $8x = 0$ or $x = 0$.

When the excluded value is 0.

ii. The expression $\frac{7p+2}{8p^2+13p+5}$ is undefined when $8p^2 + 13p + 5 = 0$ that is

$(8p+5)(p+1) = 0$ $p = \frac{-5}{8}$, $p = -1$.

The excluded values are $\frac{-5}{8}$ and -1.

5. Find the excluded values, if any of the following expressions.

i) $\frac{y}{y^2-25}$ ii) $\frac{t}{t^2-5t+6}$

iii) $\frac{x^2+6x+8}{x^2+x-2}$ iv) $\frac{x^3-27}{x^3+x^2-6x}$

Solution:

i. The expression $\frac{y}{y^2-25}$ is undefined

when $y^2 - 5^2 = 0$

$y^2 - 5^2 = 0$

$(y+5)(y-5) = 0$

$y+5 = 0, y-5 = 0$

$y = -5, y = 5$

Hence the excluded values are -5 and 5.

ii. The expression $\frac{t}{t^2-5t+6}$ is undefined

when $t^2 - 5t + 6 = 0$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t-2 = 0, t-3 = 0$

$t = 2, t = 3$

Hence the excluded values are 2 and 3.

iii. $\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+4)(x+2)}{(x+2)(x-1)} = \frac{x+4}{x-1}$

The expression $\frac{x+4}{x-1}$ is undefined when

$x - 1 = 0$. Hence the excluded value is 1.

iv. $\frac{x^3-27}{x^3+x^2-6x} = \frac{(x-3)(x^2+3x+9)}{x(x^2+x-6)}$

$= \frac{(x-3)(x^2+3x+9)}{(x)(x+3)(x-2)}$

The expression $\frac{x^3-27}{x^3+x^2-6x}$ is undefined

when $x^3 + x^2 - 6x = 0$

$\Rightarrow (x)(x+3)(x-2) = 0$

$\Rightarrow x = 0$ or $x = -3$ or $x = 2$

Hence the excluded values are 0, –3, 2

6. Find the square root of the following rational expression.

$\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

Solution:

$\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4} = \sqrt{\frac{4y^8z^{12}}{x^4}} = 2\left|\frac{y^4z^6}{x^2}\right|$

7. Find the square root of the following expressions

i) $256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

ii) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

Solution:

i. $\sqrt{\left(256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}\right)}$

$= 16\,|(x-a)^4(x-b)^2(x-c)^8(x-d)10|$

ii. $\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

8. Find the square root of the following rational expression.

$$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$$

Solution:

$$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4} =$$

$$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}}$$

$$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$$

9. Determine the quadratic equations, whose sum and product of roots are

(i) –9, 20 (ii) $\frac{5}{3}$, 4 **SEP-21**

Solution:

i. –9, 20

$x^2 - [\alpha + \beta]x + \alpha\beta = 0$

$x^2 - [-9]x + 20 = 0 \Rightarrow x^2 + 9x + 20 = 0$

ii. $\frac{5}{3}$, 4

Required Quadratic Equations

x^2 – (Sum of the roots)x + product of the roots = 0

$x^2 - \frac{5}{3}x + 4 = 0$

Multiply 3 on both sides

$3x^2 - 5x + 12 = 0$

10. Find the sum and product of the roots for each of the following quadratic equations

(i) $x^2 + 3x - 28 = 0$ (ii) $x^2 + 3x = 0$

Solution:

i. $x^2 + 3x - 28 = 0$

a = 1, b = 3, c = –28

Sum of the roots = $\alpha+\beta = -\frac{b}{a} = -\frac{3}{1} = -3$

Product of the roots = $\alpha\beta = \frac{c}{a}$

$= -\frac{28}{1} = -28$

ii. $x^2 + 3x = 0$

a = 1, b = 3, c = 0

Sum of the roots = $\alpha+\beta = -\frac{b}{a} = -\frac{3}{1} = -3$

Product of the roots = $\alpha\beta = \frac{c}{a} = \frac{0}{1} = 0$

11. In the matrix A = $\begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$, write

(i) The number of elements

(ii) The order of the matrix

(iii) Write the elements a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}.

Solution:

i) Number of elements = 4 × 4 = 16

ii) Order of matrix = 4 × 4

iii) $a_{22} = \sqrt{7}$; $a_{23} = \frac{\sqrt{3}}{2}$; $a_{24} = 5$;

$a_{34} = 0$; $a_{43} = -11$; $a_{44} = 1$

12. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution:

Matrix having 18 elements 1 × 18 (or) 2 × 9 (or) 3 × 6 (or) 6 × 3 (or) 9 × 2 (or) 18 × 1

Matrix having 6 elements 1 × 6 (or) 2 × 3 (or) 3 × 2 (or) 6 × 1

13. Construct a 3×3 matrix whose elements are given by

(i) $a_{ij} = |i - 2j|$ (ii) $a_{ij} = \frac{(i+j)^3}{3}$

Solution:

i. $a_{ij} = |i - 2j|$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{bmatrix} |1-2| & |1-4| & |1-6| \\ |2-2| & |2-4| & |2-6| \\ |3-2| & |3-4| & |3-6| \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$$

ii. $a_{ij} = \frac{(i+j)^3}{3}$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$$

14. Construct a 3×3 matrix whose elements are $a_{ij} = i^2j^2$

Solution:

The general 3 × 3 matrix is given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = 1^2 \times 1^2 = 1\times1 = 1$; $a_{12} = 1^2 \times 2^2 = 1\times4 = 4$;

$a_{13} = 1^2 \times 3^2 = 1\times9 = 9$; $a_{21} = 2^2 \times 1^2 = 4\times1 = 4$;

$a_{22} = 2^2 \times 2^2 = 4\times4 = 16$; $a_{23} = 2^2 \times 3^2 = 4\times9 = 36$

$a_{31} = 3^2 \times 1^2 = 9\times1 = 9$; $a_{32} = 3^2 \times 2^2 = 9\times4 = 36$;

$a_{33} = 3^2 \times 3^2 = 9\times9 = 81$

Hence the required matrix is $A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$

15. If $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

Solution:

$$A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix} \quad A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

16. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of –A. **SEP-20**

Solution:

$$A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix} \quad -A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

17. If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$

Solution:

$$A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$\therefore (A^T)^T = A$

18. Find the values of *x*, y and z from the following equations

(i) $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution:

i. $\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$\Rightarrow$ 12 = y; 3 = z; $x = 3$

ii. $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

$\Rightarrow$ $5 + z = 5$ $x + y = 6$;

$z = 5 - 5$ $y = 6 - x$;

$z = 0$

$xy = 8$

$x(6-x) = 8$

$6x - x^2 - 8 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x - 2 = 0$ (or) $x - 4 = 0$

$x = 2$ (or) $x = 4$

If $x = 2$ then $y = \frac{8}{x} = \frac{8}{2} = 4$;

If $x = 4$ then $y = \frac{8}{4} = 2$

iii. $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

$x+y+z=9$ ..... (1)

$x+z=5$ ..... (2)

$y+z=7$ ..... (3)

Substitute (3) in (1)

$x+7=9 \Rightarrow x=9-7=2$

Substitute x =2 in (2)

$2+z=5 \Rightarrow z=5-2=3$

Substitute z = 3 in (3)

$y+3=7 \Rightarrow y=7-3 \Rightarrow y=4$

19. If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$

then Find 2A+B.

Solution:

$2A+B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$

$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$

$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix}$

$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$

20. If $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$,

find 4A–3B.

Solution:

$4A-3B = 4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix} - 3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$

$= \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} + \begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}$

$= \begin{pmatrix} 20+21 & 16-12 & -8+9 \\ 2-\frac{3}{4} & 3-\frac{21}{2} & 4\sqrt{2}-9 \\ 4-15 & 36+18 & 16-27 \end{pmatrix}$

$= \begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$

21. If $A = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$ **then verify that (i) A+B = B +A**

(ii) A+(–A) = (–A)+A =O.

Solution:

i. $A+B=B+A$

L.H.S.

$A+B = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$ ..... (1)

R.H.S.

$B+A = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$

$= \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$ ..... (2)

(1), (2) $\Rightarrow A+B=B+A$

ii. $A+(-A)=(-A)+A=0$

$A+(-A) = \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ ..... (1)

$(-A)+A = \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ ..... (2)

(1), (2) $\Rightarrow A+(-A)=(-A)+A=0$

22. If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ find the value of (i) B – 5A (ii) 3A – 9B

Solution:

$A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$

i. B – 5A

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - 5\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} + \begin{pmatrix} 0 & -20 & -45 \\ -40 & -15 & -35 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$$

ii. 3A – 9B

$$= 3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} - 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} + \begin{pmatrix} -63 & -27 & -72 \\ -9 & -36 & -81 \end{pmatrix}$$

$$= \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

5 Marks

STAGE 1

1. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

Solution:

		8	–1	1		
8	64	–16	17	–2	1	
	(–) 64					
16 –1		–16	17			
	(+)	–16	(–) 1			
16 –2 1			16	–2	1	
			(–) 16	(+) –2	(–) 1	
				0		

Required Square root = $|8x^2 - x + 1|$

2. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution:

	3	2	4		
3	9	12	28	a	b
	(–) 9				
6 2		12	28		
	(–)	12	(–) 4		
6 4 4			24	a	b
			(–) 24	(–) 16	(–) 16

a = 16, b = 16

3. Find the square root of the following polynomials by division method

Solution:

i. $x^4 - 12x^3 + 42x^2 - 36x + 9$

	1	–6	3		
1	1	–12	42	–36	9
	(–) 1				
2 –6		–12	42		
	(+)	–12	(–) 36		
2 –12 3			6	–36	9
			(–) 6	(+) –36	(–) 9
				0	

Required Square root = $|x^2 - 6x + 3|$

ii. $37x^2 - 28x^3 + 4x^4 + 42x + 9$

	2	–7	–3		
2	4	–28	37	42	9
	(–) 4				
4 –7		–28	37		
	(+)	–28	(–) 49		
4 –14 –3			–12	42	9
			(+) –12	(–) 42	(–) 9
				0	

Required Square root = $|2x^2 - 7x - 3|$

iii. $16x^4 + 8x^2 + 1$

	4	0	1		
4	16	0	8	0	1
	(–) 16				
8 0		0	8		
		0	0		
8 0 1			8	0	1
			(–) 8	(–) 0	(–) 1
			0		

Required Square root = $|4x^2 + 1|$

iv. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

	11 –9 –12
11	121 –198 –183 216 144
	(–)121
22 –9	– 198 –183
	(+) –198 (–)81
22 –18 –12	– 264 216 144
	(+) –264 (–)216 (–)144
	0

Required Square root $= |11x^2 - 9x - 12|$

4. Find the values of a and b if the following polynomials are perfect squares

i. $4x^4 - 12x^3 + 37x^2 + bx + a$

Solution:

	2 –3 7
2	4 –12 37 b a
	(–) 4
4 –3	–12 37
	(+) –12 (–) 9
4 –6 7	28 b a
	(–) 28 (+)– 42 (–) 49
	a = 49 , b = –42

ii. $ax^4 + bx^3 + 361x^2 + 220x + 100$

Solution:

	10 11 12
10	100 220 361 b a
	(–)100
20 11	220 361
	(–) 220 (–)121
20 22 12	240 b a
	(–) 240 (–)264 (–)144
	a = 144 , b = 264

5. Find the values of m and n if the following polynomials are perfect sqaures

i. $36x^4 - 60x^3 + 61x^2 - mx + n$ MAY-22

Solution:

	6 –5 3
6	36 –60 61 –m n
	(–) 36
12 –5	–60 61
	(+) –60 (–)25
12 –10 3	36 –m n
	(–) 36 (+)– 30 (–) 9
	–m = –30, m = 30
	n = 9

ii. $x^4 - 8x^3 + mx^2 + nx + 16$

Solution:

	1 –4 4
1	1 –8 m n 16
	(–) 1
2 –4	–8 m
	(+) –8 (–)16
2 –8 4	m–16 n 16
	(–) 8 (+)– 32 (–) 16
	0

$\frac{m-16}{2} = 4$

m – 16 = 8, n = –32

m = 8 + 16

m = 24

6. If $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$ then verify that A + (B + C) = (A + B) + C.

Solution:

$$A + (B + C) = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \left(\begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 5 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad \text{..... (1)}$$

$(A + B) + C$

$$= \left(\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}\right) + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix} \quad \text{..... (2)}$$

From (1) & (2) LHS = RHS

7. If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ verify that A(B + C) = AB + AC.

Solution:

$$B + C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-7 & 2+6 \\ -4+3 & 2+2 \end{pmatrix} = \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

LHS = A (B + C)

$$= \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

RHS = AB + AC

$$= \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-4 & 4+8 \\ -13+16 & 4+0 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$\therefore$ LHS = RHS

8. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^T A^T$ SEP-20

Solution:

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$$AB^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$\therefore$ LHS = RHS

9. Given that $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$ verify that A(B +C) = AB + AC.

Solution:

$$A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

To verify that A (B + C) = AB + AC

LHS

$$B + C = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$A(B + C) = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 2 & 2 & 4 \\ -1 & 6 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 2-3 & 2+18 & 4+15 \\ 10+1 & 10-6 & 20-5 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad .....(1)$$

$$AB = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+9 & -1+15 & 2+6 \\ 5-3 & -5-5 & 10-2 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 14 & 8 \\ 2 & -10 & 8 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -1 & 20 & 19 \\ 11 & 4 & 15 \end{pmatrix} \quad .....(2)$$

(1), (2) $\Rightarrow$ A (B + C) = AB + AC.

10. If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$

verify that $(AB)^T = B^TA^T$

Solution:

$$A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}, B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \; A^T = \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

(1), (2) $\Rightarrow (AB)^T = B^TA^T$

11. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ **show that** $A^2 - 5A + 7I_2 = 0$

Solution:

$$A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

$A^2 - 5A + 7I_2$

$$= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} -15 & -5 \\ 5 & -10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Hence, $A^2 - 5A + 7I_2 = 0$

FORMULAE AND DEFINITIONS

1.	**Simultaneous linear equation in three variables**	General form of a linear equation in three variable is $ax + by + cz + d = 0$
		A linear equation in three variables represents a plane.
		A system of equation can have unique solution (or) infinitely many solution (or) no solution
		The system of equation has **no solution** if any step comes as **0 = 1** while solving.
		The system of equation has infinitely **many solution** if any step comes as **0 = 0** while solving.
2.	Excluded Value	A value that makes a rational expression undefined is called an Excluded value.

3.	Relationship between LCM and GCD		$f(x) \times g(x) = LCM \times GCD$
4.	General Form of Quadratic Equation		$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$
			$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
5.	Formula for finding roots (solution) of a Quadratic Equation		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
6.	If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$, then	Sum of roots	$\alpha+\beta = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$
		Product of roots	$\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

NATURE OF ROOTS OF A QUADRATIC EQUATION		
Values of Discriminant $\Delta = b^2 - 4ac$		**Nature of roots**
7.	$\Delta > 0$	Real and unequal roots
8.	$\Delta = 0$	Real and equal roots
9.	$\Delta < 0$	No real roots

SOME RESULTS INVOLVING α and β		**ALGEBRAIC IDENTITES**	
10.	$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$	18.	$(a+b)^2 = a^2 + b^2 + 2ab$
11.	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	19.	$(a-b)^2 = a^2 + b^2 - 2ab$
12.	$\alpha^2 - \beta^2 = (\alpha+\beta)(\sqrt{(\alpha+\beta)^2 - 4\alpha\beta})$	20.	$(a+b)(a-b) = a^2 - b^2$
13.	$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$	21.	$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
14.	$\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$	22.	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
15.	$\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$	23.	$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
16.	$\alpha^4 - \beta^4 = (\alpha+\beta)(\alpha-\beta)(\alpha^2+\beta^2)$	24.	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
17.	$(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$	25.	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

26.	Variation	Direct Variation	x increases / decreases y also increases / decreases Equation $y = kx$
		Indirect Variation	x increases, y decreases (OR) x decreases, y increases Equation $xy = k$
27.	If A, B, C, are in three matrices		(i) A(B + C) = AB + AC (ii) A(BC) = (AB)C (iii) (AB)T = BT.AT

***

4 Geometry

Exercise 4.5

Multiple choice Questions

1. **If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when**

   A) $\angle B = \angle E$ B) $\angle A = \angle D$
   C) $\angle B = \angle D$ D) $\angle A = \angle F$

2. **In ΔLMN, $\angle L = 60°$, $\angle M = 50°$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is** SEP-20

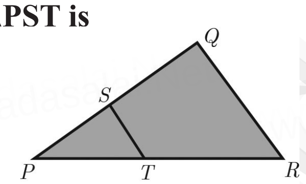
   A) 40° B) 70° C) 30° D) 110°

3. **If ΔABC is an isosceles triangle with $\angle C = 90°$ and AC = 5 cm, then AB is** MAY-22

   A) 2.5 cm B) 5 cm
   C) 10 cm D) $5\sqrt{2}$ cm

4. **In a given figure ST || QR, PS = 2 cm and SQ = 3 cm. Then the ratio of the area of ΔPQR to the area of ΔPST is**

   A) 25 : 4
   B) 25 : 7
   C) 25 : 11
   D) 25 : 13



5. **The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ = 10 cm, then the length of AB is**

   A) $6\frac{2}{3}$ cm B) $\frac{10\sqrt{6}}{3}$ cm
   C) $66\frac{2}{3}$ cm D) 15 cm

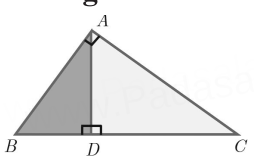
6. **If in ΔABC, DE || BC. AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is** SEP-21

   A) 1.4 cm B) 1.8 cm
   C) 1.2 cm D) 1.05 cm

7. **In a ΔABC, AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is** MAY-22

   A) 6 cm B) 4 cm C) 3 cm D) 8 cm

8. **In the adjacent figure $\angle BAC = 90°$ and $AD \perp BC$ then**

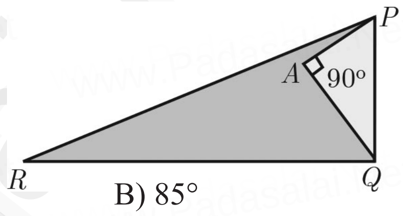


   A) $BD \cdot CD = BC^2$ B) $AB \cdot AC = BC^2$
   C) $BD \cdot CD = AD^2$ D) $AB \cdot AC = AD^2$

9. **Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?**

   A) 13 m B) 14 m C) 15 m D) 12.8 m

10. **In the given figure, PR = 26 cm, QR = 24 cm, $\angle PAQ = 90°$, PA = 6 cm and QA = 8 cm. Find $\angle PQR$**



   A) 80° B) 85°
   C) 75° D) 90°

11. **A tangent is perpendicular to the radius at the**

    A) centre B) point of contact
    C) infinity D) chord

12. **How many tangents can be drawn to the circle from an exterior point?** SEP-21

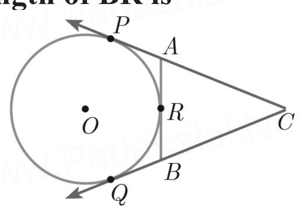
    A) one B) two
    C) infinite D) zero

13. **The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70°$ then the value of $\angle AOB$ is**

    A) 100° B) 110° C) 120° D) 130°

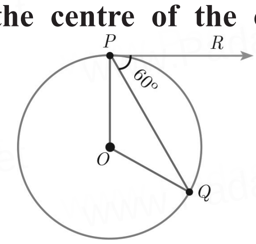
14. **In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then the length of BR is**

    A) 6 cm
    B) 5 cm
    C) 8 cm
    D) 4 cm



15. **In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is** SEP-20

    A) 120°
    B) 100°
    C) 110°
    D) 90°



2 Marks STAGE 1

1. If ΔABC is similar to ΔDEF such that BC = 3 cm, EF= 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Solution:

Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

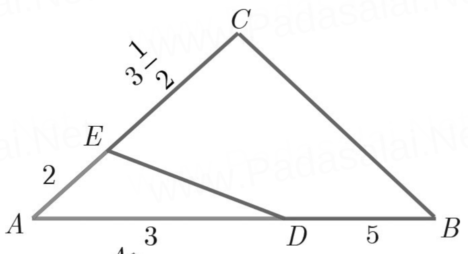
$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

gives $$\frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$$

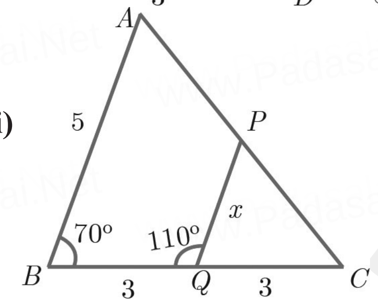
$$\text{Area } (\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

2. Check whether the which triangles are similar and find the value of *x*.

i)



ii)



Solution:

i. From the figure, in ΔABC and ADE

$$\frac{AC}{AE} = \frac{3\frac{1}{2}+2}{2} = \frac{\frac{7}{2}+2}{2} = \frac{\frac{7+4}{2}}{2}$$

$$= \frac{11}{2} \times \frac{1}{2} = \frac{11}{4} \quad \text{..... (1)}$$

$$\frac{AB}{AD} = \frac{3+5}{3} = \frac{8}{3} \quad \text{..... (2)}$$

From (1), (2) $\Rightarrow \frac{AC}{AE} \neq \frac{AB}{AD}$

∴ ΔABC and ΔADE are not similar

ii. From the figure, in ΔABC and ΔPQC

∠ABC= ∠PQC = 70° ..... (1)

(Corresponding angles are equal)

∠C = ∠C (Common Angles) ..... (2)

∴ ∠A = ∠QPC (∵ AAA criterian)

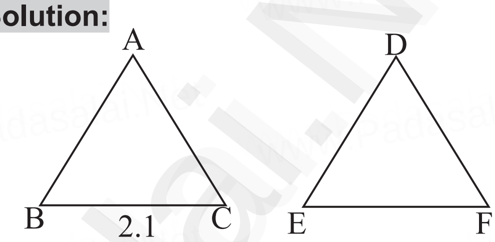
Hence, ΔABC and ΔPQC are similar triangles

Then, $\frac{AB}{PQ} = \frac{BC}{QC} \Rightarrow \frac{5}{x} = \frac{6}{3} = 2$

$\therefore x = \frac{5}{2}$ =2.5

3. If ΔABC ~ ΔDEF such that area of ΔABC is 9 cm² and the area of ΔDEF is 16 cm² and BC = 2.1 cm. Find the length of EF.

Solution:



Given ΔABC ~ ΔDEF

$$\frac{Area\,of\,(\Delta ABC)}{Area\,of\,(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$= \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

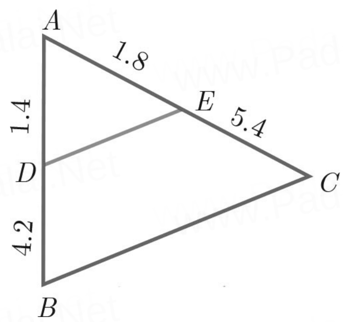
$$\Rightarrow \quad \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow \quad EF^2 = (2.1)^2 \times \frac{16}{9}$$

$$\Rightarrow \quad EF = 2.1 \times \frac{4}{3} = 2.8 \text{ cm}$$

4. D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC .

Solution:



AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

BD = AB – AD = 5.6 – 1.4 = 4.2 cm and

EC = AC – AE = 7.2 – 1.8 = 5.4 cm

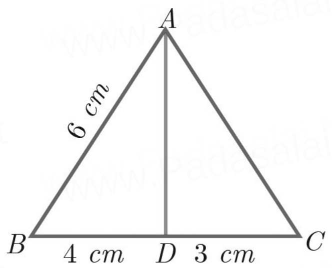
$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, by converse of Basic Proportionality Theorem, we have DE is parallel to BC.

Hence Proved.

5. In the Figure, AD is the bisector of ∠A. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC. **MAY-22**



Solution:

In ΔABC, AD is the bisector of ∠A.

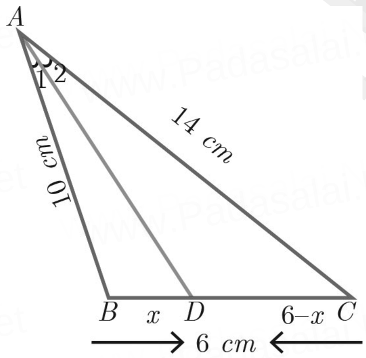
Therefore by Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{4}{3} = \frac{6}{AC} \text{ gives } 4AC = 18$$

$$\text{Hence } AC = \frac{9}{2} = 4.5 \text{ cm}$$

6. In the Figure, AD is the bisector of ∠BAC, if AB = 10 cm, AC = 14 cm and BC = 6 cm. Find BD and DC.



Solution:

AD is the bisector of ∠BAC

AB =10 cm, AC = 14 cm, BC = 6 cm

By Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{x}{6-x} = \frac{10}{14}$$

$$\frac{x}{6-x} = \frac{5}{7}$$

$$7x = 30 - 5x$$

$$12x = 30$$

$$x = \frac{30}{12} = 2.5 \text{ cm}$$

∴ BD = 2.5 cm DC = 3.5 cm

7. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC **SEP-21**

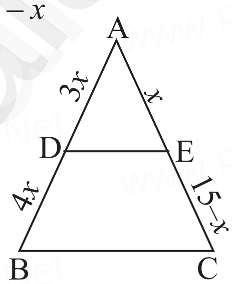
(i) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm find AE.

(ii) If AD = 8x − 7, DB = 5x − 3, AE = 4x − 3 and EC = 3x − 1, find the value of x.

Solution:

i. If $\frac{AD}{DB} = \frac{3}{4}$, AC = 15 cm , AE = x,

EC = 15 – x



DE || BC then by basic proportionality theorem.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{3}{4} = \frac{x}{15-x}$$

$$3(15-x) = 4x$$

$$45 - 3x = 4x$$

$$45 = 7x$$

$$x = \frac{45}{7} = 6.43 \text{ cm}$$

ii. Given AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1

By basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

$$\Rightarrow (8x-7)(3x-1) = (5x-3)(4x-3)$$

$$\Rightarrow 24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

$$24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$

$$\Rightarrow 4x^2 - 2x - 2 = 0$$

$\Rightarrow \quad 2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = 1, x = -\frac{1}{2}$ (Not Admissible).

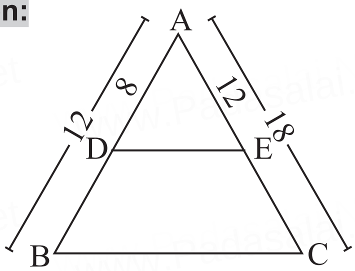
$\therefore x = 1$

8. In ΔABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC.

(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm.

Solution:



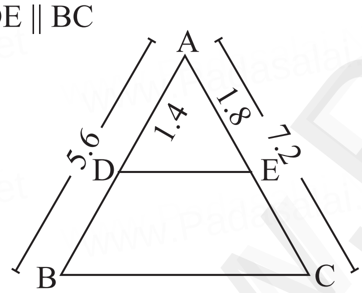
i. AB = 12 cm, AD = 8 cm AE = 12 cm and AC = 18cm

$\frac{AD}{AB} = \frac{8}{12} = \frac{2}{3}$ ..... (1)

$\frac{AE}{AC} = \frac{12}{18} = \frac{2}{3}$ ..... (2)

From (1) & (2) $\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\therefore$ DE || BC



ii. AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

$\frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4}$ ..... (1)

$\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4}$ ..... (2)

(1), (2) $\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

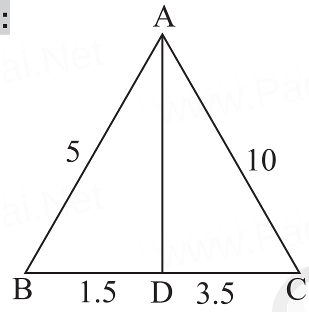
$\therefore$ DE || BC

9. Check whether AD is bisector of ∠A of ΔABC in each of the following

(i) AB = 5 cm, AC = 10 cm, BD= 1.5 cm and CD= 3.5 cm. **SEP-20**

(ii) AB= 4 cm, AC = 6 cm, BD = 1.6 cm and CD= 2.4 cm.

Solution:



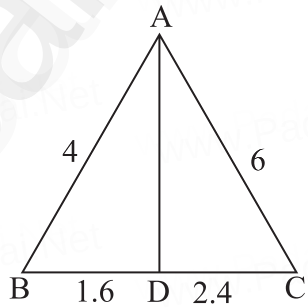
i. AB = 5cm , AC =10 cm, BD =1.5 cm and CD = 3.5 cm

$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$ ..... (1)

$\frac{BD}{CD} = \frac{1.5}{3.5} = \frac{3}{7}$ ..... (2)

(1), (2) $\Rightarrow \frac{AB}{AC} \neq \frac{BD}{CD}$ ($\because$ By ABT)

AD is not a bisector of ∠A in ΔABC



ii. AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$ ..... (1)

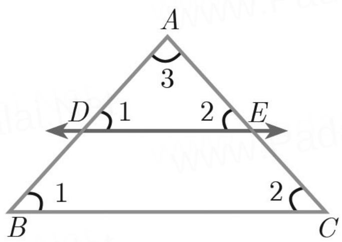
$\frac{BD}{CD} = \frac{1.6}{2.4} = \frac{2}{3}$ ..... (2)

(1), (2) $\Rightarrow \frac{AB}{AC} = \frac{BD}{CD}$ ($\because$ By ABT)

AD is a bisector of ∠A in ΔABC

5 Marks STAGE 1

1. State and Prove Basic Proportionality Theorem (BPT) or Thales Theorem. **MAY-22**



Statement:

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

Given:

In ΔABC, D is a point on AB and E is a point on AC

To Prove:

$\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line DE || BC

No.	Statement	Reason
1.	$\angle ABC = \angle ADE \longrightarrow 1$	Corresponding angles are equal because DE || BC
2.	$\angle ACB = \angle AED \longrightarrow 2$	Corresponding angles are equal because DE || BC
3.	$\angle DAE = \angle BAC \longrightarrow 3$	Both triangles have a common angle.
	$\Delta ABC \sim \Delta ADE$	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E
	$1+\frac{DB}{AD} = 1+\frac{EC}{AE}$	On Simplication
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
	Hence Proved	

2. State and Prove Angle Bisector Theorem.

Statement: SEP-20

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

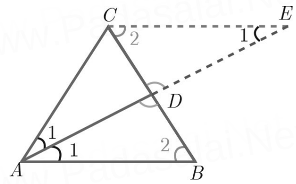
Proof

Given:

In ΔABC, AD is the internal bisector

To Prove:

$\frac{AB}{AC} = \frac{BD}{CD}$.



Construction:

Draw a line through C parallel to AB.

Extend AD to meet line through C at E.

No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles AC = CE …. (1)	In ΔACE $\angle CAE = \angle CEA$.
3.	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE. Hence Proved.

3. State and Prove Pythagoras Theorem.

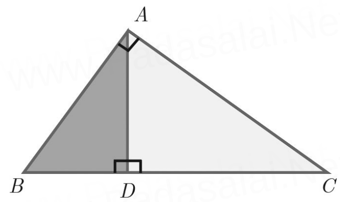
Statement:

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof

Given:

In ΔABC, $\angle A = 90°$



To Prove:

$AB^2 + AC^2 = BC^2$

Construction: Draw AD ⊥ BC

No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ..... (1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity

2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity

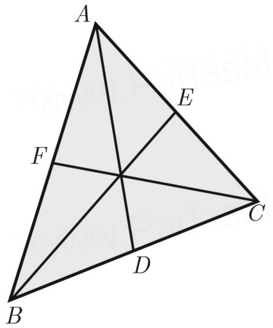
Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$
$$= BC\,(BD + DC)$$
$$AB^2 + AC^2 = BC \times BC = BC^2$$

Hence the theorem is proved.

