QUARTERLY EXAMINATION - 2022

Reg. No.

XII - BUSINESS MATHEMATICS & STATISTICS

Time Allowed : 3.00 Hrs. Trichy-Dt. Maximum Marks: 90

INSTRUCTIONS : 1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2. Use **Blue** or **Black** ink to write and underline and **Pencil** to draw diagrams.

PART - I

Note i) ***Answer all the questions***

ii) ***Choose the most appropriat answer from the given four alternatives and write the option code and the corresponding answer.*** ***(20×1=20)***

1. $\nabla \equiv$ ______

a) 1 + E b) 1 - E c) $1-E^{-1}$ d) $1+E^{-1}$

2. If h = 1 then $\Delta(x^2) =$ ______

a) 2x b) 2x - 1 c) 2x + 1 d) 1

3. If C is a constant then the value of E(C) ______

a) 0 b) 1 c) c f (c) d) c

4. If E(x) = 5 and E(y) = -2 then the value of E(x-y)

a) 3 b) 5 c) 7 d) -2

5. If $|A_{nxn}| = 3$, $|adjA|=243$ then the value of n

a) 4 b) 5 c) 6 d) 7

6. If A= (1 2 3) then the rank of AA^T

a) 0 b) 2 c) 3 d) 1

7. The value of $\int_{-\pi/2}^{\pi/2} \cos x\, dx$ a) 0 b) 2 c) 1 d) 4

8. $\int\sqrt{e^x}\, dx =$ ______

a) $\sqrt{e^x} + c$ b) $2\sqrt{e^x} + c$ c) $\frac{1}{2}\sqrt{e^x} + c$ d) $\frac{1}{2\sqrt{e^x}} + c$

9. The profit of a function p(x) is maximum when

a) MC - MR = 0 b) MC = 0 c) MR = 0 d) MC + MR = 0

10. The complementary function of $(D^2 + 4) y = e^{2x}$ is

a) $(Ax + B)e^{2x}$ b) $(Ax + B)e^{-2x}$ c) Acos2x + Bsin2x d) $Ae^{-2x}+ Be^{2x}$

11. The degree of differential equation $\frac{d^4y}{dx^4} - \left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} = 3$

a) 1 b) 2 c) 3 d) 4

12. The demand and supply functions are given by $D(x) = 16 - x^2$ and $S(x) = 2x^2 + 4$ are under perfect competition, then the equilibrium price x is

a) 2 b) 3 c) 4 d) 5

13. If $A = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$ then $\rho(A) =$ ______

a) 0 b) 1 c) 2 d) n

14. The value of $\int_0^\infty x^4 e^{-x} dx$

a) 12 b) 4 c) 4 ! d) 64

15. Area bounded by $y = |x|$ between the limits 0 and 2

a) 1 sq.units b) 3 sq.units c) 2 sq.units d) 4 sq.units

16. The integrating factor of $x\frac{dy}{dx} - y = x^2$ is

a) $\frac{-1}{x}$ b) $\frac{1}{x}$ c) log x d) x

17. If $f(x) = x^2 + 2x + 2$ and $h = 1$ then $\Delta f(x) =$ ______

a) 2x - 3 b) 2x + 3 c) x + 3 d) x- 3

18. If P(x)= 1/10, x = 10 then the value of E (x)

a) 0 b) 6/8 c) 1 d) -1

19. $\Delta(\log ax) =$ ______

a) $\log\frac{h}{x}$ b) $\log\left(1+\frac{h}{x}\right)$ c) log 1 d) 0

20. $\int_0^{\pi/3} \tan x dx =$ ______

a) log 2 b) 0 c) $\log\sqrt{2}$ d) 2log2

PART - II

Answer any seven questions. Question Number 30 is compulsory.

