

HINDU HIGHER SECONDARY SCHOOL, ALWARTHIRUNAGARAI.
TIRUCHENDUR EDUCATION DISTRICT.

QUARTERLY MODEL EXAM 2022-2023

Date : 08-Sep-22

Reg.No. :

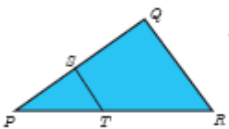
Exam Time : 03:00:00 Hrs

PART - A

Total Marks : 100

14 x 1 = 14

- Let f and g be two functions given by
 $f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$
 $g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$ then the range of $f \circ g$ is
 (a) $\{0,2,3,4,5\}$ (b) $\{-4,1,0,2,7\}$ (c) $\{1,2,3,4,5\}$ (d) $\{0,1,2\}$
- Composition of functions is commutative
 (a) Always true (b) Never true (c) Sometimes true
- If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then the value of m is
 (a) 4 (b) 2 (c) 1 (d) 3
- If 6 times of 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P. is
 (a) 0 (b) 6 (c) 7 (d) 13
- If $A = 2^{65}$ and $B = 2^{64} + 2^{63} + 2^{62} + \dots + 2^0$ Which of the following is true?
 (a) B is 2^{64} more than A (b) A and B are equal (c) B is larger than A by 1
 (d) A is larger than B by 1
- If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
 (a) 3 (b) 5 (c) 6 (d) 8
- The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are
 (a) 100, 120 (b) 10, 12 (c) -120, 100 (d) 12, 10
- If the roots of the equation $qx^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in _____.
 (a) A.P (b) G.P (c) Both A.P and G.P (d) none of these
- If in triangles ABC and EDF , $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when
 (a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$
- In a given figure $ST \parallel QR$, $PS = 2\text{cm}$ and $SQ = 3\text{ cm}$. Then the ratio of the area of $\triangle PQR$ to the area $\triangle PST$ is



- (a) 25 : 4 (b) 25 : 7 (c) 25 : 11 (d) 25 : 13
- The straight line given by the equation $x = 11$ is
 (a) parallel to X axis (b) parallel to Y axis (c) passing through the origin
 (d) passing through the point (0,11)
- The slope of the line joining $(12, 3)$, $(4, a)$ is $\frac{1}{8}$. The value of 'a' is
 (a) 1 (b) 4 (c) -5 (d) 2

13) (2, 1) is the point of intersection of two lines.

- (a) $x - y - 3 = 0$; $3x - y - 7 = 0$ (b) $x + y = 3$; $3x + y = 7$ (c) $3x + y = 3$; $x + y = 7$
 (d) $x + 3y - 3 = 0$; $x - y - 7 = 0$

14) The value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ equal to

- (a) $\tan^2\theta$ (b) 1 (c) $\cot^2\theta$ (d) 0

PART - B

14 x 2 = 28

Answer any 10 questions. Question No. 28 is compulsory

15) Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

16) Check whether the following sequences are in A.P. or not?

$$x + 2, 2x + 3, 3x + 4, \dots$$

17) Find the 19th term of an A.P. -11, -15, -19,....

18) Find the first term of a G.P. in which $S_6 = 4095$ and $r = 4$

19) If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17. find k

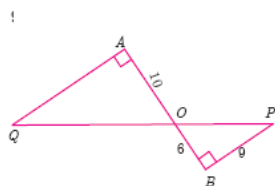
20) If α, β are the roots of the equation $3x^2 + 7x - 2 = 0$, find the values of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

21) If α and β are the roots of $x^2 + 7x + 10 = 0$ find the values of

$$\alpha^2 + \beta^2$$

22) QA and PB are perpendiculars to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ.