4. Show that in a triangle, the medians are concurrent. SEP-21

Solution:



Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

Since D is midpoint of BC,

$BD = DC$. So $\frac{BD}{DC} = 1$ ..... (1)

Since E is midpoint of CA,

$CE = EA$. So $\frac{CE}{EA} = 1$ ..... (2)

Since F is midpoint of AB,

$AF = FB$. So $\frac{AF}{FB} = 1$ ...... (3)

Thus, multiplying (1), (2), (3) we get

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB}$$
$$= 1 \times 1 \times 1 = 1$$

And so, Ceva's theorem is satisfied.

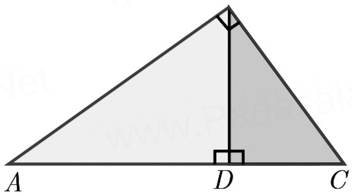
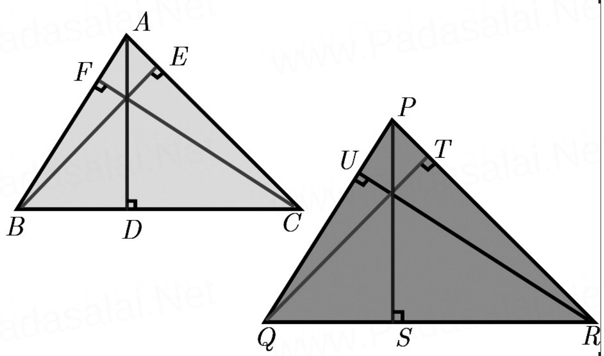
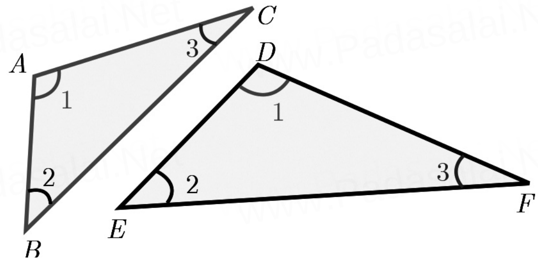
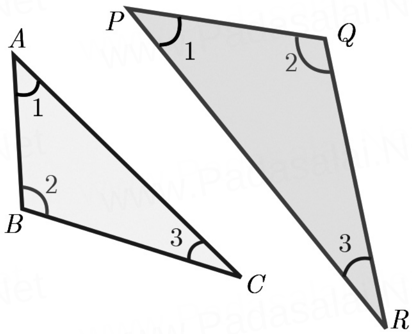
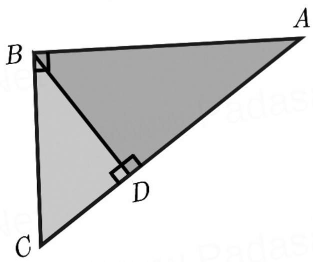
Hence the Medians are concurrent.

***

FORMULAE AND DEFINITIONS

Congruent triangles	Similar triangles
$\Delta ABC \cong \Delta PQR$	$\Delta ABC \sim \Delta PQR$
$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R.$	$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R.$
$AB = PQ, BC = QR, CA = RP$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$	$AB \neq PQ, BC \neq QR, CA \neq RP$ but $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} > 1$ or < 1
Same shape and same size.	Same shape but not same size.

Some useful results on similar triangles:

1.	A perpendicular line drawn from the vertex of a right angled triangle divides the triangle into two triangles similar to each other and also to original triangle. ΔADB ~ ΔBDC, ΔABC ~ ΔADB, ΔABC ~ ΔBDC	
2.	If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of their corresponding altitudes. i.e. if ΔABC ~ ΔPQR then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AD}{PS} = \frac{BE}{QT} = \frac{CF}{RU}$	
3.	If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of the corresponding perimeters. ΔABC ~ ΔDEF then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{AB + BC + CA}{DE + EF + FD}$	
4.	The ratio of the area of two similar triangles are equal to the ratio of the squares of their corresponding sides. $\frac{\text{area } (\Delta ABC)}{\text{area } (\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$	
5.	If two triangles have common vertex and their bases are on the same straight line, the ratio between their areas is equal to the ratio between the length of their bases. Here, $\frac{\text{area } (\Delta ABD)}{\text{area } (\Delta BDC)} = \frac{AD}{DC}$	

- A tangent to a circle will be perpendicular to the radius at the point of contact.
- Two tangents can be drawn from any exterior point of a circle.
- The lengths of the two tangents drawn from an exterior point to a circle are equal.
- Two direct common tangents drawn to two circles are equal in length.

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5 Coordinate Geometry

Exercise 5.5

Multiple choice Questions

1. **The area of triangle formed by the points (–5, 0) , (0, –5) and (5, 0) is** SEP-21
   A) 0 sq.units B) 25 sq.units
   C) 5 sq.units D) none of these

2. **A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is**
   A) $x = 10$ B) $y = 10$
   C) $x = 0$ D) $y = 0$

3. **The straight line given by the equation $x = 11$ is** SEP-20
   A) parallel to X axis
   B) parallel to Y axis
   C) passing through the origin
   D) passing through the point (0, 11)

4. **If (5,7), (3,p) and (6,6) are collinear, then the value of p is** MAY-22
   A) 3 B) 6 C) 9 D) 12

5. **The point of intersection of $3x - y = 4$ and $x + y = 8$ is**
   A) (5, 3) B) (2, 4)
   C) (3, 5) D) (4, 4)

6. **The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is**
   A) 1 B) 4 C) –5 D) 2

7. **The slope of the line which is perpendicular to a line joining the points (0, 0) and (–8, 8) is** MAY-22
   A) –1 B) 1
   C) $\frac{1}{3}$ D) –8

8. **If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is**
   A) $\sqrt{3}$ B) $-\sqrt{3}$
   C) $\frac{1}{\sqrt{3}}$ D) 0

9. **If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissae is 5 then the equation of the line AB is**
   A) $8x + 5y = 40$ B) $8x - 5y = 40$
   C) $x = 8$ D) $y = 5$

10. **The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is**
    A) $7x - 3y + 4 = 0$ B) $3x - 7y + 4 = 0$
    C) $3x + 7y = 0$ D) $7x - 3y = 0$

11. **Consider four straight lines**
    (i) $l_1 : 3y = 4x + 5$; (ii) $l_2 : 4y = 3x - 1$
    (iii) $l_3 : 4y + 3x = 7$ (iv) $l_4 : 4x + 3y = 2$
    Which of the following statement is true?
    A) l_1 and l_2 are perpendicular
    B) l_1 and l_4 are parallel
    C) l_2 and l_4 are perpendicular
    D) l_2 and l_3 are parallel

12. **A straight line has equation $8y = 4x + 21$. Which of the following is true?**
    A) The slope is 0.5 and the y intercept is 2.6
    B) The slope is 5 and the y intercept is 1.6
    C) The slope is 0.5 and the y intercept is 1.6
    D) The slope is 5 and the y intercept is 2.6

13. **When proving that a quadrilateral is a trapezium, it is necessary to show**
    A) Two sides are parallel.
    B) Two parallel and two non-parallel sides.
    C) Opposite sides are parallel.
    D) All sides are of equal length.

14. **When proving that a quadrilateral is a parallelogram by using slopes you must find**
    A) The slopes of two sides
    B) The slopes of two pair of opposite sides
    C) The lengths of all sides
    D) Both the lengths and slopes of two sides

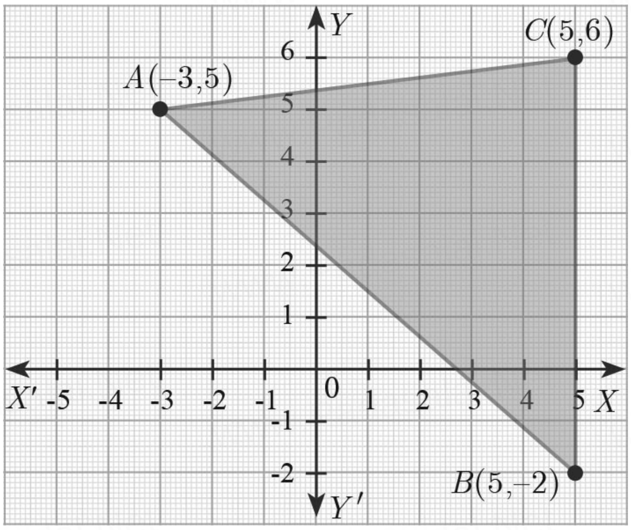
15. **(2, 1) is the point of intersection of two lines.**
    A) $x - y - 3 = 0$; $3x - y - 7 = 0$
    B) $x + y = 3$; $3x + y = 7$
    C) $3x + y = 3$; $x + y = 7$
    D) $x + 3y - 3 = 0$; $x - y - 7 = 0$

2 Marks

STAGE 1

1. Find the area of the triangle whose vertices are (–3, 5), (5, 6) and (5, –2)

Solution:



A(–3, 5), B(5, –2), C(5,6)

↓ ↓ ↓

x_1y_1 x_2y_2 x_3y_3

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -3 & 5 \\ 5 & -2 \\ 5 & 6 \\ -3 & 5 \end{vmatrix}$$

$$= \frac{1}{2}[(6+30+25) - (25-10-18)]$$

$$= \frac{1}{2}[61+3]$$

$$= \left|\frac{64}{2}\right| = 32 \text{ sq. units.}$$

2. Show that the points P (–1.5, 3), Q (6, –2), R (–3, 4) are collinear. **MAY-22**

Solution:

Area of ΔPQR = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} -1.5 & 3 \\ 6 & -2 \\ -3 & 4 \\ -1.5 & 3 \end{vmatrix} = 0$$

$$\frac{1}{2}[(3+24-9) - (18+6-6)] = 0$$

$$\frac{1}{2}[18 - 18] = 0$$

∴ Therefore, the given points are collinear.

3. If the area of the triangle formed by the vertices A (–1, 2), B (*k*, –2) and C (7, 4) (taken in order) is 22 sq. units, find the value of *k*.

Solution:

The vertices are A (–1, 2), B (k, –2) and C (7, 4)

Area of ΔABC is 22 sq.units

$$\frac{1}{2}\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 22$$

$$\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 44$$

$$\{(2 + 4k + 14) - (2k - 14 - 4)\} = 44$$

$$4k + 16 - 2k + 18 = 44$$

$$2k + 34 = 44$$

$$2k = 10$$

$$\text{Therefore } k = 5$$

4. Find the area of the triangle formed by the points (i) (1, –1), (–4, 6) and (–3, –5) (ii) (ii) (–10, –4), (–8, –1) and (–3, –5)

Solution:

i. $$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$$= \frac{1}{2}[(6+20+3) - (4-18-5)]$$

$$= \frac{1}{2}[6+20+3-4+18+5]$$

$$= \frac{1}{2}[(6+20+3+18+5)-4]$$

$$= \frac{1}{2}[52-4]$$

$$= \frac{1}{2}[48] = 24 \text{ sq.units.}$$

ii. $$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -10 & -4 \\ -8 & -1 \\ -3 & -5 \\ -10 & -4 \end{vmatrix}$$

$$= \frac{1}{2}[(10+40+12)-(32+3+50)]$$

$$= \frac{1}{2}[62 - 85]$$

$$= \frac{1}{2}[-23] = -11.5 \text{ sq.units.}$$

∴ Area of the Triangle = 11.5 sq.units

5. Determine whether the sets of points are collinear?

(i) $\left(-\frac{1}{2}, 3\right)$ **(–5, 6) and (–8, 8)**

Solution:

$\left(-\frac{1}{2}, 3\right)$ (–5, 6) and (–8, 8)

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -\frac{1}{2} & 3 \\ -5 & 6 \\ -8 & 8 \\ -\frac{1}{2} & 3 \end{vmatrix}$$

$$= \frac{1}{2}[(-3-40-24)-(-15-48-4)]$$

$$= \frac{1}{2}[(-67)-(-67)] = 0$$

∴ The given points are collinear.

(ii) (a, b+c), (b, c+a) and (c, a+b)

Solution:

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b+c \\ b & c+a \\ c & a+b \\ a & b+c \end{vmatrix}$$

$$= \frac{1}{2}[(ac+a^2+ab+b^2+bc+c^2) - (b^2+bc+c^2+ca+a^2+ab]$$

$$= \frac{1}{2}[ac+a^2+ab+b^2+bc+c^2-b^2-bc-c^2-ca-a^2-ab]$$

$$= \frac{1}{2}[0] = 0 \text{ sq.units.}$$

Aliter:

(a, b+c), (b, c+a), (c, a+b)

x_1, y_1 x_2, y_2 x_3, y_3

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ b+c-c-a & b+c-a-b \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ -(a-b) & -(a-c) \end{vmatrix}$$

$$= \frac{1}{2}[(a-b)(a-c)+(a-b)(a-c)]$$

$$= \frac{1}{2}[0] = 0$$

∴ The given points are collinear.

6. Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p'.

S.No.	Vertices	Area (sq.units)
(i)	(0, 0), (p, 8), (6, 2)	20
(ii)	(p, p), (5, 6), (5, –2)	32

Solution:

i. A (0, 0), B (p, 8), C (6, 2)

Area of ΔABC = 20 sq.units.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \text{Area of } \Delta ABC$$

$$\frac{1}{2}\begin{vmatrix} 0 & 0 \\ p & 8 \\ 6 & 2 \\ 0 & 0 \end{vmatrix} = 20$$

$(0+2p+0)-(0+48+0) = 40$

$2p - 48 = 40$

$2p = 88$

$p = 44$

ii. A (p, p), B (5, 6), C (5, –2)

Area of Δ = 32 sq.units

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 32$$

$$\frac{1}{2}\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 32$$

$$\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 64$$

$(6p-10+5p)-(5p+30-2p) = 64$

$6p - 10 + 5p - 5p - 30 + 2p = 64$

$8p - 40 = 64$

$\Rightarrow 8p = 64 + 40$

$8p = 104$

$\Rightarrow p = \frac{104}{8}$

$\Rightarrow p = 13$

7. In each of the following, find the value of 'a' for which the given points are collinear.

(i) (2, 3), (4, a) and (6, –3)

(ii) (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a)

Solution:

i. (2, 3), (4, a) and (6, –3)

$\Delta = 0$

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} 2 & 3 \\ 4 & a \\ 6 & -3 \\ 2 & 3 \end{vmatrix} = 0$$

$[(2a-12+18) - (12+6a-6)] = 0$

$2a - 12 + 18 - 12 - 6a + 6 = 0$

$-4a = 0$

$\therefore a = 0$

ii. (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a)

$\Delta = 0$ sq.units.

$(2a^2-6a+2a^2+6-2a-8+8a-2a+2a^2) -$

$(-2a+2a^2+2-2a-8a-2a^2+6a-2a^2) = 0$

$\Rightarrow (6a^2-2a-2) - (-2a^2-6a+2) = 0$

$\Rightarrow 8a^2 + 4a - 4 = 0 \div 4$

$2a^2 + a - 1 = 0$

$(a+1)(2a-1) = 0$

$\Rightarrow \therefore a = +\frac{1}{2}$ and $a = -1$

Aliter:

(a, a–2a), (–a+1, 2a), (–4–a, 6–2a)

x_1, y_1 x_2, y_2 x_3, y_3

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a+a-1 & a+4+a \\ 2-2a-2a & 2-2a-6+2a \end{vmatrix} = 0$$

$$\begin{vmatrix} 2a-1 & 2a+4 \\ 2-4a & -4 \end{vmatrix} = 0$$

$-4(2a-1) - (2-4a)(2a+4) = 0$

$-8a+4 - [4a+8-8a^2-16a] = 0$

$-8a+4-4a-8+8a^2+16a = 0$

$8a^2+4a-4 = 0$

$2a^2+a-1 = 0$

$(a+1)(2a-1) = 0$

$a = -1$ (or) $a = \frac{1}{2}$

5 Marks

STAGE 1

1. The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (–3, 2), (–1, –1) and (1, 2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Solution:

Vertices of one triangular tile are at

(–3, 2), (–1, –1) (1, 2)

$$\text{Area of this tile} = \frac{1}{2}\begin{vmatrix} -3 & 2 \\ -1 & -1 \\ 1 & 2 \\ -3 & 2 \end{vmatrix}$$

$= \frac{1}{2}\{(3-2+2)-(-2-1-6)\}$

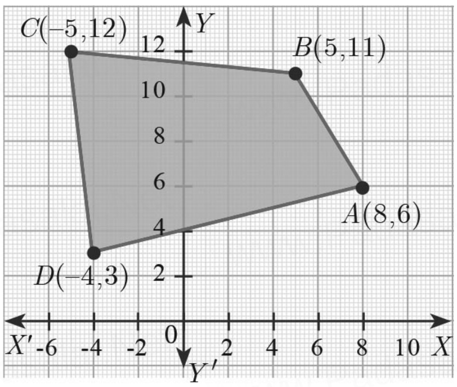
$= \frac{1}{2}(12) = 6$ sq.units

Since the floor is covered by 110 triangle shaped identical tiles,

Area of the floor = 110 × 6 = 660 sq. units

2. Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (–5, 12) and (–4, 3).

Solution:



Before determining the area of the quadrilateral, plot the vertices in a graph A (8, 6), B (5, 11), C (–5, 12) and D (–4, 3).

Therefore, area of the quadrilateral ABCD

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & 6 \\ 5 & 11 \\ -5 & 12 \\ -4 & 3 \\ 8 & 6 \end{vmatrix}$$

$= \frac{1}{2}[(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)]$

$= \frac{1}{2}[88 + 60 - 15 - 24 - 30 + 55 + 48 - 24]$

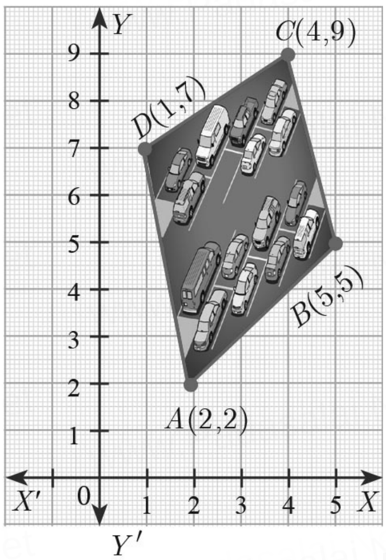
$= \frac{1}{2}[88 + 60 + 55 + 48 - 15 - 24 - 30 - 24]$

$= \frac{1}{2}[251 - 93]$

$= \frac{1}{2}[158] = 79$ sq.units.

3. The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

Solution:



The parking lot is a quadrilateral whose vertices A (2, 2), B (5, 5), C (4, 9) and D (1, 7).

Therefore, Area of parking lot is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 2 \\ 5 & 5 \\ 4 & 9 \\ 1 & 7 \\ 2 & 2 \end{vmatrix}$$

$= \frac{1}{2}[(10 + 45 + 28 + 2) - (10 + 20 + 9 + 14)]$

$= \frac{1}{2}[85 - 53]$

$= \frac{1}{2}[32] = 16$ sq.units

So, Area of parking lot = 16 sq.feets.

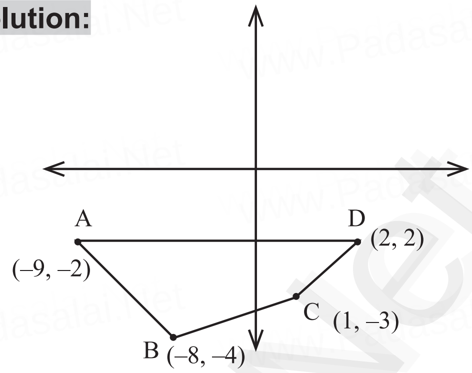
Construction rate per square fee = ₹ 1300

Therefore, total cost for constructing the parking lot = 16 × 1300 = ₹ 20,800

4. Find the area of the quadrilateral whose vertices are at

(i) (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution:



Let A (–9, –2), B (–8, –4), C (1, –3), D (2, 2)

$$\text{Area of the quadrilateral} = \frac{1}{2}\begin{vmatrix} -9 & -2 \\ -8 & -4 \\ 1 & -3 \\ 2 & 2 \\ -9 & -2 \end{vmatrix}$$

$= \frac{1}{2}[(36+24+2-4) - (16-4-6-18)]$

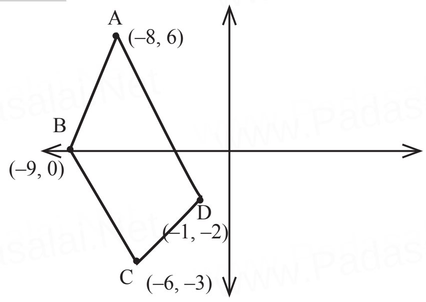
$= \frac{1}{2}[(36+24+2-4-16+4+6+18)]$

$= \frac{1}{2}[(36+24+2+4+6+18) - (4+16)]$

$= \frac{1}{2}[90-(20)] = \frac{1}{2}[70] = 35$ sq. units

(ii) (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution:



A (–8, 6), B (–9, 0), C (–6, –3), D (–1, –2)

$$\text{Area of the quadrilateral} = \frac{1}{2}\begin{vmatrix} -8 & 6 \\ -9 & 0 \\ -6 & -3 \\ -1 & -2 \\ -8 & 6 \end{vmatrix}$$

$= \frac{1}{2}[(0+27+12-6) - (-54+0+3+16)]$

$= \frac{1}{2}[27+12-6+54-3-16]$

$= \frac{1}{2}[(27+12+54) - (6+3+16)]$

$= \frac{1}{2}[93-25] = \frac{1}{2}[68] = 34$ sq. units

Aliter:

A(–8, 6), B(–9, 0), C(–6, –3), D(–1, –2)

x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -8-(-6) & -9-(-1) \\ 6-(-3) & 0-(-2) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -8+6 & -9+1 \\ 6+3 & 0+2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -8 \\ 9 & 2 \end{vmatrix}$$

$= \frac{1}{2}[-4+72] = \frac{1}{2}[68] = 34$ sq. units

5. Find the value of k, if the area of a quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3) SEP-20

Solution:

$$\frac{1}{2}\begin{vmatrix} -4 & -2 \\ -3 & k \\ 3 & -2 \\ 2 & 3 \\ -4 & -2 \end{vmatrix} = 28$$

$\Rightarrow (-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12) = 56$

$\Rightarrow (11 - 4k) - (3k - 10) = 56$

$\Rightarrow 11 - 4k - 3k + 10 = 56$

$\Rightarrow 21 - 7k = 56$

$\Rightarrow 7k = -35$

$\Rightarrow k = -5$

6. If the points A (–3, 9), B (a, b) and C (4, –5) are collinear and if a + b = 1, then find a and b.

Solution:

Given A (–3, 9), B(a, b), C(4, –5) are collinear and a + b = 1 ..... (1)

Area of the triangle formed by 3 points = 0

$$\frac{1}{2}\begin{vmatrix} -3 & 9 \\ a & b \\ 4 & -5 \\ -3 & 9 \end{vmatrix} = 0$$

$\Rightarrow (-3b - 5a + 36) - (9a + 4b + 15) = 0$

$\Rightarrow -5a - 3b + 36 - 9a - 4b - 15 = 0$

$\Rightarrow -14a - 7b + 21 = 0$

$\Rightarrow -14a - 7b = -21$

$\Rightarrow 14a + 7b = 21 \ (\div 7)$

$\Rightarrow 2a + b = 3$ ..... (2)

Given a + b = 1 ..... (1)

(1) – (2) ⇒ a = 2 b = –1

7. A triangular shaped glass with vertices at A (–5, –4) , B (1, 6) and C (7, –4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution:

The required number of buckets = $\frac{\text{Area of the } \Delta ABC}{\text{Area of the paint covered by one bucket}}$

$$\text{Area of the } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -5 & -4 \\ 1 & 6 \\ 7 & -4 \\ -5 & -4 \end{vmatrix}$$

$= \frac{1}{2}[(-30 - 4 - 28) - (-4 + 42 + 20)]$

$= \frac{1}{2}[-62 - 58]$

$= \frac{1}{2}[-120]$

= 60 sq. units.

∴ The required number of buckets = $\frac{60}{6} = 10$

FORMULAE AND DEFINITIONS

NATURE OF ROOTS OF A QUADRATIC EQUATION		
1.	Distance between two points	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
2.	Mid point	$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
3.	Centroid	$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$
4.	Section Formula (Internal Division)	$\left(\frac{lx_2+mx_1}{l+m},\frac{ly_2+my_1}{l+m}\right)$
5.	Section Formula (External Division)	$\left(\frac{lx_2-mx_1}{l-m},\frac{ly_2-my_1}{l-m}\right)$
6.	Area of Triangle	$\frac{1}{2}\begin{pmatrix}x_1 & x_2 & x_3 & x_1\\ y_1 & y_2 & y_3 & y_1\end{pmatrix}$
7.	Area of quadrilateral	$\frac{1}{2}\begin{pmatrix}x_1 & x_2 & x_3 & x_4 & x_1\\ y_1 & y_2 & y_3 & y_4 & y_1\end{pmatrix}$
		$\frac{1}{2}\begin{pmatrix}x_1-x_3 & y_1-y_3\\ x_2-x_4 & y_2-y_4\end{pmatrix}$
FORMULA FOR SLOPE		
8.	If angle is given	$m=\tan\theta$
9.	If two points are given	$m=\frac{y_2-y_1}{x_2-x_1}$
10.	Slope of the straight line $ax+by+c=0$	$m=\frac{-a}{b}=\frac{-\text{coefficient of } x}{\text{coefficient of } y}$
11.	y - intercept	$c=\frac{-c}{a}=\frac{-\text{constant term}}{\text{coefficient of } y}$
COLLINEARITY OF THREE POINTS		
12.	If three points A, B, C are collinear.	$\begin{pmatrix}x_1 & x_2 & x_3 & x_1\\ y_1 & y_2 & y_3 & y_1\end{pmatrix}=0$
		Slope of AB = Slope of BC (OR) Slope of AC

CONDITION FOR PARALLELISM AND PERPENDICULARITY			
13	If two lines $a_1x+b_1y+c_1=0$	Parallel	$m_1=m_2$ (OR) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$
		Perpendicular	$m_1\times m_2=-1$ (OR) $a_1a_2+b_1b_2=0$

14.	Equation of x axis	$y = 0$
15.	Equation of x axis	$x = 0$
16.	Equation of a straight line parallel to x axis	$y = \pm b$
17.	Equation of a straight line parallel to y axis	$x = \pm a$
18.	Equation of straight line parallel to $ax + by + c = 0$	$ax + by + k = 0$
19.	Equation of a straight line perpendicular to $ax + by + c = 0$	$bx - ay + k = 0$
20.	Equation of straight line passing through origin	$y = mx$
21.	Equation of straight line (slope - y - intercept form)	$y = mx + c$
22.	Equation of straight line (slope - x intercept form)	$y = m(x - d)$
23.	Equation of straight line (point - slope form)	$y - y_1 = m(x - x_1)$
24.	Equation of straight line (two point form)	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
25.	Equation of straight line (intercept form)	$\frac{x}{a} + \frac{y}{b} = 1$

***

6 Trigonometry

Exercise 6.5

Multiple choice Questions

1. The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ is equal to
   A) $\tan^2\theta$ B) 1
   C) $\cot^2\theta$ D) 0

2. $\tan\theta\,\mathrm{cosec}^2\theta - \tan\theta$ is equal to
   A) $\sec\theta$ B) $\cot 2\theta$
   C) $\sin\theta$ D) $\cot\theta$

3. If $(\sin\alpha + \mathrm{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$, then the value of k is equal to
   A) 9 B) 7 C) 5 D) 3

4. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \mathrm{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to
   A) 2a B) 3a C) 0 D) 2ab

5. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then $x^2 - \frac{1}{x^2}$ is equal to
   A) 25 B) $\frac{1}{25}$ C) 5 D) 1

6. If $\sin\theta = \cos\theta$, then $2\tan^2\theta + \sin^2\theta - 1$ is equal to
   A) $\frac{-3}{2}$ B) $\frac{3}{2}$ C) $\frac{2}{3}$ D) $\frac{-2}{3}$

7. If $x = a\tan\theta$ and $y = b\sec\theta$ then
   A) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
   C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

8. $(1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\theta)$ is equal to
   A) 0 B) 1 C) 2 D) –1

9. $a\cot\theta + b\,\mathrm{cosec}\theta = p$ and $b\cot\theta + a\,\mathrm{cosec}\theta = q$ then $p^2 - q^2$ is equal to
   A) $a^2 - b^2$ B) $b^2 - a^2$
   C) $a^2 + b^2$ D) $b - a$

10. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$, then the angle of elevation of the sun has measure **SEP-21**
    A) 45° B) 30° C) 90° D) 60°

11. The electric pole subtends an angle of 30° at a point on the same level as its foot. At a second point 'b' metres above the first, the depression of the foot of the pole is 60°. The height of the pole (in metres) is equal to
    A) $\sqrt{3}\,b$ B) $\frac{b}{3}$ C) $\frac{b}{2}$ D) $\frac{b}{\sqrt{3}}$

12. A tower is 60 m heigh. Its shadow is x metres shorter when the sun's altitude is 45° than when it has been 30°, then x is equal to
    A) 41.92 m B) 43.92 m **MAY-22**
    C) 43 m D) 45.6 m

13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are 30° and 60° respectively. The height of the multistoried building and the distance between two buildings (in metres) is
    A) $20, 10\sqrt{3}$ B) $30, 5\sqrt{3}$
    C) 20, 10 D) $30, 10\sqrt{3}$

14. Two persons are standing 'x' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is
    A) $\sqrt{2}\,x$ B) $\frac{x}{2\sqrt{2}}$
    C) $\frac{x}{\sqrt{2}}$ D) $2x$

15. The angle of elevation of a cloud from a point h metres above a lake is b. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is
    A) $\frac{h(1+\tan\beta)}{1-\tan\beta}$ B) $\frac{h(1-\tan\beta)}{1+\tan\beta}$
    C) $h\tan(45° - \beta)$ D) none of these

2 Marks STAGE 1

1. Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Solution:

$$\frac{\sin A}{1+\cos A} = \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A(1-\cos A)}{1-\cos^2 A}$$

$$= \frac{\sin A(1-\cos A)}{\sin^2 A} = \frac{1-\cos A}{\sin A}$$

2. Prove that $1+\dfrac{\cot^2\theta}{1+cosec\theta}$ = cosecθ

Solution:

$$1 + \frac{\cot^2\theta}{1+cosec\theta} = 1+\frac{cosec^2\theta-1}{cosec\theta+1}$$

$[\because cosec^2\theta - 1 = \cot^2\theta]$

$$= 1+\frac{(cosec\theta+1)(cosec\theta-1)}{cosec\theta+1}$$

$= 1+(cosec\theta - 1) = cosec\theta$

3. Prove that $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$ = cosecθ + cotθ

Solution:

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$= \sqrt{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$$

LHS = cosecθ + cotθ

∴ LHS = RHS

4. Prove the following identities.

(i) cotθ + tanθ = secθ cosecθ

Solution:

LHS = cotθ + tanθ

$$= \frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

= secθ cosecθ

∴ LHS = RHS.

(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Solution:

LHS = $\tan^4\theta + \tan^2\theta = \tan^2\theta(\tan^2\theta + 1)$

$= \tan^2\theta(\sec^2\theta)$ $(\because 1 + \tan^2\theta = \sec^2\theta)$

$= (\sec^2\theta - 1)(\sec^2\theta)$

$(\because \tan^2\theta = \sec^2\theta - 1)$

$= \sec^4\theta - \sec^2\theta$

∴ LHS = RHS

5. Prove the following identities.

(i) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}$ = secθ tanθ SEP-20

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}\times\sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

= secθ + tanθ = RHS

Hence Proved.

(ii) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$ = 2secθ

Solution:

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

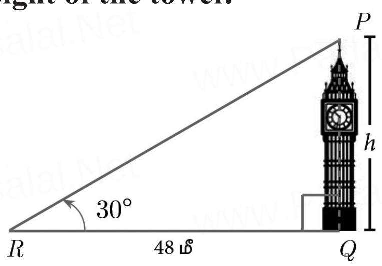
$$= \frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$$

= 2secθ

Hence Proved.