(7x2=14)

21. Find the rank of matrix $\begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix}$

22. Solve : 3x + 5y = 9, 2x + 3y = 7

23. Evaluate : $\int \left(x+\frac{1}{x}\right)^2 dx$

24. Find the order and degree of differential equation $\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{1/2} = 0$

25. If f(x) = $x^2 + 3x$ and h= 1 then prove that $\Delta f(x) = 2x + 4$

26. Construct cumulative distribution function for the given probability distribution.

X	0	1	2	3
P(X=x)	0.3	0.2	0.4	0.1

27. Write any two properties of Mathematical expectation.

28. Let X be a random variable and Y = 2x + 1. What is the variance of Y if variance of X is 5?

29. Evaluate : $\int_0^\infty e^{-2x} x^5 dx$

30. Find the area bounded by y = 4x + 3 with x axis between the lines x = 1 and x = 4.

PART - III

Answer any seven questions. Question Number 40 is compulsory. ***(7x3=21)***

31. If $U_0 = 1$, $U_1 = 11$, $U_2 = 21$, $U_3 = 28$ and $U_4 = 29$ then find $\Delta^4 U_0$

32. A continuous random variable x has p.d.f. $f(x) = 5x^4$ $0 \le x \le 1$ find a_1 such that $P[x \le a_1] = P[x > a_1]$

33. In a business venture a man can make a profit of ₹2000 with a probability of 0.4 or have a loss of ₹1000 with a probability of 0.6. What is his expected, variance and standard deviation of profit?

34. Solve by rank method x + y + z = 9, 2x + 5y + 7z = 52, 2x + y - z = 0

35. If f'(x) = 1/x and f(1) = $\pi/4$ then find f(x)

36. Evaluate by using property $\int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} dx$

37. Solve the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

38. Determine the cost of producing 200 air conditioners if the marginal cost (is per unit) is $C'(x) = \frac{x^2}{200} + 4$.

39. Find the area of the parabola $y^2 = 8x$ bounded by its latus rectum.

40. Find the missing term.

x	0	1	2	3	4
y	1	3	9	-	81

PART - IV

Answer all the questions. ***(7x5=35)***

41. a. Investigate for what values of 'a' and 'b' the following system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ have i) no solution ii) a unique solution iii) an infinite number of solutions. **(OR)**

b. Integrate : $\frac{4x^2+2x+6}{(x+1)^2(x-3)}$

42. a. Find the consumer's surplus and producer's surplus for the demand function $P_d = 25 - 3x$ and supply function $P_s = 5 + 2x$ **(OR)**

b. Solve : $(3D^2 + D - 14)y = 4 - 13e^{-7/3x}$

43. a. If x = 32 then find f(x) from the following table.

x	30	35	40	45	50
f(x)	15.9	14.9	14.1	13.3	12.5

(OR)

b. Find a polynomial of degree two which takes the values.

x	0	1	2	3	4	5	6	7
f(x)	1	2	4	7	11	16	22	29

44. a. Solve the differential equation $\frac{dy}{dx} = \frac{x-y}{x+y}$ **(OR)**

b. Find the area of the circle whose center is at the origin and the radius is 'a' unit by using integration method.

45. a. Evaluate : $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}dx$ **(OR)**

b. Elasticity of a function $\frac{Ey}{Ex}$ is given by $\frac{Ey}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$. Find the function when x = 2, y = 3/8

46. a. A continuous random variable x has the following probability function

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find k

ii) Evaluate $P(x < 6)$, $P(x \geq 6)$ and $P(0<x<5)$

iii) If $P(x \leq x) > 1/2$ then find the minimum value of k **(OR)**

b. A total of ₹8500 was invested in three interest earning accounts. The interest rates were 2%, 3% and 6% if the total simple interest for one year was ₹380 and the amount invested at 6% was equal to the sum of the amounts in the other two accounts, then how much was invested in each account. (use Cramer's rule) **(OR)**

47. a. If f(x) is defined by $f(x) = Ke^{-2x}$ $0 \leq x < \infty$ is a density function. Determine the constant, k, mean and variance. **(OR)**

b. If h = 1 then Evaluate $\Delta\left[\frac{5x+12}{x^2+5x+6}\right]$

Dalmia Hr. Sec. School, Dalmiapuram - Trichy - Dt.

