1. Inverse Trigonometric Functions

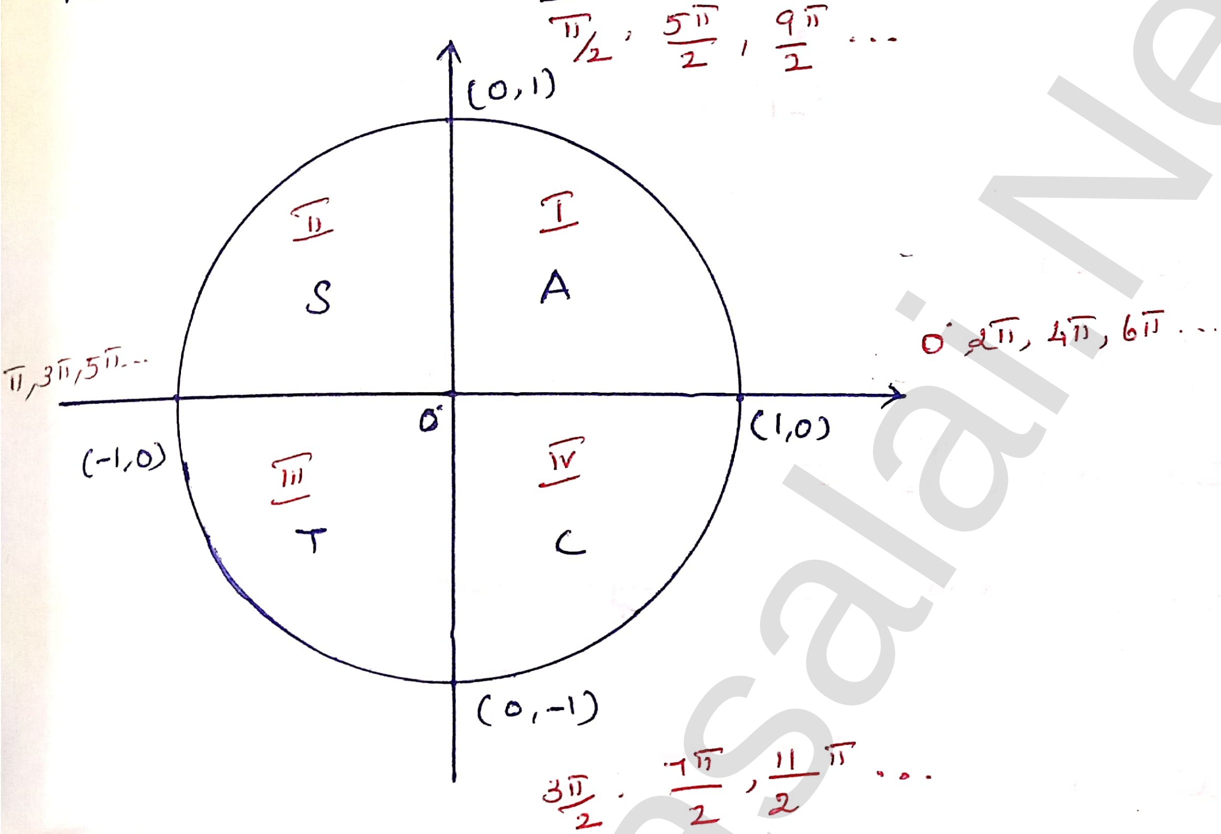
Introduction

$\Rightarrow$ $y = \sin x$

here sin followed by Angle $\Rightarrow$ result is value

$y = \sin^{-1} x$

here $\sin^{-1}$ followed by value $\Rightarrow$ result is angle.



General term

$\Rightarrow$

x axis $\Rightarrow n\pi$

x axis (+ve) $\Rightarrow 2n\pi$

x axis (-ve) $\Rightarrow (2n+1)\pi$

y axis $\Rightarrow (2n+1)\frac{\pi}{2}$

y axis (+ve) $\Rightarrow (4n+1)\frac{\pi}{2}$

y axis (-ve) $\Rightarrow (4n-1)\frac{\pi}{2}$

Note

On the horizontal line

$\rightarrow$ No change

On the vertical line

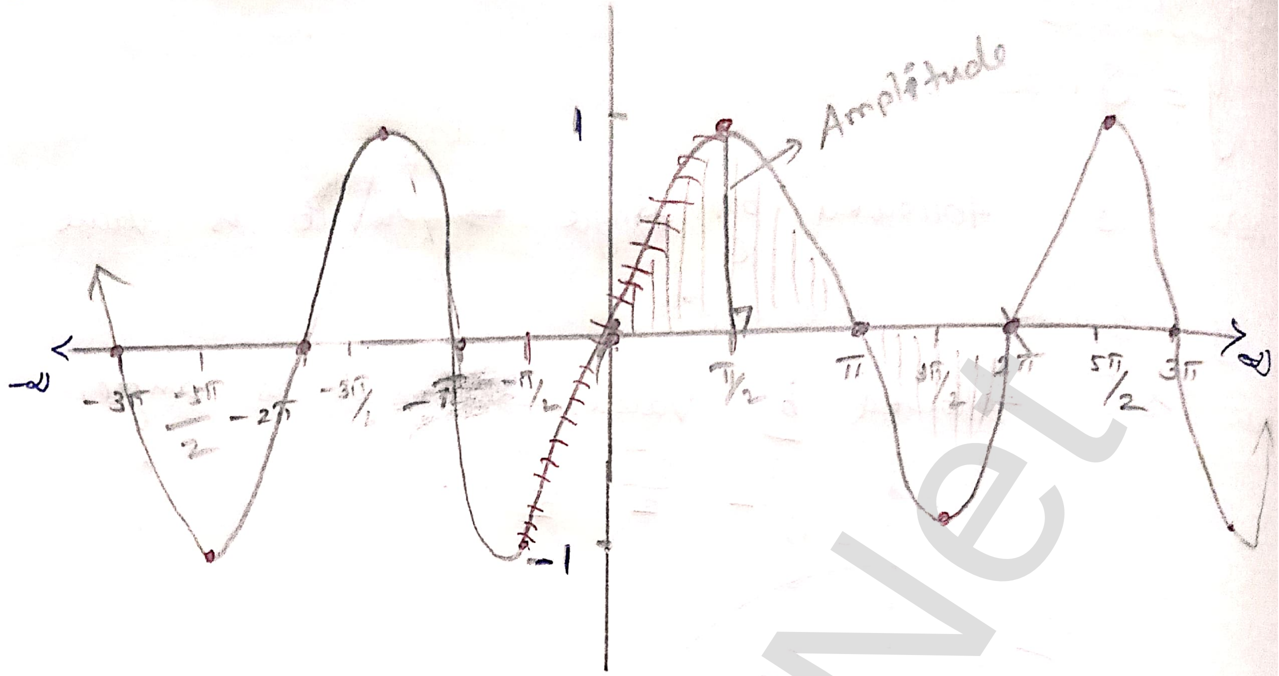
$\rightarrow$ change

$\sin \longleftrightarrow \cos$

$\tan \longleftrightarrow \cot$

$\sec \longleftrightarrow \operatorname{cosec}$

Y = Sinx



Domain : $(-\infty, \infty)$

Range : $[-1, 1]$

Principal Domain : $[-\pi/2, \frac{\pi}{2}]$

ie Principal domain is always subset of a domain

Amplitude : 1

period : 2π

Formula:-

Amplitude: $A \sin \alpha x$

Amplitude : $|A|$

period : $\frac{2\pi}{|\alpha|}$

Eg :-

$y = \sin x$

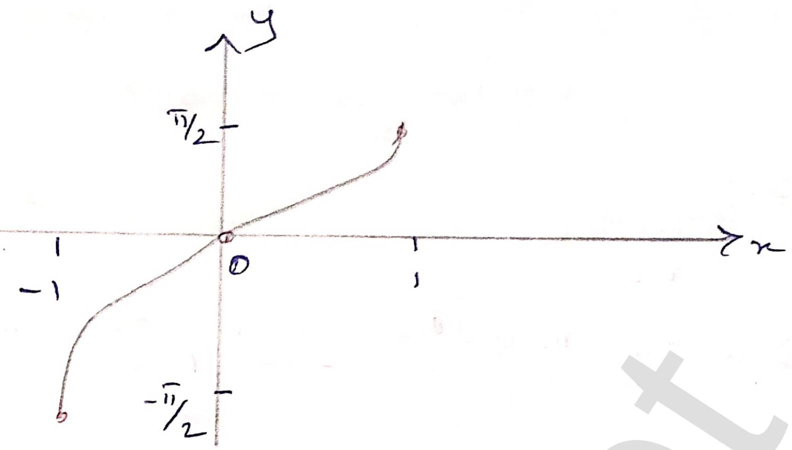
$y = 1 \sin 1x$

Amplitude: $|A| = |1| = 1$

period: $\frac{2\pi}{|\alpha|} = \frac{2\pi}{|1|} = 2\pi$

$y = \sin^{-1} x$

$\Rightarrow \quad x = \sin y$



To find points.