23) Find the equation of a line through the given pair of points $(2, \frac{2}{3})$ and $(\frac{-1}{2}, 2)$

24) Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

25) What is the inclination of a line whose slope is 1

26) Show that the given vertices form a right angled triangle and check whether it satisfies Pythagoras theorem L(0, 5), M(9,12) and N(3,14)

27) prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec} \theta + \cot \theta$

28) prove the following identities

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = 2\sec\theta$$

PART - C

14 x 5 = 70

Answer any 10 questions. Question No. 42 is compulsory

29) If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} 2x + 7, & x < -2 \\ x^2 - 2, & -2 \leq x < 3 \\ 3x - 2, & x \geq 3 \end{cases}$$

(i) $f(4)$

(ii) $f(-2)$

(iii) $f(4) + 2f(1)$

(iv) $\frac{f(1)-3f(4)}{f(-3)}$

30) Find the sum to n terms of the series $5 + 55 + 555 + \dots$

31) Find the GCD of the polynomials $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

32) Find the square root of the following expressions

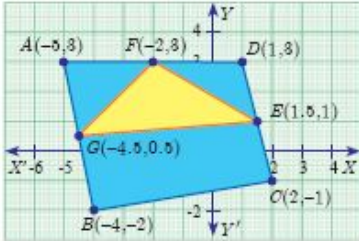
$$16x^2 + 9y^2 - 24xy + 24x - 18y + 9$$

33) Find the square root of the expression $\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$

34) The roots of the equation $x^2 + 6x - 4 = 0$ are α, β . Find the quadratic equation whose roots are α^2 and β^2

35) In trapezium ABCD, $AB \parallel DC$, E and F are points on non-parallel sides AD and BC respectively, such that $EF \parallel AB$. Show that $AE \frac{AE}{ED} = \frac{BF}{FC}$

36) In the figure, find the area of triangle AGF



37) If the points A(2, 2), B(-2, -3), C(1, -3) and D(x, y) form a parallelogram then find the value of x and y.

38) Find the equation of the median and altitude of ΔABC through A where the vertices are A(6, 2), B(-5, -1) and C(1, 9)

39) Prove that $\sin^2 A \cos^2 B + \cos^2 A \sin^2 B + \cos^2 A \cos^2 B + \sin^2 A \sin^2 B = 1$

40) if $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then prove that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$

41) If $\frac{\cos \alpha}{\cos \beta} = m$ and $\frac{\cos \alpha}{\sin \beta} = n$, then prove that $(m^2 + n^2) \cos^2 \beta = n^2$

42) if $\frac{\cos \theta}{1 + \sin \theta} = \frac{1}{a}$, then prove that $\frac{a^2 - 1}{a^2 + 1} = \sin \theta$

PART - D

4 x 8 = 32

Answer any one from 43 & 44 and any one from 45 & 46.

43) Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

44) A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find

(i) the constant of variation

(ii) how far will it travel in $\frac{1}{2}$

(iii) the time required to cover a distance of 300 km from the graph.

45) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)

46) Construct a ΔPQR which the base $PQ = 4.5$ cm, $\angle R = 35^\circ$ and the median RG R to PG is 6 cm



நன்றி!

வினையாட்டுத்துறையும், கணிதத்துறையும் ஒன்று விடா முயற்சி+கடின பயிற்சி = வெற்றி



10
Maths

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PART - A

- 1) (d) $\{0,1,2\}$
- 2) (a) Always true
- 3) (c) 1
- 4) (a) 0
- 5) (d) A is larger than B by 1
- 6) (b) 5
- 7) (c) -120, 100
- 8) (b) G.P
- 9) (c) $\angle B = \angle D$
- 10)
- (a) 25 : 4
- 11)
- (b) parallel to Y axis
- 12)
- (d) 2
- 13)
- (b) $x + y = 3$; $3x + y = 7$
- 14)
- (b) 1

PART - B

14 x 2 = 28

Answer any 10 questions. Question No. 28 is compulsory

15)

$$f(x) = 2x + 1, g(x) = x^2 - 2$$

$$f \circ g(x) = f(g(x)) = f(x^2 - 2) = 2(x^2 - 2) + 1 = 2x^2 - 3$$

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2 = 4x^2 + 4x - 1$$

Thus $f \circ g = 2x^2 - 3$, $g \circ f = 4x^2 + 4x - 1$. From the above, we see that $f \circ g \neq g \circ f$.

16)

To check that the given sequence is in A.P., it is enough to check if the differences between the consecutive terms are equal or not.

$$t_2 - t_1 = (2x + 3) - (x - 2) = x + 1$$

$$t_3 - t_1 = (3x + 4) - (2x + 3) = x + 1$$

$$t_2 - t_1 = t_3 - t_2$$

Thus, the differences between consecutive terms are equal.

