CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	MODEL EXAMINATION :1(VOL.1)	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : **(20× 1 = 20)**

1. If $|\text{adj}\ (\text{adj}\ A)| = |A|^9$, then the order of the square matrix A is

   (1) 3 (2) 4 (3) 2 (4) 5

2. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then $A =$

   (1) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ (3) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ (4) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

3. If $A = \begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$, then $9I_2 - A =$

   (1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

4. The value of $\sum_{n=1}^{13}(i^n + i^{n-1})$ is

   (1) $1 + i$ (2) i (3) 1 (4) 0

5. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

   (1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $\frac{-1}{i-2}$ (4) $\frac{1}{i-2}$

6. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

   (1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

7. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   (1) Z (2) $\bar{Z}$ (3) $\frac{1}{Z}$ (4) 1

8. A zero of $x^3 + 64$ is

   (1) 0 (2) 4 (3) $4i$ (4) -4

9. A polynomial equation in x of degree n always has

   (1) n distinct roots (2) n real roots (3) n complex roots (4) at most one root

10. According to the rational root theorem, which number is not possible rational zero of

    $4x^7 + 2x^4 - 10x^3 - 5$?

    (1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

11. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

12. If $\sin^{-1} x = 2\sin^{-1} \alpha$ has a solution, then

(1) $|\alpha| \le \frac{1}{\sqrt{2}}$ (2) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

13. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

(1) $-\pi \le x \le 0$ (2) $0 \le x \le \pi$ (3) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (4) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

14. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

15. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

16. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is

(1) 8 (2) 6 (3) 10 (4) 12

17. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath} + \hat{\jmath}, \hat{\imath} + 2\hat{\jmath}, \hat{\imath} + \hat{\jmath} + \pi\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

18. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) π

19. Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

(1) 0° (2) 45° (3) 60° (4) 90°

20. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 5\hat{k}$, $\vec{c} = 3\hat{\imath} + 5\hat{\jmath} - \hat{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(1) $-17\hat{\imath} + 21\hat{\jmath} - 97\hat{k}$ (2) $17\hat{\imath} + 21\hat{\jmath} - 123\hat{k}$

(3) $-17\hat{\imath} - 21\hat{\jmath} + 97\hat{k}$ (4) $-17\hat{\imath} - 21\hat{\jmath} - 97\hat{k}$

PART-II

(i) Answer any SEVEN questions. **($7\times 2 = 14$)**

(ii) Qn.No.30 is compulsory

21. If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.
22. Simplify : $\sum_{n=1}^{102} i^n$
23. Show that $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real
24. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.
25. Find the value of $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right)$
26. A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form.
27. A concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write the equation of the parabolic arch.
28. A particle is acted upon by the forces $3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $2\hat{\imath} + \hat{\jmath} - \hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.
29. Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x - 3y + 5z + 7 = 0$. Also, find the distance between the two planes.
30. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

PART-III

(i) Answer any SEVEN questions. **($7\times 3 = 21$)**

(ii) Qn.No.40 is compulsory

31. Find a matrix A if adj $A = \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.
32. Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a,\ x + 2y + 3z = b,\ 3x + 5y + 7z = c$.
33. Find the value of the real numbers x and y, if the complex number $(2+i)x + (1-i)y + 2i - 3$ and $x + (-1+2i)y + 1 + i$ are equal.
34. Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$
35. Find the domain of $\sin^{-1}(2 - 3x^2)$
36. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$.
37. Find the equation of the hyperbola whose Centre $(2,1)$, one of the foci $(8,1)$ and corresponding directrix $x = 4$.
38. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then prove that the vectors $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are also coplanar.
39. Show that the straight lines $\vec{r} = (5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}) + s(4\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ and $\vec{r} = (8\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + t(7\hat{\imath} + \hat{\jmath} + 3\hat{k})$ are coplanar. Find the vector equation of plane in which they lie.
40. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) (OR)

b) Let z_1, z_2, and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$.

42. a) Solve $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3: 2. (OR)

b) Find the equation of the circle passing through the points $(1,1), (2,-1)$, and $(3,2)$.