$y = 0 \Rightarrow x = 0$

$y = \pi/2 \Rightarrow x = 1$

$y = -\pi/2 \Rightarrow x = -1$

Range: $[-\pi/2, \frac{\pi}{2}]$

Principal Domain: $[-1, 1]$

Properties:

angle

① $\sin^{-1}(\sin x) = x \quad ; \quad x \in [-\pi/2, \pi/2]$

angle

② $\sin^{-1}(\sin x) = \pi - x \quad ; \quad x \in [\pi/2, \frac{3\pi}{2}]$

value

③ $\sin^{-1}(-x) = -\sin^{-1}(x) \quad ; \quad x \in [-1, 1]$

Prove that $\sin^{-1}(\sin x) = x$

Proof.

$y = \sin x$

$\sin^{-1} y = x$

$\boxed{\sin^{-1}(\sin x) = x}$

Prove that $\sin^{-1}(-x) = -\sin^{-1} x$

$y = \sin^{-1}(-x)$

$\sin y = -x$

$-\sin y = x$

$\sin(-y) = x$

$-y = \sin^{-1} x$

$y = -\sin^{-1} x$

$\boxed{\sin^{-1}(-x) = -\sin^{-1} x}$

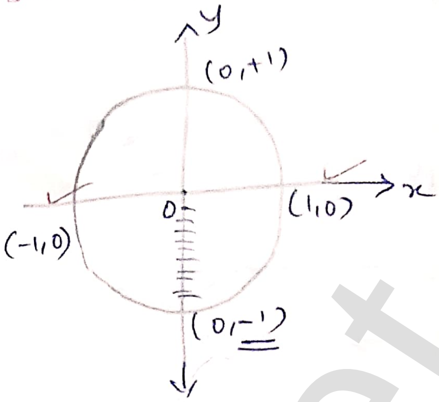
① Find all the values of x such that

(i) $-10\pi \le x \le 10\pi$ and $\sin x = 0$.

Solution.

(i) $\sin x = 0$

$\Rightarrow \boxed{x = n\pi, \quad n = 0, \pm 1, \pm 2 \ldots \pm 10}$



(ii) $-3\pi \le x \le 3\pi$ and $\sin x = -1$

Solution

$\sin x = -1$

$\boxed{x = (4n-1)\frac{\pi}{2}, \quad n = 0, \pm 1}$

y axis (-ve) $= (4n-1)\frac{\pi}{2}$

$-3\pi = -3 \times 180 = -540$

$3\pi = 540$

2. Find the period and amplitude of

(i) $y = \sin 7x$

solution

$\boxed{y = A \sin \alpha x}$

$\Rightarrow y = \sin 7x$

$\Rightarrow A = 1$

$\alpha = 7$

Amplitude: $|A|$

$= |1|$

$\boxed{\text{Amplitude} = 1}$

Period: $\frac{2\pi}{|\alpha|}$

$= \frac{2\pi}{|7|}$

$\boxed{\text{period} = \frac{2\pi}{7}}$

(ii) $y = -\sin\left(\frac{1}{3}x\right)$

solution.

$y = A \sin \alpha x$

$\Rightarrow$

$A = -1$

$\alpha = \frac{1}{3}$

Amplitude: $|A|$

$= |-1|$

$\boxed{\text{Amplitude} = 1}$

Period: $\frac{2\pi}{|\alpha|}$

$= \frac{2\pi}{|1/3|}$

$= \frac{2\pi}{1/3}$

$= \frac{3(2\pi)}{1}$

$\boxed{\text{Period} = 6\pi}$

(iii) $y = 4 \sin(-2x)$

Solution,

$y = A \sin \alpha x$

$\Rightarrow A = 4$

$\alpha = -2$

Amplitude: $|A|$

$= |4|$

$\boxed{\text{Amplitude} = 4}$

Period: $\frac{2\pi}{|\alpha|}$

$= \frac{2\pi}{|-2|}$

$= \frac{2\pi}{2}$

$\boxed{\text{Period} = \pi}$

③ Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$ for $0 \le x \le 6\pi$

solution

$y = \sin\left(\frac{1}{3}x\right)$

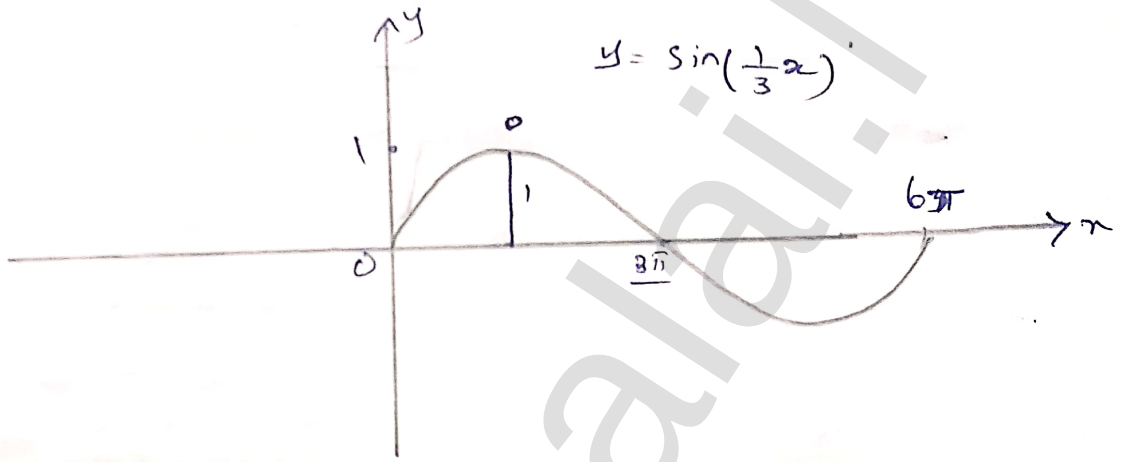
w.k.T $y = A \sin \alpha x$

$\Rightarrow A = 1$

$\alpha = \frac{1}{3}$

Amplitude: $|A| = |1| = 1$

period: $\frac{2\pi}{|\alpha|} = \frac{2\pi}{\left|\frac{1}{3}\right|} = \frac{2\pi}{\frac{1}{3}} = \frac{3(2\pi)}{1} = 6\pi$



④ Find the value of

(i) $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right]$

R.w

$\sin^{-1}(\sin x) = x, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin x) = \pi - x, \quad x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

Solution:-

$\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$

$\frac{2\pi}{3} = 2(60^\circ) = 120^\circ$

$120 \in \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow$

$\frac{2\pi}{3} \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \Rightarrow$

$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3}$

$= \frac{3\pi - 2\pi}{3}$

$\boxed{\text{Ans} = \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

$\frac{5\pi}{4} = 5(45^\circ)$

Solution

$\frac{5\pi}{4} \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \Rightarrow \pi - x$

$\Rightarrow$

$$\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right] = \pi - \frac{5\pi}{4}$$

$$= \frac{4\pi - 5\pi}{4}$$

$$\boxed{\text{Ans} = \frac{-\pi}{4} \in \left[-\pi/2, \pi/2\right]}$$

5) For what value of x does $\sin x = \sin^{-1} x$?