6. A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.



Solution:

In $\Delta PQR \quad \tan\theta = \frac{PQ}{QR}$

$\tan 30° = \frac{h}{48}$

$\frac{1}{\sqrt{3}} = \frac{h}{48}$

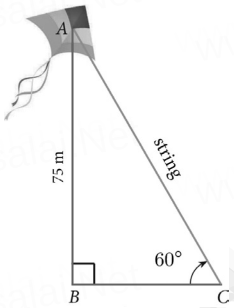
$h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{3} = 16\sqrt{3}$

Therefore the height of the tower is,

$h = 16\sqrt{3}$ m

7. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution:



In $\Delta ABC \sin\theta = \frac{AB}{AC}$

$\sin 60° = \frac{75}{AC}$

$\frac{\sqrt{3}}{2} = \frac{75}{AC}$

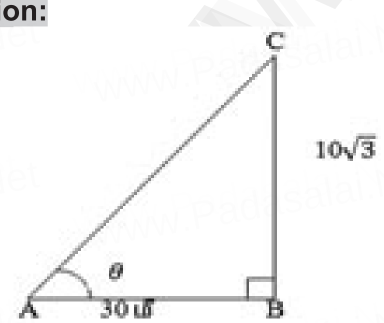
$AC = \frac{75 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3}$

$AC = 50\sqrt{3}$ m

$\therefore$ Hence, the length of the string is $50\sqrt{\ }$ m.

8. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m. **SEP-21**

Solution:



In ΔABC

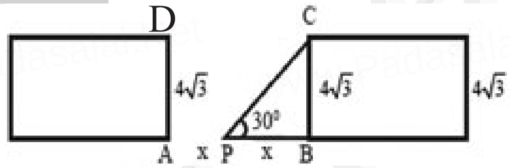
$\tan\theta = \frac{opposite\ side}{Adjacent\ side} \Rightarrow \tan\theta = \frac{10\sqrt{3}}{30}$

$\tan\theta = \frac{\sqrt{3}}{3} \qquad \Rightarrow \tan\theta = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}} \qquad \theta = 30°$

9. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.

Solution:



In the figure, BC – House, AB – Width of Road, P – Median of Road

$AP = PB = x$

In ΔPBC, $\tan 30° = \frac{BC}{PB}$

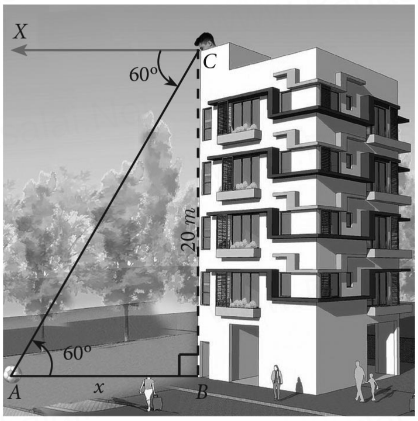
$\Rightarrow \tan 30° = \frac{4\sqrt{3}}{PB} \quad \Rightarrow \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{PB}$

$PB = 4\sqrt{3} \times \sqrt{3} = 4 \times 3 = 12$

Hence, Width of Road

$= AP + PB = 12 + 12 = 24$ m

10. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3} = 1.732$)



Solution:

Let BC be the height of the tower and A be the position of the ball lying on the ground.

Then, BC = 20 m and

$\angle XCA = 60° = \angle CAB$

Let AB = x metres.

In the right angled triangle ABC,

$\tan 60° = \frac{20}{AB}$

$$\sqrt{3} = \frac{20}{AB}$$

$$AB = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

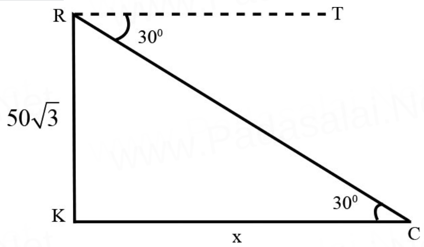
$$AB = \frac{20\sqrt{3}}{3} = \frac{20 \times 1.732}{}$$

$$= \frac{34.640}{3} = 11.54 \text{ m}$$

Hence, the distance between the foot of the tower and the ball is 11.55 m.

11. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. **MAY-22**

Solution:



In ΔABC, $\tan\theta = \frac{opposite\ side}{Adjacent\ side}$

$$\tan 30° = \frac{50\sqrt{3}}{BC}$$

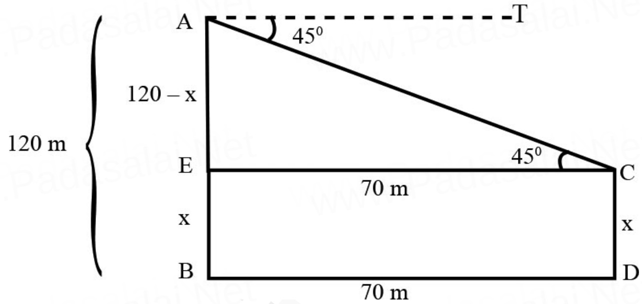
$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}$$

$$BC = 50\sqrt{3} \times \sqrt{3}$$

$$= 50\,(3) = 150 \text{ m}$$

12. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building. ($\sqrt{3}$ = 1.732)

Solution:



CD – First Building,

AB – Second Building

From the figure AB = 120 m,

EB = CD = x, AE=120 – x,

EC = BD = 70 m

In ΔACE, $\tan 45° = \frac{AE}{EC}$

$$\Rightarrow \quad 1 = \frac{120 - x}{70}$$

$$\Rightarrow \quad 120 - x = 70 \text{ m}$$

$$\therefore x = 50 \text{ m}$$

1.	$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$	$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
2.	$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}}$	$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$	$\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

	θ°	**30°**	**45°**	**60°**	**90°**
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	unde-fined
cosec θ	unde-fined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	unde-fined
cot θ	unde-fined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Identity	**Another results**
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$
$\sec^2\theta + \tan^2\theta = 1$	$\sec^2\theta = 1 + \tan^2\theta$ $\tan^2\theta = \sec^2\theta - 1$
$\operatorname{cosec}^2\theta - \cot^2\theta = 1$	$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\sin(90-\theta) = \cos\theta$	$\cos(90-\theta) = \sin\theta$	$\tan(90-\theta) = \cot\theta$
$\operatorname{cosec}(90-\theta) = \sec\theta$	$\sec(90-\theta) = \operatorname{cosec}\theta$	$\cot(90-\theta) = \tan\theta$

***

7 Mensuration

Exercise 7.5

Multiple choice Questions

1. **The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is**
   A) 60π cm^2 B) 68π cm^2
   C) 120π cm^2 D) 136π cm^2

2. **If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is** MAY-22
   A) $4\pi r^2$ sq. units B) $6\pi r^2$ sq. units
   C) $3\pi r^2$ sq. units D) $8\pi r^2$ sq. units

3. **The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be** SEP-21
   A) 12 cm B) 10 cm
   C) 13 cm D) 5 cm

4. **If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is**
   A) 1 : 2 B) 1 : 4
   C) 1 : 6 D) 1 : 8

5. **The total surface area of a cylinder whose radius is 1/3 of its height is**
   A) $\frac{9\pi h^2}{8}$ sq.units B) $24\pi h^2$ sq.units
   C) $\frac{8\pi h^2}{9}$ sq.units D) $\frac{56\pi h^2}{9}$ sq.units

6. **In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is**
   A) 5600π cm^3 B) 11200π cm^3
   C) 56π cm^3 D) 3600π cm^3

7. **If the radius of the base of a cone is tripled and the height is doubled then the volume is**
   A) made 6 times B) made 18 times
   C) made 12 times D) unchanged

8. **The total surface area of a hemi-sphere is how much times the square of its radius.** SEP-21
   A) π B) 4π C) 3π D) 2π

9. **A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is**
   A) $3x$ cm B) x cm C) $4x$ cm D) $2x$ cm

10. **A frustum of a right circular cone is of height 16cm with radii of its ends as 8cm and 20cm. Then, the volume of the frustum is**
    A) 3328π cm^3 B) 3228π cm^3
    C) 3240π cm^3 D) 3340π cm^3

11. **A shuttle cock used for playing badminton has the shape of the combination of**
    A) a cylinder and a sphere
    B) a hemisphere and a cone
    C) a sphere and a cone
    D) frustum of a cone and a hemisphere

12. **A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r^2 units. Then $r_1 : r_2$ is** SEP-20
    A) 2 : 1 B) 1 : 2
    C) 4 : 1 D) 1 : 4

13. **The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is**
    A) $\frac{4}{3}\pi$ B) $\frac{10}{3}\pi$
    C) 5π D) $\frac{20}{3}\pi$

14. **The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 := 1 : 2$ then $r_2 : r_1$ is**
    A) 1 : 3 B) 1 : 2
    C) 2 : 1 D) 3 : 1

15. **The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is**
    A) 1 : 2 : 3 B) 2 : 1 : 3
    C) 1 : 3 : 2 D) 3 : 1 : 2

2 Marks STAGE 1

1. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution:

l = 5 cm, R = 4 cm, r = 1 cm

C.S.A of the frustum = π (R + r)l sq.units

$= \frac{22}{7}(4+1) \times 5$

$= \frac{22 \times 5 \times 5}{7}$

$= \frac{550}{7}$

$= 78.57 \text{ cm}^2$

2. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Solution:

r : h = 5 : 7 ⇒ r = 5x cm , h = 7x cm

CSA = 5500 sq.cm

$2\pi rh = 5500 \Rightarrow 2 \times \frac{22}{7} \times 5x \times 7x = 5500$

$x^2 = \frac{5500}{2 \times 22 \times 5} = 25 \Rightarrow x = 5$

Hence, Radius = 5 × 5 = 25 cm,

Height = 7 × 5 = 35 cm

3. The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights. **|May 22|**

Solution:

Ratio of the volumes of two cones

$= \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2$

$= h_1 : h_2$

= 3600 : 5040

= 360 : 504

= 40 : 56

= 5 : 7

4. If the ratio of radii of two spheres is 4:7, find the ratio of their volumes.

Solution:

The ratio of radii of two spheres = 4 : 7

Let radius of first sphere is 4x,

that is $r_1 = 4x$

Let radius of second sphere is 7x,

that is $r_2 = 7x$

The ratio of their volumes

$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{(4x)^3}{(7x)^3} = \frac{4^3 \times x^3}{7^3 \times x^3}$

$= \frac{4^3}{7^3} = \frac{64}{343}$

Hence the ratio of the volumes is 64 : 343

5. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$:4.

Solution:

Given

Total Surface Area of a solid Sphere

= Total surface Area of a solid hemisphere

$\Rightarrow 4\pi R^2 = 3\pi r^2$

$\Rightarrow \therefore \frac{R^2}{r^2} = \frac{3}{4} \Rightarrow \therefore \frac{R}{r} = \frac{\sqrt{3}}{2}$

∴ Ratio of their volumes

$= \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{2R^3}{r^3} = 2\left[\frac{R}{r}\right]^3 = 2\left[\frac{\sqrt{3}}{2}\right]^3$

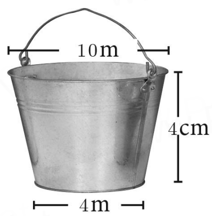
$\Rightarrow 2 \times \frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}$

∴ Ratio of their volumes = $3\sqrt{3}$: 4

5 Marks STAGE 1

1. An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

Solution:



Let h, l, R and r be the height, slant height, outer radius and inner radius of the frustum.

Given that, diameter of the top =10 m;

radius of the top R = 5 m.

diameter of the bottom = 4 m;

radius of the bottom r = 2 m, height h = 4 m

Now, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{4^2 + (5-2)^2}$

$l = \sqrt{16+9} = \sqrt{25} = 5$ m

C.S.A. $= \pi(R + r)l$ sq. units

$= \frac{22}{7}(5 + 2) \times 5$

$= \frac{22}{\not 7} \times \not 7 \times 5$

$= 110 \text{ m}^2$

T.S.A. $= \pi(R + r)l + \pi R^2 + \pi r^2$ sq. units

$= \pi[(R+r)l + R^2+r^2]$

$= \frac{22}{7}[(5+2)5+5^2+2^2]$

$= \frac{22}{7}(35+25+4)$

$= \frac{1408}{7} = 201.14\text{m}^2$

Therefore, C.S.A. = 110 m^2 and

T.S.A. = 201.14 m^2

2. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Solution:

From the given figure, r = 6m, R = 12m and h = 8m.

But, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{10} = 10$

$l = 10$ m

The required total arc of table lamp

= CSA of frustrum + Area of the top

$= \pi(R + r)l + \pi r^2$

$= \frac{22}{7} \times 18 \times 10 + \frac{22}{7} \times 6 \times 6$

$= \frac{22}{7} \times 6[30+6] = \frac{22}{7} \times 6 \times 36$

$= 678.86 \text{ m}^2$

Cost of painting for 1 sq.m. is ₹ 2.

∴ The total cost of painting

= 678.86 × 2 = ₹1357.72.

3. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹40 per litre. **MAY-22**

Solution:

h = 16 cm, r = 8 cm, R = 20 cm,

Volume of the frustum

$= \frac{1}{3} \pi h[R^2 + Rr + r^2]$ cu. units

$= \frac{1}{3} \times \frac{22}{7} \times 16\ [20^2 + 20(8) + 8^2]$

$= \frac{1}{3} \times \frac{22}{7} \times 16\ [400 + 160 + 64]$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 624$

$= 10459 \text{ cm}^3$

= 10.459 litre

The cost of milk is ₹ 40 per litre

The cost of 10.459 litres milk = 10.459 × 40

= ₹ 418.36

4. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum. **SEP-21**

Solution:

height of the frustum, h = 45 cm,

bottom radii, R = 28 cm,

top radii, r = 7 cm

Volume of the frustum

$= \frac{1}{3} \pi h\ [R^2 + Rr + r^2]$ cu.units

$= \frac{1}{3} \times \frac{22}{7} \times 45\ [28^2 + 28 \times 7 + 7^2]$

$= \frac{1}{3} \times \frac{22}{7} \times 45\ [784 + 196 + 49]$

$= \frac{1}{3} \times \frac{22}{7} \times 45 \times 1029$

$= 22 \times 15 \times 147 = 48510 \text{ cm}^3$

FORMULAE AND DEFINITIONS

	SHAPES	CSA / LSA	TSA	VOLUME
1.	**Cube**	$4a^2$	$6a^2$	a^3
2.	**Cuboid**	$2(l+b)h$	$2(lb+bh+lh)$	$l \times b \times h$
3.	**Solid Cylinder**	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
4.	**Hollow Cylinder**	$2\pi(R+r)h$	$2\pi(R+r)(R-r+h)$	$\pi(R^2-r^2)h$
5.	**Solid Sphere**	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
6.	**Hollow Sphere**	$4\pi R^2$	$4\pi(R^2+r^2)$	$\frac{4}{3}\pi(R^3-r^3)$
7.	**Hemisphere**	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
8.	**Hollow Hemisphere**	$2\pi(R^2+r^2)$	$\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
9.	**Solid Cone**	πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
10.	**Frustum**	$\pi(R+r)l$	$\pi(R+r)l+\pi R^2+\pi r^2$	$\frac{1}{3}\pi h[R^2+r^2+Rr]$
11.	**Slant height of cone** $l=\sqrt{r^2+h^2}$	Slant height of frustum $l=\sqrt{h^2+(R-r)^2}$		
12.	**Radius of cone** $r=\sqrt{l^2-h^2}$	Area of Circle $= \pi r^2$		
13.	**Height of cone** $h=\sqrt{l^2-r^2}$	Circumference of Circle $= 2\pi r$		
	SECTOR AND CONE			
14.	Area of sector $\left(\frac{\theta}{360^0}\times\pi R^2\right)$ = CSA of Cone (πrl)			
15.	Length of arc of sector $\left(\frac{\theta}{360^0}\times 2\pi R\right)$ = Circumference of base of the Cone ($2\pi r$)			
16.	Radius of sector (R) = Slant height of cone (l)			
	CONVERSIONS			
17.	$1\ m^3 = 1000$ litres	$1\ d.m^3 = 1$ litre	$1000\ cm^3 = 1$ litre	$1\ kl = 1000$ litres
18.	$1\ cm = 10\ mm$	$1\ m = 100\ cm$	$1\ km = 1000\ m$	

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8 Statistics and Probability

Exercise 8.5

Multiple choice Questions

1. Which of the following is not a measure of dispersion?

A) Range B) Standard deviation
C) Arithmetic mean D) Variance

2. The range of the data 8, 8, 8, 8, 8. . . 8 is

A) 0 B) 1 C) 8 D) 3

3. The sum of all deviations of the data from its mean is

A) Always positive B) always negative
C) zero D) non-zero integer

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is SEP-20

A) 40000 B) 160900
C) 160000 D) 30000

5. Variance of first 20 natural numbers is

A) 32.25 B) 44.25 C) 33.25 D) 30

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is

A) 3 B) 15 C) 5 D) 225

7. If the standard deviation of x, y, z is p then the standard deviation of $3x + 5$, $3y + 5$, $3z + 5$ is

A) $3p + 5$ B) $3p$
C) $p + 5$ D) $9p + 15$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is

A) 3.5 B) 3 C) 4.5 D) 2.5

9. Which of the following is incorrect?

A) $P(A) > 1$ B) $0 \le P(A) \le 1$
C) $P(\varphi) = 0$ D) $P(A) + P(\overline{A}) = 1$

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

A) $\frac{q}{p+q+r}$ B) $\frac{p}{p+q+r}$
C) $\frac{p+q}{p+q+r}$ D) $\frac{p+r}{p+q+r}$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is SEP-21

A) $\frac{3}{10}$ B) $\frac{7}{10}$
C) – D) $\frac{7}{9}$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is MAY-22

A) 2 B) 1 C) 3 D) 1.5

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

A) 5 B) 10 C) 15 D) 20

14. If a letter is chosen at random from the English alphabets {a, b, ..., z}, then the probability that the letter chosen precedes x SEP-20

A) $\frac{12}{13}$ B) $\frac{1}{13}$
C) $\frac{23}{26}$ D) $\frac{3}{26}$

15. A purse contains 10 notes of ₹ 2000, 15 notes of ₹ 500, and 25 notes of ₹ 200. One note is drawn at random. What is the probability that the note is either a ₹ 500 note or ₹ 200 note?

A) $\frac{1}{5}$ B) $\frac{3}{10}$ C) $\frac{2}{3}$ D) $\frac{4}{5}$

2 Marks STAGE 1

1. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:

Largest value L = 67; Smallest value S = 18

Range R = L − S = 67 − 18 = 49

$$\text{Coefficient of range} = \frac{L-S}{L+S}$$

$$\text{Coefficient of range} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

2. Find the range of the following distribution.

Age (in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution:

Here

Largest value, L = 28

Smallest Value, S = 18

Range R = L – S

R = 28 – 18 = 10 Years.

3. The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution:

Range R = 13.67

Largest value L = 70.08

Range R = L – S

13.67 = 70.08 – S

S = 70.08 – 13.67 = 56.41

Therefore, the smallest value is 56.41

4. Find the range and coefficient of range of following data SEP-20

(i) 63, 89, 98, 125, 79, 108, 117, 68

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

i. 63, 89, 98, 125, 79, 108, 117, 68

L = 125, S = 63

Range, R = L – S = 125 – 63 = 62

$$\text{Coefficient of Range} = \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{188} = 0.33$$

ii. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4, S = 13.6

Range, R = L – S = 61.4 – 13.6 = 47.8

$$\text{Coefficient of Range} = \frac{L-S}{L+S} = \frac{47.8}{61.4+13.6} = \frac{47.8}{75.0} = 0.64$$

5. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution:

Range, R = 36.8

Smallest Value, S = 13.4

Largest Value, L = R + S

= 36.8 + 13.4 = 50.2

6. Calculate the range of the following data.

Income	400-450	450-500	500-550
Number of workers	8	12	30
Income	550-600	600-650	
Number of workers	21	6	

Solution:

Given: Largest Value, L = 650

Smallest Value,, S = 400

∴ Range = L – S = 650 – 400 = 250

7. Find the standard deviation of first 21 natural numbers.

Solution:

Standard Deviation of first 21 natural numbers,

$$\sigma = \sqrt{\frac{n^2-1}{12}}$$

$$= \sqrt{\frac{(21)^2-1}{12}} = \sqrt{\frac{441-1}{12}}$$

$$= \sqrt{\frac{440}{12}} = \sqrt{36.66} = 6.05$$

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution:

standard deviation of a data, σ = 4.5

each value of the data decreased by 5,

the new standard deviation does not change and it is also 4.5.

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution:

The new standard deviation of a data is 3.6, and each of the data is divided by 3 then the new standard deviation is also divided by 3.

The new standard deviation = $\frac{3.6}{3} = 1.2$

The new variance = (Standard Deviation)^2

$= \sigma^2 = (1.2)^2 = 1.44$

10. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution:

Mean $\bar{x} = 25.6$

Coefficient of variation, C.V. = 18.75

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$18.75 = \frac{\sigma}{25.6} \times 100$$

$$\sigma = \frac{18.75 \times 25.6}{100} = 4.8$$

11. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution:

Co-efficient of variation C.V. = $\frac{\sigma}{\bar{x}} \times 100$.

$\sigma = 6.5$, $\bar{x} = 12.5$

$$CV = \frac{\sigma}{\bar{x}} \times 100 = \frac{6.5}{12.5} \times 100$$

$$= \frac{6500}{125} = 52\ \%$$

12. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution:

$\bar{x} = 15$, C.V. = 48,

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$\sigma = \frac{C.V \times \bar{x}}{100} = \frac{48 \times 15}{100} = \frac{720}{100} = 7.2$$

13. If n = 5 , x = 6 , x^2 = 765, then calculate the coefficient of variation.

Solution:

$n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{765}{5} - (6)^2}$$

$$= \sqrt{153 - 36} = \sqrt{117}$$

$= 10.8$

$$CV = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{10.8}{6} \times 100 = \frac{1080}{6} = 180\%$$

14. A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution:

Total number of possible outcomes

n(S) = 5 + 4 = 9

i) Let A be the event of getting a blue ball.

Number of favourable outcomes for the event A. Therefore, n(A) = 5

Probability that the ball drawn is blue.

Therefore, $P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$

ii) A will be the event of not getting a blue ball.

So $P(\bar{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$

15. Two coins are tossed together. What is the probability of getting different faces on the coins? MAY-22

Solution:

When two coins are tossed together, the sample space is

S = {HH,HT,TH,TT}; n (S) = 4

Let A be the event of getting different faces on the coins.

A = {HT,TH}; n (A) = 2

Probability of getting different faces on the coins is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

16. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

n(S) = 8

Event A :

Two Consecutive tails = {HTT, TTH, TTT}

n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

17. What is the probability that a leap year selected at random will contain 53 Saturdays.

Solution:

A leap year has 366 days.

So it has 52 full weeks and 2 days.

52 Saturdays must be in 52 full weeks.

S = {(Sun - Mon, Mon - Tue, Tue - Wed, Wed - Thu, Thu - Fri, Fri - Sat, Sat - Sun)}

n(S) = 7

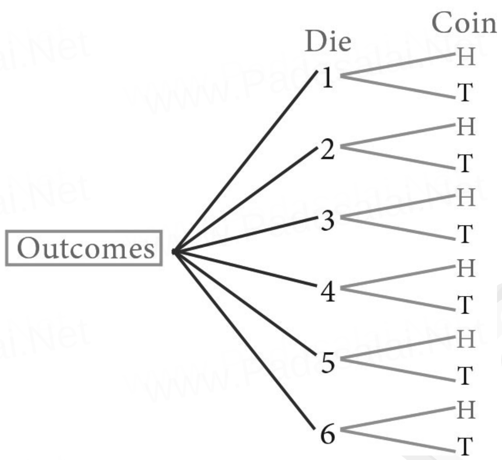
Let A be the event of getting 53^{rd} Saturday.

Then A = {Fri - Sat, Sat - Sun} n(A) = 2

Probability of getting 53 Saturdays in a leap year is $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

18. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head. SEP-21

Solution:



Sample space

S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T};

n(S) = 12

Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H}; n(A) = 3

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

19. If P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09 then find P(A∪B) .

Solution:

P(A) = 0.37 , P(B) = 0.42 , P(A∩B) = 0.09

P(A∪B) = P(A) + P(B) − P(A∩B)

P(A ∪ B) = 0.37 + 0.42 − 0.09 = 0.7

20. What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution:

Total number of cards = 52

A = Number of King cards

n(A) = 4 $\quad P(A) = \frac{4}{52}$

B = Number of Queen cards = 4

n(B) = 4 $\quad P(B) = \frac{4}{52}$

Both the events of drawing a king and a queen are mutually exclusive P(A ∪ B) = P(A) + P(B)

∴ Probability of drawing either a king or a queen $= \frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

21. If P(A) = 23, P(B) =25, P(A∪B) = 13 then find P(A∩B).

Solution:

$P(A) = \frac{2}{3}, P(B) = \frac{2}{5}, P(A\cup B) = \frac{1}{3}$

$$P(A\cap B) = P(A) + P(B) - P(A\cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$

$$= \frac{10+6-5}{15}$$

$$P(A\cap B) = \frac{11}{15}$$

22. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\overline{A})+P(\overline{B})$.

Solution:

Given P(A∪B) = 0.6, P(A∩B) = 0.2

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A) + P(B) = P(A\cup B) + P(A\cap B)$$

$$= 0.6 + 0.2$$

$$= 0.8$$

$$\therefore P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B)$$

$$= 2 - [P(A) + P(B)]$$

$$= 2 - 0.8$$

$$= 1.2$$

5 Marks STAGE 1

1. Find the mean and variance of the first *n* natural numbers.

Solution:

$$\text{Mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+...+n}{n} = \frac{n(n+1)}{2\times n}$$

$$\bar{x} = \frac{n+1}{2}$$

Variance σ^2

$$= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \left| \begin{array}{l} \Sigma x_i^2 = 1^2+2^2+3^2+...+n^2 \\ (\Sigma x_i)^2 = (1+2+3+...+n)^2 \end{array} \right|$$

$$= \frac{n(n+1)(2n+1)}{6\times n} - \left|\frac{n(n+1)}{2\times n}\right|^2$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

Variance $\sigma^2 = \frac{n+1}{2}\left[\frac{n-1}{6}\right] = \frac{n^2-1}{12}$

2. Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13 SEP-21

Solution:

When we roll two dice, the sample space is given by

S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) };

n (S) = 36

i) Let A be the event of getting the sum of outcome values equal to 4.

Then A = {(1, 3), (2, 2), (3, 1)}; n(A)= 3.

Probability of getting the sum of outcomes equal to 4 is

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

ii) Let B be the event of getting the sum of outcome values greater than 10.

Then B = {(5, 6), (6, 5), (6, 6)}; n(B) = 3

Probability of getting the sum of outcomes greater than 10 is

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

iii) Let C be the event of getting the sum of outcomes less than 13. Here all the outcomes have the sum value less than 13. Hence C = S .

Therefore, n(C) = n(S) = 36

Probability of getting the total value less than 13 is

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

3. From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Solution:

Suits of playing cards	Spade ♠	Heart ♥	Clavor ♣	Diamond ♦
Cards of each suit	A	A	A	A
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9
	10	10	10	10
	J	J	J	J
	Q	Q	Q	Q
	K	K	K	K
Set of playing cards in each suit	13	13	13	13

n(S) = 52

i) Let A be the event of getting a red card.

n(A) = 26

Probability of getting a red card is

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

ii) Let B be the event of getting a heart card.

n(B) = 13

Probability of getting a heart card is

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king. n(C) = 2

Probability of getting a red king card is

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen (Q), and King (K).

$n(D) = 4\times3 = 12$

Probability of getting a face card is

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10.

$n(E) = 4\times9 = 36$

Probability of getting a number card is

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

4. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

$n(S) = 36$

When two dice are rolled together, there will be $6\times6 = 36$ outcomes.

Let S be the sample space. Then $n(S) = 36$

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then $A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$B = \{(1,3), (2,2), (3,1)\}$

$\therefore A\cap B = \{(2,2)\}$

Then, $n(A) = 6$, $n(B) = 3$, $n(A\cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P (getting a doublet or a total of 4) = $P(A\cup B)$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{6}{36}+\frac{3}{36}-\frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

5. If A is an event of a random experiment such that $P(A) : P(\bar{A})=17:15$ and $n(S) = 640$ then find (i) $P(\bar{A})$ (ii) $n(A)$.

Solution:

Given $n(S) = 640$

$$\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$$

$$\frac{1-P(\bar{A})}{P(\bar{A})} = \frac{17}{15}$$

$$15[1-P(\bar{A})] = 17P(\bar{A})$$

$$15-15P(\bar{A}) = 17P(\bar{A})$$

$$15 = 15P(\bar{A}) + 17P(\bar{A})$$

$$32P(\bar{A}) = 15$$

$$P(\bar{A}) = \frac{15}{32}$$

$$P(A) = 1-P(\bar{A})$$

$$= 1-\frac{15}{32}$$

$$= \frac{32-15}{32} = \frac{17}{32}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\frac{17}{32} = \frac{n(A)}{640}$$

$$n(A) = \frac{17\times \overset{20}{\cancel{640}}}{\cancel{32}}$$

$$n(A) = 340$$

6. Two unbiased dice are rolled once. Find the probability of getting
(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number
(iv) the sum as 1 SEP-20

Solution:

$n(S) = 36$

i) A = Probability of getting Doublets (Equal numbers on both dice)

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$$n(A) = 6;\ P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

B = Probability of getting the product of the prime number

ii) $B = \{(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)\}$

$$n(B) = 6;\ P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

C = Probability of getting sum of the prime number.

iii) $C =\{(1,1), (2,1), (1,2), (1,4), (4,1), (1,6), (6,1), (2,3), (2,5), (3,2), (3,4), (4,3), (5,2), (5,6), (6,5)\}$

$n(C) = 14;\ P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

iv) D = Probability of getting the sum as 1

$n(D) = 0;\ P(D) = \frac{n(D)}{n(S)} = 0$

7. Three fair coins are tossed together. Find the probability of getting
(i) all heads
(ii) atleast one tail
(iii) atmost one head
(iv) atmost two tails

Solution:

Possible Outcomes = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

No.of possible outcomes,

$n(S) = 2 \times 2 \times 2 = 8$

i) A = Probability of getting all heads

A = {HHH} $n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$

ii) B = Probability of getting atleast one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

$n(B) = 7 \quad P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$

iii) C = Probability of getting atmost one head.

C = {TTT, TTH, THT, HTT}

$n(C) = 4 \quad P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

iv) D = Probability of getting atmost two tails.

D = {TTH, THT, HTT, HHT, HTH, THH, HHH}

$n(D) = 7 \quad P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$

8. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
(i) white
(ii) black or red
(iii) not white
(iv) neither white nor black

Solution:

S = {5 Red, 6 White, 7 Green, 8 Black}

$n(S) = 26$

i) A – probability of getting white balls

$n(A) = 6;\ P(A) = \frac{6}{26} = \frac{3}{13}$

ii) B – Probability of getting black (or) red balls

$n(B) = 8 + 5 = 13;\ P(B) = \frac{13}{26} = \frac{1}{2}$

iii) C – Probability of not getting white balls

$n(C) = 20;\ P(C) = \frac{20}{26} = \frac{10}{13}$

iv) D – Probability of getting of neither white nor black

$n(D) = 12;\ P(D) = \frac{12}{26} = \frac{6}{13}$

9. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Solution:

In a box there are 20 non – defective and x defective bulbs

$n(S) = x + 20$

Let A – probability of getting Defective Bulbs

$n(A) = x$

$P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+20}$

From Given data

$$\frac{x}{x+20} = \frac{3}{8}$$

$$8x = 3x + 60$$

$$5x = 60$$

$$x = 12$$

∴ Number of defective bulbls = 12

10. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Solution:

Removed cards:

The King and Queen of diamonds,

Queen and Jack of hearts, and King of spades (i.e) remaining number of cards

$= 52 - 6 = 46$

$n(S) = 46$

i) A is probability of getting Clavor Cards

$n(A) = 13 \quad P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$

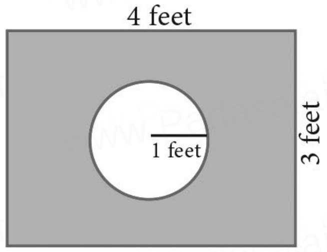
ii) B is probability of getting a queen of red card.