43. a) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$. (OR)

b) Solve: $(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$

44. a) Solve the systems of linear equation by Cramer's rule:

$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0,\ \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0,\ \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ (OR)

b) Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$.

45. a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7,\ x + y + \lambda z = \mu,\ x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. (OR)

b) Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio $p:q:r$.

46. a) Identify the type of conic and find centre, foci, vertices, and directrices :

$18x^2 + 12y^2 - 144x + 48y + 120 = 0$ (OR)

b) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

47. a) Solve:

(i) $2\tan^{-1} x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}, a > 0, b > 0$

(ii) $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}, x > 0.$ (OR)

b) Using vector method, prove that $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	MODEL EXAMINATION :2(VOL.1)	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : (20× 1 = 20)

1. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$
(1) A (2) B (3) I_3 (4) B^T

2. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is
(1) 15 (2) 12 (3) 14 (4) 11

3. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$
(1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

4. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
(1) 0 (2) 1 (3) -1 (4) i

5. The area of the triangle formed by the complex numbers z, iz, and $z + iz$ in the Argand's diagram is
(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

6. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is
(1) $\sqrt{3} - 2$ (2) $\sqrt{3} + 2$ (3) $\sqrt{5} - 2$ (4) $\sqrt{5} + 2$

7. If $\left|z - \frac{3}{z}\right| = 2$, then the least value of $|z|$ is
(1) 1 (2) 2 (3) 3 (4) 5

8. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
(1) mn (2) $m + n$ (3) m^n (4) n^m

9. If α, β, and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is
(1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

10. The value of $\sin^{-1}(\cos x), 0 \leq x \leq \pi$ is
(1) $\pi - x$ (2) $x - \frac{\pi}{2}$ (3) $\frac{\pi}{2} - x$ (4) $x - \pi$

11. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is
(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

12. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has
(1) no solution (2) unique solution
(3) two solutions (4) infinite number of solutions

13. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3).
(1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

14. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
(1) 3 (2) -1 (3) 1 (4) 9

15. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(1) $4(a^2+b^2)$ (2) $2(a^2+b^2)$ (3) a^2+b^2 (4) $\frac{1}{2}(a^2+b^2)$

16. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

17. If the planes $\vec{r}\cdot(2\hat{\imath}-\lambda\hat{\jmath}+\hat{k})=3$ and $\vec{r}\cdot(4\hat{\imath}+\hat{\jmath}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2},-2$ (2) $-\frac{1}{2},2$ (3) $-\frac{1}{2},-2$ (4) $\frac{1}{2},2$

18. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then

(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

19. Distance from the origin to the plane $3x-6y+2z+7=0$ is

(1) 0 (2) 1 (3) 2 (4) 3

20. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then (α,β) is

(1) $(-5,5)$ (2) $(-6,7)$ (3) $(5,-5)$ (4) $(6,-7)$

PART-II

(i) Answer any SEVEN questions. (7× 2 = 14)

(ii) Qn.No.30 is compulsory

21. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

22. $A=\begin{bmatrix}3 & 5\\ 1 & 2\end{bmatrix}$, $B=\text{adj } A$ and $C=3A$, find $\frac{|\text{adj } B|}{|C|}$.

23. If $z=x+iy$, find in rectangular form $\text{Im}\,(3z+4\bar{z}-4i)$

24. Examine for the rational roots of $x^8-3x+1=0$.

25. Show that, if p,q,r are rational, the roots of the equation $x^2-2px+p^2-q^2+2qr-r^2=0$ are rational.

26. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

27. Find the equation of the tangent and normal to the circle $x^2+y^2-6x+6y-8=0$ at (2,2).

28. Prove by vector method that the median to the base of an isosceles triangle is perpendicular to the base.

29. Find the angle between the line $\vec{r}=(2\hat{\imath}-\hat{\jmath}+\hat{k})+t(\hat{\imath}+2\hat{\jmath}-2\hat{k})$ and the plane $\vec{r}\cdot(6\hat{\imath}+3\hat{\jmath}+2\hat{k})=8$

30. If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is non-singular, find A^{-1}.

PART-III

(i) Answer any SEVEN questions. (7× 3 = 21)

(ii) Qn.No.40 is compulsory

31. Find the inverse by Gauss-Jordan method: $\begin{bmatrix}2 & -1\\ 5 & -2\end{bmatrix}$

32. Determine the values of λ for which the following system of equations

$(3\lambda-8)x+3y+3z=0, 3x+(3\lambda-8)y+3z=0, 3x+3y+(3\lambda-8)z=0$ has a non-trivial solution.