Solution: -

$\sin x = \sin^{-1} x$

At $x = 0$

$\sin x = \sin^{-1} x.$

R.W

$\sin 0 = 0 \to ①$

$0 = \sin^{-1} 0 \to ②$

from ① & ② $\Rightarrow$

$\Rightarrow \sin 0 = \sin^{-1} 0$

6) Find the domain of the following

1) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Solution:

$f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$

$-1 \le \frac{x^2+1}{2x} \le 1 \quad (x \neq 0)$

$\times 2x^2$

$-1 \times 2x^2 \le \frac{(x^2+1)\cancel{2}x^{\cancel{2}}}{\cancel{2x}} \le 1 \times 2x^2$

$-2x^2 \le x(x^2+1) \le 2x^2$

R.W

Domain:

Angle $\to [-\pi/2, \pi/2]$

Value $\to [-1, 1]$

$\sin^{-1}(x) \to$ value

$\sin(x) \to$ angle

$-2x^2 \le x(x^2+1)$

$-2x^2 \le x^3 + x$

$x(x^2+1) \le 2x^2$

$x^3 + x \le 2x^2$

$0 \le x^3 + x + 2x^2$

$0 \le x^3 + 2x^2 + x$

$0 \le x(x^2 + 2x + 1)$

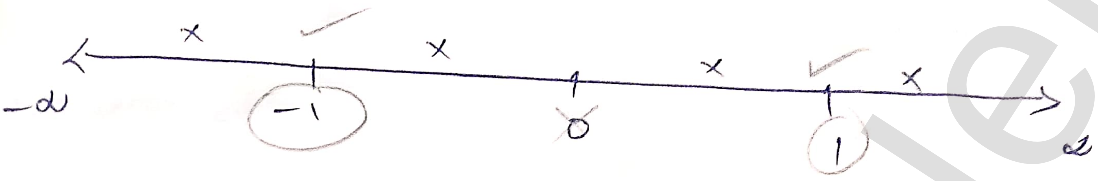
$0 \le x(x+1)^2$

$\Rightarrow x(x+1)^2 \ge 0$

$x^3 - 2x^2 + x \le 0$

$x(x^2 - 2x + 1) \le 0$

$x^2(x-1)^2 \le 0$



$\therefore$ Domain : $x = \{-1, 1\}$

(ii) $g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$

Solution:-

$g(x) = 2\sin^{-1}(2x-1) - \pi/4$

$-1 \le 2x-1 \le 1$

$-1+1 \le 2x \le 1+1$

$0 \le 2x \le 2$

$\div 2 \quad 0 \le x \le 1$

$x \in [0,1]$

$\boxed{\therefore \text{Domain is } [0,1]}$

R.W

$[-1,1] \to$ value

$[-\pi/2, \pi/2] \to$ Angle

$\sin^{-1}x \to x$ value

$\sin x \to x$ Angle

(7) Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

Solution:-

w.k.t

$\sin(A+B) = \sin A\cos B + \cos A \sin B.$

$\therefore \sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

$= \sin^{-1}\left[\sin\left(\frac{5\pi}{9}+\frac{\pi}{9}\right)\right]$

$= \sin^{-1}\left[\sin\left(\frac{6\pi}{9}\right)\right]$

$= \sin^{-1}\left[\sin\left(2\frac{\pi}{3}\right)\right]$

$= \pi - x$

$= \pi - \frac{2\pi}{3}$

$= \frac{3\pi - 2\pi}{3}$

$\boxed{\text{Ans} = \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

R.w

$\sin^{-1}(\sin x) = x$

$\Rightarrow x \in [-\pi/2, \pi/2]$

$= \pi - x$

$\Rightarrow x \in [\pi/2, 3\pi/2]$

$2\pi/3 = 2(60°) = 120°$

$120° \in [\pi/2, 3\pi/2]$

$\Rightarrow \pi - x$

Eg: 4.1 page no: 136

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ (in radians and degree)

Solution:-

$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{\sin^{-1}\left(-\frac{1}{2}\right) = -30° \text{ (or) } -\frac{\pi}{6}}$

R.w

$\sin^{-1}(-x) = -\sin^{-1}x$

if $x \in [-1, 1]$

$\frac{1}{2} \in [-1, 1]$

$\sin 30° = \frac{1}{2}$

$30° = \sin^{-1}\left(\frac{1}{2}\right)$

Eg: 4.2 page: 136

Find the principal value of $\sin^{-1}(2)$ if it exists.

Solution

$\sin^{-1}(2)$

$\Rightarrow 2 \notin [-1, 1]$

$\therefore \sin^{-1}(2)$ does not exist.

R.w

$\sin^{-1}x \to x$ value

$x \in [-1, 1]$

① Find the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Solution:-

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

R.W

$\frac{1}{\sqrt{2}} \in [-1,1]$

(ii) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

Solution:-

$-\frac{\pi}{3} \in \left[\frac{-\pi}{2}, \pi/2\right] \Rightarrow \sin^{-1}(\sin x) = x$

$\therefore \boxed{\sin^{-1}(\sin(-\pi/3)) = -\pi/3}$

R.W

$\sin^{-1}(\sin x) = x$ if $x \in \left[-\pi/2, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin x) = \pi - x$ if $x \in [\pi/2, 3\pi/2]$

(iii) $\sin^{-1}\left(\sin \frac{5\pi}{6}\right)$

Solution.

$\sin^{-1}\left(\sin \frac{5\pi}{6}\right) = \pi - \frac{5\pi}{6}$

$= \frac{6\pi - 5\pi}{6}$

$\boxed{\text{Ans} = \frac{\pi}{6} \in \left[-\pi/2, \frac{\pi}{2}\right]}$

$\frac{5\pi}{6} = 5(30°) = 150°$

$150° \in [\pi/2, 3\pi/2]$

$\Rightarrow \sin^{-1}(\sin x) = \pi - x.$

Eg:4.4 page: 137

Find the domain of $\sin^{-1}(2-3x^2)$

Solution:-

$\sin^{-1}(2-3x^2)$

$-1 \le 2-3x^2 \le 1$

$-1-2 \le -3x^2 \le 1-2$

$-3 \le -3x^2 \le -1$

$\div 3 \quad -1 \le -x^2 \le \frac{-1}{3}$

$\sin^{-1} x \to x$ value

$\Rightarrow$ Domain $= [-1,1]$

$X-1$

$$1 \geq x^2 \geq \frac{1}{3}$$

$$\frac{1}{3} \leq x^2 \leq 1$$

Taking square root on both side

$$\frac{1}{\sqrt{3}} \leq |x| \leq 1$$

$$x \in \left[-1, \frac{-1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right]$$