$n(B) = 0 \quad P(B) = \frac{n(B)}{n(S)} = \frac{0}{46} = 0$

iii) C is probability of getting King of black card.

$n(C) = 1 \quad P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$

11. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game? ($\pi = 3.14$)



Solution:

Total Region $= 4 \times 3 = 12$ sq.ft

$\therefore n(S) = 12$

Winning Region = Area of circle

$= \pi r^2 = \pi(1)^2$

$= \pi = 3.14$ sq. unit

$n(A) = 3.14$

$P(\text{Winning the Game}) = \frac{n(A)}{n(S)}$

$= \frac{3.14}{12} = \frac{314}{1200}$

$= \frac{157}{600}$

12. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution:

$\sigma = 1.2, \;\; CV = 25.6, c = ?$

$CV = \frac{\sigma}{\bar{x}} \times 100$

$\bar{x} = \frac{\sigma}{C.V} \times 100 = \frac{1.2}{25.6} \times 100 = \frac{1200}{256}$

$\bar{x} = 4.7$

13. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on

(i) the same day

(ii) different days

(iii) consecutive days?

Solution:

$n(S) = 36$

i) A be the Probability of Priya and Amuthan to visit shop on same day

A = {(Mon, Mon), (Tue, Tue), (Wed, Wed), (Thurs, Thurs), (Fri, Fri), (Sat, Sat)}

$n(A) = 6$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

ii) P (Priya and Amuthan Visit on Different Days)

$= P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$

C be the Probability of Priya and Amuthan to visit on Consequent days

iii) C = {(Mon, Tue), (Tue, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat) (Tue, Mon), (Wed, Tue), (Thurs, Wed), (Fri, Thurs), (Sat, Fri,)}

$n(C) = 10$

$P(C) = \frac{n(C)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

14. In a game, the entry fee is ₹ 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

$n(S) = 8$

i) For Receiving double entry Fees have to get Three Heads

A = Probability of Getting three Heads

A = {HHH}

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

ii) For getting Entry Fess getting atleast one Head

B = Probability of Getting One or Two Heads

B = {TTH, THT, HTT, HHT, HTH, THH}

n(B) = 6

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

iii) To loss the entry fees, she have to get no Heads

C = Probability of Getting No Heads

C = {TTT}

n(C) = 1

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

15. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

n(S) = 36

When two dice are rolled together, there will be 6 × 6 = 36 outcomes.

Let S be the sample space.

Then n(S) = 36

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then A = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

B = {(1,3),(2,2),(3,1)}

$\therefore A \cap B = \{(2,2)\}$

Then, n(A) = 6 , n(B) = 3 , $n(A \cap B) = 1$.

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$\therefore$ P (getting a doublet or a total of 4) = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

16. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and P(A and B) = $\frac{1}{8}$, find

(i) P (A or B)

(ii) P(not A and not B).

Solution:

i. $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$$P(A \text{ or } B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$$

ii. $P(\text{not } A \text{ and not } B) = P(\overline{A} \cap \overline{B})$

$= P\left(\overline{A \cup B}\right)$

$= 1 - P(A \cup B)$

$$P(\text{not } A \text{ and not } B) = 1 - \frac{5}{8} = \frac{3}{8}$$

17. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution:

Total number of cards = 52 ; n(S) = 52.

Let A be the event of getting a king card.

$$n(A) = 4 ; \quad P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a heart card

$$n(B) = 13; \quad P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Let C be the event of getting a red card

$$n(C) = 26; \quad P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$$

$P(A \cap B)$ = P(getting heart king) = $\frac{1}{52}$

$P(B \cap C)$ = P(getting red and heart) = $\frac{13}{52}$

$P(A \cap C)$ = P(getting red king) = $\frac{2}{52}$

$P(A \cap B \cap C)$ = P(getting heart, king which is red) = $\frac{1}{52}$

Therefore, required probability is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$$

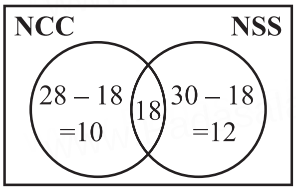
$$= \frac{28}{52} = \frac{7}{13}$$

18. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them. **MAY-22**



Solution:

Total number of students n(S) = 50

i. A : A : opted only NCC but not NSS

$P(A) = \frac{n(A)}{n(S)} = \frac{10}{50} = \frac{1}{5}$

ii. B : opted only NSS but not NCC

$P(B) = \frac{n(B)}{n(S)} = \frac{12}{50} = \frac{6}{25}$

iii. C : opted only one

$P(C) = \frac{n(C)}{n(S)} = \frac{(10+12)}{50} = \frac{22}{50} = \frac{11}{25}$

19. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) =36

A = Probability of getting an even number in the first die.

A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6))}

n(A) = 18; $P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

B = Probability of getting a total face sum is 8

B = {(2,6), (3,5), (4,4), (5,3), (6,2)}

n(B) = 5; $P(B) = \frac{n(B)}{n(S)} = \frac{}{36}$

A∩B = {(2,6), (4, 4), (6, 2)}

n(A∩B) = 3

$P(A\cap B) = \frac{3}{36}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{20}{36} = \frac{5}{9}$

20. A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution:

S = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37}

n(S) = 18

Let A = Multiple of 7

A= {7, 21, 35}, n(A) = 3

$P(A) = \frac{3}{18}$

Let B = a Prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}

n(B) = 11; $P(B) = \frac{11}{18}$

A∩B = {7} ⇒ n(A∩B) = 1

$P(A\cap B) = \frac{1}{18}$

P(Either A or B) = P(A∪B)

= P(A) + P(B) – P(A∩B)

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$

21. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

A = Probability of getting atmost 2 tails

A = {HHH, HHT, HTH, THH, TTH, THT, HTT}

n(A) = 7P(A) = –

B = Probability of getting atmost 2 heads

B = {HHT, HTH, THH, HHH}

$n(B) = 4\ P(B) = \frac{4}{8}$

$P(A \cap B) = \frac{4}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$

FORMULAE AND DEFINITIONS

1.	Probability of an event	$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of outcomes}}$ $P(A) = \frac{n(A)}{n(S)}$
2.	Probability of sure event	$P(S) = 1$
3.	Probability of impossible event	$P(\phi) = 0$
4.	Probability value always lies from	0 to 1 (OR) $0 \le P(A) \le 1$
5.	Probability of complement event	$P(\bar{A}) = 1 - P(A)$ $[\because P(A) + P(\bar{A}) = 1]$

***

MULTIPLE CHOICE QUESTIONS ANSWERS

UNIT - 1

1.C	2.C	3.A	4.B	5.C	6.D	7.D
8.A	9.C	10.A	11.A	12.D	13.C	14.B
15.D						

UNIT - 2

1.C	2.A	3.B	4.C	5.D	6.A	7.D
8.C	9.A	10.C	11.C	12.D	13.B	14.B
15.C						

UNIT - 3

1.D	2.A	3.B	4.A	5.B	6.C	7.D
8.B	9.C	10.C	11.B	12.A	13.B	14.D
15.B	16.B	17.D	18.B	19.C	20.A	

UNIT - 4

1.C	2.B	3.D	4.A	5.D	6.A	7.B
8.C	9.A	10.D	11.B	12.B	13.B	14.D
15.A						

UNIT - 5

1.B	2.A	3.B	4.C	5.C	6.D	7.B
8.A	9.C	10.C	11.C	12.A	13.B	14.B
15.B						

UNIT - 6

1.B	2.D	3.B	4.A	5.B	6.B	7.A
8.C	9.B	10.D	11.B	12.B	13.D	14.B
15.A						

UNIT - 7

1.D	2.A	3.A	4.B	5.C	6.B	7.B
8.C	9.C	10.A	11.D	12.A	13.A	14.B
15.D						

UNIT - 8

1.C	2.A	3.C	4.B	5.C	6.D	7.B
8.A	9.A	10.B	11.B	12.B	13.C	14.C
15.D						

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GOVT QUESTION PAPER - SEPTEMBER 2020

CLASS: X

MATHEMATICS

Time allowed: 3.00 Hours Maximum Marks: 100

Instructions : 1. Check the question paper for fairness of printing. If there is any lack of fairness inform the hall supervisor immediately.

2. Use Blue or Black ink to write and underline and pencil to draw diagrams.

Note: This question paper contains **four** parts.

PART - I

Note : (i) Answer all the questions. **14×1=14**

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}, then state which of the following statement is true?

a) $(A\times C) \subset (B\times D)$ b) $(B\times D) \subset (A\times C)$ c) $(A\times B) \subset (A\times D)$ d) $(D\times A) \subset (B\times A)$

2. Let $f(x) = x^2 - x$, then $f(x-1) - f(x+1)$ is

a) $4x$ b) $2 - 2x$ c) $2 - 4x$ d) $4x - 2$

3. Using Euclid's division lemma, if the cube of any positive integer is divided by 9, then the possible remainders are

a) 0, 1, 8 b) 1, 4, 8 c) 0, 1, 3 d) 1, 3, 5

4. If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \ldots\ldots + 2^0$, which of the following is true?

a) B is 2^{64} more than A b) A and B are equal c) B is larger than A by 1 d) A is larger than B by 1

5. $\dfrac{a^2}{a^2-b^2} + \dfrac{b^2}{b^2-a^2} =$

a) $a - b$ b) $a + b$ c) $a^2 - b^2$ d) 1

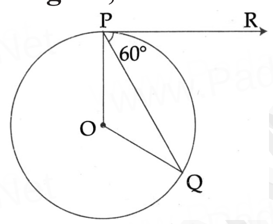
6. Transpose of a column matrix is

a) unit matrix b) diagonal matrix c) column matrix d) row matrix

7. In ΔLMN, ∠L=60°, ∠M = 50°. If ΔLMN ~ ΔPQR, then the value of ∠R is

a) 40° b) 70° c) 30° d) 110°

8. In the figure, if PR is tangent to the circle at P and O is the centre of the circle, then ∠POQ is



a) 120°

b) 100°

c) 110°

d) 90°

9. The straight line given by the equation $x = 11$ is

a) Parallel to x-axis b) Parallel to y-axis

c) Passing through the origin d) Passing through the point (0, 11)

10. If tanθ + cotθ = 2, then the value of tan²θ + cot²θ is

a) 0 b) 1 c) 2 d) 4

11. A child reshapes a cone made up of clay of height 24 cm and radius 6 cm into a sphere, then the radius of sphere is

a) 24 cm b) 12 cm c) 6 cm d) 48 cm

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

a) 2 : 1 b) 1 : 2 c) 4 : 1 d) 1 : 4

13. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

a) 40000 b) 160900 c) 160000 d) 30000

14. If a letter is chosen at random from the English alphabets (a, b, c, ..., z), then the probability that the letters chosen precedes *x*, is

a) $\frac{12}{13}$ b) $\frac{1}{13}$ c) $\frac{23}{26}$ d) $\frac{3}{26}$

PART - II

Answer any 10 questions. Question No. 28 is compulsory. 10×2=20

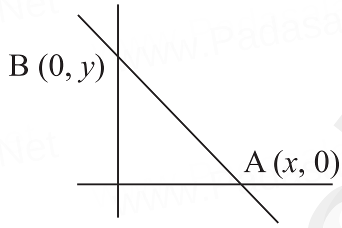
15. If A × B = {(3, 2) (3, 4) (5, 2) (5, 4)}, then find A and B.
16. Show that the function $f: N \rightarrow N$ defined by $f(m) = m^2 + m + 3$ is one-one function.
17. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ end in 5?
18. Find the 3rd and 4th terms of a sequence, if $a_n = \begin{cases} n^2 & if \quad n \text{ is odd} \\ \frac{n^2}{2} & if \quad n \text{ is even} \end{cases}$
19. Find the value of $1^2 + 2^2 + 3^2 + \ldots\ldots + 10^2$ and hence deduce $2^2 + 4^2 + 6^2 + \ldots\ldots + 20^2$.
20. Find the value of k for which the equation $9x^2 + 3kx + 4 = 0$ has real and equal roots.
21. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of –A.
22. Check whether AD is bisector of ∠A of ΔABC in the following:
    AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm.
23. Find the slope of a line joining the points (14, 10) and (14, –6).
24. Prove $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$
25. Find the diameter of a sphere whose surface area is 154 m^2.
26. If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area.
27. Find the range and coefficient of range of the data. 63, 89, 98, 125, 79, 108, 117, 68.
28. Find the volume of the iron used to make a hollow cylinder of height 9cm and whose internal and external radii are 3 cm and 5 cm respectively.

PART - III

Answer any 10 questions. Question No. 42 is compulsory. 10×5=50

29. Let A = The set of all natural numbers less than 8
    B = The set of all prime numbers less than 8
    C = The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$
30. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let $f: A \rightarrow B$ be a function given by $f(x) = 3x - 1$. Represent this function i) by Arrow diagram ii) in a table form iii) as a set of ordered pairs iv) in a graphical form
31. Find the sum of all natural numbers between 100 and 1000 which are divisible by 11.
32. Solve: $6x + 2y - 5z = 13;\ 3x + 3y - 2z = 13;\ 7x + 5y - 3z = 26$
33. Find the GCD of the polynomials, $x^4 + 3x^3 - x - 3$ and $x^3 + x^2 - 5x + 3$.
34. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$
35. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^T A^T$.

36. State and prove Angle Bisector theorem.
37. Find the value of k, if the area of a quadrilateral is 29 sq. units, whose vertices are (–4, –2), (–3, k), (3, –2) and (2, 3).
38. From the top of a tower 60m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. (tan 38° = 0.7813, $\sqrt{3}$ = 1.732)
39. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small non-hollow cylindrical metal of radius 5 cm and height 4 cm is immersed in it completely. Calculate the rise of water in the glass.
40. The scores of a cricketer in 7 matches are 70, 80, 60, 50, 40, 90, 95. Find the standard deviation.
41. Two unbiased dice are rolled once. Find the probability of getting:
    i) a doublet (equal numbers on both dice) ii) the product as a prime number
    iii) the sum as a prime number iv) the sum as 1
42. A straight line AB cuts the co-ordinate axes at A and B. If the mid-point of AB is (2, 3), find the equation of AB.



PART - IV

Answer all the questions. **2×8=16**

43. a) Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC. $\left(scale\ factor\ \frac{6}{5}\right)$

OR

b) Draw two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also measure the lengths of the tangents.

44. a) Graph the quadratic equation $x^2 - 8x + 16 = 0$ and state the nature of their solution.

OR

b) A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount.

Hence find

(i) the marked price when a customer gets a discount of ₹ 3250 (from graph)

(ii) the discount when the marked price is ₹ 2500

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STAGE - II

1 Relations and Functions

2 Marks

STAGE 2

1. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if fog = gof, then find the value of k.

Solution:

$f(x) = 3x - 2 \qquad g(x) = 2x + k$

$fog = f[g(x)] \qquad gof = g[f(x)]$

$= f[2x + k] \qquad gof = g[3x - 2]$

$= 3(2x + k) - 2 \qquad = 2(3x - 2) + k$

$= 6x + 3k - 2 \qquad = 6x - 4 + k$

$fog = gof \Rightarrow 6x + 3k - 2 = 6x - 4 + k$

$\Rightarrow 3k - k = -4 + 2 \Rightarrow 2k = -2 \Rightarrow k = -1$

2. Find k if $fof(k) = 5$ where $f(k) = 2k - 1$.

Solution:

$$fof(k) = 5$$
$$fo(2k - 1) = 5$$
$$(2x - 1)\ o\ (2k - 1) = 5$$
$$2(2k - 1) - 1 = 5$$
$$4k - 2 = 5 + 1$$
$$4k - 2 = 6$$
$$4k = 8$$
$$k = 2$$

3. Using the functions f and g given below, find fog and gof . Check whether fog = gof .

(i) $f(x) = x - 6$, $g(x) = x^2$

(ii) $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

(iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

(iv) $f(x) = 3 + x$, $g(x) = x - 4$

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

Solution:

i. $f(x) = x - 6$, $g(x) = x^2$

$(f\ o\ g)(x) = f(g(x)) = f(x^2) = x^2 - 6$

$(g\ o\ f)(x) = g(f(x)) = g(x - 6)$

$= (x - 6)^2 = x^2 - 12x + 36$

$\therefore fog \neq gof$

ii. $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

$(fog)(x) = f(g(x)) = f(2x^2 - 1) = \frac{2}{2x^2 - 1}$

$(gof)(x) = g(f(x))$

$= g\left(\frac{2}{x}\right) = 2\left(\frac{2}{x}\right)^2 - 1$

$= 2\left(\frac{4}{x^2} - 1\right) = \frac{8}{x^2} - 2$

$\therefore fog \neq gof$

iii. $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

$(fog)(x) = f(g(x)) = f(3 - x) = \frac{(3-x)+6}{3}$

$= \frac{9-x}{3}$

$(gof)(x) = g(f(x)) = g\left(\frac{x+6}{3}\right)$

$= 3 - \frac{x+6}{3} = \frac{9-x-3}{3} = \frac{6-x}{3}$

$\therefore fog \neq gof$

iv. $f(x) = 3 + x$, $g(x) = x - 4$

$(fog)(x) = f(g(x)) = f(x - 4)$

$= 3 + (x - 4) = x - 1$

$(gof)(x) = g(f(x)) = g(3 + x)$

$= 3 + x - 4 = x - 1$

$\therefore fog = gof$

v. $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

$(fog)(x) = f(g(x))$

$= f(1 + x)$

$= 4(1 + x^2 + 2x) - 1$

$= 4x^2 + 8x + 3$

$(gof)(x) = g(f(x))$

$= g(4x^2 - 1)$

$= 1 + 4x^2 - 1$

$= 4x^2$

$\therefore fog \neq gof$

4. Find the value of k, such that fog = gof

(i) $f(x) = 3x + 2$, $g(x) = 6x - k$

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Solution:

i) $f(x) = 3x + 2 \quad g(x) = 6x - k$

$$(3x + 2)\ o\ (6x - k) = (6x - k)\ o\ (3x + 2)$$
$$3(6x - k) + 2 = 6(3x + 2) - k$$
$$18x - 3k + 2 = 18x + 12 - k$$
$$-3k + k = 12 - 2$$

$$-2k = 10$$
$$k = \frac{-10}{2}$$
$$k = -5$$

ii. f(x) = 2x – k g(x) = 4x + 5

$$fog = gof$$
$$(2x - k) \text{ o } (4x + 5) = (4x + 5) \text{ o } (2x - k)$$
$$2(4x + 5) - k = 4(2x - k) + 5$$
$$8x + 10 - k = 8x - 4k + 5$$
$$-k + 4k = 5 - 10$$
$$3k = -5$$
$$k = \frac{-5}{3}$$

5. If f (x) = 2x – 1, g(x)= $\frac{x+1}{2}$, show that fog = gof = x

Solution:

∴ fog = (fog)(x) = f(g(x))

$$fog = gof$$
$$(2x - 1) \text{ o } \left(\frac{x+1}{2}\right) = \left(\frac{x+1}{2}\right) \text{ o } (2x - 1)$$
$$2 = \left(\frac{x+1}{2}\right) - 1 = \frac{2x - 1 + 1}{2}$$
$$x + 1 - 1 = \frac{2x}{2}$$
$$x = x$$

$\therefore fog = gof = x$

6. If f(x) = x^2 – 1, g(x) = x – 2 find a, if go f(a) = 1.

Solution:

$$g \text{ o } f = 1$$
$$(x - 2) \text{ o } (a^2 - 1) = 1$$
$$a^2 - 1 - 2 = 1$$
$$a^3 - 3 = 1$$
$$a^2 = 4$$
$$\therefore \quad a = \pm 2$$

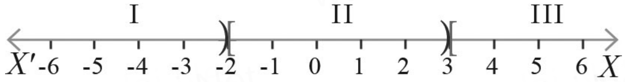
5 Marks STAGE 2

1. If the function f : R → R is defined by

$$f(x) = \begin{cases} 2x+7; & x < -2 \\ x^2 - 2; & -2 \le x < 3 \\ 3x - 2; & x \ge 3 \end{cases}$$

then find the values of (i) f(4) (ii) f(–2) (iii) f(4) + 2f(1) (iv) $\frac{f(1) - 3f(4)}{f(-3)}$

Solution:



$f(x) = 2x+7$ $f(x) = x^2-2$ $f(x) = 3x-2$

The function f is defined by three values in I, II, III as shown by the side

For a given value of x = a, find out the interval at which the point a is located, there after find f(a) using the particular value defined in that interval.

i) First, we see that, x = 4 lie in the third interval.
Therefore, f(x) = 3x – 2;
f(4) = 3(4) – 2 = 10

ii) x = –2 lies in the second interval.
Therefore, f(x) = x^2 – 2;
f(–2) = $(-2)^2$ – 2 = 2

iii) From (i), f(4) = 10. To find f(1), first we seet that, x = 1 lies in the second interval.
Therefore, f(x) = x^2 – 2 ⇒ f(1) = 1^2 – 2 = –1
So, f(4) + 2f(1) =10 + 2(–1) = 8

iv) We know that f(1) = –1 and f(4) = 10. Find finding f(–3), we see that x = –3 lies in the first interval.
Therefore, f(x) = 2x + 7,
thus f(–3) = 2(–3) + 7 = 1

Hence, $\frac{f(1) - 3f(4)}{f(-3)} = \frac{-1 - 3(10)}{1} = -31$

2. If the function f is defined by

$$f(x) = \begin{cases} x + 2; & x > 1 \\ 2; & -1 \le x \le 1 \\ x - 1 & -3 < x < -1 \end{cases}$$

find the values of

(i) f (3) (ii) f (0) (iii) f (–1 5) (iv) f (2) + f (–2)

Solution:

i) f(3) = x + 2 = 3 + 2 = 5

ii) f(0) = 2

iii) f(–1.5) = x – 1 = –1.5 – 1 = –2.5

iv) f(2) + f(–2)
= [x+2] + [x–1]
= [2+2] + [–2–1] = 4 + [–3] = 4 – 3 = 1

3. A function f:[–5, 9] → R is defined as follows:

$$f(x) = \begin{cases} 6x + 1; & -5 \le x < 2 \\ 5x^2 - 1; & 2 \le x < 6 \\ 3x - 4; & 6 \le x \le 9 \end{cases}$$

Find (i) f (–3) + f (2) (ii) f (7) – f (1)

(iii) 2f (4) + f (8) (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

Solution:

i) f(–3) + f(2)

= [6x + 1] + [5x² – 1]

= [6(–3) + 1] + [5(2)² – 1]

= [–18 + 1] + [5(4) –1] = –17 + [20 – 1]

= –17 + 19 = 2

ii) f(7) – f(1)

= [3x – 4] – [6x + 1]

= [3(7) – 4] – [6(1) + 1]

= [21 – 4] – [6 + 1] = 17 – 7 = 10

iii) 2f(4) + f(8)

= 2[5x² – 1] + [3x – 4]

= 2[5(4)² – 1] + [3(8) – 4]

= 2[5(16) –1] + [24 – 4]

= 2[80 –1] + [20] = 2[79] + 20

= 158 + 20 = 178

iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

$= \frac{2[6x+1]-[3x-4]}{[5x^2-1]+[6x+1]}$

$= \frac{2[6(-2)+1]-[3(6)-4]}{[5(4)^2-1]+[6(-2)+1]}$

$= \frac{2[-12+1]-[18-4]}{[5(16)-1]+[-12+1]}$

$= \frac{2[-11]-[14]}{[80-1]+[-11]}$

$= \frac{-22-14}{79-11}$

$= \frac{-36}{68} = \frac{-9}{17}$

4. The distance S an object travels under the influence of gravity in time t seconds is given by S(t) = $\frac{1}{2}$gt + at + b where, (g is the acceleration due to gravity), a, b are constants. Verify whether the function S(t) is one-one or not.

Solution:

$s(t) = \frac{1}{2}gt^2 + at + b$

Let t be 1, 2, 3,…… seconds

$s(t_1) = s(t_2)$

$\frac{1}{2}gt_1^2 + at_1 + b = \frac{1}{2}gt_2^2 + at_2 + b$

$\frac{1}{2}gt_1^2 + at_1 + b - \frac{1}{2}gt_2^2 - at_2 - b = 0$

$\frac{1}{2}g(t_1^2 - t_2^2) + a(t_1 - t_2) = 0$

$\Rightarrow \frac{1}{2}g[(t_1 - t_2)(t_1 + t_2) + a(t_1 - t_2)] = 0$

$(t_1 - t_2)[\frac{1}{2}g(t_1 + t_2) + a] = 0$

$\Rightarrow t_1 - t_2 = 0$

$[\because \frac{1}{2}g[(t_1 + t_2) + a \neq 0]$

$t_1 = t_2$

∴ s(t) it is one – one function

5. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C) = F where F = $\frac{9}{5}$C + 32. Find, (i) t(0) (ii) t(28) (iii) t(–10) (iv) the value of C when t(C) = 212 (v) the temperature when the Celsius value is equal to the Farenheit value.

Solution:

$t(c) = F = \frac{9}{5}C + 32$

i) $t(0) = \frac{9}{5}(0) + 32 = 32°F$

ii) $t(28) = \frac{9}{5}(28) + 32 = 50.4 + 32 = 82.4°F$

iii) $t(-10) = \frac{9}{5}(-10) + 32 = -18 + 32 = 14°F$

iv) t(c) = 212

$212 = \frac{9}{5}C + 32 \Rightarrow \frac{9}{5}C + 32 = 212$

$\frac{9}{5}C = 212 - 32 \Rightarrow \frac{9}{5}C = 180$

$\Rightarrow C = 180 \times \frac{5}{9} = 100°C$

v) Celsius Value = Farenheit Value

$C = \frac{9}{5}C + 32 \Rightarrow 5C = 9C + 160$

$\Rightarrow 9C - 5C = -160 \Rightarrow 4C = -160;$

$\Rightarrow C = \frac{-160}{4} = -40°$

6. If f $(x) = 2x + 3$, g$(x) = 1 - 2x$ and h$(x) = 3x$. Prove that fo(goh) = (fog)oh

Solution:

$f(x) = 2x + 3$, $g(x) = 1 - 2x$, $h(x) = 3x$

Now, $(fog)(x) = f(g(x))$

$= f(1 - 2x) = 2(1-2x) + 3$

$= 2 - 4x + 3 = 5 - 4x$

Then,

$(fog) \circ h(x) = (fog)(h(x)) = (fog)(3x)$

$= 5 - 4(3x)$

$= 5 - 12x$ ..... (1)

$(goh)(x) = g(h(x)) = g(3x) = 1 - 2(3x)$

$= 1 - 6x$

So,

$fo(goh)(x) = f(1 - 6x) = 2(1 - 6x) + 3$

$= 2 - 12x + 3 = 5 - 12x$ ..... (2)

From (1) and (2),

we get (fog)oh = fo(goh)

Hence Proved.

7. Find x if gff(x) = fgg(x), given f$(x) = 3x + 1$ and g$(x) = x + 3$.

Solution:

$gff(x) = g[f\{f(x)\}] = g[f(3x + 1)]$

$= g[3(3x + 1) + 1] = g(9x + 4)$

$g(9x+4) = [(9x + 4) + 3] = 9x + 7$

$fgg(x) = f[g\{g(x)\}] = f[g(x+3)]$

$= f[(x+3) + 3] = f(x+6)$

$f(x+6) = [3(x+6) + 1] = 3x + 19$

$gff(x) = fgg(x)$

Thus two quantities begin equal we get

$9x + 7 = 3x + 19$

$9x - 3x = 12 \Rightarrow 6x = 12 \Rightarrow x = 3$

8. Consider the functions f(x), g(x), h(x) as given below. Show that (fog)oh = fo(goh) in each case.

(i) f $(x) = x - 1$, g$(x) = 3x + 1$ and h$(x) = x^2$

(ii) f$(x) = x^2$, g$(x) = 2x$ and h$(x) = x + 4$

(iii) f$(x) = x - 4$, g$(x) = x^2$ and h$(x) = 3x - 5$

Solution:

i. $(fog)oh = fo(goh)$

$[(x-1) \circ (3x+1)] \circ x^2 = (x-1) \circ [(3x+1) \circ x^2]$

$[3x + 1 - 1] \circ x^2 = (x - 1) \circ [3x^2 + 1]$

$[3x] \circ x^2 = 3x^2 + 1 - 1$

$3x^2 = 3x^2$

$(fog)oh = fo(goh)$

Hence proved.

ii. $(fog)oh = fo(goh)$

$[(x-4) \circ x^2] \circ (3x-5) = (x-4) \circ [x^2 \circ (3x-5)]$

$[x^2 - 4] \circ [3x - 5] = (x - 4) \circ [(3x - 5)^2]$

$[3x - 5]^2 - 4 = [3x - 5]^2 - 4$

[∵(a–b)2 = a2 – 2ab + b2]

$(fog)oh = fo(goh)$

Hence proved.

iii. $(fog)oh = fo(goh)$

$[(x^2 \circ 2x] \circ (x + 4) = (x^2 \circ [2x \circ (x + 4)]$

$[2x]^2 \circ (x + 4] = x^2 \circ [2(x + 4)]$

$[2(x + 4)]^2 = [2(x+4)]^2$

$(fog)oh = fo(goh)$

Hence proved.

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2 Numbers and Sequences

2 Marks

STAGE 2

1. If the Highest Common Factor of 210 and 55 is expressible in the form 55*x* – 325 , find *x*.

Solution:

Using Euclid's Division Algorithm, let us find the HCF of given numbers

$210 = 55 \times 3 + 45$

$55 = 45 \times 1 + 10$

$45 = 10 \times 4 + 5$

$10 = 5 \times 2 + 0$

Remainder = 0

So, the last divisor 5 is the Highest Common Factor.

Since, HCF is expressible in the form

$55x - 325 = 5$

gives $55x = 330$

Hence, $x = 6$

2. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of (i) 340 and 412 (ii) 867 and 255 (iii) 10224 and 9648

Solution:

By Euclid's Division Algorithm

a = bq + r

i) To find HCF of 340 and 412

$412 = 340(1) + 72$

$340 = 72(4) + 52$

$72 = 52(1) + 20$

$52 = 20(2) + 12$

$20 = 12(1) + 8$

$12 = 8(1) + 4$

$8 = 4(2) + 0$ Remainder 0

The remainder is 0, when the last divisor is 4.

∴ HCF of 340 and 412 is 4

ii) To find HCF of 867 and 255

$867 = 255(3) + 102$

$255 = 102(2) + 51$

$102 = 51(2) + 0$ Remainder 0

∴ HCF of 340 and 412 is 51

iii) To find HCF of 10224 and 9648

$10224 = 9648(1) + 576$

$9648 = 576(16) + 432$

$576 = 432(1) + 144$

$432 = 144(3) + 0$

∴ HCF of 10224 and 9648 is 144

3. If d is the Highest Common Factor of 32 and 60, find *x* and *y* satisfying d = 32*x* + 60y.

Solution:

Applying Euclid's Division Lemma,

a = bq + r

$60 = 32 \times 1 + 28$

$\Rightarrow 32 = 28 \times 1 + 4$

$28 = 4 \times 7 + 0$

∴ H.C.F. of 32 and 60 is 4

That is d = 4. $d = 32x + 60y$

$\Rightarrow 4 = 32x + 60y$

$4 = 32(2) + 60(-1)$

$\Rightarrow \therefore x = 2, y = -1$

4. Find the remainders when 70004 and 778 is divided by 7.

Solution:

Since 70000 is divisible by 7

$70000 \equiv 0 \pmod 7$

$70000 + 4 \equiv 0 + 4 \pmod 7$

$70004 \equiv 4 \pmod 7$

Therefore, the remainder when 70004 is divisded by 7 is 4

∵ 777 is divisible by 7

$777 \equiv 0 \pmod 7$

$777 + 1 \equiv 0 + 1 \pmod 7$

$778 \equiv 1 \pmod 7$

Therefore, the remainder when 778 is divisible by 7 is 1.