33. Obtain the Cartesian form of the locus of , $|2z-3-i|=3$

34. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$.

35. Solve the cubic equation : $2x^3-x^2-18x+9=0$. if sum of two of its roots vanishes.

36. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$.

37. Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

38. Find the value of $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$

39. Show that the four points $(6,-7,0), (16,-19,-4), (0,3,-6), (2,-5,10)$ lie on a same plane.

40. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) Find the value of k for which the equations $kx - 2y + z = 1,\ x - 2ky + z = -2,\ x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution (OR)

b) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

42. a) Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$. (OR)

b) Simplify : $(-\sqrt{3} + 3i)^{31}$.

43. a) Form the equation whose roots are the squares of the roots of the equation $x^3 + ax^2 + bx + c = 0$. (OR)

b) Identify the type of conic and find centre, foci, vertices, and directrices : $9x^2 - y^2 - 36x - 6y + 18 = 0$

44. a) If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P., prove that $9pqr = 27r^2 + 2q^3$ Assume $p, q, r \neq 0$ (OR)

b) Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$.

45. a) Simplify : (i) $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$ (ii) $\sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$ (OR)

b) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16 m, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be?

46. a) Show that the lines $\vec{r} = (6\hat{\imath} + \hat{\jmath} + 2\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) + t(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ are skew lines and hence find the shortest distance between them. (OR)

b) Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$

47. a) By using Gaussian elimination method, balance the equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$ (OR)

b) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0), (2,2-1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	MODEL EXAMINATION:03 (VOL.1)	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : **(20× 1 = 20)**

1. If $A=\begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|\text{adj }(AB)|=$

   (1) -40 (2) -80 (3) -60 (4) -20

2. If A^TA^{-1} is symmetric, then $A^2=$

   (1) A^{-1} (2) $(A^T)^2$ (3) A^τ (4) $(A^{-1})^2$

3. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ and $A(\text{ adj } A)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then $k=$

   (1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

4. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is

   (1) 17 (2) 14 (3) 19 (4) 21

5. If z is a complex number such that $z\in\mathbb{C}\setminus\mathbb{R}$ and $z+\frac{1}{z}\in\mathbb{R}$, then $|z|$ is

   (1) 0 (2) 1 (3) 2 (4) 3

6. The principal argument of $\frac{3}{-1+i}$ is

   (1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

7. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

   (1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

8. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

   (1) -2 (2) -1 (3) 1 (4) 2

9. A zero of x^3+125 is

   (1) 0 (2) 5 (3) $5i$ (4) -5

10. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x-2\sin^2 x+1$ is

    (1) 2 (2) 4 (3) 1 (4) ∞

11. The number of positive zeros of the polynomial $\sum_{j=0}^{n}{}^nC_r(-1)^rx^r$ is

    (1) 0 (2) n (3) $<n$ (4) r

12. If the function $f(x)=\sin^{-1}(x^2-3)$, then x belongs to

    (1) $[-1,1]$ (2) $[\sqrt{2},2]$ (3) $[-2,-\sqrt{2}]\cup[\sqrt{2},2]$ (4) $[-2,-\sqrt{2}]$

13. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then $\cos 2u$ is equal to

(1) $\tan^2\alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$

14. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

15. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

16. An ellipse has OB as semi minor axes, F and F' its foct and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$

17. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11,2). the coordinates of the other end are

(1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

18. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

(1) $\frac{\sqrt{7}}{2\sqrt{2}}$ (2) $\frac{7}{2}$ (3) $\frac{\sqrt{7}}{2}$ (4) $\frac{7}{2\sqrt{2}}$

19. If the distance of the point (1,1,1) from the origin is half of its distance from the plane $x + y + z + k = 0$, then the values of k are

(1) ± 3 (2) ± 6 (3) $-3,9$ (4) $3,-9$

20. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

PART-II

(i) Answer any SEVEN questions. **($7\times 2 = 14$)**

(ii) Qn.No.30 is compulsory

21. Solve the following systems of linear equations by Cramer's rule: $\frac{3}{x} + 2y = 12,\ \frac{2}{x} + 3y = 13$

22. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

23. Find: $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

24. If $|z| = 2$, show that $8 \le |z + 6 + 8i| \le 12$.

25. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

26. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as root.

27. Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

28. Find the value of the expression in terms of x, with the help of a reference triangle. $\sin\left(\cos^{-1}(1-x)\right)$

29. Find the equation of the parabola which passes through $(2,-3)$ and symmetric about y-axis.

30. Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.

PART-III

(i) Answer any SEVEN questions. **$(7\times 3 = 21)$**

(ii) Qn.No.40 is compulsory

31. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

32. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

33. Prove the following properties:

    (i) z is real if and only if $z = \bar{z}$ (ii) $\mathrm{Re}\,(z) = \frac{z+\bar{z}}{2}$ and $\mathrm{Im}\,(z) = \frac{z-\bar{z}}{2i}$

34. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.

35. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

36. Solve the equation: $x^4 - 14x^2 + 45 = 0$.

37. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$ for $x > 0$.

38. Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to $10x - 3y + 9 = 0$.

39. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$.

40. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. (OR)

    b) Simplify: $(1+i)^{18}$

42. a) Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$. (OR)

    b) If $ax^2 + bx + c$ is divided by $x+3, x-5$, and $x-1$, the remainders are 21,61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

43. a) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ (OR)

    b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

44. a) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$. (OR)

b) Prove that $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$.

45. a) Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship. (OR)

b) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

46. a) Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent. (OR)

b) Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

47. a) At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin. (OR)

b) Find the coordinates of the point where the straight line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + t(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ intersects the plane $x - y + z - 5 = 0$.

CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	MODEL EXAMINATION NO.:4 (VOL.1)	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : (20× 1 = 20)

1. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$
   (1) $\left(\cos^2\frac{\theta}{2}\right)A$ (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (3) $(\cos^2\theta)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

2. If adj $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj $B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is
   (1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

3. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is
   (1) 1 (2) 2 (3) 4 (4) 3

4. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is
   (1) -2 (2) -1 (3) 1 (4) 2

5. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals
   (1) (1,0) (2) $(-1,1)$ (3) (0,1) (4) (1,1)

6. If z is a complex number such that $z \in \mathbb{C} \setminus \mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$, then $|z|$ is
   (1) 0 (2) 1 (3) 2 (4) 3

7. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is
   (1) mn (2) $m+n$ (3) m^n (4) n^m

8. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is
   (1) 2 (2) 4 (3) 1 (4) ∞

9. If $\sin^{-1} x = 2\sin^{-1}\alpha$ has a solution, then
   (1) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (2) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

10. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to
    (1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{3}}{2}$

11. $\sin(\tan^{-1} x), |x| < 1$ is equal to
    (1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

12. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$
    (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

13. The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is
    (1) (4,7) (2) (7,4) (3) (9,4) (4) (4,9)

14. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is
    (1) 2 (2) 3 (3) 1 (4) 4

15. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
    (1) $2ab$ (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

16. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is
(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

17. If the planes $\vec{r} \cdot (2\hat{\imath} - \lambda\hat{\jmath} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{\imath} + \hat{\jmath} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are
(1) $\frac{1}{2}, -2$ (2) $-\frac{1}{2}, 2$ (3) $-\frac{1}{2}, -2$ (4) $\frac{1}{2}, 2$

18. The vector equation $\vec{r} = (\hat{\imath} - 2\hat{\jmath} - \hat{k}) + t(6\hat{\jmath} - \hat{k})$ represents a straight line passing through the points
(1) $(0,6,-1)$ and $(1,-2,-1)$ (2) $(0,6,-1)$ and $(-1,-4,-2)$
(3) $(1,-2,-1)$ and $(1,4,-2)$ (4) $(1,-2,-1)$ and $(0,-6,1)$