Hence the sequence $x + 2, 2x + 3, 3x + 4, \dots$ is in A.P

Here First term $a = -11$

Common difference $d = t_2 - t_1 = -15 - (-11)$

$$= -15 + 11$$

$$d = -4$$

n^{th} term of an A.P. is $t_n = a + (n - 1)d$

$$19^{\text{th}} \text{ term } (t_{19}) = -11 + (19 - 1)(-4)$$

$$= -11 + 18(-4)$$

$$= -11 + (-72) = -83$$

19th term of -11, -15, -19, ... is -83.

18)

Common ratio $= 4 > 1$, sum of first 6 terms $S_6 = 4095$

$$\text{Hence, } S_6 = \frac{a(r^n - 1)}{r - 1} = 4095$$

$$\text{Since, } r = 4, \frac{a(4^6 - 1)}{4 - 1} = 4095 \text{ gives } a \times \frac{4095}{3} = 4095$$

First term $a = 3$.

19)

$$x^2 - 13x + k = 0 \text{ here, } a = 1, b = -13, c = k$$

Let α, β be the roots of the equation. Then

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-13)}{1} = 13 \dots\dots (1) \text{ also } \alpha - \beta = 17 \dots\dots (2)$$

$$(1) + (2) \text{ we get, } 2\alpha = 30 \text{ gives } \alpha = 15$$

$$\text{Therefore, } 15 + \beta = 13 \text{ (from (1)) gives } \beta = -2$$

$$\text{But, } \alpha\beta = \frac{c}{a} = \frac{k}{1} \text{ gives } 15 \times (-2) = k \text{ we get, } k = -30$$

20)

$$3x^2 + 7x - 2 = 0 \text{ here, } a = 3, b = 7, c = -2$$

since, α, β are the roots of the equation

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{3}, \alpha\beta = \frac{c}{a} = \frac{-2}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-7}{3}\right)^2 - 2\left(\frac{-2}{3}\right)}{\frac{-2}{3}} = \frac{-61}{6}$$

21)

$$x^2 + 7x + 10 \text{ here, } a = -1, b = 7, c = 10$$

if α and β are roots of the equation then,

$$\alpha + \beta = \frac{-b}{a} = \frac{-7}{-1} = 7; \alpha\beta = \frac{c}{a} = \frac{10}{-1} = -10$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (7)^2 - 2 \times (-10) = 49 + 20 = 69$$

22)

$$\triangle AOQ \text{ and } \triangle BOP, \angle OAQ = \angle OBP = 90^\circ$$

$$\angle AOQ = \angle BOP \text{ (Vertically opposite angles)}$$

Therefore, by AA Criterion of similarity,

$$\triangle AOQ \sim \triangle BOP$$

$$\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$$

$$\frac{10}{6} = \frac{AQ}{9} \text{ gives } AQ = \frac{10 \times 9}{6} = 15 \text{ cm}$$

23)

Equation of the line passing through (x_1, y_1) and (x_2, y_2)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}} = \frac{x-2}{-\frac{1}{2}-2}$$

$$\frac{3y-2}{-6-2} = \frac{2x-4}{-1-4}$$

$$-5(3y-2) = -8(2x-4)$$

$$-15y+10 = -16x+32$$

$$16x-15y-22=0$$

24)

Show that the straight lines $x-2y+3=0$ is

$$m_1 = \frac{-1}{-2} = \frac{1}{2}$$

Slope of the straight line $6x+3y+8=0$ is

$$m_2 = \frac{-6}{3} = -2$$

$$\text{Now, } m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$$

Hence, the two straight lines are perpendicular

25)

Slope 'm' = 1

$$\tan \theta = 1 = \tan 45^\circ$$

$$\theta = 45^\circ$$

26)

L (0, 9), M (9, 12) and N (3, 14)

$$\text{Slope of a line} = \frac{y_1-y_2}{x_1-x_2}$$

$$\text{Slope of LM} = \frac{5-12}{0-9} = \frac{-7}{-9} = \frac{7}{9}$$

$$\text{Slope of MN} = \frac{12-14}{9-3} = \frac{-2}{6} = \frac{-1}{3}$$

$$\text{Slope of LN} = \frac{5-14}{0-3} = \frac{-9}{-3} = 3$$

$$(\text{Slope of MN}) \times (\text{Slope of LN}) = \left(-\frac{1}{3}\right) \times 3 = -1$$

MN is perpendicular to LN.

