

12M

Common Quarterly Examination - September 2022



TENKASI

DISTRICT

Standard - 12

MATHEMATICS

Time Allowed: 3.00 Hours

Maximum Marks: 90

PART - A

20×1=20

Choose the correct answer:

1) If $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then $A =$

1) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

2) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

3) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

2) A is a non-singular matrix of order 3, $|A| = 5$ then $|\text{adj } A| =$

1) 125

2) 5

3) 25

4) 15

3) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$,

Then the system has infinitely many solutions if

1) $\lambda = 5, \mu \neq -5$

2) $\lambda = -7, \mu = 5$

3) $\lambda \neq 7, \mu \neq -5$

4) $\lambda = 7, \mu = -5$

4) The principal argument of $\frac{3}{-1+i}$ is

1) $\frac{-5\pi}{6}$

2) $\frac{-2\pi}{3}$

3) $\frac{-3\pi}{4}$

4) $\frac{-\pi}{2}$

5) z_1, z_2 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is

1) 3

2) 2

3) 1

4) 0

6) If $z = \frac{(\sqrt{3} + i)^3 (3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to

1) 0

2) 1

3) 2

4) 3

7) A zero of $x^3 + 64$ is

1) 0

2) 4

3) 4i

4) -4

8) The polynomial $x^3 - Kx^2 + 9x$ has three real zeros if and only if, K satisfies

1) $|K| \geq 6$

2) $|K| \leq 6$

3) $K = 0$

4) $|K| > 6$

9) The value of $\sin^{-1}(\cos x)$, $0 \leq x \leq \pi$ is

1) $\pi - x$

2) $x - \frac{\pi}{2}$

3) $\frac{\pi}{2} - x$

4) $x - \pi$

10) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

1) $\frac{1}{2} \cos^{-1}\left(\frac{3}{5}\right)$

2) $\frac{1}{2} \sin^{-1}\left(\frac{3}{5}\right)$

3) $\frac{1}{2} \tan^{-1}\left(\frac{3}{5}\right)$

4) $\tan^{-1}\left(\frac{1}{2}\right)$

11) If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

1) $\frac{1}{2}$

2) $\frac{1}{\sqrt{5}}$

3) $\frac{2}{\sqrt{5}}$

4) $\frac{\sqrt{3}}{2}$

- 12) $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$ is
- 1) $\frac{\pi}{4}$ 2) $-\frac{\pi}{4}$ 3) $\frac{3\pi}{4}$ 4) $\frac{7\pi}{4}$
- 13) The centre of the circle inscribe in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is
- 1) (4, 7) 2) (7, 4) 3) (9, 4) 4) (4, 9)
- 14) If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of 'k' is
- 1) 3 2) -1 3) 1 4) 9
- 15) The sum of the focal distance of any point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is
- 1) 9 2) 4 3) 6 4) 36
- 16) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is
- 1) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ 2) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 3) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 4) $(3\sqrt{3}, -2\sqrt{2})$
- 17) The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}$, $\hat{i} + 2\hat{j}$, $\hat{i} + \hat{j} + \pi\hat{k}$ is
- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) π 4) $\frac{\pi}{4}$
- 18) The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}$, $z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is
- 1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$
- 19) If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$, $\lambda > 0$ is $1/5$ then the value of λ is
- 1) $2\sqrt{3}$ 2) $3\sqrt{2}$ 3) 0 4) 1
- 20) Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is
- 1) 0 2) 1 3) 2 4) 3

PART - B**Answer any SEVEN questions.****7×2=14****Question No. 30 is compulsory.**

21) Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

22) Simplify: $\left(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}\right)^{18}$

23) Obtain the cartesian form of the locus of $z = x + iy$ in $|z+i| = |z-1|$.

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24) If α , β and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$.

25) Find the principal value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$.

26) Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$.

27) Find the equation of the parabola whose vertex is $(5, -2)$ and focus $(2, -2)$.

28) If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2+y^2 = 16$, find the value of 'c'.

29) Show that the vectors $\hat{i} + 2\hat{j} - 3\hat{k}$, $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ are coplanar.

30) Show that the points $(2, 3, 4)$, $(-1, 4, 5)$ and $(8, 1, 2)$ are collinear.

PART - C

Answer any SEVEN questions.

7×3=21

Question No. 40 is compulsory.

31) Solve: $x-y+z = -9$

$$2x-2y+2z = -18$$

$$3x-3y+3z = -27$$

32) Find the matrix A for which $A \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

33) If $|z| = 3$, show that $7 \leq |z+6-8i| \leq 13$.

34) Show that the value of $\sum_{k=1}^8 \left[\cos \frac{2k\pi}{9} + i \sin \frac{2k\pi}{9} \right]$ is -1 .

35) Solve the cubic equation $2x^3-x^2-18x+9=0$ if sum of two of its roots vanishes.

36) Prove that $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{12}$.

37) Find the vertices, foci of the hyperbola $9x^2-16y^2 = 144$.

38) Prove that the point of intersection of the tangents at " t_1 " and " t_2 " on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1+t_2)]$.

39) Prove that $|\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}| = 2|\vec{a} \vec{b} \vec{c}|$.

40) Show that $\vec{r} = (6\hat{i} + \hat{j} + 2\hat{k}) + s(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (3\hat{i} + 2\hat{j} - 2\hat{k}) + t(2\hat{i} + 4\hat{j} - 5\hat{k})$ are skew lines.

PART - D

Answer ALL the questions:

7×5=35

41) Prove by vector method, $\sin(\alpha-\beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$.

(OR)

Find the vector parametric and cartesian equation of the plane passing through the points $(-1, 2, 0)$, $(2, 2, -1)$ and parallel to the straight line

$$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$$

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- 42) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

(OR)

If $z = x+iy$ and $\arg\left[\frac{z-i}{z+2}\right] = \frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2 = 0$.

- 43) Solve: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

(OR)

If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer co-efficients whose roots are $\alpha+\beta+\gamma+\delta$ are $\alpha\beta\gamma\delta$.

- 44) If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products of AB and BA and

hence solve the equation $x+y+2z = 1, 3x+2y+z = 7, 2x+y+3z = 2$.

(OR)

Determine the values of λ for which the equations $x+y+3z = 0, 4x+3y+\lambda z = 0, 2x+y+2z = 0$ has (i) a unique solution (ii) a non-trivial solution.

- 45) Find the value of $\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$.

(OR)

Solve: $\tan^{-1}\left[\frac{x-1}{x-2}\right] + \tan^{-1}\left[\frac{x+1}{x+2}\right] = \frac{\pi}{4}$

- 46) A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Show that the height of the arch 6m from the centre on either sides is 8.4m.

(OR)

Find the foci, vertices of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$.

- 47) If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that

$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = |\vec{a} \ \vec{b} \ \vec{d}| \vec{c} - |\vec{a} \ \vec{b} \ \vec{c}| \vec{d}.$$

(OR)

An ellipse whose centre is of origin and its major axis is along x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, find the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse.

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