19. The coordinates of point where line $\vec{r} = (6\hat{\imath} - \hat{\jmath} - 3\hat{k}) + t(-\hat{\imath} + 4\hat{k})$ meets plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 3$ are
(1) $(2,1,0)$ (2) $(7,-1,-7)$ (3) $(1,2,-6)$ (4) $(5,-1,1)$

20. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is
(1) $(-5,5)$ (2) $(-6,7)$ (3) $(5,-5)$ (4) $(6,-7)$

PART-II

(i) Answer any SEVEN questions. **($7 \times 2 = 14$)**

(ii) Qn.No.30 is compulsory

21. If adj $A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}

22. Solve the system of linear equation by matrix inversion method:
$2x + 5y = -2, x + 2y = -3$

23. Show that the equation $z^2 = \bar{z}$ has four solutions.

24. Show that the given equation represent a circle and find its centre and radius $|3z - 6 + 12i| = 8$.

25. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.

26. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.

27. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$.

28. Find the equation of the tangent at $t = 2$ to the parabola $y^2 = 8x$. (Hint: use parametric form)

29. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

30. Show that the points $(2,3,4)$, $(-1,4,5)$ and $(8,1,2)$ are collinear.

PART-III

(i) Answer any SEVEN questions. **($7 \times 3 = 21$)**

(ii) Qn.No.40 is compulsory

31. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

32. Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$ (i) real (ii) purely imaginary.

33. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that
$\frac{z_1 + z_2}{1 + z_1 z_2}$ is a real number.

34. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.

35. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

36. Simplify: $\sin^{-1}[\sin 10]$

37. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.
38. Prove that $[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}] = 0$.
39. Find the coordinate of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r} = (\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$. Also, find the shortest distance from the given point to the straight line.
40. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}, -1 < x < 1$.

PART-IV

Answer the following questions: **$(7\times 5 = 35)$**

41. a) Find the inverse by Gauss-Jordan method: $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ (OR)

b) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

42. a) Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3. (OR)

b) If the system of equations $px + by + cz = 0, ax + qy + cz = 0,\ ax + by + rz = 0$ has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

43. a) Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} - \hat{\jmath} + 4\hat{k})$ and perpendicular to plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 8$. (OR)

b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}.$$

44. a) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

(ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$ (OR)

b) Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$.

45. a) Solve the equation : $x^4 + 3x^3 - 3x - 1 = 0$ (OR)

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2 .

46. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection. (OR)

b) Test for consistency of the following system of linear equations and if possible solve:

$4x - 2y + 6z = 8,\ x + y - 3z = -1,\ 15x - 3y + 9z = 21.$

47. a) Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also, find the equation of the perpendicular. (OR)

b) Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

CENTUM ACHIEVERS' ACADEMY

56,KASTHURI BAI 4TH STREET,GANAPATHY, CBE-06.PH.NO.7667761819

XII STD(MATHS)	MODEL EXAMINATION NO.:5(VOL.1)	TIME : 2 ½ Hrs MARKS : 90

PART-I

Choose the correct answer from the given four alternatives : (20× 1 = 20)

1. If $\rho(A) = \rho([A \mid B])$, then the system $AX = B$ of linear equations is

   (1) consistent and has a unique solution (2) consistent

   (3) consistent and has infinitely many solution (4) inconsistent

2. If $0 \leq \theta \leq \pi$ and the system of equations $x + (\sin\ \theta)y - (\cos\ \theta)z = 0, (\cos\ \theta)x - y + z = 0,$

   $(\sin\ \theta)x + y - z = 0$ has a non-trivial solution then θ is

   (1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

3. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

   (1) 0 (2) -2 (3) -3 (4) -1

4. If A, B and C are invertible matrices of some order, then which one of the following is not true?

   (1) $\text{adj}\ A = |A|A^{-1}$ (2) $\text{adj}(AB) = (\text{adj}\ A)(\text{adj}\ B)$

   (3) $\det\ A^{-1} = (\det\ A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