Hence, the given vertices form a right angled triangle

$$\text{Distance formula} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$LM = \sqrt{(0-9)^2 + (5-12)^2} = \sqrt{81+49} = \sqrt{130}$$

$$LM^2 = 130$$

$$MN = \sqrt{(9-3)^2 + (12-14)^2} = \sqrt{36+4} = \sqrt{40}$$

$$MN^2 = 40$$

$$LN = \sqrt{(0-3)^2 + (5-14)^2} = \sqrt{9+81} = \sqrt{90}$$

$$LN^2 = 90$$

$$LN^2 + MN^2 = LM^2$$

Hence, the Pythagoras theorem is satisfied.

27)

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} \text{ [multiply numerator and denominator by the conjugate of } 1-\cos\theta]$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos\theta)^2}} = \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} \text{ [since } \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1+\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta$$

$$\sqrt{\frac{1+\sin \theta}{1-\sin \theta}} = \sec \theta + \tan \theta$$

$$\text{LHS} = \sqrt{\frac{1+\sin \theta}{1-\sin \theta}}$$

$$= \sqrt{\frac{1+\sin \theta}{1-\sin \theta} \times \frac{1-\sin \theta}{1-\sin \theta}}$$

[Multiplying the Numerator and denominator by $\sqrt{1-\sin \theta}$]

$$= \sqrt{\frac{1^2 - \sin^2 \theta}{(1-\sin \theta)^2}} \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \sqrt{\frac{\cos^2 \theta}{(1-\sin \theta)^2}} \quad [\because 1 - \sin^2 \theta = \cos^2 \theta]$$

$$= \frac{\cos \theta}{1-\sin \theta}$$

$$= \frac{\cos \theta}{1-\sin \theta} \times \frac{1+\sin \theta}{1+\sin \theta}$$

[Multiplying Numerator and denominator by $1 + \sin \theta$]

$$= \frac{\cos \theta(1+\sin \theta)}{1^2 - \sin^2 \theta} = \frac{\cos \theta(1+\sin \theta)}{\cos^2 \theta}$$

$$[\because (a+b)(a-b) = a^2 - b^2] \quad [1 - \sin^2 \theta = \cos^2 \theta]$$

$$= \frac{1+\sin \theta}{\cos \theta} = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \sec \theta + \tan \theta = \text{RHS}$$

PART - C

14 x 5 = 70

Answer any 10 questions. Question No. 42 is compulsory

29)

The function f is defined by three values in intervals I, II, III as shown by the side.

For a given value of $x = a$, find out the interval at which the point a is located, there after find

$f(a)$ using the particular value defined in that interval.

(i) First, we see that, $x = 4$ lie in the third interval.

Therefore, $f(x) = 3x - 2$; $f(4) = 3(4) = 10$

(ii) $x = -2$ lies in the second interval

Therefore, $f(x) = x^2 - 2$; $f(-2) = (-2)^2 - 2 = 2$

(iii) From (i), $f(4) = 10$.

To find $f(1)$ first we see that $x = 1$ lies in the second interval.

Therefore, $f(x) = x^2 - 2 \Rightarrow f(1) = 1^2 - 2 = -1$

So, $f(4) + 2f(1) = 10 + 2(-1) = 8$

(iv) We know that $f(1) = -1$ and $f(4) = 10$

For finding $f(-3)$, we see that $x = -3$, lies in the first interval.