Std-12 Quarterly Examination - Trichy dt.

Business Maths and Statistics

Answer Key Prepared By

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Part-I (onemark)

1. (c) $1-e^{-1}$
2. (c) $2x+1$
3. (d) c
4. (c) 7
5. (c) 6
6. (d) 1
7. (b) 2
8. (b) $2\sqrt{e^x}+c$
9. (a) $MC-MR=0$
10. (c) $A\cos 2x + B\sin 2x$
11. (a) 1
12. (a) 2
13. (c) 2
14. (c) 4!
15. (c) 2 Sq. units
16. (b) $\frac{1}{x}$
17. (b) $2x+3$
18. (c) 1
19. (b) $\log\left(1+\frac{h}{x}\right)$
20. (a) $\log 2$.

Part-II (Two Marks)

21) $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}_{3\times 3}$

$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{vmatrix} = 22 \neq 0$

$\therefore \rho(A) = 3$

22) $\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10-9 = 1 \neq 0$

$\Delta_x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 8$

$\Delta_y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = -3$

By Cramer's rule,

$x = \frac{\Delta_x}{\Delta} = \frac{8}{1} = 8$

$y = \frac{\Delta_y}{\Delta} = \frac{-3}{1} = -3$

(23)

$$\int \left(x+\frac{1}{x}\right)^2 dx$$

$$= \int \left[x^2+\frac{1}{x^2}+2\right] dx$$

$$= \frac{x^3}{3} - \frac{1}{x} + 2x + C$$

(24)

$$\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{1/2} = 0$$

$$\frac{d^3y}{dx^3} = \left(\frac{dy}{dx}\right)^{1/2}$$

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{dy}{dx}\right)$$

Order (வரிசை) = 3

Degree (படி) = 2

(25)

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x) = \Delta(x^2+3x)$$

$$= \left[(x+1)^2 + 3(x+1)\right] - \left[x^2+3x\right]$$

$$= x^2+2x+1+3x+3-x^2-3x$$

$$= 2x+4$$

(26) $F(x) = P(X \le x)$

$F(0) = P(X \le 0) = 0.3$

$F(1) = P(X \le 1) = 0.3 + 0.2 = 0.5$

$F(2) = P(X \le 2) = 0.3 + 0.2 + 0.4$

$= 0.9$

$F(3) = P(X \le 3)$

$= 0.3 + 0.2 + 0.4 + 0.1$

$= 1$

$$\therefore F(x) = \begin{cases} 0 & ; x < 0 \\ 0.3 & ; 0 \le x < 1 \\ 0.5 & ; 1 \le x < 2 \\ 0.9 & ; 2 \le x < 3 \\ 1 & ; x \le 3 \end{cases}$$

(27) (i) $E(a) = a$, $a \to$ constant

(ii) $E(X+Y) = E(X) + E(Y)$

(iii) $E(aX) = aE(X)$

(iv) $E(aX+b) = aE(X)+b$

, $a, b \to$ constants

(28) $V(Y) = V(2X+1)$

$= V(2X) + V(1)$

$= 2^2 V(X) + 0$

$= 4V(X)$

$= 4(5)$

$= 20$

(29)