5. Determine the value of d such that 15 ≡ 3 (mod d)

Solution:

$15 \equiv 3 \pmod d$ means

$15 - 3 = kd$, for some integer k.

12 = kd gives d divides 12.

The divisors of 12 are 1, 2, 3, 4, 6, 12.

But d should be larger than 3 and so the possible values for d are 4, 6, 12.

6. Find the least positive value of x such that (i) $67 + x \equiv 1 \pmod 4$ (ii) 98 $(x + 4) \pmod 5$

Solution:

i) $67 + x \equiv 1 \pmod 4$

$67 + x - 1 = 4n$

$66 + x = 4n$

$66 + x$ is a multiple of 4.

68 is the nearest multiple of 4 more than 66.

Therefor the least positive value of x is 2.

ii) $98 \equiv (x + 4) \pmod 5$

$98 - (x + 4) = 5n$, for some integer n.

$94 - x = 5n$

$94 - x$ is a multiple of 5.

Therefore, the least positive value of x must be 4

$\because$ 94 − 4 = 90 is the nearest multiple of 5 less than 94.

7. Solve $8x \equiv 1 \pmod{11}$

Solution:

$8x - 1 = 11n$

$\Rightarrow \quad 8x = 11n + 1$

$\Rightarrow \quad x = \frac{11n+1}{8}$

$n = 5 \quad \Rightarrow x = 7$

$n = 13 \quad \Rightarrow x = 18$.......

8. Compute x, such that $10^4 \equiv x \pmod{19}$

Solution:

$10^2 = 100 \equiv 5 \pmod{19}$

$10^1 = (10^2)^2 \equiv 5^2 \pmod{19}$

$10^4 = 25 \pmod{19}$

$10^4 = 6 \pmod{19}$ [since $25 \equiv 6 \pmod{19}$]

Therefore, $x = 6$

9. Find the number of integer solutions of $3x \equiv 1 \pmod{15}$.

Solution:

$3x \equiv 1 \pmod{15}$ can be written as

$3x - 1 = 15k$ for some integer k

$3x = 15k + 1$

$x = \frac{15k+1}{3}$

$x = 5k + \frac{1}{3}$

Since 5k is an integer, $5k + \frac{1}{3}$ cannot be an integer.

So there is no integer solution.

10. Find the least positive value of x such that

(i) $71 \equiv x \pmod 8$ (ii) $78 + x \equiv 3 \pmod 5$

(iii) $89 \equiv (x + 3) \pmod 4$

(iv) $96 \equiv \frac{x}{7} \pmod 5$

(v) $5x \equiv 4 \pmod 6$

Solution:

i) $71 \equiv x \pmod 8 \Rightarrow 71 - x = 8k$

$\Rightarrow \quad 64 + 7 - x = 8k$

$\therefore \quad x = 7$

ii) $78 + x \equiv 3 \pmod 5$

$\Rightarrow 78 + x - 3 = 5k$

$\Rightarrow 75 + x$ is multiple of 5.

$\therefore$ The least positive value of $x = 0$

iii) $89 \equiv (x + 3) \pmod 4$.

$\Rightarrow 89 - x - 3 = 4k$

$\Rightarrow 86 - x = 4k$

$\Rightarrow 86 - x$ is multiple of 4 $\therefore x = 2$

iv) $96 \equiv \frac{x}{7} \pmod 5$

$96 - \frac{x}{7} = 5k$

$96 - \frac{x}{7}$ is multiple of 5 $\therefore x = 7$

v) $5x \equiv 4 \pmod 6$

$5x - 4 = 6k$

put, k = 1, $\Rightarrow 5x - 4 = 6; x = 2$

11. Solve: $5x \equiv 4 \pmod 6$

Solution:

$5x \equiv 4 \pmod 6$

$5x - 4 = 6k$

$5x = 6k + 4$

$x = \frac{6k+4}{5}$, k = 1, 6, 11, ...

If k = 1, $x = \frac{6(1)+4}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$

If k = 6, $x = \frac{6(6)+4}{5} = \frac{36+4}{5} = \frac{40}{5} = 8$

If k = 11, $x = \frac{6(11)+4}{5} = \frac{66+4}{5} = \frac{70}{5} = 14$

$\therefore x = 2, 8, 14, \ldots\ldots$

12. Solve: 3x − 2 ≡ 0 (mod 11)

Solution:

$3x - 2 \equiv 0 \pmod{11}$

$3x - 2 = 11k$

$3x = 11k + 2$

$x = \frac{11k+2}{3}$, k = 2, 5, 8, ….

If k = 2, $x = \frac{11(2)+2}{3} = \frac{22+2}{3} = \frac{24}{3} = 8$

If k = 5, $x = \frac{11(5)+2}{3} = \frac{55+2}{3} = \frac{57}{3} = 19$

If k = 8, $x = \frac{11(8)+2}{3} = \frac{88+2}{3} = \frac{90}{3} = 30$

$\therefore x = 8, 19, 30, \ldots\ldots$

5 Marks — STAGE 2

1. Find the HCF of 396, 504, 636.

Solution:

To find HCF of three given numbers, first we have to find HCF of the first two numbers.

To find HCF of 396 and 504.

Using Euclid's division algorithm we get

$504 = 396 \times 1 + 108$

The remainder is $108 \neq 0$

$396 = 108 \times 3 + 72$

The remainder is $72 \neq 0$

$108 = 72 \times 1 + 36$

The remainder is $36 \neq 0$

$72 = 36 \times 2 + 0$

The remainder is Zero.

Therefore HCF of 396, 504 = 36.

To find the HCF of 636 and 36.

Using Euclid's division algorithm we get

$636 = 36 \times 17 + 24$

The remainder is $24 \neq 0$

$36 = 24 \times 1 + 12$

The remainder is $12 \neq 0$

$24 = 12 \times 2 + 0$

The remainder is zero.

Therefore HCF of 636, 36 = 12.

Therefore, Highest Common Factor of 396, 504 and 636 is 12.

2. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of 84, 90 and 120.

Solution:

To find HCF of 84, 90 and 120

First to find HCF of 84 and 90

$90 = 84q + r \quad (6 \neq 0)$

$90 = 84 \times 1 + 6$

$84 = 6 \times 14 + 0$

$\therefore$ HCF of 84, 90 = 6.

Then to find HCF of 6 and 120

$120 = 6 \times 20 + 0$

$\therefore$ HCF of 84, 90,120 is 6

3. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms. SEP-21

Solution:

Let the 3 consecutive terms in an A.P. be a–d, a, a+d.

Sum of three terms

$a-d + a + a+d = 27$

$3a = 27,$

$a = \frac{27}{3}$

$a = 9$

Product of three terms

$(a-d)(a)(a+d) = 288$

$a(a^2 - d^2) = 288$

$9(9^2 - d^2) = 288$

$81 - d^2 = \frac{288}{9}$

$81 - d^2 = 32$

$49 = d^2 \therefore d = \pm 7$

$\therefore$ The three terms of A.P are

2, 9, 16 (or) 16, 9, 2

4. The ratio of 6^{th} and 8^{th} term of an A.P. is 7:9. Find the ratio of 9^{th} term to 13^{th} term.

Solution: MAY-22

$t_6 : t_8 = 7 : 9$

$\Rightarrow \frac{t_6}{t_8} = \frac{7}{9} \Rightarrow \frac{a+5d}{a+7d} = \frac{7}{9}$

$\Rightarrow 9a + 45d = 7a + 49d \Rightarrow a = 2d$

To find $t_9 : t_{13}$

$\Rightarrow \frac{t_9}{t_{13}} = \frac{a+8d}{a+12d} = \frac{10d}{14d} = \frac{5}{7}$

The required ratio is 5 : 7.

5. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.

Solution:

	1 year	2 year
Income	₹15,000	₹16,500
Expenses	₹13,000	₹13,900
Savings	₹2,000	₹2,600

$\therefore$ Annual Savings ₹2,000, ₹2,600, ₹3,200…

Here a = 2,000, d = 600, t_n = 20,000

$a+(n-1)d = 20,000$

$\Rightarrow 2000+(n-1)600 = 20,000$

$\Rightarrow (n-1)600 = 20,000 - 2000 = 18000$

$\Rightarrow n - 1 = \frac{18000}{600}$

$\Rightarrow n - 1 = 30$

$\Rightarrow n = 31$ years

The savings of Priya after 31 years is ₹ 20,000.

6. A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹4623. Find the amount received by each child.

Solution:

Let the amount received by the three children be in the form of a – d, a, a + d.

Since, sum of the amount is ₹ 207

(a – d) + a + (a + d) = 207.

$3a = 207 \Rightarrow a = 69$

It is given that product of the two least amounts is 4623.

$(a - d)a = 4623$

$(69 - d)69 = 4623;$

$69 - d = \frac{4623}{69} = 67$

$\therefore d = 2$

Therefore, amount given by the mother to her three children are ₹(69 – 2), ₹69 , ₹(69 + 2)

That is ₹67, ₹69 and ₹71.

7. Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Solution:

301 + 308 + 315 + … + 595 = ?

a = 300; d = 7; l = 595

$n = \left(\frac{l-a}{d}\right) + 1$

```
     42              8
 7 | 300         7 | 600
   | 28            | 56
   |----           |----
   |  20           |  40
   |  14           |  35
   |----           |----
      6               5
```

a = 300 + 7 – 6 $\quad$ l = 600 – 5

a = 301 $\quad$ l = 595

$n = \frac{595-300}{7} + 1$

$n = \frac{294}{7} + 1$

$n = 42 + 1$

$n = 43$

$S_n = \frac{n}{2}(a + l)$

$S_{43} = \frac{43}{2}(301 + 595)$

$= \frac{43}{2}(896) = 43 \times 448$

$S_{43} = 19264$

8. The sum of first n, 2n and 3n terms of an A.P. are S_1, S_2, and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Solution:

If S_1, S_2 and S_3 are the sum of first n, 2n and 3n terms of an A.P. respectively then

$S_1 = \frac{n}{2}[2a+(n-1)d],$

$S_2 = \frac{2n}{2}[2a+(2n-1)d],$

$S_3 = \frac{3n}{2}$ [2a+(3n–1)d],

$S_2 – S_1$

$= \frac{2n}{2}$ [2a+(2n–1)d] – $\frac{n}{2}$ [2a+(n–1)d]

$= \frac{n}{2}$ [[4a+2(2n–1)d] – [2a+(n–1)d]

$S_2 – S_1 = \frac{n}{2} \times$ [2a+(3n–1)d]

$3(S_2 – S_1) = \frac{3n}{2}$ [2a+(3n–1)d]

$3(S_2 – S_1) = S_3$

9. In a Geometric progression, the 4th term is $\frac{8}{9}$ and the 7^{th} term is $\frac{64}{243}$.
Find the Geometric Progression.

Solution:

4^{th} term $t_4 = \frac{8}{9} \Rightarrow ar^3 = \frac{8}{9}$ ..... (1)

7^{th} term $t_7 = \frac{64}{243} \Rightarrow ar^6 = \frac{64}{243}$ ..... (2)

Dividing (2) by (1)

we get $\frac{ar^6}{ar^3} = \frac{\frac{64}{243}}{\frac{8}{9}} = \frac{64}{243} \times \frac{9}{8} = \frac{8}{27}$

$r^3 = \frac{8}{27} \Rightarrow r = \frac{2}{3}$

Substituting the value of r in (1),

we get, $a \times \left(\frac{2}{3}\right)^3 = \frac{8}{9} \Rightarrow a = 3$

Therefore the Geometric Progession

a, ar, ar^2….. That is , 3, 2, $\frac{4}{3}$,….

10. Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Solution:

A.P. $\Rightarrow x$, 10, y, 24, z

That is $y = \frac{10+24}{2}$

$= \frac{34}{2} = 17$

$\therefore$ A.P = x, 10, 17, 24, z

Here we know that d = 17 – 10 = 7

$\therefore x = 10 - 7 = 3$

$z = 24 + 7 = 31$

$\therefore x = 3, y = 17, z = 31.$

11. Find the sum of all odd positive integers less than 450.

Solution:

The required answer = 1 + 3 + 5 + ... + 449

Here, a = 1, d = 2, l = 449

$n = \left(\frac{l-a}{d}\right) + 1$

$= \frac{449-1}{2} + 1 = 225$

$\Rightarrow S_n = \frac{225}{2}[1 + 449]$ $\quad \because S_n = \frac{n}{2}$ [a+1]

$= 225 \times 225$

$= 50625$

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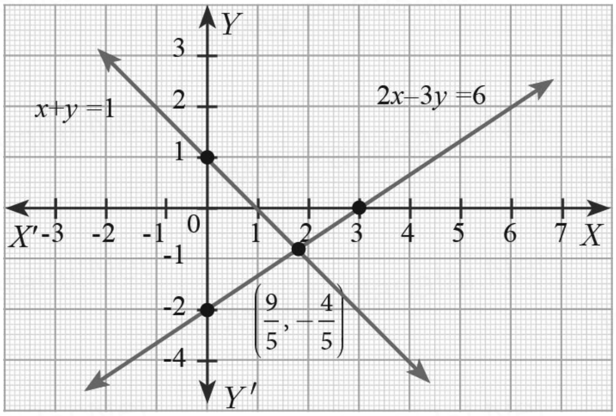
3 Algebra

2 Marks STAGE 2

1. Solve: 2x − 3y = 6, x + y = 1

Solution:



$2x - 3y = 6$ ..... (1)

$x + y = 1$ ..... (2)

$(1) \times 1 \Rightarrow \quad 2x - 3y = 6$

$(2) \times 3 \Rightarrow \quad 3x + 3y = 3(+)$

$$5x = 9 \Rightarrow x = \frac{9}{5}$$

$(2) \Rightarrow \frac{9}{5} + y = 1$

$y = 1 - \frac{9}{5}$

$y = \frac{5-9}{5} = \frac{-4}{5}$

Therefore, $x = \frac{9}{5}$, $y = \frac{-4}{5}$

2. Reduce each of the following rational expressions to its lowest form.

i) $\frac{x^2-1}{x^2+x}$ ii) $\frac{x^2-11x+18}{x^2-4x+4}$

iii) $\frac{9x^2-81x}{x^3-8x^2-9x}$ iv) $\frac{p^2-3p-40}{2p^3-24p^2-64p}$

Solution:

i) $\frac{x^2-1}{x^2+x} = \frac{x^2-1^2}{x(x+1)} = \frac{(x+1)(x-1)}{x(x+1)}$

$= \frac{(x-1)}{x}$

ii) $\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)(x-2)}{(x-2)(x-2)} = \frac{x-9}{x-2}$

iii) $\frac{9x^2+81x}{x^3-8x^2-9x} = \frac{9x(x+9)}{x(x^2+8x-9)}$

$= \frac{9x(x+9)}{(x)(x+9)(x-1)} = \frac{9}{x-1}$

iv) $\frac{p^2-3p-40}{2p^3-24p^2-64p} = \frac{(p-8)(p+5)}{2p(p-8)(p-4)}$

$= \frac{(p+5)}{2p(p-4)}$

3. Determine the nature of roots for the following quadratic equations

(i) $x^2 - x - 20 = 0$ (ii) $9x^2 - 24x + 16 = 0$

(iii) $2x^2 - 2x + 9 = 0$

Solution:

(i) $x^2 - x - 20 = 0$

Here, a = 1, b = –1, c = –20

Now, $\Delta = b^2 - 4ac$;

$\Delta = (-1)^2 - 4(1)(-20) = 81$

Here $\Delta = 81 > 0$.

So the equation will have real and unequal roots.

(ii) $9x^2 - 24x + 16 = 0$

Here, a = 9, b = –24, c = 16

Now, $\Delta = b^2 - 4ac$;

$\Delta = (-24)^2 - 4(9)(-16) = 0$

Here $\Delta = 0$.

So the equation will have real and equal roots.

(iii) $2x^2 - 2x + 9 = 0$

Here, a = 2, b = –2, c = 9

Now, $\Delta = b^2 - 4ac$;

$\Delta = (-2)^2 - 4(2)(9) = -68$

$\Delta = -68 < 0$.

So the equation will have no real roots.

4. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

Solution:

Let x be the required number

$\frac{1}{x}$ be its reciprocal

Given $x - \frac{1}{x} = \frac{24}{5}$

$\frac{x^2-1}{x} = \frac{24}{5}$

$$5x^2 - 5 = 24x$$
$$5x^2 - 24x - 5 = 0$$
$$5x^2 - 25x + x - 5 = 0$$
$$x = 5, -\frac{1}{5}$$

5. Determine the nature of the roots for the following quadratic equations

(i) $15x^2 + 11x + 2 = 0$ SEP-21

(ii) $x^2 - x - 1 = 0$

Solution:

(i) $15x^2 + 11x + 2 = 0$

$a = 15,\ b = 11,\ c = 2$

$\Delta = b^2 - 4ac$

$= 11^2 - 4 \times 15 \times 2$

$= 121 - 120 = 1 = (+)$ve

$\therefore$ The roots are real and unequal.

(ii) $x^2 - x - 1 = 0$

$a = 1,\ b = -1,\ c = -1$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(1)(-1)$

$= 1 + 4 = 5$

$\therefore$ The roots are real and unequal.

5 Marks — STAGE 2

1. Solve the following system of linear equations in three variables

$3x - 2y + z = 2,\ 2x + 3y - z = 5,\ x + y + z = 6.$

Solution:

$3x - 2y + z = 2$ ..... (1)

$2x + 3y - z = 5$ ..... (2)

$x + y + z = 6$ ..... (3)

Adding (1) and (2)

$$\begin{array}{r} 3x - 2y + z = 2 \\ 2x + 3y - z = 5\ (+) \\ \hline 5x + y = 7 \end{array}$$ ..... (4)

Adding (2) and (3)

$$\begin{array}{r} 2x + 3y - z = 5 \\ x + y + z = 6\ (+) \\ \hline 3x + 4y = 11 \end{array}$$ ..... (5)

(4) × 4 – (5)

$$\begin{array}{r} 20x + 4y = 28 \\ 3x + 4y = 11\ (-) \\ \hline 17x = 17 \end{array} \Rightarrow x = 1$$

Substituting $x = 1$ in (4), $5 + y = 7 \Rightarrow y = 2$

Substituting $x = 1, y = 2$ in (3), $1 + 2 + z = 6$

we get, $z = 3$

Therefore, $x = 1, y = 2, z = 3$

2. Solve: $x + 2y - z = 5\ ;\ x - y + z = -2;$ $-5x - 4y + z = -11$

Solution:

$x + 2y - z = 5$ ..... (1)

$x - y + z = -2$ ..... (2)

$-5x - 4y + z = -11$ ..... (3)

Adding (1) and (2) we get

$$\begin{array}{r} x + 2y - z = 5 \\ x - y + z = -2\ (+) \\ \hline 2x + y = 3 \end{array}$$ ..... (4)

Subtracting (2) and (3)

$$\begin{array}{r} x - y + z = -2 \\ -5x - 4y + z = -11\ (-) \\ \hline 6x + 3y = 9 \end{array}$$

Dividing by 3 $2x + y = 3$ ..... (5)

Subtracting (4) and (5),

$$\begin{array}{r} 2x + y = 3 \\ 2x + y = 3 \\ \hline 0 = 0 \end{array}$$

Here we arrive at an identity $0 = 0$.

Hence the system has an infinite number of solutions.

3. Solve: $3x + y - 3z = 1\ ;\ -2x - y + 2z = 1;$ $-x - y + z = 2.$

Solution:

$3x + y - 3z = 1$ ..... (1)

$-2x - y + 2z = 1$ ..... (2)

$-x - y + z = 2$ ..... (3)

Adding (1) and (2),

$$\begin{array}{r} 3x + y - 3z = 1 \\ -2x - y + 2z = 1\ (+) \\ \hline x - z = 2 \end{array}$$ ..... (4)

Adding (1) and (3),

$$\begin{array}{r} 3x + y - 3z = 1 \\ -x - y + z = 2\ (+) \\ \hline 2x - 2z = 3 \end{array}$$ ......(5)

$(5) \times 1 \Rightarrow 2x - 2z = 3$

$(4) \times 2 \Rightarrow 2x - 2z = 4\ (-)$

$0 = -1$

Here we arrive at a contradiction as $0 \neq -1$.

This means that the system is inconsistent and has no solution.

4. Solve the following system of linear equations in three variables SEP-21

$x + y + z = 5$; $2x - y + z = 9$; $x - 2y + 3z = 16$

Solution:

$x + y + z = 5$ ..... (1)

$2x - y + z = 9$ ..... (2)

$x - 2y + 3z = 16$ ..... (3)

$(1) - (3) \Rightarrow 3y - 2z = -11$ ..... (4)

$(2) \Rightarrow 2x - y + z = 9$

$(1) \times 2 \Rightarrow 2x + 2y + 2z = 10\ (-)$

$-3y - z = -1$ ..... (5)

(4) + (5)

$3y - 2z = -11$

$-3y - z = -1 (+)$

$-3z = -12$

$z = 4$

Substitute $z = 4$ in (5)

$-3y - 4 = -1$

$\Rightarrow -3y = 3$

$\Rightarrow y = -1$

Substitute $y = -1$, $z = 4$ in (1)

$\Rightarrow x - 1 + 4 = 5$

$\Rightarrow x + 3 = 5$

$\Rightarrow x = 2$

Therefore, $x = 2$, $y = -1$, $z = 4$

5. Discuss the nature of solutions of the following system of equation

$x + 2y - z = 6$; $-3x - 2y + 5z = -12$; $x - 2z = 3$

Solution:

$x + 2y - z = 6$ ..... (1)

$-3x - 2y + 5z = -12$ ..... (2)

$x - 2z = 3$ ..... (3)

$(1) + (2) \Rightarrow -2x + 4z = -6$ ..... (4)

$(4) + (3) \times 2 \Rightarrow 2x - 4z = 6$

$0 = 0$

Here we arrive at an identity $0 = 0$.

Hence the system has an infinite number of solution.

6. Vani, her father and her grand father have an average age of 53. One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now?

Solution:

Let the present age of Vani, her father, grand father be x, y, z respectively.

By data given,

$$\frac{x+y+z}{3} = 53$$

$\Rightarrow x + y + z = 159$ ..... (1)

$$\frac{1}{2}z + \frac{1}{3}y + \frac{1}{4}x = 65$$

$$\frac{6z+4y+3x}{12} = 65$$

$3x + 4y + 6z = 780$ ..... (2)

$(z - 4) = 4(x - 4)$

$\Rightarrow 4x - z = 12$ ..... (3)

From (1) & (2)

$(1)\times 4 \Rightarrow 4x + 4y + 4z = 636$

$(2) \Rightarrow 3x + 4y + 6z = 780$

(subtracting) $x - 2z = -144$ ..... (4)

From (3) & (4)

$(3)\times 2 \Rightarrow 8x - 2z = 24$

$(4) \Rightarrow x - 2z = -144$

(subtracting) $7x = 168$ ..... (5)

$$x = \frac{168}{7} = 24$$

Substitute $x = 24$ in(3)

$4(24) - z = 12$

$96 - z = 12$

$z = 84$

$(1) \Rightarrow 24 + y + 84 = 159$

$\Rightarrow y = 51$

∴ Vani's Present Age = 24 years

Father's Present Age = 51years

Grand father's Age = 84 years

7. Find the GCD of the polynomials $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

Solution:

Let $f(x) = 2x^3 - 5x^2 + 5x - 3$ and

$g(x) = x^3 + x^2 - x + 2$

$$
\begin{array}{r|l}
 & 2 \\
\hline
x^3 + x^2 - x + 2 & 2x^3 - 5x^2 + 5x - 3 \\
 & 2x^3 + 2x^2 - 2x + 4 \ (-) \\
\hline
 & -7x^2 + 7x - 7 \\
\hline
 & = -7(x^2 - x + 1)
\end{array}
$$

$-7(x^2 - x + 1) \neq 0$,

note that – 7 is not a divisor of g(x).

Now dividing g(x) = $x^3 + x^2 - x + 2$ by the new remainder $x^2 - x + 1$ (leaving the constant factor), we get

$$
\begin{array}{r|l}
 & x+2 \\
\hline
x^2 - x + 1 & x^3 + x^2 - x + 2 \\
 & x^3 - x^2 + x \quad (-) \\
\hline
 & 2x^2 - 2x + 2 \\
 & 2x^2 - 2x + 2 \\
\hline
 & 0
\end{array}
$$

Here we get zero remainder.

Therefore, GCD $(2x^3 - 5x^2 + 5x - 3,$
$x^3 + x^2 - x + 2) = x^2 - x + 1$

8. Find the GCD of $6x^3 - 30x^2 + 60x - 48$ and $3x^3 - 12x^2 + 21x - 18$.

Solution:

$f(x) = 6x^3 - 30x^2 + 60x - 48$
$= 6(x^3 - 5x^2 + 10x - 8)$

and $g(x) = 3x^3 - 12x^2 + 21x - 18$
$= 3(x^3 - 4x^2 + 7x - 6)$

Now, we shall find the GCD of
$x^3 - 5x^2 + 10x - 8$ and $x^3 - 4x^2 + 7x - 6$

$$
\begin{array}{r|l}
 & 1 \\
\hline
x^3 - 5x^2 + 10x - 8 & x^3 - 4x^2 + 7x - 6 \\
 & x^3 - 5x^2 + 10x - 8 \ (-) \\
\hline
 & x^2 - 3x + 2 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & x-2 \\
\hline
x^2 - 3x + 2 & x^3 - 5x^2 + 10x - 8 \\
 & x^3 - 3x^2 + 2x \\
\hline
 & -2x^2 + 8x - 8 \\
 & -2x^2 + 6x - 4 \ (-) \\
\hline
 & 2x - 4 \\
 & = 2(x - 2)
\end{array}
$$

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x - 2 & x^2 - 3x + 2 \\
 & x^2 - 2x \quad (-) \\
\hline
 & -x + 2 \\
 & -x + 2 \quad (-) \\
\hline
 & 0
\end{array}
$$

Here, we get zero as remainder, GCD of leading coefficients 3 and 6 is 3.

Thus, GCD $[(6x^3 - 30x^2 + 60x - 48,$
$3x^3 - 12x^2 + 21x - 18)] = 3(x - 2)$

9. Find the GCD of the given polynomials

(i) $x^4 + 3x^3 - x - 3$, $x^3 + x^2 - 5x + 3$ SEP-20

(ii) $x^4 - 1$, $x^3 - 11x^2 + x - 11$

(iii) $3x^4 + 6x^3 - 12x^2 - 24x$, $4x^4 + 14x^3 + 8x^2 - 8x$

(iv) $3x^3 + 3x^2 + 3x + 3$, $6x^3 + 12x^2 + 6x + 12$

Solution:

i. $f(x) = x^4 + 3x^3 + x - 3$ and
$g(x) = x^3 + x^2 - 5x + 3$

$$
\begin{array}{r|l}
 & x+2 \\
\hline
x^3 + x^2 - 5x + 3 & x^4 + 3x^3 + 0 - x - 3 \\
 & x^4 + x^3 - 5x^2 + 3x \\
\hline
 & 2x^3 + 5x^2 - 4x - 3 \\
 & 2x^3 + 2x^2 - 10x + 6 \ (-) \\
\hline
 & 3x^2 + 6x - 9 \\
 & = 3(x^2 + 2x - 3)
\end{array}
$$

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x^2 + 2x - 3 & x^3 + x^2 - 5x + 3 \\
 & x^3 + 2x^2 - 3x \quad (-) \\
\hline
 & -x^2 - 2x + 3 \\
 & -x^2 - 2x + 3 \quad (-) \\
\hline
 & 0
\end{array}
$$

$\therefore$ GCD of (f(x), g(x)) $= x^2 + 2x - 3$

ii. $f(x) = x^4 - 1$ and $g(x) = x^3 - 11x^2 + x - 11$

$$
\begin{array}{r|l}
 & x+11 \\
\hline
x^3 - 11x^2 + x - 11 & x^4 + 0x^3 + 0x^2 - 0x - 1 \\
 & x^4 - 11x^3 + x^2 - 11x \quad (-) \\
\hline
 & 11x^3 - x^2 + 11x - 1 \\
 & 11x^3 - 121x^2 + 11x - 121 \ (-) \\
\hline
 & 120x^2 + 120 \\
 & = 120(x^2 + 1)
\end{array}
$$

$$
\begin{array}{r|l}
 & x-11 \\
\hline
x^2 - 0x + 1 & x^3 - 11x^2 + x - 11 \\
 & x^3 + 0x^2 + x \quad (-) \\
\hline
 & -11x^2 + 0x - 11 \\
 & -11x^2 + 0x - 11 \ (-) \\
\hline
 & 0
\end{array}
$$

$\therefore$ GCD of (f(x), g(x)) $= x^2 + 1$

iii. $f(x) = 3x^4 + 6x^3 - 12x^2 - 24x$
$= 3x(x^3 + 2x^2 - 4x - 8)$

$g(x) = 4x^4 + 14x^3 + 8x^2 - 8x$

$= 2x(2x^3 + 7x^2 + 4x - 4)$

G.C.D of $(3x, 2x)$ is x

$$\begin{array}{r|l} & 2x + 3 \\ \hline x^3 + 2x^2 - 4x - 8 & 2x^3 + 7x^2 + 4x - 4 \\ & 2x^3 + 4x^2 - 8x - 16 \ (-) \\ \hline & 3x^2 + 12x + 12 \end{array}$$

$= 3(x^2 + 4x + 4)$

$$\begin{array}{r|l} & x - 2 \\ \hline x^2 + 4x + 4 & x^3 + 2x^2 - 4x - 8 \\ & x^3 + 4x^2 + 4x \ (-) \\ \hline & -2x^2 - 8x - 8 \\ & -2x^2 - 8x - 8 \ (-) \\ \hline & 0 \end{array}$$

$\therefore$ GCD of (f(x), g(x)) $= x(x^2 + 4x + 4)$

iv. $f(x) = 3x^3 + 3x^2 + 3x + 3$

$= 3(x^3 + x^2 + x + 1)$

$g(x) = 6x^3 + 12x^2 + 6x + 12$

$= 6(x^3 + 2x^2 + x + 2)$

GCD of(3, 6) = 3

$$\begin{array}{r|l} & 1 \\ \hline x^3 + x^2 + x + 1 & x^3 + 2x^2 + x + 2 \\ & x^3 + x^2 + x + 1 \ (-) \\ \hline & x^2 + 0x + 1 \end{array}$$

$$\begin{array}{r|l} & x + 1 \\ \hline x^2 + 0x + 1 & x^3 + x^2 + x + 1 \\ & x^3 + 0x^2 + x \ (-) \\ \hline & x^2 + 0x + 1 \\ & x^2 + 0x + 1 \\ \hline & 0 \end{array}$$

$\therefore$ GCD of (f(x), g(x)) $= 3(x^2 + 1)$

10. Simplify: $\dfrac{b^2 + 3b - 28}{b^2 + 4b + 4} \div \dfrac{b^2 - 49}{b^2 - 5b - 14}$

Solution:

$$\frac{b^2 + 3b - 28}{b^2 + 4b + 4} \div \frac{b^2 - 49}{b^2 - 5b - 14}$$

$$= \frac{(b-4)(b+7)}{(b+2)(b+2)} \times \frac{(b-7)(b+2)}{(b+7)(b-7)}$$

$$= \frac{b-4}{b+2}$$

11. If $A = \dfrac{2x+1}{2x-1}$, $B = \dfrac{2x-1}{2x+1}$

find $\dfrac{1}{A-B} - \dfrac{2B}{A^2 - B^2}$

Solution:

$$\frac{1}{A-B} - \frac{2B}{A^2 - B^2}$$

$$= \frac{1}{A-B} - \frac{2B}{(A+B)(A-B)}$$

$$= \frac{A+B-2B}{(A+B)(A-B)} = \frac{(A-B)}{(A+B)(A-B)}$$

$$= \frac{1}{A+B} = \frac{1}{\frac{2x+1}{2x-1} + \frac{2x-1}{2x+1}}$$

$$= \frac{1}{\frac{(2x+1)^2 + (2x-1)^2}{(2x+1)(2x-1)}}$$

$$= \frac{(2x+1)(2x-1)}{(2x+1)^2 + (2x-1)^2}$$

$$= \frac{[2x]^2 - 1^2}{4x^2 + 1 + 4x + 4x^2 + 1 - 4x}$$

$$= \frac{4x^2 - 1}{8x^2 + 2} = \frac{4x^2 - 1}{2(4x^2 + 1)}$$

12. If $A = \dfrac{x}{x+1}$, $B = \dfrac{1}{x+1}$

prove that $\dfrac{(A+B)^2 + (A-B)^2}{A \div B} = \dfrac{2(x^2+1)}{x(x+1)^2}$

Solution:

Given $A = \dfrac{x}{x+1}$, $B = \dfrac{1}{x+1}$

$$\frac{(A+B)^2 + (A-B)^2}{A \div B} = \frac{2(A^2 + B^2)}{A \div B}$$

$$A^2 + B^2 = \frac{x^2}{(x+1)^2} + \frac{1}{(x+1)^2} = \frac{x^2+1}{(x+1)^2}$$

$$A \div B = \frac{x}{x+1} \times \frac{x+1}{1} = x$$

$$\frac{2(A^2 + B^2)}{A \div B} = (2)\left(\frac{x^2+1}{(x+1)^2}\right)\left(\frac{1}{x}\right)$$

$$= \frac{2(x^2+1)}{x(x+1)^2}$$

13. Simplify:

$$\frac{1}{x^2 - 5x + 6} + \frac{1}{x^2 - 3x + 2} - \frac{1}{x^2 - 8x + 15}$$

Solution:

$$\frac{1}{x^2 - 5x + 6} + \frac{1}{x^2 - 3x + 2} - \frac{1}{x^2 - 8x + 15}$$

$$= \frac{1}{(x-2)(x-3)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x^2-6x+5)+(x^2-8x+15)-(x^2-3x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x-9}{(x-1)(x-3)(x-5)}$$

14. Find the square root of the following expressions

(i) $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$

(ii) $(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$

(iii) $\left[\sqrt{15}x^2 + \left(\sqrt{3}+\sqrt{10}\right)x + \sqrt{2}\right]\left[\sqrt{5}x^2 + \left(2\sqrt{5}+1\right)x + 2\right]\left[\sqrt{3}x^2 + \left(\sqrt{2}+2\sqrt{3}\right)x + 2\sqrt{2}\right]$

Solution:

i) $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$

$$= \sqrt{(4x)^2 + (-3y)^2 + 3^2 + 2(4x)(-3y) + 2(-3y)(3) + 2(4x)(3)}$$

$$\therefore \sqrt{(4x-3y+3)^2} = |4x - 3y + 3|$$

ii) $\sqrt{(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)}$

$$= \sqrt{(3x-1)(2x+1)(3x-1)(x+1)(2x+1)(x+1)}$$

$$= |(3x-1)(2x+1)(x+1)|$$

iii) First let us factorize the polynomials

$$\sqrt{15}x^2 + \left(\sqrt{3}+\sqrt{10}\right)x + \sqrt{2} = \sqrt{15}x^2 + \sqrt{3}x + \sqrt{10}x + \sqrt{2}$$

$$= \sqrt{3}x\left(\sqrt{5}x+1\right) + \sqrt{2}\left(\sqrt{5}x+1\right)$$

$$= \left(\sqrt{5}x+1\right)\times\left(\sqrt{3}x+\sqrt{2}\right)$$

$$\sqrt{5}x^2 + \left(2\sqrt{5}+1\right)x + 2 = \sqrt{5}x^2 + 2\sqrt{5}x + x + 2$$

$$= \sqrt{5}x(x+2) + 1(x+2) = \left(\sqrt{5}x+1\right)(x+2)$$

$$\sqrt{3}x^2 + \left(\sqrt{2}+2\sqrt{3}\right)x + 2\sqrt{2} = \sqrt{3}x^2 + \sqrt{2}x + 2\sqrt{3}x + 2\sqrt{2}$$

$$= x\left(\sqrt{3}x+\sqrt{2}\right) + 2\left(\sqrt{3}x+\sqrt{2}\right)$$

$$= (x+2)\left(\sqrt{3}x+\sqrt{2}\right)$$

Therefore,

$$\sqrt{\left[\sqrt{15}x^2+\left(\sqrt{3}+\sqrt{10}\right)x+\sqrt{2}\right]\left[\sqrt{5}x^2+\left(2\sqrt{5}+1\right)x+2\right]\left[\sqrt{3}x^2+\left(\sqrt{2}+2\sqrt{3}\right)x+2\sqrt{2}\right]}$$

$$= \sqrt{\left(\sqrt{5}x+1\right)\left(\sqrt{3}x+\sqrt{2}\right)\left(\sqrt{5}x+1\right)(x+2)\left(\sqrt{3}x+\sqrt{2}\right)(x+2)}$$

$$= \left|\left(\sqrt{5}x+1\right)\left(\sqrt{3}x+\sqrt{2}\right)(x+2)\right|$$

15. The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Solution:

Let the present age of Kumaran be x years.

Two years ago, his age is $(x - 2)$ years.

Four years from now, his age = $(x + 4)$ years

Given

$$(x-2)(x+4) = 1 + 2x$$

$$x^2 + 2x - 8 = 1 + 2x$$

$$x^2 - 9 = 0$$

gives $(x-3)(x+3) = 0$

Then, $x = \pm 3$ (Rejecting -3 as age cannot be negative).

Kumaran's present age is 3 years.

16. A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains.

Solution:

Let the average speed of passenger train be x km/hr.

Then the average speed of express train will be $(x + 20)$ km/hr.

Time taken by the passenger train to cover distance of 240 km = $\frac{240}{x}$ hr

Time taken by the express train to cover distance of 240 km = $\frac{240}{x+20}$ hr

Given,
$$\frac{240}{x} = \frac{240}{x+20} + 1$$

$$\frac{240}{x} - \frac{240}{x+20} = 1$$

$$240\left[\frac{1}{x} - \frac{1}{x+20}\right] = 1$$

$$\Rightarrow \quad 240\left[\frac{x+20-x}{x(x+20)}\right] = 1$$

$$\Rightarrow \quad 4800 = (x^2+20x)$$

$$x^2+20x-4800 = 0$$

$$\Rightarrow \quad (x+80)(x-60) = 0$$

$$\Rightarrow \quad x = -80 \text{ or } 60$$

Therefore $x = 60$ (Rejecting -80 as speed cannot be negative)

Average speed of the passenger train is 60 km/hr.

Average speed of the express train is 80 km/hr.

17. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

Solution:

Let x be the original speed.

From the given data Time taken to cover the distance in

the original speed $T_1 = \frac{90}{x}$

Time taken to cover the same distance in

the increased speed $T_2 = \frac{90}{x+15}$

Given that $T_1 - T_2 = \frac{1}{2}$

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

$$90\left(\frac{1}{x} - \frac{1}{x+15}\right) = \frac{1}{2}$$

$$90\left(\frac{x+15-x}{x(x+15)}\right) = \frac{1}{2}$$

$$90\left(\frac{15}{x^2+15x}\right) = \frac{1}{2}$$

$$x^2+15x = 2700$$

$$x^2+15x-2700 = 0$$

$$(x+60)(x-45) = 0$$

$x = -60$ is not admissible, So $x = 45$

∴ The original Speed is 45 km /hr.

18. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are real and equal prove that either a = 0 (or) $a^3 + b^3 + c^3 = 3abc$

Solution:

$(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

$A = c^2 - ab$, $B = -2(a^2 - bc)$, $C = b^2 - ac$

$\Delta = 0$

$B^2 - 4AC = 0$.

The roots are real and equal.

$[-2(a^2 - bc)]^2 - 4[c^2 - ab][b^2 - ac] = 0$

$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$4[a^4 + b^2c^2 - 2a^2bc]$

$- 4[b^2c^2 - ac^3 - ab^3 + a^2bc] = 0$

$4[a^4 + b^2c^2 - 2a^2bc - b^2c^2$

$+ ac^3 + ab^3 - a^2bc] = 0$

$a^4 + ab^3 + ac^3 - 3a^2bc = 0$

$a[a^3+b^3+c^3 - 3abc] = 0$

$a = 0$ or $a^3+b^3+c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

19. If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 find k.

Solution:

$x^2 - 13x + k = 0$,

Here $a = 1, b = -13, c = k$

Let α. β be the roots of the equation, Then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-13)}{1} = 13 \quad \text{..... (1)}$$

Also $\alpha - \beta = 17$ ..... (2)

(1) + (2) we get, $2\alpha = 30$ gives $\alpha = 15$.

Therefore, $15 + \beta = 13$

(from (1)) gives $\beta = -2$

But $\alpha\beta = \frac{c}{a} = \frac{k}{1}$,

$15 \times (-2) = k$ we get, $k = -30$

20. If α and β are the roots of $x^2 + 7x + 10 = 0$ find the values of

i) $\alpha - \beta$ ii) $\alpha^2 + \beta^2$ iii) $\alpha^3 - \beta^3$

iv) $\alpha^4 + \beta^4$ v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution:

$x^2 + 7x + 10 = 0$

Here, $a = 1, b = 7, c = 10$

If α and β are the roots of the equation then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(+7)}{1} = -7$$

$$\alpha\beta = \frac{c}{a} = \frac{10}{1} = 10$$

i) $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{(-7)^2 - 4\times 10} = \sqrt{9} = 3$

ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-7)^2 - 2 \times 10 = 29$

iii) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

$= (3)^3 + 3(10)(3) = 117$

iv) $\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

(From (ii), $\alpha^2 + \beta^2 = 29$)

Thus, $29^2 - 2 \times (10)^2 = 641$

v) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{49-20}{10} = \frac{29}{10}$

vi) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{(-343)-3(10\times(-7))}{10} = \frac{-343+210}{10}$

$= \frac{-133}{10}$

21. If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of

i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Solution:

$3x^2 + 7x - 2 = 0$

Here, a = 3, b = 7, c = –2

Since α, β are the roots of the equation

i) $\alpha+\beta = -\frac{b}{a} = \frac{-7}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-2}{3}$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{-7}{3}\right)^2-2\left(\frac{-2}{3}\right)}{\frac{-2}{3}} = \frac{\frac{49}{9}+\frac{4}{3}}{\frac{-2}{3}} = \frac{\frac{49+12}{9}}{\frac{-2}{3}}$

$= \frac{61}{9}\times\frac{3}{-2} = \frac{-61}{6}$

ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$

$= \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{\left(\frac{-7}{3}\right)^3-3\left(\frac{-2}{3}\right)\left(\frac{-7}{3}\right)}{\frac{-2}{3}}$

$= \frac{\frac{-343}{27}-\frac{42}{9}}{\frac{-2}{3}} = \frac{\frac{-343-126}{27}}{\frac{-2}{3}}$

$= \frac{217}{27}\times\frac{3}{-2}$

$= -\frac{469}{18}$

22. If α, β are the roots of the equation $2x^2 - x - 1 = 0$, then form the equation whose roots are (i) $\frac{1}{\alpha}$, — (ii) $\alpha^2\beta, \beta^2\alpha$ (iii) $2\alpha+\beta, 2\beta+\alpha$

Solution:

$2x^2 - x - 1 = 0$

Here, a = 2 , b = –1 , c = –1

$\alpha+\beta = -\frac{b}{a} = \frac{-(-1)}{2} = \frac{1}{2}$

$\alpha\beta = \frac{c}{a} = -\frac{1}{2}$

i) Given roots are $\frac{1}{\alpha}, \frac{1}{\beta}$

Sum of the roots $= \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$

Product of the roots $= \frac{1}{\alpha}\times\frac{1}{\beta} = \frac{1}{\alpha\beta}$

$= \frac{1}{-\frac{1}{2}} = -2$

The required equation is, x^2 – (Sum of the roots)x + (Product of the roots) = 0

$x^2 - (-1)x - 2 = 0 \Rightarrow x^2 + x - 2 = 0$

ii) $\alpha^2\beta, \beta^2\alpha$

Sum of the roots $= \alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta)$

$= -\frac{1}{2}\left(\frac{1}{2}\right) = -\frac{1}{4}$

Product of the roots $= (\alpha^2\beta)\times(\beta^2\alpha) = (\alpha\beta)^3$

$= \left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$

The required equation is, x^2 – (Sum of the roots)x + (Product of the roots) = 0

$x^2 - \left(-\frac{1}{4}\right)x - \frac{1}{8} = 0 \Rightarrow 8x^2 + 2x - 1 = 0$

iii) $2\alpha + \beta,\ 2\beta + \alpha$

Sum of the roots $= 2\alpha + \beta + 2\beta + \alpha$

$$= 3(\alpha+\beta) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

Product of the roots

$= (2\alpha+\beta) \times (2\beta+\alpha) = 4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta$

$= 5\alpha\beta + 2(\alpha^2+\beta^2) \quad = 5\alpha\beta + 2[(\alpha+\beta)^2-2\alpha\beta]$

$= 5\left(-\frac{1}{2}\right) + 2\left[\frac{1}{4} - 2\times -\frac{1}{2}\right]$

$= -\frac{5}{2} + \left[\frac{1}{4}+1\right]$

$= -\frac{5}{2} + \frac{1}{2} + 2 = 0$

The required equation is , x^2 – (Sum of the roots)x + (Product of the roots) = 0

$$x^2 - \frac{3}{2}x + 0 = 0$$

$\Rightarrow \quad 2x^2 - 3x = 0$

23. The roots of the equation $x^2 + 6x - 4 = 0$ are α, β. Find the quadratic equation whose roots are (i) α^2 and β^2 (ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$ (iii) $\alpha^2\beta$ and $\beta^2\alpha$

Solution:

i) α^2 and β^2

$x^2 + 6x - 4 = 0$

a = 1, b = 6, c = –4

$\alpha + \beta = -\frac{6}{1} = -6,\ \ \alpha\beta = \frac{-4}{1} = -4$

Sum of the roots

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-6)^2 - 2(-4)$

$= 36 + 8$

$= 44$

Product of the roots

$\alpha^2\beta^2 = (\alpha\beta)^2 = (-4)^2 = 16$

x^2 – (Sum of the roots)x + (Product of the roots) = 0

$\therefore x^2 - 44x + 16 = 0$

ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

Sum of the roots $= \frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\alpha+2\beta}{\alpha\beta}$

$= \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{2(-6)}{-4} = \frac{-12}{-4} = 3$

Product of the roots $= \frac{2}{\alpha} \times \frac{2}{\beta} = \frac{4}{\alpha\beta}$

$= \frac{4}{-4} = -1$

x^2 – (Sum of the roots)x + (Product of the roots) = 0

$\therefore x^2 - 3x - 1 = 0$

iii) $\alpha^2\beta$ and $\beta^2\alpha$

Sum of the roots $= \alpha^2\beta + \beta^2\alpha$

$= \alpha\beta(\alpha+\beta) = (-4)(-6) = 24$

Product of the roots $= (\alpha^2\beta)(\beta^2\alpha) = \alpha^3\beta^3$

$= (\alpha\beta)^3 = (-4)^3 = -64$

x^2 –(Sum of the roots)x + (Product of the roots) = 0

$\therefore x^2 - 24x - 64 = 0$

24. If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \frac{-13}{7}$. Find the values of a. MAY-22

Solution:

$7x^2 + ax + 2 = 0 \Rightarrow \alpha + \beta = \frac{-a}{7}$ ..... (1)

$\alpha\beta = \frac{2}{7}$; $\beta - \alpha = \frac{-13}{7}$

$\Rightarrow \alpha - \beta = \frac{13}{7}$ ..... (2)

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\left(\frac{13}{7}\right)^2 = \left(\frac{-a}{7}\right)^2 - 4\left(\frac{2}{7}\right)$

$\frac{169}{49} = \frac{a^2}{49} - \frac{8}{7}$

$\frac{169}{49} = \frac{a^2-56}{49}$

$a^2 - 56 = 169$

$a^2 = 225$

$\Rightarrow \quad a = \pm 15$

25. If one root of the equation $2y^2 - ay + 64 = 0$ is twice the other then find the values of a.

Solution:

$2y^2 - ay + 64 = 0$

Here, a = 2, b = –a, c = 64

$\alpha + \beta = \frac{a}{2}$ ..... (1)

$\alpha\beta = \frac{64}{2} = 32$ ..... (2)

Now, $\alpha = 2\beta$

(2) $\Rightarrow \alpha\beta = 32 \Rightarrow 2\beta^2 = 32$

$\Rightarrow \beta^2 = 16 \Rightarrow \ \beta = \pm 4$

Substitute β = 4 in eqn (2)

$\Rightarrow 4\alpha = 32$ then, α = 8,

Substitute $\beta = -4$ in eqn (2)

$\Rightarrow -4\alpha = 32$ then, $\alpha = -8$,

$(1) \Rightarrow 4 + 8 = \frac{a}{2}$

$\Rightarrow \quad 12 = \frac{a}{2}$

$a = 24$

$\therefore a = 24$ and $a = -24$

26. If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k.

Solution:

Given $3x^2 + kx + 81 = 0$

Here $a = 3, b = k, c = 81$

$\alpha + \beta = -\frac{k}{3}$ ..... (1)

$\alpha\beta = 27$ ..... (2)

But $\alpha = \beta^2$

From equation (2)

$\beta^3 = 27$

$\beta = 3$

$\therefore \quad \alpha = 9$

$(1) \Rightarrow 9 + 3 = -\frac{k}{3} \Rightarrow 12 = -\frac{k}{3}$

$k = -36$

27. Find X and Y if $X + Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

Solution:

$X + Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$ ..... (1) and

$X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ ..... (2)

$(1) + (2) \Rightarrow 2X = \begin{pmatrix} 10 & 0 \\ 3 & 9 \end{pmatrix}$

$\therefore X = \begin{pmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix}$

$(1) \Rightarrow Y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$

28. Find x and y if $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Solution:

$x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 4x \\ -3x \end{pmatrix} + \begin{pmatrix} -2y \\ 3y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 4x - 2y \\ -3x + 3y \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$4x - 2y = 4 \Rightarrow 2x - y = 2$ ..... (1)

$-3x + 3y = 6 \Rightarrow -x + y = 2$ ..... (2)

$(1) + (2) \Rightarrow \quad x = 4,$

$(2) \quad \Rightarrow -4 + y = 2$

$y = 6$

29. Find the non-zero values of x satisfying the matrix equation

$$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = \begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$

Solution:

$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2x^2 & 2x \\ 3x & x^2 \end{pmatrix} + \begin{pmatrix} 16 & 10x \\ 8 & 8x \end{pmatrix} = 2\begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8x \end{pmatrix} = 2\begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$

$\therefore 12x = 48 \Rightarrow x = 4$

$3x+8 = 20 \Rightarrow 3x = 12 \Rightarrow x = 4$

$x^2+8x = 12x$

$\Rightarrow x^2-4x = 0$

$\Rightarrow x(x-4) = 0,$

$x = 0, x = 4$

$\therefore x = 4$

30. Solve for x, y $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

Solution:

$\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$\begin{pmatrix} x^2 + (-4x) \\ y^2 + (-2y) \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$\Rightarrow x^2 - 4x - 5 = 0$ ..... (1)

$y^2 - 2y - 8 = 0$ ..... (2)

$(1) \Rightarrow (x-5)(x+1) = 0$ ($\because$ By Factorization)

$\therefore x = 5, x = -1$

$(2) \Rightarrow (y-4)(y+2) = 0$ ($\because$By Factorization)

$\therefore y = 4, y = -2$

31. If A = (1 −1 2), B = $\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ and C = $\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ show that(AB)C = A(BC).

Solution:

$$AB = (1\ -1\ \ 2)\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$$

$$= (1-2+2 \quad -1-1+6)$$

$$= (1 \quad 4)$$

$$(AB)\,C = (1 \quad 4)\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$= (1+8 \quad 2-4)$$

$$= (9 \quad -2)$$

$$BC = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2 & 2+1 \\ 2+2 & 4-1 \\ 1+6 & 2-3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$A(BC) = (1 \quad -1 \quad 2)\begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$= (-1-4+14 \quad 3-3-2)$$

$$= (9 \quad -2)$$

∴ LHS = RHS

***

4 Geometry

2 Marks STAGE 2

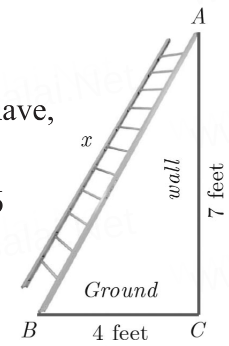
1. What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Solution:

Let x be the length of the ladder.

BC = 4 ft.

AC = 7 ft.



By Pythagoras Theorem we have,

$AB^2 = AC^2 + BC^2$

$x^2 = 7^2 + 4^2$ gives $x^2 = 49 + 16$

$x^2 = 65$

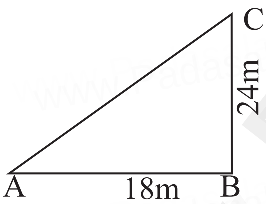
Hence $x = \sqrt{65}$

$= 8.1$

Therefore the length of the ladder is approximately 8.1 ft.

2. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?

Solution:



In ΔABC, $AC^2 = AB^2 + BC^2$

$AC^2 = (18)^2 + (24)^2 = 324 + 576$

$AC^2 = 900$

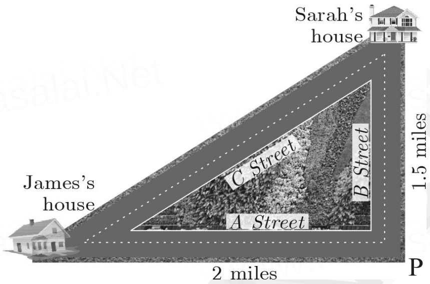
$AC = \sqrt{900}$

$AC = 30$ m

$\therefore$ The distance from the starting point is 30 m

3. There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take B street and then A street. How much shorter is the direct path along C street? (Using figure).

Solution:



While Going through Street C,

$SJ = \sqrt{(1.5)^2 + (2)^2}$

$= \sqrt{2.25 + 4} = \sqrt{6.25}$

$= 2.5$ miles

If one chooses A street and B street he has to go

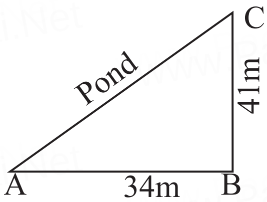
$SP + PJ = 1.5 + 2 = 3.5$ miles

Required Shorter Distance along

C street = $3.5 - 2.5 = 1$ mile

4. To get from point A to point B you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?

Solution:



To make a Straight way through the pond

$AC^2 = AB^2 + BC^2$

$= (34)^2 + (41)^2$

$= 1156 + 1681 = 2837$

$AC^2 = 2837 \Rightarrow AB = \sqrt{2837} = 53.26$ m

Through C one must walk

AC = AB + BC

$= 34 + 41 = 75$ m

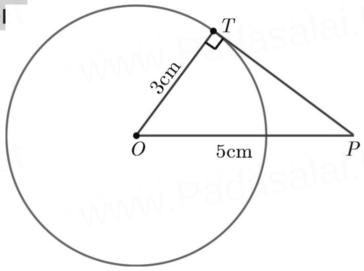
walking through a pond one must comes only 53.2m.

The difference is $(75 - 53.26)$ m = 21.74 m

To the nearest, one can save 21.74 m

5. Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

Solution:



Given OP = 5 cm, radius r = 3 cm

To find the length of tangent PT.

In right angled Triangle OTP

$OP^2 = OT^2 + PT^2$

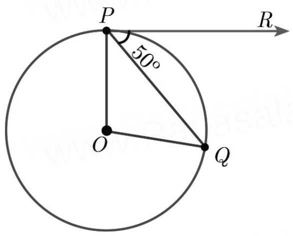
(By Phythagorous Theorem)

$5^2 = 3^2 + PT^2$

$PT^2 = 25 - 9 = 16$

Length of the tangent PT = 4 cm.

6. In Figure, O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ.



Solution:

$\angle OPQ = 90° - 50° = 40°$

(Angle between the radius and tangent is 90°)

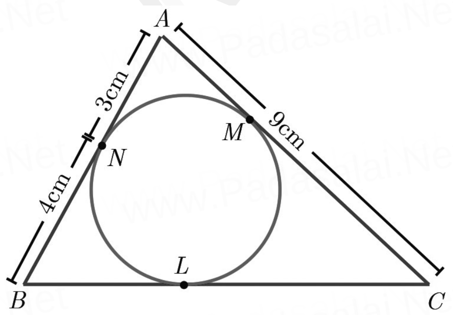
OP = OQ (Radii of a circle are equal)

$\angle OPQ = \angle OQP = 40°$ (ΔOPQ) is isosceles

$\angle POQ = 180° - \angle OPQ - \angle OQP$

$\angle POQ = 180° - 40° - 40° = 100°$

7. In Figure, ΔABC is circumscribing a circle. Find the length of BC.



Solution:

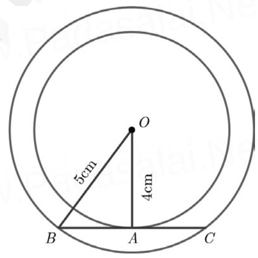
AN = AM = 3 cm

(Tangents drawn from same external point are equal)

$$\begin{aligned} BN &= BL = 4 \text{ cm} \\ CL &= CM = AC - AM \\ &= 9 - 3 = 6 \text{ cm} \\ BC &= BL + CL \\ &= 4 + 6 \\ &= 10 \text{ cm} \end{aligned}$$

8. If radii of two concentric circles are 4cm and 5cm then find the length of the chord of one circle which is a tangent to the other circle.

Solution:



OA = 4 cm,

OB = 5 cm,

also OA ⊥ BC

$OB^2 = OA^2 + AB^2$

$5^2 = 4^2 + AB^2$

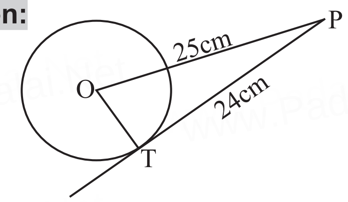
$AB^2 = 25 - 16 = 9$

Therefore AB = 3 cm, BC = 2AB

hence, BC = 2 × 3 = 6 cm

9. The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

Solution:



From the figure, $r = \sqrt{OP^2 - AP^2}$

$= \sqrt{25^2 - 24^2}$

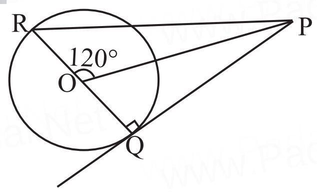
$= \sqrt{625 - 576}$

$= \sqrt{49}$

r = 7cm

10. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120°. Find ∠OPQ.

Solution:



A. SIVAMOORTHY, BT. Asst. GHS, Perumpakkam, Villupuram Dt.

From the given, we have the figure.

$\angle ROQ = 180°$

Given, $\angle ROP = 120°$

$\therefore \quad \angle POQ = 60°$

$(\because \angle ROQ = \angle ROP + \angle POQ)$

$\angle POQ + \angle OQP + \angle QPO = 180°$

(From triangle property)

then, $60° + 90° + \angle QPO = 180°$

($\angle OQP = 90°$ from tangents property)

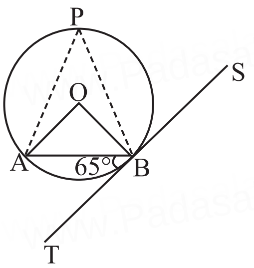
$150° + \angle QPO = 180°$

$\angle QPO = 30°$.

Hence $\quad \angle OPQ = 30°$

11. A tangent ST to a circle touches it at B. AB is a chord such that $\angle ABT = 65°$. Find $\angle AOB$, where "O" is the centre of the circle.

Solution:



From the figure

$\angle OBT = 90°$ ($\because$ OB radius, BT tangent)

$\therefore \quad \angle OBA = 90° - 65° = 25°$

and $\angle OAB = 25°$

$(\because OA = OB$, then $\angle OBA = \angle OAB)$

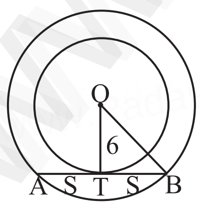
$\therefore \angle AOB = 180° - (\angle OAB + \angle OBA)$

$= 180° - 50°$

$\angle AOB = 130°$

12. In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

Solution:



AB = 16 cm and OC = 6 cm

But $OC \perp AB$ and C is divided two equal parts

($\because$ by circles theorem)

then, AC = CB = 8cm

To find OB.(OB is radius of larger circle)

By Pythagoras,

$OB = \sqrt{OC^2 + BC^2}$

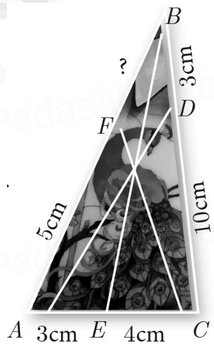
$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

OB = 10 cm

13. An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.

Solution:



From the figure,

AE = 3 cm,

BF = x,

BD = 3 cm, EC = 4 cm,

FA = 5 cm,

CD = 10 cm

By Ceva's Theorem ,

$$\frac{BF}{FA} \times \frac{CD}{DB} \times \frac{AE}{EC} = 1$$

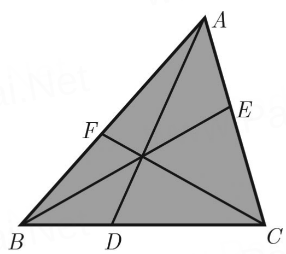
$$\Rightarrow \quad \frac{x}{5} \times \frac{10}{3} \times \frac{3}{4} = 1$$

$$\frac{x}{2} = 1$$

$x = 2$ cm

Hence, the required is 2 cm.

14. CEVA'S Theorem



Statement:

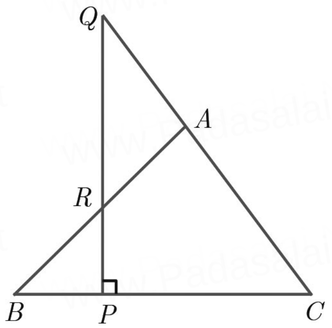
Let ABC be a triangle and let D, E, F be points on lines BC, CA, AB respectively.

Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$

where the lengths are directed.

This also works for the reciprocal of each of the ratios as the reciprocal of 1 is 1.

15. MENELAUS Theorem (Without Proof)



Statement:

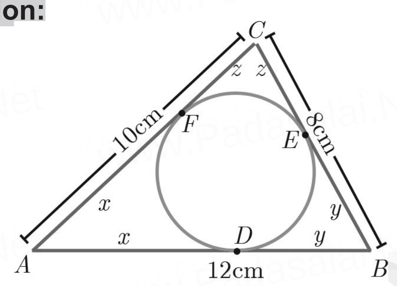
A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB (or their extension) of a triangle ABC to be collinear is that

$\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1$ where all segments in the formula are directed segments.

5 Marks **STAGE 2**

1. A circle is inscribed in ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure, Find AD, BE and CF.

Solution:



By result for tangents from external point

AD = AF = x, DB = BE = y, EC = CF = z

From the figure

x + y = AB = 12 ..... (1)

y + z = BC = 8 ..... (2)

z + x = CA = 10 ..... (3)

(1) + (2) + (3)

AB + BC + AC = 30

$\Rightarrow x+y+y+z+z+x = 30$

$\Rightarrow 2(x + y + z) = 30$

$x + y + z = 15$ .... (4)

AB = AD + BD = 12

$\Rightarrow x + y + z = 15$

$12 + z = 15$

$z = 3$

$\Rightarrow x + y + z = 15$

$x + 8 = 15$

$x = 7$

$\Rightarrow x + y + z = 15$

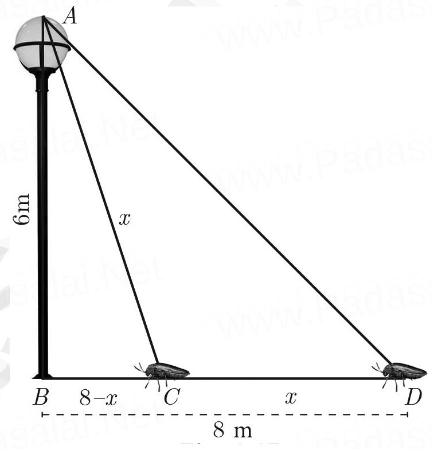
$10 + y = 15$

$y = 5$

Hence AD = 7 cm, BE = 5 cm, CF = 3 cm

2. An insect 8 m away initially from the foot of a lamp post which is 6 m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?