Therefore, $f(x) = 2x + 7$; thus, $f(-3) = 2(-3) + 7 = 1$

Hence, $\frac{f(1)-3f(4)}{f(-3)} = \frac{-2-3(10)}{1} = -31$

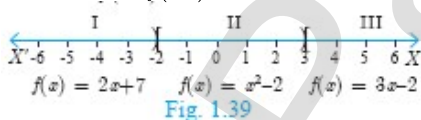


Fig. 1.39

30)

The series is neither Arithmetic nor Geometric series. So it can be split into two series and then find the sum.

$$5 + 55 + 555 + \dots + n \text{ terms} = 5 [1 + 11 + 111 + \dots n \text{ terms}]$$

$$= \frac{5}{9} [9 + 99 + 999 + \dots + n \text{ terms}]$$

$$= \frac{5}{9} [(10 - 1) + (100 - 1) + (1000 - 1) + \dots + n \text{ terms}]$$

$$= \frac{5}{9} [10 + 100 + 1000 + \dots + n \text{ terms}] - n$$

$$= \frac{5}{9} \left[\frac{10(10^n - 1)}{(10 - 1)} - n \right] = \frac{50(10^n - 1)}{81} - \frac{5n}{9}$$

31)

Let $f(x) = 2x^3 - 5x^2 + 5x - 3$ and $g(x) = x^3 + x^2 - x + 2$

$$\begin{array}{r}
 2 \\
 \overline{2x^3 - 5x^2 + 5x - 3} \\
 2x^3 + 2x^2 - 2x + 4 \quad (-) \\
 \hline
 -7x^2 + 7x - 7 \\
 \hline
 -7(x^2 - x + 1)
 \end{array}$$

$-7(x^2 - x + 1) \neq 0$, note that -7 is not a divisor of $g(x)$

Now dividing, $g(x) = x^3 + x^2 - x + 2$ by the new remainder $x^2 - x + 1$ (leaving the constant factor), we get

$$\begin{array}{r}
 x + 2 \\
 \overline{x^3 + x^2 - x + 2} \\
 x^3 - x^2 + x \quad (-) \\
 \hline
 2x^2 - 2x + 2 \\
 2x^2 - 2x + 2 \quad (-) \\
 \hline
 0
 \end{array}$$

Here, we get zero remainder

Therefore, $\text{GCD}(2x^3 - 5x^2 + 5x - 3, x^3 + x^2 - x + 2) = x^2 - x + 1$.

32)

$$\begin{aligned}
 & \sqrt{16x^2 + 9y^2 - 24xy + 24x - 18y + 9} \\
 &= \sqrt{(4x)^2 + (-3y)^2 + (3)^2 + 2(4x)(-3y) + 2(-3y)(3) + 2(4x)(3)} \\
 &= \sqrt{(4x - 3y + 3)^2} = |4x - 3y + 3|
 \end{aligned}$$

33)

$$\begin{array}{r}
 \frac{2x}{y} + 5 - \frac{3y}{x} \\
 \overline{\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}} \\
 \frac{4x^2}{y^2} \quad (-) \\
 \hline
 \frac{4x}{y} + 5 \quad \overline{\frac{20x}{y} + 13} \\
 \frac{20x}{y} + 25 \quad (-) \\
 \hline
 \frac{4x}{y} + 10 - \frac{3y}{x} \quad \overline{-12 - \frac{30y}{x} + \frac{9y^2}{x^2}} \\
 -12 - \frac{30y}{x} + \frac{9y^2}{x^2} \quad (-) \\
 \hline
 0
 \end{array}$$

Hence $\sqrt{\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}} = \left| \frac{2x}{y} + 5 - \frac{3y}{x} \right|$

34)

If the roots are given, the quadratic equation is $X^2 - (\text{sum of the roots})x + \text{product the roots} = 0$. For the given equation.

$$x^2 - 6x - 4 = 0$$

$$\alpha + \beta = -6$$

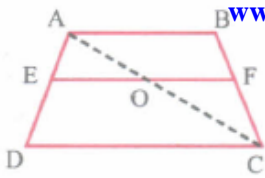
$$\alpha\beta = -4$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-6)^2 - 2(-4) = 36 + 8 = 44$$

$$\alpha^2\beta^2 = (\alpha\beta)^2 = (-4)^2 = 16$$

$$\therefore \text{The required equation} = x^2 - 44x + 16 = 0$$



Given: ABCD is a trapezium in which $DC \parallel AB$ and $EF \parallel AB$

To prove that $\frac{AE}{ED} = \frac{BF}{FC}$

Construction : join AC meeting EF at G

Proof:

In ADC, we have

$EG \parallel DC$

$$\Rightarrow \frac{AE}{ED} = \frac{AG}{GC} \text{ [By thales theorem]} \quad \dots(1)$$

In ABC, we have

$$\frac{AG}{GC} = \frac{BF}{FC} \text{ [By thales theorem]} \quad \dots(2)$$

From (1) and (2), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

36)

Area of triangle AGF

Vertices A (-5, 3), G (-4.5, 0.5) and F (-2, 3).

Area of triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. units

$$= \frac{1}{2}[-5(0.5 - 3) - 4.5(3 - 3) - 2(3 - 0.5)]$$

$$= \frac{1}{2}[12.5 - 5] = \frac{7.5}{2} = 3.75 \text{ sq. units}$$

37)



$$\text{Slope of a line} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{Slope of AB} = \frac{2+3}{2+2} = \frac{5}{4}$$

$$\text{Slope of BC} = \frac{-3+3}{-2-1} = 0$$

$$\text{Slope of CD} = -\frac{-3-y}{1-x}$$

$$\text{Slope of AD} = \frac{2-y}{2-x}$$

Since, the points form a parallelogram

AB is parallel to CD and BC is parallel to AD

Slope of AB = Slope of CD

$$\frac{5}{4} = \frac{-3-y}{1-x}$$

$$5(1-x) = 4(-3-y)$$

$$5 - 5x = -12 - 4y$$

$$5x - 4y = 17$$

Slope of BC = Slope of AD

$$0 = \frac{2-y}{2-x}$$

$$2 - y = 0$$

$$y = 2$$

Substituting in (1)

$$5x - 4(2) = 17$$

$$5x = 17 + 8 = 25$$

$$x = \frac{25}{5} = 5$$

$$x = 5, y = 2$$

38)



Given vertices are A (6, 2), B (- 5, - 1) and C (1, 9)

Median through A :

Let D be the mid point of BC

$$\text{Mid point of BC} = D \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= D \left(\frac{-5+1}{2}, \frac{-1+9}{2} \right)$$

$$= D (-2, 4)$$

Now AD is the median.

$$\text{Equation of AD } \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

$$-4y + 8 = x - 6$$

$$x + 4y - 14 = 0$$

Altitude through A



Altitude is passing through 'A' and perpendicular to BC.

Now,

$$\text{Slope of BC} = \frac{y_1-y_2}{x_1-x_2} = \frac{-1-9}{-5-1} = \frac{-10}{-6} = \frac{5}{3}$$

$$\text{Slope of Altitude} = -\frac{3}{5}$$

Equation of the altitude which is passing through A (6,2) and having slope $-\frac{3}{5}$ is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{5}(x - 6)$$

$$5y - 10 = -3x + 18$$

$$3x + 5y - 28 = 0$$

39)

$$\begin{aligned} & \sin^2 A \cos^2 B + \cos^2 A \sin^2 B + \cos^2 A + \cos^2 B + \sin^2 A \sin^2 B \\ &= \sin^2 A \cos^2 B + \sin^2 A \sin^2 B + \cos^2 A + \cos^2 B + \sin^2 A \sin^2 B \\ &= \sin^2 A (\cos^2 B + \sin^2 B) + \cos^2 A (\sin^2 B + \cos^2 B) \\ &= \sin^2 A (1) + \cos^2 A (1) \quad (\text{since } \sin^2 B + \cos^2 B = 1) \\ &= \sin^2 A + \cos^2 A = 1 \end{aligned}$$

40)