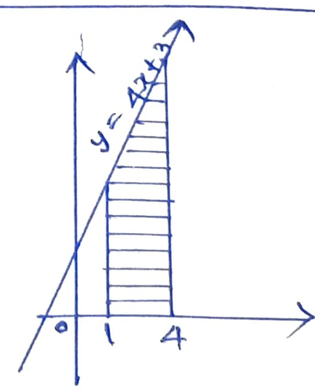
$$\int_0^\infty e^{-2x} x^5 \, dx = \frac{n!}{a^{n+1}}$$

$$= \frac{5!}{2^{5+1}} = \frac{120}{64} = \frac{15}{8}$$

(30)

$$\text{Area} = \int_1^4 y \, dx$$

$$= \int_1^4 (4x+3) \, dx$$



$$= \left(\frac{4x^2}{2} + 3x\right)_1^4$$

$$= \left[\frac{4(4)^2}{2} + 3(4)\right] - \left[\frac{4(1)^2}{2} + 3(1)\right]$$

$$= [32+12] - [2+3]$$

$$= 44 - 5 = 39 \text{ sq. units.}$$

Part - III (Three marks)

31

$\Delta^4 U_0 = (E-1)^4 U_0$

$= (E^4 - 4E^3 + 6E^2 - 4E + 1) U_0$

$= E^4 U_0 + 4E^3 U_0 + 6E^2 U_0$
$\quad - 4E^1 U_0 + U_0$

$= U_4 - 4U_3 + 6U_2 - 4U_1 + U_0$

$= 29 - 4(28) + 6(21) - 4(11) + 1$

$= 156 - 156$

$= 0$

32 $P(X \le a_1) = P(X > a_1)$

$P(X \le a_1) = 1/2$

$\int_0^{a_1} f(x)\, dx = 1/2$

$\int_0^{a_1} 5x^4 dx = 1/2$

$\cancel{5}\left(\frac{x^5}{\cancel{5}}\right)_0^{a_1} = 1/2$

$a_1^5 = 0.5$

$a_1 = (0.5)^{1/5}$

33 The p.d.f is

X :	-1000	2000
$P(X)$:	0.6	0.4

$E(X) = \sum x\, p(x)$

$= -1000 \times 0.6 + 2000 \times 0.4$

$= -600 + 800$

$E(X) = 200$

$E(X^2) = \sum x^2 p(x)$

$E(X^2) = (-1000)^2 (0.6) + (2000)^2 \times (0.4)$

$= 600000 + 1600000$

$E(X^2) = 2200000$

$V(X) = E(X^2) - [E(X)]^2$

$= 2200000 - 40000$

$V(X) = 2160000$

$S.D = \sqrt{V(X)} = \sqrt{2160000}$

$S.D = ₹1469.69.$

34 $AX = B$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix}$$

The augmented matrix is

$$(A|B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{array}\right] R_3 \to 3R_3 + R_2$$

$\therefore \rho(A|B) = 3, \quad \rho(A) = 3, \; n = 3$

$\therefore \rho(A|B) = \rho(A) = n = 3$

$\therefore$ The system is consistent and it has unique solution.

$\therefore AX = B \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 34 \\ -20 \end{pmatrix}$

$x + y + z = 9$

$3y + 5z = 34$

$-4z = -20$

$\therefore z = 5, \quad y = 3, \quad x = 1$

35

$f(x) = \int f'(x)\,dx + c$

$= \int \frac{1}{x}\,dx + c$

$f(x) = \log x + c \rightarrow (1)$

Put $x = 1$, $f(1) = \pi/4$ in (1)

$\therefore \frac{\pi}{4} = \log 1 + c$

$c = \frac{\pi}{4}$

$\therefore f(x) = \log x + \frac{\pi}{4}$

36

$$I = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x}\,dx \rightarrow (1)$$

$$I = \int_0^{\pi/2} \frac{\sin^7\left(\frac{\pi}{2}-x\right)}{\sin^7\left(\frac{\pi}{2}-x\right) + \cos^7\left(\frac{\pi}{2}-x\right)}\,dx$$

$$I = \int_0^{\pi/2} \frac{\cos^7 x}{\cos^7 x + \sin^7 x}\,dx \rightarrow (2)$$

$(1) + (2) \Rightarrow$

$$2I = \int_0^{\pi/2} \frac{\sin^7 x + \cos^7 x}{\sin^7 x + \cos^7 x}\,dx$$

$$2I = \int_0^{\pi/2} dx = (x)_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