Solution:



Distance between the insect and the foot of the lamp post BD = 8 m.

The height of the lamp post, AB = 6 m.

After moving a distance of xm, let the insect be at C.

Let, AC = CD = x.

Then BC = BD – CD = 8 – x

In ΔABC, ∠B = 90°

$AC^2 = AB^2 + BC^2$ gives $x^2 = 6^2 + (8 - x)^2$

$x^2 = 36 + 64 - 16x + x^2$

$16x = 100, x = 6.25$

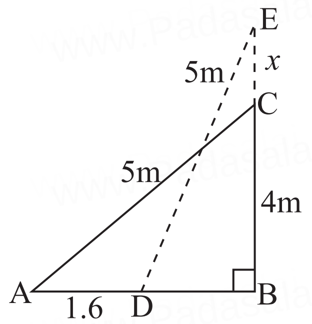
Then BC = 8 – x

= 8 – 6.25 = 1.75 m

Therefore, the insect is 1.75 m away from the foot of the lamp post.

3. 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:



Length of the Ladder, AC = 5 cm,

Height of Wall, BC = 4cm, AD = 1.6 cm,

Let EC = X

From ΔABC, By Pythagorous theorem

$$AB = \sqrt{AC^2 - BC^2}$$

$$= \sqrt{25-16} = \sqrt{9}$$

$$AB = 3 \text{ m}$$

From the figure we have,

$$AB = AD + BD$$

$$3 = 1.6 + BD$$

$$\Rightarrow BD = 1.4 \text{ m}$$

In ΔDBE, By Pythagorous theorem

$$(BE)^2 = (DE)^2 - (BD)^2$$

$$(4 + x)^2 = 5^2 - (1.4)^2$$

$$(4 + x)^2 = 23.04$$

$$4 + x = \sqrt{23.04}$$

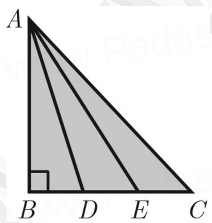
$$= 4.8$$

$$\therefore \quad x = 0.8$$

The distance by which top of the slide moves upwards is 0.8m.

4. In the adjacent figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$

Solution:



D, E trisect BC.

Let BD = DE = EC = k, BC = 3k, BE = 2k

In ΔABC, by Pythagoras

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = AB^2 + (3k)^2$

$AB^2 = AC^2 - 9k^2$ ..... (1)

In ΔABE, by Pythagoras

$\Rightarrow AE^2 = AB^2 + BE^2$

$\Rightarrow AE^2 = AB^2 + (2k)^2$

$AB^2 = AE^2 - 4k^2$ ..... (2)

In ΔABD, by Pythagoras

$AD^2 = AB^2 + BD^2$

$\Rightarrow AB^2 = AD^2 - k^2$ ..... (3)

(1), (2) ⇒

$AC^2 - AE^2 = 5k^2$ (∵ (1)=(2)) ..... (4)

(2), (3) ⇒ $AE^2 - AD^2 = 3k^2$ ..... (5)

(4) × 3 – (5) × 5

$\Rightarrow \quad 3AC^2 - 3AE^2 = 15k^2$

$5AE^2 - 5AD^2 = 15k^2$

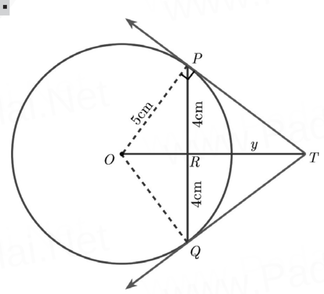
$3AC^2 - 8AE^2 + 5AD^2 = 0$

$\therefore 8AE^2 = 3AC^2 + 5AD^2$

Hence the proof.

5. PQ is a chord of length 8 cm to a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of the tangent TP.

Solution:



Let TR = y

Since, OT is perpendicular bisector of PQ.

PR = QR = 4 cm

In ΔORP $OP^2 = OR^2 + PR^2$

$OR^2 = 5^2 + 4^2$

$= 25 - 16 = 9$

$\Rightarrow$ OR = 3 cm

OT = OR + RT = 3 + y ..... (1)

In ΔPRT, $TP^2 = TR^2 + PR^2$ ..... (2)

and ΔOPT we have,

$OT^2 = TP^2 + OP^2$

$OT^2 = (TR^2 + PR^2) + OP^2$

(Substiture for TP^2 from (2))

$(3 + y)^2 = y^2 + 4^2 + 5^2$

(Substiture for OT from (1))

$9 + 6y + y^2 = y^2 + 16 + 25$

$6y = 41 - 9$

we get $y = \frac{16}{3}$

From (2) $TP^2 = TR^2 + PR^2$

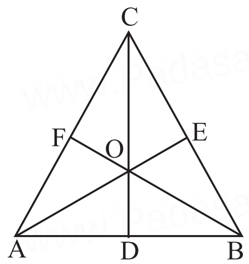
$$TP^2 = \left(\frac{16}{3}\right)^2 + 4^2$$

$$= \frac{256}{9} + 16 = \frac{400}{9}$$

So $TP = \frac{20}{3}$ cm

6. Show that the angle bisectors of a triangle are concurrent.

Solution:



Let O be any point inside a triangle ABC.

The bisector of CD, AE and BF meet the sides AB, BC, CA at point D, E and F respectively.

In ΔAOB, OD is the bisector of ∠AOB

$\therefore \frac{OA}{OB} = \frac{AD}{DB}$

(by angle bisector theorem) ..... (1)

In ΔBOC, OE is the bisector of ∠BOC

$\therefore \frac{OB}{OC} = \frac{BE}{EC}$ ..... (2)

In ΔCOA, OF is the bisector of ∠COA

$\therefore \frac{OC}{OA} = \frac{CF}{FA}$ ..... (3)

(1)×(2)×(3) ⇒

$$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

$$\Rightarrow \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA} = 1$$

But if AE, BF and CD are the bisectors of ∠A, ∠B and ∠C, then

$$\frac{AB}{AC} = \frac{BE}{EC}, \frac{BC}{AB} = \frac{CF}{FA}, \frac{CA}{CB} = \frac{AD}{DB}$$

Hence from the above 3 equations, we get

$$\frac{BE}{EC} \times \frac{CF}{FA} \times \frac{AD}{DB}$$

$$= \frac{AB}{AC} \times \frac{BC}{AB} \times \frac{CA}{CB} = 1 \text{ (from (4))}$$

Hence, O is point of concurrence of the angle bisectors.

***.

5 Coordinate Geometry

2 Marks

STAGE 2

1. Find the slope of a line joining the given points (i) (–6, 1) and (–3, 2) (ii) (14, 10) and (14, –6) SEP-20

Solution:

i) (–6, 1) and (–3, 2)

Slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-3-(-6)}$

$= \frac{2-1}{-3+6}$

$\therefore$ Slope, $m = \frac{1}{3}$

ii) (14, 10) and (14, –6)

Slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-10}{14-14} = \frac{-16}{0}$

$\therefore$ Slope, $m = \frac{-16}{0}$

The slope is undefined.

2. Find the slope of a line joining the points
(i) (5, $\sqrt{5}$) with the origin
(ii) (sin θ, –cosθ) and (–sinθ, cosθ)

Solution:

i) Given points are (5, $\sqrt{5}$) and (0, 0)

Slope = m

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-\sqrt{5}}{0-5}$

$= \frac{\sqrt{}}{} = \frac{1}{\sqrt{5}}$

ii) Given points are

(sinθ, –cosθ) and (–sinθ, cosθ)

Slope = m

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\cos\theta-(-\cos\theta)}{-\sin\theta-\sin\theta}$

$= \frac{2\cos\theta}{-2\sin\theta} = -\cot\theta$

3. The line r passes through the points (–2, 2) and (5, 8) and the line s passes through the points (–8, 7) and (–2, 0). Is the line r perpendicular to s?

Solution:

The slope of line r is $m_1 = \frac{8-2}{5+2} = \frac{6}{7}$

The slope of line r is $m_2 = \frac{0-7}{-2+8} = \frac{-7}{6}$

The product of slopes $= \frac{6}{7} \times \frac{-7}{6} = -1$

That is, $m_1 \, m_2 = -1$

Therefore, the line r is perpendicular to line s.

4. What is the slope of a line perpendicular to the line joining A(5, 1) and P where P is the mid-point of the segment joining (4, 2) and (–6, 4).

Solution:

P is the midpoint of the segment joining (4, 2) and (–6, 4)

$P(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{4+(-6)}{2}, \frac{2+4}{2}\right)$

A (5, 1) and P (–1, 3)

Slope of AP $= \frac{y_2 - y_1}{x_2 - x_1}$

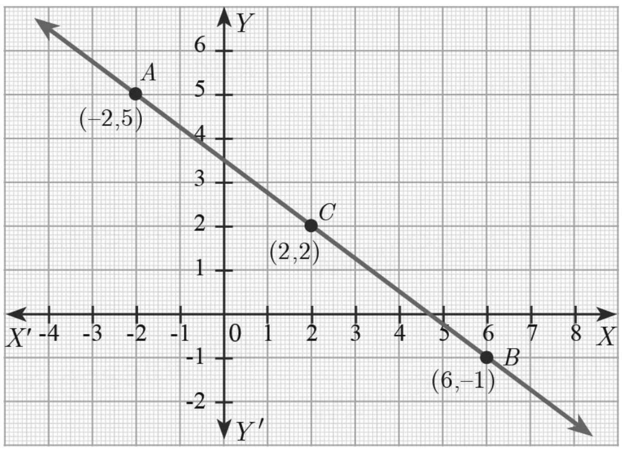
$= \frac{3-1}{-1-5} = \frac{2}{-6} = \frac{-1}{3}$

Slope of the line perpendicular to AP

$= \frac{-1}{slop\ of\ AP} = \frac{-1}{\frac{-1}{3}} = 3$

5. Show that the points (–2, 5) , (6, –1) and (2, 2) are collinear.

Solution:



Slope of AB $= \frac{-1-5}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Slope of BC $= \frac{2+1}{2-6} = \frac{3}{-4} = \frac{-3}{4}$

Slope of AB = Slope of BC

Therefore, the points A, B, C all lie in a same straight line.

Hence A, B and C are collinear.

6. Show that the given points are collinear: (–3, –4) , (7, 2) and (12, 5)

Solution:

Slope of AB $= \frac{2-(-4)}{7-(-3)} = \frac{6}{10} = \frac{3}{5}$ .....(1)

Slope of BC $= \frac{5-2}{12-7} = \frac{3}{5}$ .....(2)

Slope of AC $= \frac{5-(-4)}{12-(-3)} = \frac{9}{15} = \frac{3}{5}$ .....(3)

From (1), (2), (3) ⇒ the given points A, B, C are collinear.

7. If the three points (3, –1) , (a, 3) and (1, –3) are collinear, find the value of a.

Solution:

Let the given points A (3, –1), B (a, 3) and C (1, –3) and given A, B and C are collinear.

∴ Slope of AB = Slope of BC

$\Rightarrow \quad \frac{3-(-1)}{a-3} = \frac{-3-3}{1-a}$

$\Rightarrow \quad \frac{4}{a-3} = \frac{-6}{1-a}$

$\Rightarrow \quad 4 - 4a = -6a + 18$

$\Rightarrow \quad 2a = 14$

$\Rightarrow \quad a = 7$

8. The line through the points (–2, a) and (9, 3) has slope –12. Find the value of a.

Solution:

The slope of the points (–2, a) and (9, 3)

$= -\frac{1}{2}$

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-a}{9+2} = \frac{3-a}{11}$

$\therefore \quad \frac{3-a}{11} = -\frac{1}{2}$

$6 - 2a = -11$

$2a = 17$

$a = \frac{17}{2}$

9. The line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (*x*, 24) . Find the value of *x*.

Solution:

Slope of line joining (–2, 6), (4, 8)

$m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$

Slope of line joining (8, 12) (*x*, 24)

$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

Since two lines are perpendicular

$m_1 \times m_2 = -1$

$\Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$

$\Rightarrow \quad \frac{4}{x-8} = -1$

$\Rightarrow \quad x - 8 = -4$

$\Rightarrow \quad x = 4$

10. The line r passes through the points (–2, 2) and (5, 8) and the line s passes through the points (–8, 7) and (–2, 0). Is the line r perpendicular to s?

Solution:

The slope of line r is $m_1 = \frac{8-2}{5+2} = \frac{6}{7}$

The slope of line r is $m_2 = \frac{0-7}{-2+8} = \frac{-7}{6}$

The product of slopes $= \frac{6}{7} \times \frac{-7}{6} = -1$

That is, $m_1\ m_2 = -1$

Therefore, the line r is perpendicular to line s.

11. The line p passes through the points (3, –2), (12, 4) and the line q passes through the points (6, –2) and (12, 2) . Is p parallel to q? MAY-22

Solution:

The slope of line p is $m_1 = \frac{4+2}{12-3} = \frac{6}{9} = \frac{2}{3}$

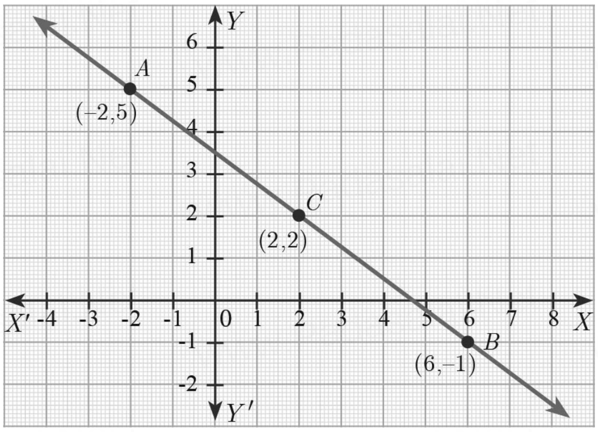
The slope of line q is $m_2 = \frac{2 \quad 2}{12 \quad 6} = \frac{4}{6} = \frac{2}{3}$

Thus, slope of line p = slope of line q.

Therefore, line p is parallel to the line q.

12. Show that the points (–2, 5) , (6, –1) and (2, 2) are collinear.

Solution:



Slope of AB $= \frac{-1-5}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Slope of BC $= \frac{2+1}{2-6} = \frac{3}{-4} = \frac{-3}{4}$

Slope of AB = Slope of BC

Therefore, the points A, B, C all lie in a same straight line.

Hence A, B and C are collinear.

13. Find the equation of a straight line whose

(i) Slope is 5 and y intercept is –9

(ii) Inclination is 45° and y intercept is 11

Solution:

i) Given Slope, m = 5, y intercept, c = –9

Therefore, equation of a straight line is,

$$y = mx + c$$
$$y = 5x - 9$$
$$0 = 5x - y - 9$$

∴ Required equation is $5x - y - 9 = 0$

ii) Given, θ=45°, y intercept, c = 11

Slope, m = tan θ

m = tan 45°

Slope, m = 1

y intercept, C = 11

Therefore, equation of a straight line is,

$$y = mx + C$$
$$y = 1x + 11$$
$$0 = x - y + 11$$

∴ Required equation is $x - y + 11 = 0$

14. Calculate the slope and y intercept of the straight line $8x - 7y + 6 = 0$ SEP-21

Solution:

$$8x - 7y + 6 = 0$$
$$8x + 6 = 7y$$
$$(\div 7)\quad \frac{8}{7}x + — = \frac{7}{7}y$$
$$\frac{8}{7}x + \frac{6}{7} = y$$

Comparing y = mx + C

Slope, m $= \frac{8}{7}$

y intercept, C $= \frac{6}{7}$

15. Find the equation of a line passing through the point (3, –4) and having slope $\frac{-5}{7}$

Solution:

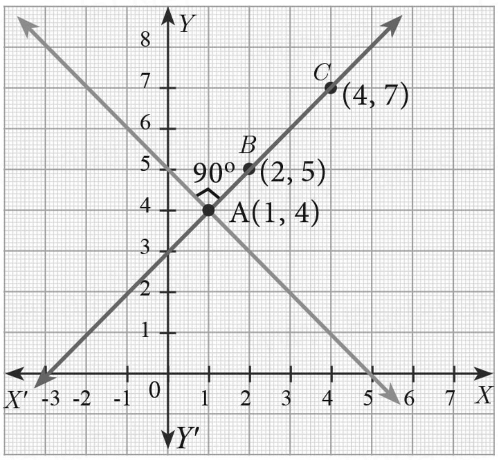
$(x_1, y_1) = (3, -4)$

Slope, $m = -\frac{5}{7}$

Equation of the straight line

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{5}{7}(x - 3)$$
$$7(y + 4) = -5(x - 3)$$
$$7y + 28 = -5x + 15$$
$$5x + 7y + 28 - 15 = 0$$
$$5x + 7y + 13 = 0$$

16. Find the equation of a line passing through the point A (1, 4) and perpendicular to the line joining points (2, 5) and (4, 7).



Solution:

Let the given points be A (1, 4), B (2, 5) and C (4, 7)

Slope of line BC $= \frac{7-5}{4-2} = \frac{2}{2} = 1$

Let m be the slope of the required line.

Since the required line is perpendicular to BC.

$m \times 1 = -1$

$m = -1$

The required line also pass through the point A (1, 4)

The equation of the required straight line is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -1(x - 1)$$
$$y - 4 = -x + 1$$

We get, $x + y - 5 = 0$

17. Find the equation of a line which passes through (5, 7) and makes intercepts on the axes equal in magnitude but opposite in sign.

Solution:

Let the x intercept be 'a' and y intercept be '$-a$'

The equation of the line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ (Here } b = -a)$$

$\therefore \quad x - y = a \quad$ ..... (1)

Since (1) passes through (5, 7)

Therefore, $5 - 7 = a \Rightarrow a = -2$

Thus the required equation of the straight line is $x - y = -2$; or $x - y + 2 = 0$

18. Find the intercepts made by the line $4x - 9y + 36 = 0$ on the coordinate axes.

Solution:

Equation of the given line is $4x - 9y + 36 = 0$

We write it as $4x - 9y = -36$ (bringing it to the normal form)

Dividing by -36 we get, $\frac{x}{-9} + \frac{y}{4} = 1$ ..... (1)

Comparing (1) with intercept form,

we get x intercept $a = -9$;

my intercept to $b = 4$

19. Find the equation of a straight line passing through the mid-point of a line segment joining the points (1, –5) , (4, 2) and parallel to (i) X axis (ii) Y axis

Solution:

Let M be the midpoint of a line segment joining the points (1, –5) and (4, 2).

$$\therefore M(x, y) = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$$

i) Equation parallel to Y – axis is y = b.

It passes through the points $\left(\frac{5}{2}, \frac{-3}{2}\right)$

$$\therefore y = -\frac{3}{2}$$
$$\Rightarrow \quad y + \frac{3}{2} = 0$$
$$\Rightarrow \quad 2y + 3 = 0$$

ii) Equation parallel to X – axis is x = a.

It passes through the points $\left(\frac{5}{2}, \frac{-3}{2}\right)$

$$\therefore x = \frac{5}{2}$$
$$\Rightarrow \quad x - \frac{5}{2} = 0$$
$$\Rightarrow \quad 2x - 5 = 0$$

20. Find the equation of a line through the given pair of points

(i) $\left(2, \frac{2}{3}\right)\left(\frac{-1}{2}, -2\right)$ (ii) (2, 3) and (–7, –1)

Solution:

Equation of the straight line 'Two points Form' is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

i) $\left(2, \frac{2}{3}\right)$ and $\left(\frac{-1}{2}, -2\right)$

The required equation

$$\frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x - 2}{-\frac{1}{2} - 2}$$

$$\Rightarrow \quad \frac{\frac{3y-2}{3}}{\frac{-8}{3}} = \frac{x-2}{\frac{-5}{2}}$$

$$\Rightarrow \quad \frac{3y-2}{-8} = \frac{2x-4}{-5}$$

$$\Rightarrow \quad -15y + 10 = -16x + 32$$

$$\Rightarrow 16x - 15y - 22 = 0$$

ii) (2, 3) and (–7, –1)

The required equation

$$\Rightarrow \quad \frac{y-3}{-1-3} = \frac{x-2}{-7-2}$$

$$\Rightarrow \quad \frac{y-3}{-4} = \frac{x-2}{-9}$$

$$\Rightarrow \quad -9y + 27 = -4x + 8$$

$$\Rightarrow 4x - 9y + 19 = 0$$

21. A cat is located at the point (–6, –4) in xy plane. A bottle of milk is kept at (5, 11). The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

Solution:

The required equation of the line joining the points (–6, –4) and (5, 11)

Two points form:

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{-(-\ )}{11-(-4)} = \frac{x-(-6)}{5-(-6)}$$

$$\Rightarrow \frac{y+4}{15} = \frac{x+6}{11}$$

$$\Rightarrow 11y + 44 = 15x + 90$$

$$\Rightarrow 15x - 11y + 90 - 44 = 0$$

$$\Rightarrow 15x - 11y + 46 = 0$$

22. Find the equation of a straight line which has slope –5/4 and passing through the point (–1, 2). MAY-22

Solution:

Given a point (–1, 2) and slope, $\frac{-5}{4}$

The required equation, $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 2 = \frac{-5}{4}(x-(-1))$$

$$\Rightarrow 4y - 8 = -5x - 5$$

$$\Rightarrow 5x+4y -3 = 0$$

23. Find the equation of a line whose intercepts on the x and y axes are given below.

(i) 4, –6 (ii) –5, $\frac{3}{4}$

Solution:

i) x intercept, a = 4, y intercept, b = –6

Equation of the straight line

$$\frac{x}{a}+\frac{y}{b} = 1$$

$$\frac{x}{4}+\frac{y}{-6} = 1$$

$$\frac{x}{4}-\frac{y}{6} = 1$$

$$\frac{6x-4y}{24} = 1$$

$$\frac{2(3x-2y)}{24} = 1$$

$$\frac{3x-2y}{12} = 1$$

$$3x - 2y = 12$$

$$3x - 2y - 12 = 0$$

x intercept, a = –5, y intercept, b = $\frac{3}{4}$

ii) Equation of the straight line

$$\frac{x}{a}+\frac{y}{b} = 1$$

$$\frac{x}{-5}+\frac{y}{\frac{3}{4}} = 1$$

$$\frac{x}{-5}+\frac{4y}{3} = 1$$

$$\frac{3x-20y}{-15} = 1$$

$$3x - 20y = -15$$

$$3x - 20y + 15 = 0$$

24. Find the intercepts made by the following lines on the coordinate axes. SEP-21

(i) $3x - 2y - 6 = 0$ (ii) $4x + 3y + 12 = 0$

Solution:

Intercepts form : $\frac{x}{a}+\frac{y}{b} = 1$

∵ a - x intercepts, b - y intercepts

i) $3x - 2y - 6 = 0$

$$\Rightarrow 3x - 2y = 6 \Rightarrow \frac{3x}{6} - \frac{2y}{6} = 1$$

$$\Rightarrow \frac{x}{2}+\frac{y}{-3} = 1$$

$$\Rightarrow \therefore\ a = 2,\ b = -3$$

ii) $4x + 3y + 12 = 0$

$$4x + 3y = -12\ (\div -12)$$

$$\Rightarrow \frac{4x}{-12}+\frac{3y}{-12} = 1$$

$$\Rightarrow \frac{x}{-3}+\frac{y}{-4} = 1$$

$$\Rightarrow \therefore\ a = -3, b = -4$$

25. Find the slope of the straight line $6x + 8y + 7 = 0$.

Solution:

Given $6x + 8y + 7 = 0$

$$\text{Slope } m = \frac{-\textit{coefficient of } x}{\textit{coefficient of } y} = -\frac{6}{8} = -\frac{3}{4}$$

Therefore, the slope of the straight line is $-\frac{3}{4}$.

26. Find the slope of the line which is

(i) parallel to $3x - 7y = 11$

(ii) perpendicular to $2x - 3y + 8 = 0$

Solution:

i) Given straight line is $3x - 7y = 11$

gives, $3x - 7y - 11 = 0$

Slope, $m = \frac{-3}{-7} = \frac{3}{7}$

Since parallel line have same slopes, slope of any line parallel to

$3x - 7y = 11$ is $\frac{3}{7}$

ii. Given straight line is $2x - 3y + 8 = 0$

Slope, $m = \frac{-2}{-3} = \frac{2}{3}$

Some product of slope is -1 for perpendicular lines, slope of any line perpendicular to

$2x - 3y + 8 = 0$ is $\frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

27. Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution:

Slope of the straight line $2x + 3y - 8 = 0$

$m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{2}{3}$

Slope of the straight line $4x + 6y + 18 = 0$

$m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{2}{3}$

Here $m_1 = m_2$

That is, slopes are equal.

Hence, the two straight lines are parallel.

28. Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution:

Slope of the straight line $x - 2y + 3 = 0$

$m_1 = -\left(\frac{a}{b}\right) = -\left(\frac{1}{-2}\right) = \frac{1}{2}$

Slope of the straight line $6x + 3y + 8 = 0$

$m_2 = -\left(\frac{a}{b}\right) = -\left(\frac{6}{3}\right) = -2$

$m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$

Product of the slopes $= -1$

Hence, the two straight lines are perpendicular.

29. Find the slope of the following straight lines (i) $5y - 3 = 0$ (ii) $7x - \frac{3}{17} = 0$

Solution:

i) $5y - 3 = 0$

$\therefore$ Slope, $m = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$= \frac{0}{5} = 0$

ii) $7x - \frac{3}{17} = 0$

$\Rightarrow \quad 7x = \frac{3}{17}$

$\Rightarrow \quad 0y + 7x + \frac{3}{17}$

$\therefore$ Slope , $m = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$m = \frac{-7}{0}$

$\therefore m = \infty$ (undefined)

30. Find the slope of the line which is

(i) parallel to $y = 0.7x - 11$

(ii) perpendicular to the line $x = -11$

Solution:

i) $y = 0.7x - 11$

This line parallel to $y = 0.7x - k$

$\therefore$ The slope of the required line is 0.7

ii) $x = -11$

$\Rightarrow x + 0y + 11 = 0$ ..... (1)

(1) line perpendicular to $0x - y + k = 0$

$\Rightarrow y = 0x + k$

$\therefore$ The Slope of the required line is 0.

5 Marks **STAGE 2**

1. If the points P(–1, –4), Q(b, c) and R(5, –1) are collinear and if 2b + c = 4, then find the values of b and c. SEP-21

Solution:

Since the three points P(–1, –4), Q(b, c) and R(5, –1) are collinear,

Area of Triangle PQR = 0

$$\frac{1}{2}\begin{vmatrix} -1 & -4 \\ b & c \\ 5 & -1 \\ -1 & -4 \end{vmatrix} = 0$$

$\frac{1}{2}\{(-c - b - 20) - (-4b + 5c + 1)\} = 0$

$-c - b - 20 + 4b - 5c - 1 = 0$

$3b - 6c = 21 \; (\div 3)$

$b - 2c = 7$ ..... (1)

Also, $2b + c = 4$ ..... (2)

(From given information)

Solving (1) and (2) we get $b = 3$, $c = -2$

2. If the points A(–3, 9) , B(a, b) and C(4, –5) are collinear and if a + b = 1, then find *a* and *b*.

Solution:

Given A (–3, 9), B(a, b), C(4, –5) are collinear and $a + b = 1$ ..... (1)

Area of the triangle formed by 3 points = 0

$$\frac{1}{2}\begin{vmatrix} -3 & 9 \\ a & b \\ 4 & -5 \\ -3 & 9 \end{vmatrix} = 0$$

$\Rightarrow (-3b - 5a + 36) - (9a + 4b + 15) = 0$

$\Rightarrow -5a - 3b + 36 - 9a - 4b - 15 = 0$

$\Rightarrow -14a - 7b + 21 = 0$

$\Rightarrow -14a - 7b = -21$

$\Rightarrow 14a + 7b = 21$ ($\div 7$)

$\Rightarrow 2a + b = 3$ ..... (2)

Given $a + b = 1$ ..... (1)

$(1) - (2) \Rightarrow a = 2 \quad b = -1$

3. Let P(11,7), Q(13.5, 4) and R(9.5, 4) be the midpoints of the sides AB, BC and AC respectively of ΔABC. Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR.

Solution:

P = Mid point of AB

$\Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (11, 7)$

$\Rightarrow \frac{x_1 + x_2}{2} = 11 \Rightarrow x_1 + x_2 = 22$ ..... (1)

$\Rightarrow \frac{y_1 + y_2}{2} = 7 \Rightarrow y_1 + y_2 = 14$ ..... (2)

Q = Mid point of BC

$\Rightarrow \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (13.5, 4)$

$\Rightarrow \frac{x_2 + x_3}{2} = 13.5 \Rightarrow x_2 + x_3 = 27$ ..... (3)

$\Rightarrow \frac{y_2 + y_3}{2} = 4 \Rightarrow y_2 + y_3 = 8$ ..... (4)

R = Mid point of AC

$\Rightarrow \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right) = (9.5, 4)$

$\Rightarrow \frac{x_1 + x_3}{2} = 9.5 \Rightarrow x_1 + x_3 = 19$ ..... (5)

$\Rightarrow \frac{y_1 + y_3}{2} = 4 \Rightarrow y_1 + y_3 = 8$ ..... (6)

$(1)+(3)+(5) \Rightarrow 2x_1 + x_2 + x_3 = 68$

$x_1 + x_2 + x_3 = 34$ ..... (7)

$(2)+(4)+(6) \Rightarrow 2y_1 + y_2 + y_3 = 30$

$y_1 + y_2 + y_3 = 15$ ..... (8)

$(7) - (1) \Rightarrow x_3 = 12$

$(7) - (3) \Rightarrow x_1 = 7$

$(7) - (5) \Rightarrow x_2 = 15$

$(8) - (2) \Rightarrow y_3 = 1$

$(8) - (4) \Rightarrow y_1 = 7$

$(8) - (6) \Rightarrow y_2 = 7$

A(7, 7), B(15, 7) and C(12, 1)

$$\text{Area } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 7 & 7 \\ 15 & 7 \\ 12 & 1 \\ 7 & 7 \end{vmatrix}$$

$= \frac{1}{2}[(49 + 15 + 84) - (105 + 84 + 7)]$

$= \frac{1}{2}[148 - 196]$

$= \frac{1}{2}[-48] = 24$ sq.units

($\because$ Area cannot be –ve)

$$\text{Area } \Delta PQR = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 11 & 7 \\ 13.5 & 4 \\ 9.5 & 4 \\ 11 & 7 \end{vmatrix}$$

$= \frac{1}{2}[(44 + 54 + 66.5) - (94.5 + 38 + 44)]$

$= \frac{1}{2}[164.5 - 176.5]$

$= \frac{1}{2}[-12] = 6$ sq.units

($\because$ Area cannot be –ve)

Now,

Area of ΔPQR = 6 sq.units

Area of ΔABC = 24 sq.units

$\therefore$ Area of ΔABC = 4 × Area of ΔPQR

4. A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through (–3, 8). Find its equation.

Solution:

If a and b are the intercepts then

$a + b = 7$ or $b = 7 - a$

By intercept form $\frac{x}{a}+\frac{y}{b}=1$

We have , $\frac{x}{a}+\frac{y}{7-a}=1$

As this line pass through the point (–3, 8), we have

$$\frac{-3}{a}+\frac{8}{7-a}=1$$

$\Rightarrow$ $-3(7-a)+8a = a(7-a)$

$-21+3a+8a = 7a-a^2$

So, $a^2-7a+11a-21 = 0$

Solving this equation

$(a-3)(a+7) = 0$

$a = 3$ or $a = -7$

Since a is positive, we have a = 3 and $b = 7-a = 7-3 = 4$

Hence, $\frac{x}{3}+\frac{y}{4}=1$

Therefore, $4x + 3y - 12 = 0$ is the required equation.