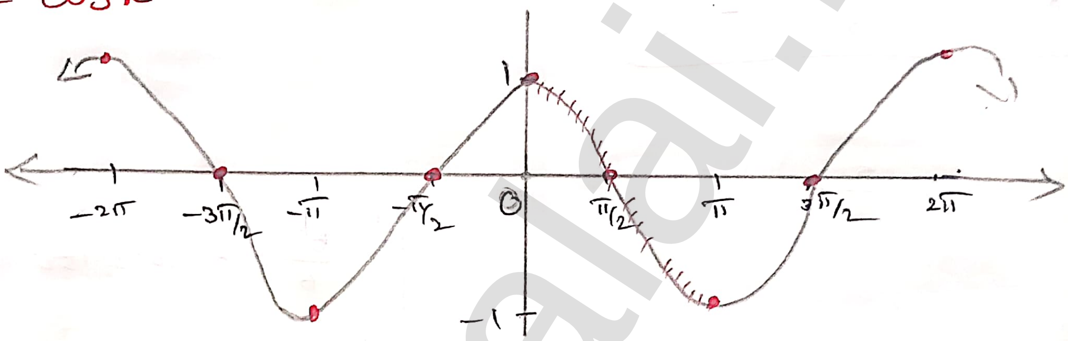
R.W

$a \leq |x| \leq b$

$x \in [-b, -a] \cup [a, b]$

Exercise - 4.2

$Y = \cos x$



Domain : $(-\infty, \infty)$

Range : $[-1, 1]$

Principal Domain : $[0, \pi]$

$Y = \cos^{-1} x$

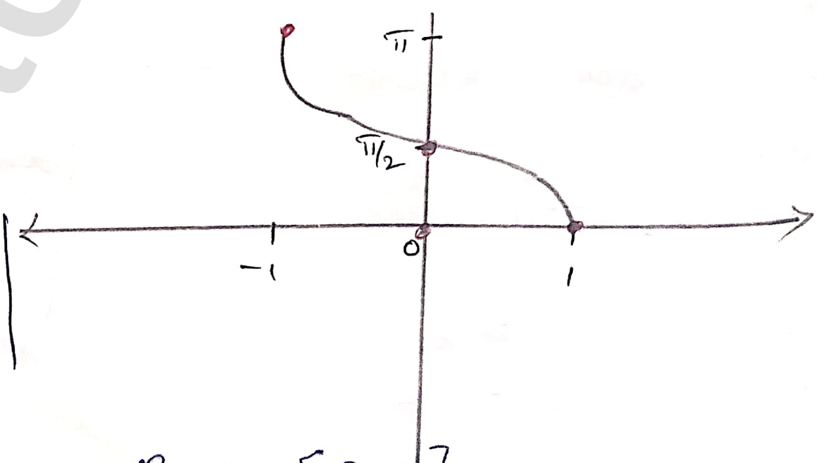
$\Rightarrow x = \cos y$

To find points:

$Y = 0 \Rightarrow x = 1$

$Y = \pi/2 \Rightarrow x = 0$

$Y = \pi \Rightarrow x = -1$



Range : $[0, \pi]$

Principal domain : $[-1, 1]$

Properties :-

① $\cos^{-1}(\cos x) = x \quad , x \in [0, \pi]$

② $\cos^{-1}(\cos x) = 2\pi - x \quad , x \in [\pi, 2\pi]$

③ $\cos^{-1}(-x) = \pi - \cos^{-1} x$

④ $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad ; x \in [-1, 1]$

Page no: 142 Exercise-4.2

① Find all values of x such that

(i) $-6\pi \le x \le 6\pi$ and $\cos x = 0$

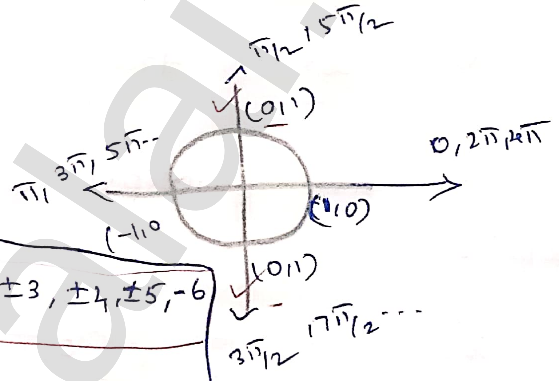
R.W

Solution:-

$\cos x = 0$

$\Rightarrow$ at y axis

$\Rightarrow \boxed{x = (2n+1)\frac{\pi}{2} \ , 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, -6}$



(ii) $-5\pi \le x \le 5\pi$ and $\cos x = 1$

Solution:-

$\cos x = 1$

$\Rightarrow$ at x axis (+ve)

$\Rightarrow \boxed{x = 2n\pi \ , \ n = 0, \pm 1, \pm 2}$

② State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$.

Solution.

$\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) \neq -\frac{\pi}{6} \notin [0, \pi]$

R.W

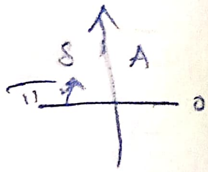
$\cos^{-1}(\cos x) = x \ ; x \in (0, \pi)$

$-\pi/6 \notin (0, \pi)$

$\notin (\pi, 2\pi)$

(3) Is $\cos(-x) = \pi - \cos^{-1}(x)$ true? Justify your ans

Solution.



Let $y = \cos^{-1}(-x)$

$\cos y = -x$

$-\cos y = x$

$\therefore \cos(\pi - y) = x$

$\pi - y = \cos^{-1} x$

$\pi - \cos^{-1} x = y$

$\pi - \cos^{-1} x = \cos^{-1}(-x)$

$\therefore \cos^{-1}(-x) = \pi - \cos^{-1} x$

Yes it is true.

(4) Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$

Solution.

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \in [0, \pi]$

R-10

$\cos 60^\circ = \frac{1}{2}$

$60^\circ = \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{3} = \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{\pi}{3} = 60^\circ \in [0, \pi]$

$\frac{1}{2} \in [-1, 1]$

(5) Find the value of

(i) $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

Solution

$2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

$= 2(60^\circ) + 30^\circ$

$= 2\frac{\pi}{3} + \frac{\pi}{6}$

$= \frac{4\pi}{6} + \frac{\pi}{6}$

$\boxed{\text{Ans} = \frac{5\pi}{6}}$

$\cos 60^\circ = \frac{1}{2}$

$\therefore 60^\circ = \cos^{-1}\left(\frac{1}{2}\right)$

$\sin 30^\circ = \frac{1}{2}$

$30^\circ = \sin^{-1}\left(\frac{1}{2}\right)$

(ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

Solution

$\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

$= 60° + (-\sin^{-1} 1)$

$= \frac{\pi}{3} - \sin^{-1}(1)$

$= \frac{\pi}{3} - \frac{\pi}{2}$

$= \frac{2\pi - 3\pi}{6}$

$\boxed{Ans = \frac{-\pi}{6}}$

R.W

$\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1, 1]$

$\cos 60° = \frac{1}{2}$

$60° = \cos^{-1}(1/2)$

$\sin \pi/2 = 1$

$\pi/2 = \sin^{-1} 1$

(iii) $\cos^{-1}\left[\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right]$

solution:-

W.K.T

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\therefore \cos^{-1}\left[\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right]$

$= \cos^{-1}\left[\cos\left(\frac{\pi}{7} + \frac{\pi}{17}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{17\pi + 7\pi}{119}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{24\pi}{119}\right)\right]$

$\boxed{Ans = \frac{24\pi}{119} \in [0, \pi]}$

Rw

$\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$

$= 2\pi - x$ if $x \in [\pi, 2\pi]$

$\frac{24\pi}{119} = 0 \cdot \ldots \pi \in [0, \pi]$

(6) Find the domain of (i) $f(x) = \sin^{-1}\left[\frac{|x|-2}{3}\right] + \cos^{-1}\left[\frac{1-|x|}{4}\right]$

(ii) $g(x) = \sin^{-1}x + \cos^{-1}x$.

Solution:

(i) $f(x) = \sin^{-1}\left[\frac{|x|-2}{3}\right] + \cos^{-1}\left[\frac{1-|x|}{4}\right]$

R-0
$\sin^{-1}x$
$\to x \in [-1,$

$-1 \le \frac{|x|-2}{3} \le 1$

$\times 3,$

$-1\times 3 \le \frac{3(|x|-2)}{3} \le 1\times 3$

$-3 \le |x|-2 \le 3$

$-3+2 \le |x| \le 3+2$

$-1 \le |x| \le 5$

$-1 \le \frac{1-|x|}{4} \le 1$

$\times 4,$

$-1\times 4 \le \frac{(1-|x|)4}{4} \le 1\times 4$

$-4 \le 1-|x| \le 4$

$-4-1 \le -|x| \le 4-1$

$-5 \le -|x| \le 3$

$5 \ge |x| \ge -3 \Rightarrow -3 \le |x| \le 5$

W.K.T

$a \le |x| \le b \Rightarrow x \in [-b,-a] \cup [a,b]$

$\Rightarrow x \in [-5,1] \cup [-1,5]$

$x \in [-5,3] \cup [5,-3]$

$\Rightarrow x \in [-5,5]$

$\therefore \boxed{\text{Domain is } [-5,5]}$

(ii) $g(x) = \sin^{-1}x + \cos^{-1}x$

$\sin^{-1}x \Rightarrow x \in [-1,1]$

$\cos^{-1}x \Rightarrow x \in [-1,1]$

$\therefore \sin^{-1}x + \cos^{-1}x \in [-1,1]$

$\therefore$ Domain is $[-1,1]$

(7) For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

Solution

$$\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$$

$$\cancel{+\cos\frac{\pi}{2} \quad \cancel{<} \; 3x-1 < \cos\pi}$$

$$\cos\frac{\pi}{2} > 3x-1 > \cos\pi$$

$$0 > 3x-1 > -1$$

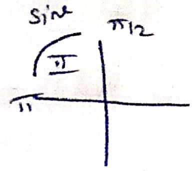
$$0+1 > 3x > -1+1$$

$$1 > 3x > 0$$

$\div 3$ $$\frac{1}{3} > x > 0$$

$$\boxed{0 < x < \frac{1}{3}}$$

sine $\pi/2$



$\pi/2 \le \cos^{-1}(3x-1) < \pi$

($\because$ It is II quadrant Sine only +ve $\Rightarrow$ cos is a negative term

(8) Find the value of

(i) $\cos\left[\cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right]$

Solution:

$$\cos\left[\cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right]$$

$$= \cos\left[\pi/2\right]$$

$$\boxed{Ans = 0}$$

R.w

$\cos^{-1}x + \sin^{-1}x = \pi/2$ if $x \in [-1,1]$

here $x = 4/5 = 0.8 \in [-1,1]$

(ii) $\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] + \cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right]$