37 The A.E is $m^2 - 6m + 8 = 0$

$(m-2)(m-4) = 0$

$m = 2$ and $m = 4$

$\therefore m_1 = 2$ and $m_2 = 4$

$\therefore CF = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{2x} + Be^{4x}$

38 $C'(x) = \frac{x^2}{200} + 4$

$C(x) = \int\left(\frac{x^2}{200} + 4\right)dx + k$

$C = \frac{x^3}{600} + 4x + k$

when $x = 0$, $c = 0 \Rightarrow k = 0$

$\therefore C(x) = \frac{x^3}{600} + 4x$

when $x = 200$,

$C = \frac{(200)^3}{600} + 4(200)$

$= \frac{8000000}{600} + 800$

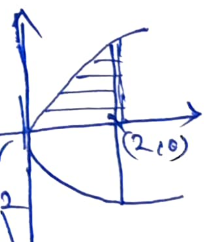
$C(x) = ₹14133.33$

39 $y^2 = 8x$ $\because y^2 = 4ax$

$4a = 8 \Rightarrow \boxed{a = 2}$

$A = 2\int_0^a y\,dx$

$= 2\int_0^2 \sqrt{8x}\,dx$



$= 2\sqrt{8}\left(\frac{x^{3/2}}{3/2}\right)_0^2$

$= \frac{4}{3} \times 2\sqrt{2}\left(2^{3/2}\right)$

$= \frac{4}{3} \times 2\sqrt{2} \times 2\sqrt{2}$

$A = \frac{32}{3}$ sq. units.

40 $\Delta^4 y_0 = 0$

$(E-1)^4 y_0 = 0$

$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$81 - 4y_3 + 54 - 12 + 1 = 0$

$y_3 = 31$

Part - IV (five marks)

41 (a)

$AX = B$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ b \end{pmatrix}$$

$$(A|B) = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{pmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{pmatrix} R_3 \to R_3 - R_2$$

case (i) No solution

$\rho(A) \neq \rho(A|B)$

$\therefore a-3 = 0$ and $b-10 \neq 0$

$\therefore a = 3$, $b \neq 10$

case (ii) unique solution

$\rho(A) = \rho(A|B)$

$\therefore a-3 \neq 0$ $\rho(A) = \rho(A|B) = 3$

$\therefore a \neq 3$ and $b \in R$

case (iii) Many solution

$\rho(A) = \rho(A|B) < n = 3$

$\therefore a-3 = 0$, $b-10 = 0$

$a = 3$ and $b = 10$.

41 (b)

By partial fraction :

$$\frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \to (1)$$

After solving,

$A = 3$, $B = 1$, $C = -2$

$$\therefore \frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{3}{x-3} + \frac{1}{x+1} - \frac{2}{(x+1)^2}$$

So, $$\int \frac{4x^2+2x+6}{(x+1)^2(x-3)}\,dx$$

$$= \int \frac{3}{x-3}\,dx + \int \frac{1}{x+1}\,dx - \int \frac{2}{(x+1)^2}\,dx$$

$$= 3\log|x-3| + \log|x+1| - \frac{2(x+1)^{-1}}{(-1)} + C$$

$$= 3\log|x-3| + \log|x+1| + \frac{2}{x+1} + C$$

42(a)

$P_d = 25-3x$

$P_s = 5+2x$

under market equilibrium,

$P_d = P_s$

$25-3x = 5+2x$

$25-5 = 2x+3x$

$5x = 20$

$x = 4$

when $x=4$, $P_0 = 25-3(4)$

$P_0 = 13$

$CS = \int_0^{x_0} f(x)\,dx - P_0x_0$

$= \int_0^4 (25-3x)\,dx - 52$

$= \left(25x - \frac{3x^2}{2}\right)_0^4 - 52$

$= \left\{\left[25(4) - \frac{3(4)^2}{2}\right] - 0\right\} - 52$

$= 100 - 24 - 52$

$= 24$ units.