5. If $(1+i)(1+2i)(1+3i)\cdots(1+ni) = x + iy$, then $2 \cdot 5 \cdot 10 \cdots (1+n^2)$ is

   (1) 1 (2) i (3) $x^2 + y^2$ (4) $1 + n^2$

6. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

   (1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

7. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

   (1) 1 (2) 2 (3) 3 (4) 4

8. A zero of $x^2 + 64$ is

   (1) 0 (2) $4i$ (3) $\pm 4i$ (4) -4

9. The polynomial $x^3 + 2x + 3$ has

   (1) one negative and two imaginary zeros (2) one positive and two imaginary zeros

   (3) three real zeros (4) no zeros

10. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

    (1) $[1,2]$ (2) $[-1,1]$ (3) $[0,1]$ (4) $[-1,0]$

11. If $\cot^{-1}(\sqrt{\sin\ \alpha}) + \tan^{-1}(\sqrt{\sin\ \alpha}) = u$, then $\cos\ 2u$ is equal to

    (1) $\tan^2\ \alpha$ (2) 0 (3) -1 (4) $\tan\ 2\alpha$

12. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

13. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

14. The equation of the circle passing through (1,5) and (4,1) and touching y-axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to

(1) $0, -\frac{40}{9}$ (2) 0 (3) $\frac{40}{9}$ (4) $\frac{-40}{9}$

15. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

16. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$, then the value of $(a+b)$ is

(1) 2 (2) 4 (3) 0 (4) -2

17. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath} + \hat{\jmath}, \hat{\imath} + 2\hat{\jmath}, \hat{\imath} + \hat{\jmath} + \pi\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

18. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) π

19. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

20. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1, \lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4)

PART-II

(i) Answer any SEVEN questions. **($7 \times 2 = 14$)**

(ii) Qn.No.30 is compulsory

21. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

22. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form

23. Find the square root of $-5 - 12i$.

24. Prove that a straight line and parabola cannot intersect at more than two points.

25. If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

26. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.
27. Find the equation of the ellipse whose foci $(0, \pm 4)$ and end points of major axis are $(0, \pm 5)$.
28. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.
29. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$, $\hat{b} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{c} = -3\vec{\imath} + \vec{\jmath} + 4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.
30. A concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write the equation of the parabolic arch.

PART-III

(i) Answer any SEVEN questions. **$(7\times 3 = 21)$**

(ii) Qn.No.40 is compulsory

31. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem).
32. Find z^{-1}, if $z = (2 + 3i)(1 - i)$.
33. Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P.
34. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$
35. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.
36. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.
37. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.
38. Determine whether the pair of straight lines $\vec{r} = (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$,
    $\vec{r} = (2\hat{\jmath} - 3\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ are parallel. Find the shortest distance between them.
39. Find the image of the point whose position vector is $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) = 38$.
40. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus of z is real axis.

PART-IV

Answer the following questions. **$(7\times 5 = 35)$**

41. a) The prices of three commodities A, B and C are ₹x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.) (OR)

    b) Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in G.P..Assume $a, b, c, d \neq 0$
42. a) Determine the values of λ for which the following system of equations

    $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$ (OR)

b) Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.

43. a) An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum. The total annual income is ₹4,800. The income from the third bond is ₹600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.) (OR)

b) If $z=x+iy$ and arg $\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$.

44. a) Find the number of solutions of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$. (OR)

b) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when (i) $\theta=\frac{\pi}{3}$ (ii) $\theta=\frac{2\pi}{3}$ (iii) $\theta=\frac{3\pi}{2}$.

45. a) Find the value of (i) $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ (ii) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$. (OR)

b) If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2', then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

46. a) Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x+2y+3z=2$. (OR)

b) Solve the equation $(2x-3)(6x-1)(3x-2)(x-2)-5=0$.

47. a) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? (OR)

b) If $\vec{a}=\hat{\imath}-\hat{\jmath}, \vec{b}=\hat{\imath}-\hat{\jmath}-4\hat{k}, \vec{c}=3\hat{\jmath}-\hat{k}$ and $\vec{d}=2\hat{\imath}+5\hat{\jmath}+\hat{k}$, verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$