Solution

$$\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right] + \cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right]$$

$$= 2\pi - \frac{4\pi}{3} + 2\pi - \frac{5\pi}{4}$$

$$= 4\pi - \frac{4\pi}{3} - \frac{5\pi}{4}$$

R.w

$\cos^{-1}(\cos x) = x$, $x \in [0,\pi]$

$\cos^{-1}(\cos x) = 2\pi - x$, $x \in [\pi, 2\pi]$

$4\left(\frac{\pi}{3}\right) = 4(60) = 240$

$240 \in [\pi, 2\pi]$

$5(\pi/4) = 5(45) = 225$

$225 \in [\pi, 2\pi]$

$$= \frac{48\pi}{12} - \frac{16\pi}{12} - \frac{15\pi}{12}$$

$$= \frac{48\pi - 16\pi - 15\pi}{12}$$

$$= \frac{48\pi - 31\pi}{12}$$

$$\boxed{Ans = \frac{17\pi}{12}}$$

Eg: 4.5 page: 141

① Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Solu

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \in [0, \pi]$$

R.W

$\frac{\sqrt{3}}{2} \in [-1, 1]$

$\pi/6 \in [0, \pi]$

Eg: 4.6 page: 142

Find (i) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Solution:

W.K.T

$$\boxed{\cos^{-1}(-x) = \pi - \cos^{-1}x \quad \text{if } x \in [-1, 1]}$$

$\frac{1}{\sqrt{2}} \in [-1, 1]$

$$\Rightarrow \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{4\pi - \pi}{4}$$

$$\boxed{Ans = \frac{3\pi}{4}}$$

(ii) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

Solution

$\cos(-\theta) = \cos\theta$

$\therefore \cos^{-1}(\cos(-\pi/3)) = \cos^{-1}(\cos \pi/3)$

w.k.T

$$\boxed{\cos^{-1}(\cos x) = x \text{ if } x \in [0, \pi]}$$

$\pi/3 = 60°$

$\pi/3 \in [0, \pi]$

$$\Rightarrow \boxed{\cos^{-1}(\cos \pi/3) = \pi/3 \in [0, \pi]}$$

(iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

solution

$7\pi/6 = 7(30°) = 210°$

$210 \in [\pi, 2\pi]$

w.k.T

$$\boxed{\cos^{-1}(\cos x) = 2\pi - x \text{ if } x \in [\pi, 2\pi]}$$

here

$\frac{7\pi}{6} \in [\pi, 2\pi]$

$$\Rightarrow \cos^{-1}\left(\cos \frac{7\pi}{6}\right) = 2\pi - \frac{7\pi}{6}$$

$$= \frac{12\pi - 7\pi}{6}$$

$$\boxed{\text{Ans} = \frac{5\pi}{6} \in [0, \pi]}$$

Eg: 4.7 page: 142

Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

solution.

$$\cos^{-1}\left(\frac{2+\sin x}{3}\right)$$

$\cos^{-1} x \to x \in [-1, 1]$

$$-1 \le \frac{2+\sin x}{3} \le 1$$

$\times 3$,

$$-3 \le 2 + \sin x \le 3$$

$$-3-2 \le \sin x \le 3-2$$

$y = \sin x$

range $[-\pi/2, \pi/2]$ Domain $[-1, 1]$

$-5 \le \sin x \le 1$

$\times$ $-1 \le \sin x \le 1$

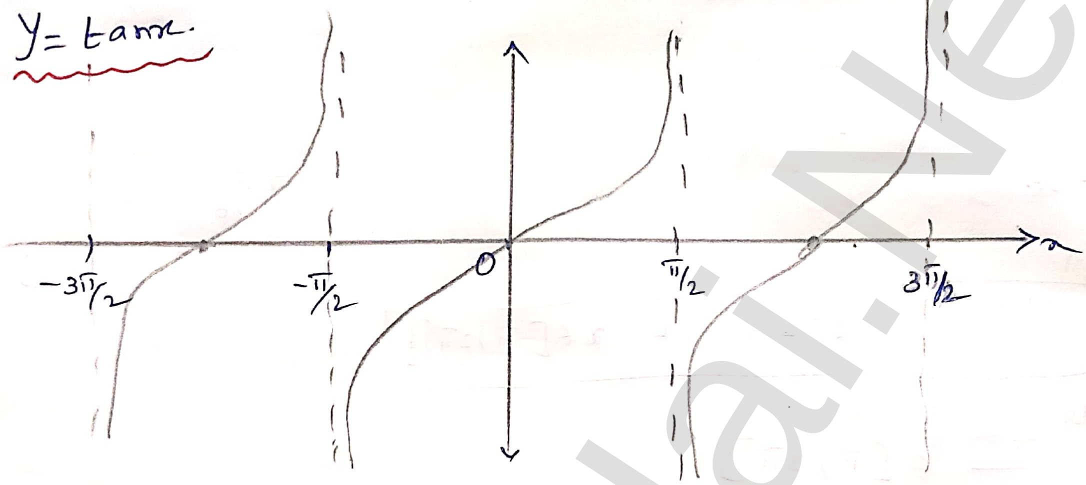
$-\pi/2 \le x \le \pi/2$ (or)

$x \in [-\pi/2, \pi/2]$ (or)

$\therefore$ Domain is $[-\pi/2, \pi/2]$

Exercise-4.3

$y = \tan x$



$y = \tan x$

$0 = \tan 0$

$\infty = \tan \pi/2$

$-\infty = \tan(-\pi/2) = -\tan \pi/2$

Domain: $R - \{(2n+1)\pi/2\}$

Range: $(-\infty, \infty)$ or R

Principal domain: $(-\pi/2, \frac{\pi}{2})$

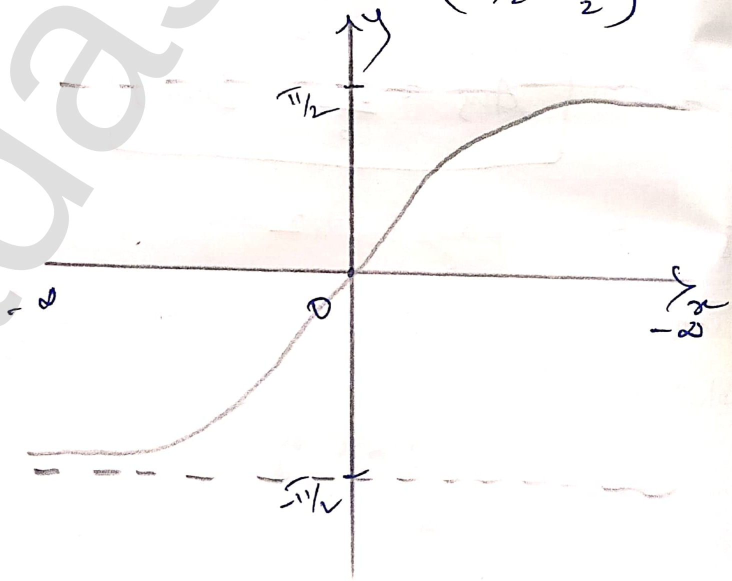
$y = \tan^{-1} x$

$\tan y = x$

$\tan \pi/2 = \infty$

$\tan 0 = 0$

$\tan(-\pi/2) = -\tan \pi/2 = -\infty$



Principal domain: R

Range: $(-\pi/2, \pi/2)$

Properties:

① $\tan^{-1}(\tan x) = x$ if $x \in (-\pi/2, \pi/2)$

② $\tan^{-1}(\tan x) = x - \pi$ if $x - \pi \in (-\pi/2, \pi/2)$

③ $\tan(\tan^{-1} x) = x$ if $x \in R$ (or) $(-\infty, \infty)$

④ $\tan^{-1}(-x) = -\tan^{-1} x$ if $x \in R$ (or) $(-\infty, \infty)$

⑤ $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

⑥ $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right]$

Page: 147 Exercise 4.3

① Find the domain of the following function:-

(i) $\tan^{-1}\left(\sqrt{9-x^2}\right)$

Solution

$\tan^{-1}\sqrt{9-x^2}$

$\sqrt{9-x^2} \in R$

$9 - x^2 \geq 0$

$-x^2 \geq -9$

$x^2 \leq 9$

$|x| \leq 3$

$-3 \leq x \leq 3$

$\therefore \boxed{x \in [-3, 3]}$

Domain is $[-3, 3]$

(ii) $\frac{1}{2}\tan^{-1}(1-x^2) - \frac{\pi}{4}$

$\tan^{-1} x \to x$ value $x \in R$

Solution.

$\frac{1}{2}\tan^{-1}(1-x^2) - \frac{\pi}{4}$

$1 - x^2 \in R$

$\boxed{x \in R}$

$\therefore$ Domain is 'R'

② Find the value of (i) $\tan^{-1}\left(\tan \frac{5\pi}{4}\right)$

Solution.

$\tan^{-1}\left(\tan \frac{5\pi}{4}\right)$

$\frac{5\pi}{4} \notin [-\pi/2, \pi/2]$

$\therefore \Rightarrow \frac{5\pi}{4} - \pi$

$= \frac{5\pi - 4\pi}{4}$

$\boxed{x = \frac{\pi}{4} \in [-\pi/2, \pi/2]}$

R.W

$\tan^{-1}(\tan x) = x$ if $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$= x - \pi$ if $x - \pi \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\frac{5\pi}{4} = 5(45) = 225 \notin [-\pi/2, \pi/2]$

(ii) $\tan^{-1}(\tan(-\pi/6))$

Solution:-

$\tan^{-1}(\tan(-\pi/6))$

$\therefore \tan^{-1}(\tan^{-1}(-\pi/6)) = -\pi/6$

R.W

$\tan^{-1}(\tan x) = x$ if $x \in (-\pi/2, \pi/2)$

$-\pi/6 \in (-\pi/2, \pi/2)$

(3) Find the value of

(i) $\tan\left[\tan^{-1}\left(\frac{7\pi}{4}\right)\right]$

Solution:

$\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right) = \frac{7\pi}{4} \in R$

R.W

$\frac{7\pi}{4} \in R.$

$\tan(\tan^{-1} x) = x$ if $x \in R$

(ii) $\tan(\tan^{-1}(1947))$

Solution

$\tan(\tan^{-1}(1947)) = 1947 \in R$

(iii) $\tan(\tan^{-1}(-0.2021))$

Solution

$\tan(\tan^{-1}(-0.2021)) = -0.2021 \in R$

(4) Find the value of (i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

Solution.

$$\tan^{-1}\left(\cos^{-1}\left(\tfrac{1}{2}\right)-\sin^{-1}\left(-\tfrac{1}{2}\right)\right)$$
$$=\tan^{-1}\left(\cos^{-1}\left(\tfrac{1}{2}\right)-\left(-\sin^{-1}\left(\tfrac{1}{2}\right)\right)\right)$$
$$=\tan^{-1}\left(\cos^{-1}\tfrac{1}{2}+\sin^{-1}\tfrac{1}{2}\right)$$
$$=\tan^{-1}\left(\pi/2\right)$$

$\boxed{\text{Ans} = \infty}$

R.W

$\sin^{-1}(-x) = -\sin^{-1}x$ if $x\in[-1,1]$

here $x = 1/2 \in [-1,1]$

$\sin^{-1}x+\cos^{-1}x = \pi/2$ if $x\in[-1,1]$

(ii) ~~tan/(/tA~~ $\sin\left[\tan^{-1}\left(\tfrac{1}{2}\right)-\cos^{-1}\left(\tfrac{4}{5}\right)\right]$

Solution:-

$$\sin\left[\tan^{-1}\left(\tfrac{1}{2}\right)-\cos^{-1}\left(\tfrac{4}{5}\right)\right]$$
$$=\sin\left(\sin^{-1}\left(\tfrac{1}{\sqrt5}\right)-\sin^{-1}\left(\tfrac{3}{5}\right)\right)$$

w.k.T

$$\boxed{\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]}$$

$$=\sin\left[\sin^{-1}\left(\tfrac{1}{\sqrt5}\sqrt{1-\left(\tfrac{3}{5}\right)^2}-\tfrac{3}{5}\sqrt{1-\left(\tfrac{1}{\sqrt5}\right)^2}\right)\right]$$
$$=\sin\left[\sin^{-1}\left(\tfrac{1}{\sqrt5}\sqrt{1-\tfrac{9}{25}}-\tfrac{3}{5}\sqrt{1-\tfrac{1}{5}}\right)\right]$$
$$=\sin\left[\sin^{-1}\left(\tfrac{1}{\sqrt5}\sqrt{\tfrac{25-9}{25}}-\tfrac{3}{5}\sqrt{\tfrac{5-1}{5}}\right)\right]$$
$$=\sin\left[\sin^{-1}\left(\tfrac{1}{\sqrt5}\sqrt{\tfrac{16}{25}}-\tfrac{3}{5}\sqrt{\tfrac{4}{5}}\right)\right]$$
$$=\sin\left[\sin^{-1}\left(\tfrac{1}{\sqrt5}\cdot\tfrac{4}{5}-\tfrac{3}{5}\cdot\tfrac{2}{\sqrt5}\right)\right]$$
$$=\sin\left[\sin^{-1}\left(\tfrac{4}{5\sqrt5}-\tfrac{6}{5\sqrt5}\right)\right]$$

R.W

$\tan^{-1}\left(\tfrac{op}{ad}\right)$

$hy^2 = opp^2+adj^2$

$\tan^{-1}\left(\tfrac{1}{2}\right)$

$hy^2 = 1^2+2^2$

$hy^2 = 1+4$

$hy^2 = 5$

$\boxed{hy = \sqrt5}$

$\Rightarrow \sin^{-1}\left(\tfrac{op}{hy}\right) = \sin^{-1}\left(\tfrac{1}{\sqrt5}\right)$

$\cos^{-1}\left(\tfrac{adj}{hy}\right)$

$\cos^{-1}\left(\tfrac{4}{5}\right)$

$hy^2 = op^2+adj^2$

$5^2 = op^2+4^2$

$25 = op^2+16$

$25-16 = op^2$

$9 = op^2$

$\boxed{op = 3}$

$\sin^{-1}\left(\tfrac{3}{5}\right)$

$= \sin\left(\sin^{-1}\left(-\frac{2}{5\sqrt5}\right)\right)$

$= \frac{-2}{5\sqrt5}$

$= \frac{-2}{5\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}$

$= \frac{-2\sqrt5}{5\sqrt5\sqrt5}$

$= \frac{-2\sqrt5}{5(5)}$

$\boxed{\text{Ans} = \frac{-2\sqrt5}{25}}$

$\sin(\sin^{-1}x) = x$

$x \in [-1,$

$\frac{-2}{5\sqrt5} = 0\ldots \in [-1,$

iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$

Solution

$\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$

$= \cos\left(\cos^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$

W.K.T.