$PS = P_0x_0 - \int_0^{x_0} g(x)\,dx$

$= 52 - \int_0^4 [5x+x^2]_0^4$

$= 52 - [5(4)+4^2]$

$= 52 - 36$

$PS = 16$ units

42 (b)

The A.E is $3m^2+m-14=0$

$(3m+7)(m-2)=0$

$m = \frac{-7}{3}, 2$

$CF = Ae^{-7/3\,x} + Be^{2x}$

$P.I_1 = \frac{1}{3D^2+D-14} \cdot 4$

$= \frac{4}{-14}$

$PI_1 = \frac{-2}{7}$

$PI_2 = \frac{1}{3D^2+D-14} \cdot -13e^{\frac{-7}{3}x}$

$= \frac{1}{(3D+7)(D-2)}(-13e^{\frac{-7}{3}x})$

$= \frac{x}{3\left(-\frac{7}{3}-2\right)} \cdot -13e^{-7/3\,x}$

$= \frac{x}{\cancel{3}\left(\frac{-13}{\cancel{3}}\right)} \cdot -13e^{\frac{-7}{3}x}$

$= \frac{x}{-13} \cdot (-13e^{\frac{-7}{3}x})$

$PI_2 = xe^{\frac{-7}{3}x}$

The general solution is

$y = Ae^{-7/3\,x} + Be^{2x} - \frac{2}{7} + xe^{\frac{-7}{3}x}$

43(a)

$$n = \frac{x - x_0}{h} = \frac{32 - 30}{5} = \frac{2}{5}$$

$$\boxed{n = 0.4}$$

Forward interpolation Table,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$\boxed{30}$	$\boxed{15.9}$				
		$\boxed{-1}$			
35	14.9		$\boxed{0.2}$		
		-0.8		$\boxed{-0.2}$	
40	14.1		0		$\boxed{0.2}$
		-0.8		0	
45	13.3		0		
		-0.8			
50	12.5				

Forward interpolation formula,

$$y = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0 + \dots$$

$$y = 15.9 + \frac{(0.4)}{1!}(-1) + \frac{(0.4)(0.4-1)}{2!}(0.2) + \frac{(0.4)(0.4-1)(0.4-2)}{3!}(-0.2) + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{4!}(0.2)$$

$$y = 15.9 - 0.4 - 0.024 - 0.0128 - 0.00832$$

$$y = 15.45$$

43(b)

$$n = \frac{x - x_n}{h} = \frac{x-7}{1} = x - 7$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		1		
1	2		1	
		2		0
2	4		1	
		3		0
3	7		1	
		4		0
4	11		1	
		5		0
5	16		1	
		6		0
6	22		1	
		7		
7	29			

$$y = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \dots$$

$$y = 29 + (x-7)\,7 + \frac{(x-7)(x-6)}{2}(1).$$

$$= 29 + 7x - 49 + \frac{1}{2}[x^2 - 13x + 42]$$

$$= \frac{1}{2}[58 + 14x - 98 + x^2 - 13x + 42]$$

$$y = \frac{1}{2}[x^2 + x + 2]$$

44(a)