Squaring both sides,

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2} \cos\theta)^2$$

$$\cos^2\theta + \sin^2\theta + 2\sin\theta \cos\theta = 2\cos^2\theta$$

$$2\cos^2\theta - \cos^2\theta - \sin^2\theta = 2\sin\theta \cos\theta$$

$$\cos^2\theta - \sin^2\theta = 2\sin\theta \cos\theta$$

$$(\cos\theta + \sin\theta)(\cos\theta + \sin\theta) = 2\sin\theta \cos\theta$$

$$\cos\theta - \sin\theta = \frac{2\sin\theta \cos\theta}{\cos\theta + \sin\theta} = \frac{2\sin\theta \cos\theta}{\sqrt{2}\cos\theta} \quad [\text{since } \cos\theta + \sin\theta = \sqrt{2} \cos\theta]$$

$$= \sqrt{2} \cos\theta$$

$$\text{Therefore } \cos\theta - \sin\theta = \sqrt{2} \cos\theta$$

41)

Given

$$\frac{\cos \alpha}{\cos \beta} = m$$

$$\frac{\cos \alpha}{\sin \beta} = n$$

$$\begin{aligned} \text{LHS} &= (m^2 + n^2) \cos^2 \beta \\ &= \left(\frac{\cos^2 \alpha}{\cos^2 \beta} + \frac{\cos^2 \alpha}{\sin^2 \beta} \right) \cos^2 \beta \\ &= \frac{(\cos^2 \alpha \sin^2 \beta + \cos^2 \alpha \cos^2 \beta)}{\cos^2 \beta \sin^2 \beta} \cos^2 \beta \\ &= \frac{\cos^2 \alpha (\sin^2 \beta + \cos^2 \beta)}{\sin^2 \beta} \\ &= \frac{\cos^2 \alpha}{\sin^2 \beta} (1) \\ &= \left(\frac{\cos \alpha}{\sin \beta} \right)^2 \\ &= n^2 = \text{RHS} \end{aligned}$$

42)

$$\text{Given } \frac{\cos \theta}{1 + \sin \theta} = \frac{1}{a}$$

$$\therefore a = \frac{1 + \sin \theta}{\cos \theta}$$

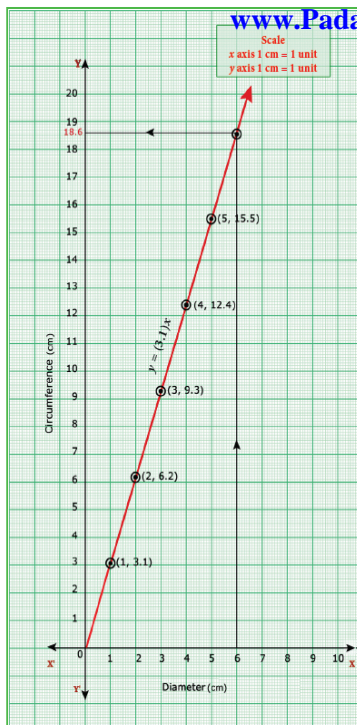
$$\begin{aligned} \text{LHS} &= \frac{a^2 - 1}{a^2 + 1} \\ &= \frac{\left(\frac{1 + \sin \theta}{\cos \theta} \right)^2 - 1}{\left(\frac{1 + \sin \theta}{\cos \theta} \right)^2 + 1} \\ &= \frac{\frac{1^2 + \sin^2 \theta + 2 \sin \theta}{\cos^2 \theta} - 1}{\frac{1^2 + \sin^2 \theta + 2 \sin \theta}{\cos^2 \theta} + 1} \\ &= \frac{\frac{1 + \sin^2 \theta + 2 \sin \theta - \cos^2 \theta}{\cos^2 \theta}}{\frac{1 + \sin^2 \theta + 2 \sin \theta + \cos^2 \theta}{\cos^2 \theta}} \\ &= \frac{(1 - \cos^2 \theta) + \sin^2 \theta + 2 \sin \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{1 + (\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta} \\ &= \frac{\sin^2 \theta + \sin^2 \theta + 2 \sin \theta}{1 + 1 + 2 \sin \theta} \\ &= \frac{2 \sin^2 \theta + 2 \sin \theta}{2 + 2 \sin \theta} \\ &= \frac{2 \sin \theta (\sin \theta + 1)}{2(1 + \sin \theta)} \\ &= \sin \theta = \text{RHS} \end{aligned}$$

PART - D

4 x 8 = 32

Answer any one from 43 & 44 and any one from 45 & 46.

43)



From the table, we found that as x increases, y also increases. Thus, the variation is a direct variation.