$\boxed{\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right]}$

$= \cos\left[\cos^{-1}\left(\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \sqrt{1-\left(\frac{3}{5}\right)^2}\sqrt{1-\left(\frac{4}{5}\right)^2}\right)\right]$

$= \cos\left[\cos^{-1}\left(\frac{12}{25} + \sqrt{1-\frac{9}{25}}\sqrt{1-\frac{16}{25}}\right)\right]$

$= \cos\left[\cos^{-1}\left(\frac{12}{25} + \sqrt{\frac{25-9}{25}}\sqrt{\frac{25-16}{25}}\right)\right]$

$= \cos\left[\cos^{-1}\left(\frac{12}{25} + \sqrt{\frac{16}{25}}\sqrt{\frac{9}{25}}\right)\right]$

R.W

$\sin^{-1}\left(\frac{4}{5}\right) - \sin^{-1}\left(\frac{op}{hy}\right)$

$hy^2 = op^2 + adj^2$

$5^2 = 4^2 + adj^2$

$25 = 16 + adj^2$

$25 - 16 = adj^2$

$9 = adj^2$

$adj = 3$

$\cos^{-1}\left(\frac{3}{5}\right)$

$\tan^{-1}\left(\frac{3}{4}\right) = \tan\left(\frac{op}{adj}\right)$

$hy^2 = op^2 + adj^2$

$hy^2 = 3^2 + 4^2$

$= 9 + 16$

$hy^2 = 25$

$hy = 5$

$\cos^{-1}\left(\frac{4}{5}\right)$

$= \cos\left[\cos^{-1}\left(\frac{12}{25} + \frac{4}{5}\left(\frac{3}{5}\right)\right)\right]$

$= \cos\left[\cos^{-1}\left(\frac{12}{25} + \frac{12}{25}\right)\right]$

$= \cos\left[\cos^{-1}\left(\frac{24}{25}\right)\right]$

R.w

$\cos(\cos^{-1}x) = x$ if $x \in [-1,1]$

$\frac{24}{25} = 0.... \in [-1,1]$

$\boxed{\text{Ans} = \frac{24}{25}}$

Eg: 4.8 page: 146

Find the principal value of $\tan^{-1}\sqrt{3}$

Solution:-

$\tan^{-1}\sqrt{3} = \frac{\pi}{3} \in (-\pi/2, \pi/2)$

Eg: 4.9 page: 147

Find (i) $\tan^{-1}(-\sqrt{3})$

Solution:-

$\tan^{-1}(-\sqrt{3}) = -\tan^{-1}\sqrt{3}$

R.w

$\tan^{-1}(-x) = -\tan^{-1}x$ if $x \in R$

$\boxed{\text{Ans} = -\pi/3 \in (-\pi/2, \pi/2)}$

(ii) $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$

Solution

$\tan^{-1}(\tan 3\pi/5)$

here $3\pi/5 \notin (-\pi/2, \pi/2)$

$\therefore \tan^{-1}(\tan 3\pi/5) = 3\pi/5 - \pi$

R.w

$\tan^{-1}(\tan x) = x$ if $x \in (-\pi/2, \pi/2)$

$\frac{3 \times \cancel{180}^{36}}{\cancel{5}} = 108 \notin (-\pi/2, \pi/2)$

$3\pi/5$

$$= \frac{3\pi - 5\pi}{5}$$

$$- \frac{2 \times \overset{36}{\cancel{180}}}{\cancel{5}} = -72 \in \left(-\pi/2, \pi/2\right)$$

$$\boxed{\text{Ans} = -\frac{2\pi}{5} \in \left(-\pi/2, \pi/2\right)}$$

(iii) $\tan\left(\tan^{-1}(2019)\right)$

Solution

$$\tan\left(\tan^{-1}(2019)\right) = 2019 \in R$$

$\tan\left(\tan^{-1}x\right) = x$ if $x \in R$

$2019 \in R$

Eg: 4.10 page no: 147

Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Solution:-

$$\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= -\tan^{-1} 1 + \cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)$$

$$= -\pi/4 + \pi/3 - \pi/6$$

$$= \frac{-3\pi}{12} + \frac{4\pi}{12} - \frac{2\pi}{12}$$

$$= \frac{-3\pi + 4\pi - 2\pi}{12}$$

$$= \frac{4\pi - 5\pi}{12}$$

$$\boxed{\text{Ans} = \frac{-\pi}{12}}$$

R.W

$\tan^{-1}(-x) = -\tan^{-1}x$ if $x \in R$

here $-1 \in R$

$\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1,1]$

here $1/2 \in [-1,1]$

Eg: 4.11 page: 147

Prove that $\tan\left(\sin^{-1}x\right) = \frac{x}{\sqrt{1-x^2}}$, $-1 < x < 1$

Proof

$$\tan\left(\sin^{-1}(x)\right) = \tan\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right)$$

W.K.T

$$\boxed{\tan(\tan^{-1}x) = x \quad \text{if} \quad x \in R}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$= R.H.S$$

$$\therefore \boxed{\tan\left(\sin^{-1}(x)\right) = \frac{x}{\sqrt{1-x^2}}}$$

Hence proved.

R.W

$\sin^{-1} x = \sin^{-1}\left(\frac{x}{1}\right)$

$\sin^{-1}\left(\frac{op}{hy}\right)$

$hy^2 = op^2 + adj^2$

$1^2 = x^2 + adj^2$

$1 = x^2 + adj^2$

$1 - x^2 = adj^2$

$adj = \sqrt{1-x^2}$

$\tan = \frac{\cancel{adj}\ op}{\cancel{o}\ adj}$

$\tan = \frac{x}{\sqrt{1-x^2}}$

Page: 154 Ex-4.4

① Find the principal value of

(i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Solution:

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{Ans = \pi/6 \in [0, \pi]}$$

(ii) $\cot^{-1}(\sqrt{3})$

Solution

$$\cot^{-1}(\sqrt{3}) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\boxed{Ans = \pi/6 \in \left(-\pi/2, \pi/2\right)}$$

(iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Solution:-

$\operatorname{cosec}^{-1}(-\sqrt{2}) = \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$= -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\boxed{\text{Ans} = -\pi/4 \in [-\pi/2, \pi/2]}$

$\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1,1]$

$\frac{-1}{\sqrt{2}} = -0\ldots \in [-1,1]$

(2) Find the value of

(i) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

Solution

$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \pi/3 - \cos^{-1}\left(-\frac{1}{2}\right)$

$= \pi/3 - \left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right)$

$= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi - 3\pi}{3} + \frac{\pi}{3}$

$= \frac{-2\pi}{3} + \frac{\pi}{3}$

$= \frac{-2\pi + \pi}{3}$

$\boxed{\text{Ans} = \frac{-\pi}{3}}$

R.w

$\pi/3 \in (-\pi/2, \pi/2)$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$ if $x \in [-1,1]$

$\frac{-1}{2} \in [-1,1]$

(ii) $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

Solution

$\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

$= -\sin^{-1}1 + \pi/3 + \tan^{-1}\left(\frac{1}{2}\right)$

R.w

$\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1,1]$

$-1 \in [-1,1]$

$\pi/3 \in [-\pi/2, \pi/2]$

$= -\sin^{-1} 1 + \pi/3 + \cot^{-1} 2$

$= -\pi/2 + \pi/3 + \cot^{-1} 2$

$= \frac{-3\pi + 2\pi}{6} + \cot^{-1} 2$

$= \frac{-\pi}{6} + \cot^{-1} 2$

$\boxed{\text{Ans} = \cot^{-1} 2 - \pi/6}$

(iii) $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

Solution.