$$\frac{dy}{dx} = \frac{x-y}{x+y} \rightarrow (1)$$

Put $y = vx$ then

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (2)$$

Sub (2) in (1),

$$v + x\frac{dv}{dx} = \frac{x - vx}{x + vx}$$

$$x\frac{dv}{dx} = \frac{1-v}{1+v} - v$$

$$x\frac{dv}{dx} = \frac{1-v-v-v^2}{1+v}$$

$$x\frac{dv}{dx} = \frac{1-2v-v^2}{1+v}$$

$$\frac{1+v}{1-2v-v^2}dv = \frac{1}{x}dx$$

$$\int \frac{2+2v}{1-2v-v^2}dv = \int \frac{1}{x}dx$$

$$\int \frac{2+2v}{v^2+2v-1}dv = -\int \frac{1}{x}dx$$

$$\log(v^2+2v-1) = -2\log x + \log c$$

$$\log(v^2+2v-1) + 2\log x = \log c$$

$$\log(v^2+2v-1) + \log x^2 = \log c$$

$$\log(v^2+2v-1)x^2 = \log c$$

$$(v^2+2v-1)x^2 = c$$

$$\left(\frac{y^2}{x^2} + \frac{2y}{x} - 1\right)x^2 = c$$

$$(y^2 + 2xy - x^2) = c$$

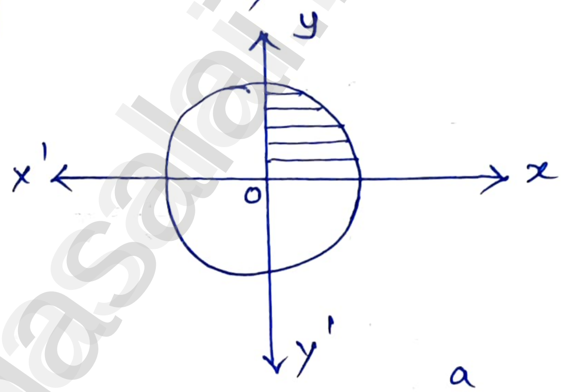
44(b)

Equation of a circle is

$$x^2 + y^2 = a^2$$

Put $y = 0$, $x^2 = a^2$

$$\Rightarrow x = \pm a$$



$$\text{Required area} = 4\int_0^a y\,dx.$$

$$A = 4\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[0 + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right]$$

$$= 4\,\frac{a^2}{2}\cdot\frac{\pi}{2}$$

$A = \pi a^2$ sq. units

45(a)

$$I=\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}\,dx \rightarrow (1)$$

$$=\int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{2+5-x}+\sqrt{7-(2+5-x)}}\,dx$$

$$I=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}\,dx$$

$(1)+(2) \Rightarrow$

$$2I=\int_2^5 \left[\frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}+\frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}\right]dx$$

$$=\int_2^5 \frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$$

$$=\int_2^5 dx$$

$$=(x)_2^5$$

$$2I=5-2$$

$$2I=3$$

$$I=\frac{3}{2}$$

45(b)

$$\frac{Ey}{Ex}=\frac{-7x}{(1-2x)(2+3x)}$$

$$\frac{x}{y}\frac{dy}{dx}=\frac{-7x}{(1-2x)(2+3x)}$$

$$\frac{1}{y}dy=\frac{-7x}{x(1-2x)(2+3x)}dx$$

$$\frac{1}{y}dy=\frac{-7}{(1-2x)(2+3x)}dx$$

$$\frac{1}{y}dy=\frac{7}{(2x-1)(3x+2)}dx$$

$$\int\frac{1}{y}dy=\int\left[\frac{2}{2x-1}-\frac{3}{3x+2}\right]dx$$

$$\log y=\log(2x-1)-3\log(3x+2)+\log c$$

$$\log y=\log\left[\frac{c(2x-1)}{3x+2}\right]$$

$$y=\frac{(2x-1)c}{3x+2} \rightarrow (1)$$

when $x=2,\ y=3/8$ in (1)

$$\frac{3}{8}=\frac{(4-1)c}{8}$$

$$c=1$$

$$\therefore y=\frac{2x-1}{3x+2}$$

46 (a)