5. Show that the given points form a right angled triangle and check whether they satisfies Pythagoras theorem.

i) A (1, –4), B (2, –3) and C (4, –7)

ii) L (0, 5), M (9, 12) and N (3, 14)

Solution:

i) A(1, –4), B(2, –3) and C(4, –7)

Slope of AB $=\frac{-3-(-4)}{2-1}=\frac{1}{1}=1$

Slope of BC $=\frac{-7-(-3)}{4-2}=\frac{-4}{2}=-2$

Slope of AC $=\frac{-7+4}{4-(+1)}=\frac{-3}{3}=-1$

(Slope of AB)×(Slope of AC)

$= 1\times(-1) = -1$

$\therefore$ ΔABC is a right angled triangle

($\because$ AB ⊥ AC)

Using Pythagoras theorem,

$AB^2 + AC^2 = BC^2$

($\because d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$)

$AB^2 = (2-1)^2+(-3+4)^2$

$= (1)^2+(1)^2 = 2$

$AC^2 = (4-1)^2+(-7+4)^2$

$= (3)^2+(-3)^2 = 18$

$BC^2 = (4-2)^2+(-7+3)^2$

$= (2)^2+(-4)^2$

$= 4+16 = 20$

$AB^2 + AC^2 = 2 + 18 = 20 = BC^2$

Hence it is satisfied.

ii) L(0, 5), M(9, 12) and N(3, 11)

Slope of LM $=\frac{12-5}{9-0}=\frac{7}{9}$

Slope of MN $=\frac{14-12}{3-9}=\frac{2}{-6}=-\frac{1}{3}$

Slope of LN $=\frac{14-5}{3-0}=\frac{9}{3}=3$

(Slope of MN) × (Slope of LN)

$=\left(-\frac{1}{3}\right)\times(3)=-1$

$\therefore$ MN ⊥ LN.

Δ LMN is a right angled triangle.

By Pythagoras theorem,

$MN^2 + LN^2 = LM^2$

$MN^2 = (3-9)^2 + (14-12)^2$

$= (-6)^2 + (2)^2$

$= 36 + 4 = 40$

$LN^2 = (3-0)^2 + (14-5)^2$

$= (3)^2 + (9)^2$

$= 9 + 81 = 90$

$LM^2 = (9-0)^2 + (12-5)^2$

$= (9)^2 + (7)^2$

$= 81 + 49 = 130$

$MN^2 + LN^2 = 40 + 90$

$= 130 = LM^2$.

Hence it is satisfied.

6. Show that the given points form a parallelogram :

A(2.5, 3.5) , B(10, –4), C(2.5, –2.5) and D(–5, 5)

Solution:

A (2.5, 3.5) B (10, –4),

Slope of AB $=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-4-3.5}{10-2.5}=-\frac{7.5}{7.5}=-1$

C(2.5, –2.5), D(–5, 5),

Slope of CD $=\frac{5-(-2.5)}{-5-2.5}$

$$= \frac{5+2.5}{-7.5} = \frac{7.5}{-7.5} = -1$$

∴ Slope of AB = Slope of CD.

So AB || CD.

B (10, –4), C (2.5, –2.5),

$$\text{Slope of BC} = \frac{-2.5-(-4)}{2.5-10}$$

$$= \frac{-2.5+4}{7.5} = \frac{1.5}{-7.5} \times \frac{10}{10}$$

$$= \frac{15}{-75} = -\frac{1}{5}$$

A (2.5, 3.5), D (–5, 5),

$$\text{Slope of AD} = \frac{5-(3.5)}{-5-2.5}$$

$$= \frac{1.5}{-7.5} = \frac{1.5}{-7.5} \times \frac{10}{10}$$

$$= \frac{15}{-75} = -\frac{1}{5}$$

∴ Slope of BC = Slope of AD.

So BC || AD .

∴ The given points form a parallelogram.

7. If the points A(2, 2), B(–2, –3), C(1, –3) and D(*x*, y) form a parallelogram then find the value of *x* and y.

Solution:

Given points A (2, 2), B(–2, –3) , C (1, –3) and D(*x*, y) are form a parallelogram.

Then AB || CD and BC || AD

∴ Slope of AD = Slope of BC

$$\Rightarrow \frac{y-2}{x-2} = \frac{-3+3}{1+2} \Rightarrow \frac{y-2}{x-2} = 0$$

$\Rightarrow$ y – 2 = 0

$\Rightarrow$ y = 2

Slope of CD = Slope of AB

$$\Rightarrow \frac{y-(-3)}{x-1} = \frac{-3-2}{-2-2}$$

$$\Rightarrow \frac{y+3}{x-1} = \frac{-5}{-4}$$

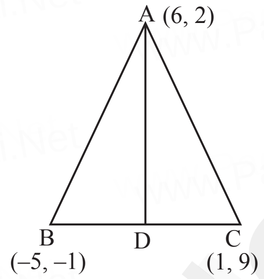
$$\Rightarrow \frac{5}{x-1} = \frac{5}{4}$$

$\Rightarrow$ $x - 1 = 4 \Rightarrow x = 5$

∴ $x = 5$, y = 2

8. Find the equation of the median and altitude of ΔABC through A where the vertices are A(6, 2) , B(–5, –1) and C(1, 9) SEP-21

Solution:



To find the equation of median through A

$$\text{Midpoint of BC} = D\left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$$

= D (–2, 4)

Equation of AD A(6, 2), D(–2, 4)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\Rightarrow \frac{y-2}{2} = \frac{x-6}{-8}$$

$$\Rightarrow \frac{y-2}{1} = \frac{x-6}{-4}$$

$\Rightarrow$ $x - 6 = -4y + 8$

$\Rightarrow$ $x + 4y - 14 = 0$

To find the equation of Altitude through A

B(–5, –1), C(1, 9)

$$\text{Slope, BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{9+1}{1+5}$$

$$= \frac{10}{6} = \frac{5}{3}$$

Hence, AD ⊥ BC,

Slope, AD = $\frac{-3}{5}$ and A (6, 2)

Equation of Altitude AD is

$$y - y_1 = m(x - x_1)$$

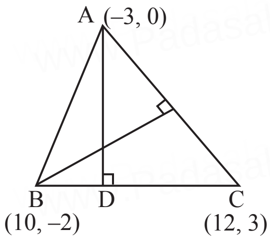
$$\Rightarrow y - 2 = \frac{-3}{5}(x-6)$$

$\Rightarrow$ $5y - 10 = -3x + 18$

$\Rightarrow$ $3x + 5y - 28 = 0$

9. A(–3, 0) B(10, –2) and C(12, 3) are the vertices of ΔABC . Find the equation of the altitude through A and B.

Solution:



B(10, –2) C(12, 3)

Slope of BC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+2}{12-10} = \frac{5}{2}$

BC ⊥ AD

∴ Slope of AD $= -\frac{2}{5}$ A (–3, 0)

The equation of the perpendicular line drawn from A to the opposite side of the triangle

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{2}{5}(x + 3)$$

$$5y = -2x - 6$$

$$2x + 5y + 6 = 0$$

A(–3, 0) C(12, 3)

Slope of AC $= \frac{3-0}{12+3} = \frac{3}{15} = \frac{1}{5}$

AC ⊥ BE

B(10, –2) Slope of BE = –5

The equation of the perpendicular line drawn from B to the opposite side of the triangle

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -5(x - 10)$$

$$y + 2 = -5x + 50$$

$$5x + y - 48 = 0$$

10. Without using Pythagoras theorem, show that the points (1, –4) , (2, –3) and (4, –7) form a right angled triangle.

Solution:

Let the points be A(1, –4), B(2, –3) and C(4, –7).

The Slope of AB $= \frac{-3+4}{2-1} = \frac{1}{1} = 1$

The Slope of BC $= \frac{-7+3}{4-2} = \frac{-4}{2} = -2$

The Slope of AC $= \frac{-7+4}{4-1} = \frac{-3}{3} = -1$

Slope of AB × Slope of AC = (1) (–1)

= –1

AB is perpendicular to AC. ∠A=90°

Therefore, ΔABC is a right angle triangle.

11. Find the equation of the perpendicular bisector of the line joining the points A(–4, 2) and B(6, –4).

Solution:

To find equation of the line joining the points A(–4, 2) and B(6, –4)

$$\Rightarrow \quad \frac{y-2}{-4-2} = \frac{x-(-4)}{6-(-4)}$$

$$\Rightarrow \quad \frac{y-2}{-6} = \frac{x+4}{10}$$

$$\Rightarrow \quad 10y - 20 = -6x - 24$$

$$\Rightarrow 6x + 10y + 4 = 0$$

$$\Rightarrow \quad 3x + 5y + 2 = 0 \qquad \text{..... (1)}$$

Equation (1) is perpendicular to

$$5x - 3y + k = 0 \qquad \text{..... (2)}$$

Equation (2) is passing though the midpoints of AB

Midpoint of AB $= \left(\frac{-4+6}{2}, \frac{2-4}{2}\right)$

$= (1, -1)$

∴ (2) ⇒ 5(1) – 3(–1) + k = 0

⇒ 5 + 3 + k = 0

⇒ k = –8

Hence, the Required Equation is

5x – 3y – 8 = 0

***

6 Trigonometry

5 Marks STAGE 2

1. If cosecθ + cotθ = P, then prove that $\cos\theta = \dfrac{P^2-1}{P^2+1}$

Solution:

cosecθ+cotθ = P ..... (1)

cosecθ –cotθ = 1/p ..... (2)

$(1)+(2) \Rightarrow \ 2\,\text{cosec}\theta = P+\dfrac{1}{P}$

$2\,\text{cosec}\theta = \dfrac{P^2+1}{P}$ ..... (3)

$(1)-(2) \Rightarrow \ 2\cot\theta = P-\dfrac{1}{P}$

$2\cot\theta = \dfrac{P^2-1}{P}$ ..... (4)

$(4)/(3) \Rightarrow \ \dfrac{2\cot\theta}{2cosec\theta} = \dfrac{\frac{P^2-1}{P}}{\frac{P^2+1}{P}}$

$\dfrac{\cot}{cosec} = \dfrac{P^2-1}{P^2+1}$

$\dfrac{\cos\theta}{\sin\theta}\times\sin\theta = \dfrac{P^2-1}{P^2+1}$

$\cos\theta = \dfrac{P^2-1}{P^2+1}$.

Hence Proved

2. Prove that $\left(\dfrac{\cos^3 A-\sin^3 A}{\cos A-\sin A}\right)-\left(\dfrac{\cos^3 A+\sin^3 A}{\cos A+\sin A}\right) = 2\sin A\cos A$

Solution:

$\left(\dfrac{\cos^3 A-\sin^3 A}{\cos A-\sin A}\right)-\left(\dfrac{\cos^3 A+\sin^3 A}{\cos A+\sin A}\right)$

$=\left(\dfrac{(\cos A-\sin A)(\cos^2 A+\sin^2 A+\cos A\sin A)}{\cos A-\sin A}\right)$

$-\left(\dfrac{(\cos A+\sin A)(\cos^2 A+\sin^2 A-\cos A\sin A)}{\cos A+\sin A}\right)$

$[\because a^3-b^3=(a-b)(a^2+b^2+ab)$

$a^3+b^3=(a+b)(a^2+b^2-ab)]$

= (1 + cosAsinA) – (1 – cosAsinA)

= 1 + cosAsinA – 1 + cosAsinA

= 2cosAsinA

3. Prove the following identities.

$\dfrac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\dfrac{\sin^3 A-\cos^3 A}{\sin A-\cos A}=2$

Solution:

Here x = sinA, y = cosA

$\text{LHS} = \dfrac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\dfrac{\sin^3 A-\cos^3 A}{\sin A-\cos A}$

$\because \ x^3+y^3=(x+y)(x^2-xy+y^2)$

$x^3-y^3=(x-y)(x^2+xy+y^2)$

$=\dfrac{(\sin A+\cos A)(\sin^2 A-\sin A\cos A+\cos^2 A)}{\sin A+\cos A}$

$+\dfrac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{\sin A-\cos A}$

$=\sin^2 A-\sin A\cos A+\cos^2 A+\sin^2 A+\sin A\cos A+\cos^2 A$

$=2(\sin^2 A+\cos^2 A)$ $(\because \sin^2 A+\cos^2 A=1)$

= 2 = RHS

Hence Proved.

4. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3}$ = 1.732) MAY-22

Solution:

In ΔAPB $\tan\theta = \dfrac{opposite\,side}{Adjacent\,side}$

$\tan 45° = \dfrac{30}{BP} \Rightarrow 1 = \dfrac{30}{BP}$

BP = 30m

In ΔBPC $\tan 60° = \dfrac{BC}{BP}$

$\sqrt{3} = \dfrac{h+30}{30}$

$30\sqrt{3} = h+30$

$h = 30\sqrt{3}-30$

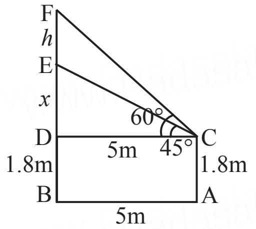
$= 30\,(1.732 - 1)$

$= 30\,(0.732)$

$= 21.960$

5. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? ($\sqrt{3} = 1.732$)

Solution:



In figure, AC – A man Standing,

EF – Window, DF – House

From the figure,

EF = h, ED = x, DF = x + h

In ΔCDE,

$\tan 45° = \dfrac{DE}{DC} \Rightarrow 1 = \dfrac{x}{5} \Rightarrow x = 5$

In ΔCDF,

$\tan 60° = \dfrac{DF}{DC} \Rightarrow \sqrt{3} = \dfrac{h+x}{5}$

$\Rightarrow h + x = \sqrt{3}\,(5)$

$\Rightarrow \quad h = (5 \times \sqrt{3}) - 5$

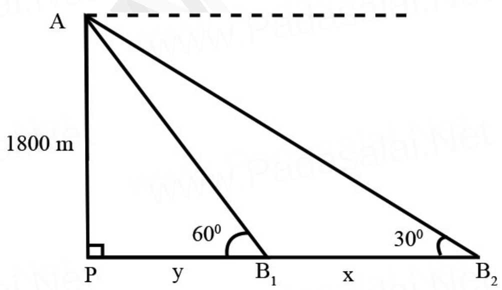
$= 5[\sqrt{3} - 1] = 5[1.732 - 1]$

$= 5[0.732] = 3.66$ m

Hence, Height of the window = 3.66m

6. An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3} = 1.732$)

Solution:



In figure, A – An Aeroplane,

B_1, B_2 are Two Boats

From the figure, AP = 1800m,

$PB_1 = y$, $B_1B_2 = x$, $PB_2 = x + y$

In ΔAPB_1, $\tan 60° = \dfrac{AP}{PB_1}$

$\Rightarrow \quad \sqrt{3} = \dfrac{1800}{y}$

$\Rightarrow \quad y = \dfrac{1800}{\sqrt{3}}$

$= \dfrac{1800}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{1800\sqrt{3}}{3}$

$= 600\sqrt{3}$ m

In ΔAPB_2, $\tan 30° = \dfrac{AP}{PB_2}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{1800}{x+y}$

$\Rightarrow \quad x + y = 1800\sqrt{3}$

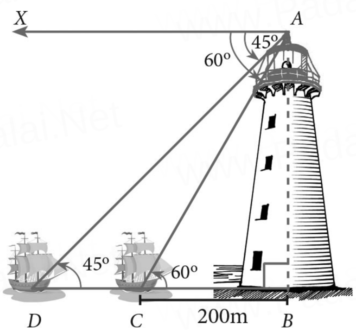
$\Rightarrow \quad x = 1800\sqrt{3} - 600\sqrt{3}$

$\Rightarrow x = 1200\sqrt{3}$ m $= 1200 \times 1.732$

$= 2078.4$ m

Hence, the distance between the boats = 2078.4 m

7. A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in km / hr), assuming that it is sailing in still water? ($\sqrt{3} = 1.732$)



Solution:

Let AB be the tower.

Let C and D be the positions of the boat

∠XAC = 60° = ∠ACB and

∠XAD = 45° = ∠ADB, BC = 200 m

In right triangle, ABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\text{gives } \sqrt{3} = \frac{AB}{200}$$

$$BC = 200\sqrt{3} \qquad \text{..... (1)}$$

In right triangle, ABD

$$\tan 45^\circ = \frac{AB}{BD}$$

$$\text{gives } 1 = \frac{200\sqrt{3}}{BD} \quad \text{[by (1)]}$$

$$\text{We get, } BD = 200\sqrt{3}$$

$$\text{Now, } CD = 200\sqrt{3} - 200$$

$$= 200(\sqrt{3} - 1) = 146.4$$

It is given that the distance CD is covered in 10 seconds.

That is, the distance of 146.4m is covered in 10 seconds.

$$\text{Therefore, speed of the boat} = \frac{distance}{time}$$

$$= \frac{146.4}{10}$$

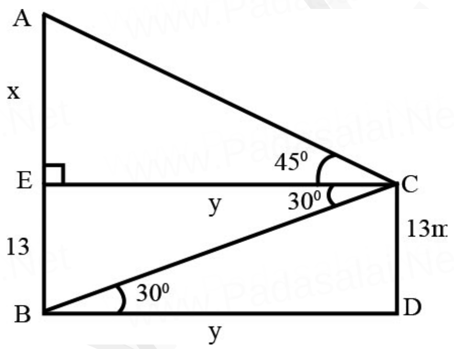
$$= 14.64 \text{ m/s}$$

$$\text{gives } 14.64 \times \frac{3600}{1000} \text{ km / hr}$$

$$= 52.704 \text{ km / hr}$$

8. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3}$ = 1.732)

Solution:



Let AB - height of second tree and

CD - height of the first tree = 13

$$\text{In } \angle AEC, \tan 45^\circ = \frac{AE}{CE}$$

$$1 = \frac{x}{y}$$

$$x = y \qquad \text{..... (1)}$$

$$\text{In } \angle BCD, \tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{13}{y}$$

$$y = 13\sqrt{3}$$

From (1) $x = y = 13\sqrt{3}$

Height of the second tree,

$$AB = AE + EB$$

$$= x + 13$$

$$= 13\sqrt{3} + 13$$

$$= 13[\sqrt{3}+1]$$

$$= 13[1 + 1.732]$$

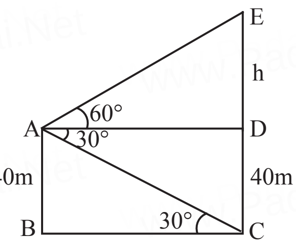
$$= 13[2.732]$$

$$= 35.52 \text{ m}$$

∴ Height of the second tree = 35.52 m

9. A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3}$ = 1.732)

Solution:



AB - Ship, CE - Hill

From the figure,

AB = CD = 40m, BC = AD = x, DE = h,

CE = 40 + h

$$\text{In } \Delta ABC \tan 30^\circ = \frac{AB}{BC} = \frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$\Rightarrow x = 40 \times \sqrt{3} \qquad \text{..... (1)}$$

$$\text{In } \Delta ADE \tan 60^\circ = \frac{DE}{AD} = \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x \times \sqrt{3} = 40 \times \sqrt{3} \times \sqrt{3} = 120 \text{ m}$$

∴ Height of the hill = 40 + 120 = 160 m

The distance of the hill from the ship is

$$\Rightarrow x = 40 \times \sqrt{3} = 69.28 \text{ m}$$

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7 Mensuration

2 Marks — STAGE 2

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the cylinder.

Solution:

Given that, C.S.A. of the cylinder

$= 88$ sq. cm

$2\pi rh = 88$

$2\times\frac{22}{7}\times r\times 14 = 88$ (h = 14 cm)

$2r = \frac{88\times 7}{22\times 14} = 2$

Therefore, diameter = 2 cm

2. A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Solution:

Given that, diameter d = 2.8 m and height = 3 m, radius r = 1.4 m

Area covered in one revolution

= curved surface area of the cylinder

= $2\pi rh$ sq. units

$= 2\times\frac{22}{7}\times 1.4\times 3$

$= 26.4$

Area covered in 1 revolution = 26.4 m²

Area covered in 8 revolutions = 8×26.4

= 211.2

Therefore, area covered is 211.2 m²

3. If the total surface area of a cone of radius 7 cm is 704 cm², then find its slant height.

Solution:

Total Surface Area = 704 cm²

$\pi r(l+r) = 704$

$\frac{22}{7}\times 7(l+7) = 704$

$l+7 = \frac{704}{22}$

$= \frac{64}{2} = 32$

$l+7 = 32 \Rightarrow l = 32-7 = 25$ cm

Therefore, slant height of the cone is 25 cm.

4. Find the diameter of a sphere whose surface area is 154 m². SEP-20

Solution:

Let r be the radius of the sphere.

Given that, surface area of sphere = 154 m²

$4\pi r^2 = 154$

$4\times\frac{22}{7}\times r^2 = 154$

gives $r^2 = \frac{154}{4}\times\frac{7}{22}$

$r^2 = \frac{49}{4}$

$r = \frac{7}{2}$

Radius of sphere $r = \frac{7}{2}$ m;

Diameter of sphere d = 7 m

5. The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases. MAY-22

Solution:

Let r_1 and r_2 be the radii of the balloons.

Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Ratio of C.S.A. of balloons

$= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Therefore, ratio of C.S.A. of balloons is 9:16.

6. Find the volume of a cylinder whose height is 2 m and whose base area is 250 m². SEP-21

Solution:

Let r and h be the radius and height of the cylinder respectively.

Given that, height $h = 2$ m, base area = 250 m²

Now, volume of a cylinder = $\pi r^2 h$ cu. units

= base area × h

= 250 × 2 = 500 m³

Therefore, volume of the cylinder = 500 m³

7. The volume of a solid right circular cone is 11088 cm³. If its height is 24 cm then find the radius of the cone.

Solution:

Let *r* and *h* be the radius and height of the cone respectively.

Given that,

volume of the cone = 11088 cm³

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = 441$$

Therefore, radius of the cone r = 21 cm

8. The ratio of the volumes of two cones is 2:3. Find the ratio of their radii if the height of second cone is double the height of the first.

Solution:

Let r_1, h_1 be the radius and height of the cone I and Let r_2, h_2 be the radius and height of the cone II.

Given that $h_2 = 2h_1$ and

$$\frac{Volume\ of\ the\ cone\ I}{Volume\ of\ the\ cone\ II} = \frac{2}{3}$$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} \times \frac{h_1}{2h_1} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} = \frac{4}{3}$$

$$\frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

∴ Ratio of their radii = $2 : \sqrt{3}$

5 Marks STAGE 2

1. A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.

Solution:

Given that, height of the cylinder h = 20 cm ; radius r = 14 cm

Now, C.S.A. of the cylinder = $2\pi rh$ sq. units

C.S.A. of the cylinder $= 2 \times \frac{22}{7} \times 14 \times 20$

$= 2 \times 22 \times 2 \times 20$

$= 1760$ cm²

T.S.A. of the cylinder $= 2\pi r\,(h + r)$ sq. units

$= 2 \times \frac{22}{7} \times 14 \times (20+14)$

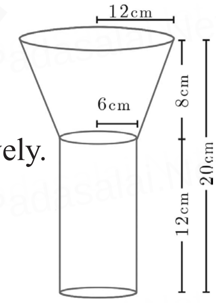
$= 2 \times \frac{22}{7} \times 14 \times 34$

$= 2992$ cm²

Therefore, C.S.A. = 1760 cm² and

T.S.A. = 2992 cm²

2. A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.



Solution:

Let h_1, h_2 be the heights of the frustum and cylinder respectively.

Let R, r be the top and bottom radii of the frustum.

Given that,

R = 12 cm, r = 6 cm, h_2 = 12 cm,

h_1 = 20 – 12 = 8 cm

Slant height of the frustum

$$l = \sqrt{(R-r)^2 + h_1^2} \text{ units}$$

$$= \sqrt{36+64}$$

$$l = 10 \text{ cm}$$

Outer Surface Area

$= 2\pi rh_2 + \pi(R+r)l$ sq.units

$= \pi(2rh_2 + (R+r)l$

$= \pi(2 \times 6 \times 12) + (18(10))$

$= \pi(144 + 180)$

$= \frac{22}{7} \times 324$

$= 1018.28$

Therefore, outer surface area of the funnel is 1018.28 cm²

3. A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?

Solution:

Let the number of small spheres obtained be n.

Let r be the radius of each small sphere and R be the radius of metallic sphere.

Here, R = 16 cm, r = 2 cm

Now, n × (Volume of a small sphere)

= Volume of big metallic sphere

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$n\left(\frac{4}{3}\pi \times 2^3\right) = \frac{4}{3}\pi \times 16^3$$

8n = 4096 gives n = 512

Therefore, there will be 512 small spheres.

4. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass? SEP-20

Solution:

In cylindrical glass r_1 = 10 cm,

Height of water raised in the glass = h_1

In cylindrical metal r_2 = 5 cm, h_2 = 4 cm

The volume of the water raised

= Volume of the cylindrical metal

$\pi r_1^2 h_1 = \pi r_2^2 h_2$

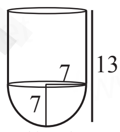
$$h_1 = \frac{r_2^2 h_2}{r_1^2} = \frac{5\times5\times4}{10\times10} = 1$$

Hence,

the height of water raised in the glass = 1cm

5. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.

Solution:



In hemisphere , r = 7 cm

In cylinder, r = 7 cm, h = 6 cm

Volume of the vessel = Volume of the cylinder + Volume of hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left(h+\frac{2}{3}r\right)$$

$$= \frac{22}{7}\times 7\times 7\times\left(6+\frac{2}{3}\times 7\right)$$

$$= 22\times 7\times\left[6+\frac{14}{3}\right]$$

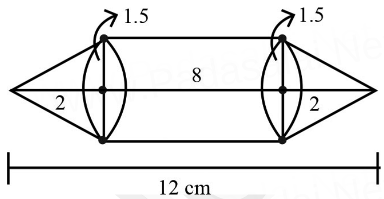
$$= 22\times 7\times\frac{32}{3}$$

$= 1642.67\ cm^3$

Hence, the capacity of the vessel is $1642.67\ cm^3$

6. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made. |May 22|

Solution:



Cylinder

Diameter d = 3 cm, Radius r = $\frac{3}{2}$ cm

Height h_1 = 12 – (2+2) = 8 cm

Cone

Radius r = $\frac{3}{2}$ cm, height h_1 = 2 cm

Volume of the model

= Volume of the cylinder + Volume of 2 cones

$$= \pi r^2 h_1 + 2\,\frac{1}{3}\pi r^2 h_2$$

$$= \pi r^2\left[h_1 + 2\,\frac{1}{3}h_2\right]$$

$$= \frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\left[8+\frac{2}{3}\times 2\right]$$

$$= \frac{\overset{11}{\cancel{22}}}{7}\times\frac{9}{\underset{2}{\cancel{4}}}\left[8+\frac{4}{3}\right]$$

$$= \frac{\overset{33}{\cancel{99}}}{\cancel{14}}\left[\frac{\overset{2}{\cancel{28}}}{\cancel{3}}\right] = 66\ cm^3$$

7. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

Solution:

In Sphere, r_1 = 12 cm,

In Cylinder, r_2 = 8 cm

Volume of the cylinder

= Volume of sphere

$$\Rightarrow \quad \pi r_2^2 h = \frac{4}{3}\pi r_1^3$$

$$\Rightarrow \quad r_2^2 h = \frac{4}{3} r_1^3$$

$$\Rightarrow \quad h = \frac{4\times12\times12\times12}{3\times8\times8} = 36\ cm$$

∴ Height of the cylinder = 36 cm

8. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.

Solution:

Diameter of the cone = 14 cm,

Radius of the cone = 7 cm,

Height of the cone h = 8 cm

Diameter of the sphere = 10 cm

$$\frac{4}{3}\pi(R^3-r^3) = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi(5^3-r^3) = \frac{1}{3}\pi \times 7 \times 7 \times 8$$

$$125 - r^3 = \frac{7\times7\times8}{4}$$

$$\Rightarrow 125 - r^3 = 98$$

$$r^3 = 27$$

$$\Rightarrow r^3 = 3^3$$

$$r = 3$$

Internal Diameter of the sphere

= 2(r) = 2(3) = 6 cm

9. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.

Solution:

Given in cone, height = 105cm;

circumference = 484 cm

$$\Rightarrow 2\pi r = 484$$

$$\Rightarrow 2\times\frac{22}{7}\times r = 484$$

$$r = \frac{484\times7}{2\times22}$$

$$= 77\text{cm}$$

$$\therefore \text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 77 \times 77 \times 105$$

$$= 652190 \text{ cm}^3$$

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7 Statistics and Probability

5 Marks STAGE 2

1. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Solution:

x	$d = x - \bar{x}$	d^2
24	–6	36
26	–4	16
33	3	9
37	7	49
29	–1	1
31	1	1
180	Σd=0	112

Mean $= \bar{x} = \frac{\Sigma x}{n} = \frac{180}{6} = 30$

Standard Deviation,

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{112}{6}} = \sqrt{18.66} = 4.32$$

Coefficient of Variation

$$= \frac{\sigma}{\bar{x}} \times 100\% = \frac{432}{30} \times 100\%$$

$= 14.4\ \%$

2. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

Solution:

Arranging the numbers in ascending order we get 38, 40, 43, 44, 46, 47, 49, 53

x	$d = x - \bar{x}$	d^2
38	–7	49
40	–5	25
43	–2	4
44	–1	1
46	1	1
47	2	4
49	4	16
53	8	64
360	Σd = 0	164

Mean $= \bar{x} = \frac{\Sigma x}{n} = \frac{360}{8} = 45$

Standard Deviation,

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{164}{8}} = \sqrt{20.5}$$

$= 4.527$

Coefficient of Variation

$$= \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{4.527}{45} \times 100\%$$

$= 10.07\ \%$

3. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

A = Exactly 2 Heads

A = {HHT, HTH, THH}

n(A) = 3

$\Rightarrow P(A) = \frac{3}{8}$

B = Atleaset one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

n(B) = 7

$\Rightarrow P(B) = \frac{7}{8}$

C = Consecutively 2 heads

C = {HHH, HHT,THH}

$n(C) = 3 \Rightarrow P(C) = \frac{3}{8}$

$P(A\cap B) = \frac{3}{8}$; $P(B\cap C) = \frac{2}{8}$

$P(A\cap C) = \frac{2}{8}$; $P(A\cap B\cap C) = \frac{2}{8}$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$$

$$= \frac{15-7}{8} = \frac{8}{8} = 1$$

4. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B) = \frac{1}{6}$, $P(B\cap C) = \frac{1}{4}$, $P(A\cap C) = \frac{1}{8}$, $P(A\cup B\cup C) = \frac{9}{10}$, $P(A\cap B\cap C) = \frac{1}{15}$, then find P(A), P(B) and P(C)?

Solution:

$P(B) = 2\,P(A)$ ..... (1)

Let $P(C) = 3\,P(A)$ ..... (2)

and $P(A\cap B) = \frac{1}{6}$,

$P(B\cap C) = \frac{1}{4}$, $P(A\cap C) = \frac{1}{8}$,

$P(A\cup B\cup C) = \frac{9}{10}$, $P(A\cap B\cap C) = \frac{1}{15}$

$\Rightarrow P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$$\Rightarrow \frac{9}{10} = P(A) + 2P(A) + 3P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$\Rightarrow \frac{9}{10} = 6P(A) - \left(\frac{4+6+3}{24}\right) + \frac{1}{15}$$

$$\Rightarrow \frac{9}{10} = 6P(A) - \frac{13}{24} + \frac{1}{15}$$

$$\Rightarrow 6P(A) = \frac{9}{10} + \frac{13}{24} - \frac{1}{15}$$

$$\Rightarrow 6P(A) = \frac{216-16+130}{240}$$

$$= \frac{330}{240} = \frac{33}{24} = \frac{11}{8}$$

$$\Rightarrow P(A) = \frac{11}{8} \times \frac{1}{6};\quad P(A) = \frac{11}{48}$$

$$(1) \Rightarrow P(B) = 2 \times \frac{11}{48} = \frac{11}{24}$$

$$(2) \Rightarrow P(C) = 3 \times \frac{11}{48} = \frac{11}{16}$$

$$\therefore P(A) = \frac{11}{48},\ P(B) = \frac{11}{24},\ P(C) = \frac{11}{16}$$

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