Let $y = kx$, where k is a constant of proportionality.

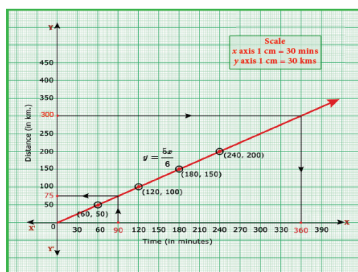
From the given values, we have

$$k = \frac{3.1}{1} = \frac{6.2}{2} = \frac{9.3}{3} = \frac{12.4}{4} = \dots = 3.1$$

When you plot the points (1, 3.1) (2, 6.2) (3, 9.3), (4, 12.4), (5, 15.5), you find the relation $y = (3.1)x$ forms a straight-line graph.

Clearly, from the graph, when diameter is 6 cm, its circumference is 18.6 cm.

44)



Let x be the time taken in minutes and y be the distance travelled in km.

Time taken x (in minutes)	60	120	180	240
Distance y (in km)	50	100	150	200

(i) Observe that as time increases, the distance travelled also increases. Therefore, the variation is a direct variation. It is of the form $y = kx$.

Constant of variation

$$k = \frac{y}{x} = \frac{50}{60} = \frac{100}{120} = \frac{150}{180} = \frac{200}{240} = \frac{5}{6}$$

Hence, the relation may be given as

$$y = kx \Rightarrow y = \frac{5}{6}x$$

(ii) From the graph, $y = \frac{5x}{6}$, if $x = 90$, then $y = \frac{5}{6} \times 90 = 75$ km

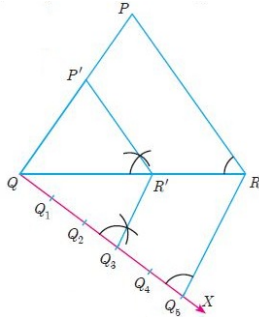
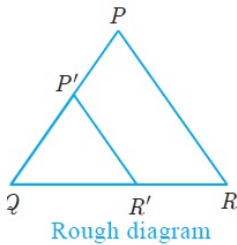
The distance travelled for $1\frac{1}{2}$ hours (i.e.,) 90 minutes is 75 km.

(iii) From the graph, $y = \frac{5x}{6}$, if $y = 300$ then $x = \frac{6y}{5} = \frac{6}{5} \times 300 = 360$ minutes (or) 6 hours.

The time taken to cover 300 km is 360 minutes, that is 6 hours.

45)

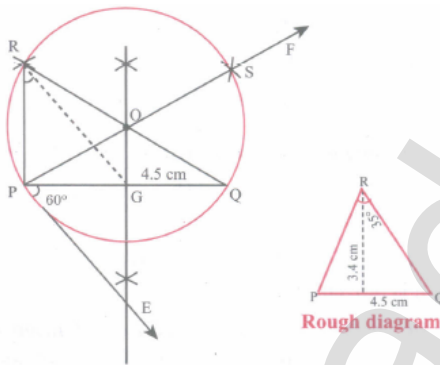
Given a triangle PQR we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR.



Steps of construction

1. Construct a $\triangle PQR$ with any measurement
 2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P .
 3. Locate 5 (the greater of 3 and 5 in $\frac{3}{5}$) points.
 Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$
 4. Join Q_5R and draw a line through Q_3 (the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q_5R to intersect QR at R' .
 5. Draw line through R' parallel to the line RP to intersect QP at P' .
- Then, $\triangle P'QR'$ is the required triangle each of whose sides is three-fifths of the corresponding sides of $\triangle PQR$.

46)



Construction:

- Step (1) Draw a line segment $PQ = 4.5$ cm
- Step (2) At P , draw PE such that $\angle QPE = 35^\circ$
- Step (3) At P , draw PF such that $\angle EPF = 90^\circ$
- Step (4) Draw \perp bisector to PQ which intersects PF at O .
- Step (5) With O centre OP as radius draw a circle.
- Step (6) From G , marked arcs of radius 6 cm on the circle marked them as R and S .
- Step (7) joined PR and RQ . Then $\triangle PQR$ is the required triangle
- Step (8) $\triangle PQS$ is the required triangle