(i) $\sum P(x) = 1$

$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$10k^2+9k-1=0$

$(k+1)(10k-1)=0$

$k=-1, \frac{1}{10}$

$\therefore \boxed{k=\frac{1}{10}}$

(ii) $P(x<6) = 0+k+2k+2k+3k+k^2$

$= 8k+k^2$

$= \frac{8}{10}+\frac{1}{100} = \frac{80}{100}+\frac{1}{100} = \frac{81}{100}$

$P(x \geq 6) = 2k^2+7k^2+k$

$= 9k^2+k$

$= \frac{9}{100}+\frac{1}{10} = \frac{9}{100}+\frac{10}{100} = \frac{19}{100}$

$P(0<x<5) = k+2k+2k+3k = 8k$

$= \frac{8}{10}$

(iii) $P(x \leq x) > \frac{1}{2}$

$P(x \leq 0) = 0$

$P(x \leq 1) = k = \frac{1}{10}$

$P(x \leq 2) = 3k = \frac{3}{10}$

$P(x \leq 3) = 5k = \frac{5}{10}$

$P(x \leq 4) = 8k = \frac{8}{10} > \frac{1}{2}$

Min val of $x = 4$.

46 (b)

Let the amount invested at 2%, 3% and 6% are x, y and z respectively.

$\therefore x+y+z = 8500$

$2x+3y+6z = 38000$

$x+y-z=0$

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = -2$

$\Delta x = \begin{vmatrix} 8500 & 1 & 1 \\ 38000 & 3 & 6 \\ 0 & 1 & -1 \end{vmatrix}$

$\Delta x = -500$

$\Delta y = \begin{vmatrix} 1 & 8500 & 1 \\ 2 & 38000 & 6 \\ 1 & 0 & -1 \end{vmatrix}$

$\Delta y = -8000$

$\Delta z = \begin{vmatrix} 1 & 1 & 8500 \\ 2 & 3 & 38000 \\ 1 & 1 & 0 \end{vmatrix}$

$\Delta z = -8500$

By Cramer's rule

$x = \frac{\Delta x}{\Delta} = \frac{-500}{-2}$

$x = 250$

$y = \frac{\Delta y}{\Delta} = \frac{-8000}{-2}$

$y = 4000$

$z = \frac{\Delta z}{\Delta} = \frac{-8500}{-2}$

$z = 4250$

47(a)

$f(x) = ke^{-2x}, \quad 0 \le x < \infty$

we know that

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$k\int_{0}^{\infty} e^{-2x} dx = 1$$

$$k\left[\frac{e^{-2x}}{-2}\right]_{0}^{\infty} = 1$$

$$\frac{k}{-2}\left[e^{-\infty} - e^{0}\right] = 1$$

$$\frac{k}{-2}[0-1] = 1$$

$$k = 2$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= 2\int_{0}^{\infty} x e^{-2x} dx$$

$$= 2\left(\frac{1!}{2^2}\right)$$

$$= \frac{2}{4}$$

Mean $= E(x) = \frac{1}{2}$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= 2\int_{0}^{\infty} x^2 e^{-2x} dx$$

$$= 2\left[\frac{2!}{2^3}\right]$$

$$= \frac{\cancel{4}}{8}$$

$$E(x^2) = \frac{1}{2}$$

$$\text{variance} = E(x^2) - [E(x)]^2$$

$$= \frac{1}{2} - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$\text{var}(x) = \frac{1}{4}$$

47(b)

$$\Delta\left(\frac{5x+12}{x^2+5x+6}\right) = \Delta\left(\frac{A}{x+3} + \frac{B}{x+2}\right)$$

$$= \Delta\left(\frac{3}{x+3} + \frac{2}{x+2}\right)$$

$$= \Delta\left(\frac{3}{x+3}\right) + \Delta\left(\frac{2}{x+2}\right)$$

$$= \frac{3}{x+4} - \frac{3}{x+3} + \frac{2}{x+3} - \frac{2}{x+2}$$

$$= \frac{3}{x+4} - \frac{1}{x+3} - \frac{2}{x+2}$$

$$= \frac{-5x-14}{(x+2)(x+3)(x+4)}$$