$\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

$= \tan^{-1}(1) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$= \tan^{-1}(1) - \sin^{-1}\frac{\sqrt{3}}{2} - \left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$= \frac{\pi}{4} - \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{4} - \frac{\pi}{3} - \pi + \frac{\pi}{4}$

$= \frac{2\pi}{4} - \frac{\pi}{3} - \pi$

$= \frac{\pi}{2} - \frac{\pi}{3} - \pi$

$= \frac{3\pi}{6} - \frac{2\pi}{6} - \frac{6\pi}{6}$

$= \frac{3\pi - 2\pi - 6\pi}{6}$

$= \frac{3\pi - 8\pi}{6}$

$\boxed{\text{Ans} = \frac{-5\pi}{6}}$

R.W

$\sin^{-1}(-x) = -\sin^{-1}x$

if $x \in [-1, 1]$

$\frac{\sqrt{3}}{2} = 0\ldots \in [-1,1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

if $x \in [-1,1]$

$\frac{1}{\sqrt{2}} = 0.. \in [-1,1]$

Find the principal value of

(i) $cosec^{-1}(-1)$

Solution :-

$cosec^{-1}(-1) = \sin^{-1}(-1)$

$= -\sin^{-1} 1$

$\boxed{Ans = -\pi/2}$

R.W

$\sin^{-1}(-x) = -\sin^{-1}x$

if $x \in [-1,1]$

$-1 \in [-1,1]$

(ii) $\sec^{-1}(-2)$

Solution :-

$\sec^{-1}(-2) = \cos^{-1}\left(\frac{-1}{2}\right)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \pi/3$

$= \frac{3\pi - \pi}{3}$

$\boxed{Ans = \frac{2\pi}{3}}$

R.W

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

if $x \in [-1,1]$

$1/2 \in [-1,1]$

Eg : 4.13 page : 154

Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

Solution :-

$\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right) = \cos^{-1}\left(\frac{-\cancel{3}^{\sqrt{3}}}{2\cancel{\sqrt{3}}}\right)$

$= \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \pi - \cos \frac{\sqrt{3}}{2}$

$= \pi - \pi/6$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

if $x \in [-1,1]$

$\frac{\sqrt{3}}{2} = 0... \in [-1,1]$

$= \frac{5\pi}{6}$

$$\boxed{Ans = \frac{5\pi}{6}}$$

Eg: 4.14 page no: 154

If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$

Solution:

$\cot^{-1}\left(\frac{1}{7}\right) = \theta$

$\cot\theta = \frac{1}{7}$

W.K.T

$\cot\theta = \frac{adj}{op.} \Rightarrow adj = 1$, $op = 7$

By pythagoras theorem

$hy^2 = op^2 + adj^2$

$hy^2 = 1^2 + 7^2$

$= 1 + 49$

$hy^2 = 50$

$hy = \sqrt{50}$

$= \sqrt{2 \times 25}$

$$\boxed{hy = 5\sqrt{2}}$$

$\cos\theta = \frac{adj}{hy}$

$$\boxed{\cos\theta = \frac{1}{5\sqrt{2}}}$$

Rw

$\cot\theta = \frac{1}{7}\left(\frac{adj}{hy}\right)$

hy^2

Eg: 4.15 page: 154

show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x$, $|x| \geq 1$

Solution: -

let

$\alpha = \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

$\cot^{-1} \frac{adj}{op}$

$\Rightarrow adj = 1$, $op = \sqrt{x^2-1}$

By pythagoras theorem

$hy^2 = op^2 + adj^2$

$hy^2 = \sqrt{x^2-1}^2 + 1^2$

$= x^2 - 1 + 1$

$hy^2 = x^2$

$\boxed{hy = x}$

$\sec\alpha = \frac{hy}{adj}$

$\sec\alpha = \frac{x}{1}$

$\alpha = \sec^{-1}(x)$

$$\boxed{\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x}$$

Hence showed

Properties of inverse trigonometric Functions.

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A \cot B}$
8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot A \cot B}$

1. $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$
2. $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$
3. $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$
4. $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$
5. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$ $\cancel{x}$.
6. $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$
7. $\cot^{-1}x + \cot^{-1}y = \cot^{-1}\left(\dfrac{yx-1}{y+x}\right)$
8. $\cot^{-1}x - \cot^{-1}y = \cot^{-1}\left(\dfrac{yx+1}{y-x}\right)$

① Find the value, if it exists, if not, give the reason for non-existence.

(i) $\sin^{-1}(\cos\pi)$

Solution

$\sin^{-1}(\cos\pi) = \sin^{-1}(-1)$

$= -\sin^{-1} 1$

$\boxed{\text{Ans} = -\pi/2}$

R.W

$\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1,1]$

$-1 \in [-1,1]$

(ii) $\tan^{-1}\left(\sin\left(\frac{-5\pi}{2}\right)\right)$

Solution

$\tan^{-1}\left(\sin\left(\frac{-5\pi}{2}\right)\right) = \tan^{-1}\left(-\sin\frac{5\pi}{2}\right)$

$= \tan^{-1}(-\sin\pi/2)$

$= \tan^{-1}(-1)$

$= -\tan^{-1} 1$

$\boxed{\text{Ans} = -\pi/4}$

R.W

$5\pi/2 = 5(90) = 450°$

$\cancel{450 = 360 + 90}$

$\cancel{\frac{5\pi}{2}} = \pi/2$

$360 \overline{)450}$; 1 ; 360 ; 90

$\tan^{-1}(-x) = -\tan^{-1}x$

$x \in R$

$-1 \in R$

(iii) $\sin^{-1}[\sin 5]$

Solution:

$\sin^{-1}[\sin 5] \neq 5 \quad \because 5 \notin [-\pi/2, \pi/2]$

$\sin^{-1}(\sin 5) = \sin^{-1}[\sin(2\pi - (2\pi - 5))]$

$= \sin^{-1}(-\sin(2\pi - 5))$

R.W

$\sin^{-1}(\sin x) = x$ if $x \in [-\pi/2, \pi/2]$

$\pi = 3.14$

$5 = \pi - \pi + 5$

$= \pi - (\pi - 5)$ ✗

$= 2\pi - (2\pi - 5)$ ✓

$= 3\pi - (3\pi - 5)$

$= \sin^{-1}(\sin(-(2\pi - 5)))$

$= \sin^{-1}(\sin(5 - 2\pi))$

$\boxed{\text{Ans} = 5 - 2\pi \in [-\pi/2, \pi/2]}$

$5 - 2\pi \in -\pi/2, \pi/2$

$-1.57, 1.57$

(2) Find the value of the expression in terms of x, with the help of a reference triangle.

(i) $\sin(\cos^{-1}(1-x))$

Solution:

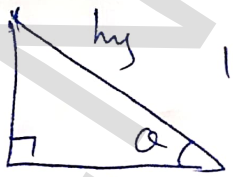
$\sin(\cos^{-1}(1-x))$

$= \sin\left(\cos^{-1}\frac{(1-x)}{1}\right)$

$= \sin\left(\sin^{-1}\frac{\sqrt{2x-x^2}}{1}\right)$

$\boxed{\text{Ans} = \sqrt{2x-x^2}}$

R.W



$hy^2 = op^2 + adj^2$

$1^2 = op^2 + (1-x)^2$

$1 = op^2 + 1^2 + x^2 - 2x$

$\cancel{1} = op^2 + \cancel{1} + x^2 - 2x$

$op^2 = 2x - x^2$

$op = \sqrt{2x - x^2}$

$\sin^{-1}\frac{op}{hy} = \frac{\sqrt{2x-x^2}}{1}$

(ii) $\cos\left(\tan^{-1}\left(\frac{3x-1}{1}\right)\right)$

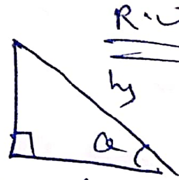
Solution:

$\cos(\tan^{-1}(3x-1)) = \cos\left(\tan^{-1}\left(\frac{3x-1}{1}\right)\right)$

$= \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{9x^2-6x+2}}\right)\right)$

$\boxed{\text{Ans} = \frac{1}{\sqrt{9x^2-6x+2}}}$

R.W



$hy^2 = op^2 + adj^2$

$= (3x-1)^2 + 1^2$

$= 9x^2 + 1 - 6x + 1$

$hy^2 = 9x^2 - 6x + 2$

$hy = \sqrt{9x^2 - 6x + 2}$

iii) $\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)$

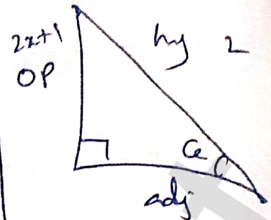
Solution:

$$\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)=\tan\left(\sin^{-1}\left(\frac{2x+1}{2}\right)\right)$$

$$=\tan\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3-4x-4x^2}}\right)\right]$$

$$\boxed{\text{Ans} = \frac{2x+1}{\sqrt{3-4x-4x^2}}}$$

R.w



$hy^2 = op^2 + adj^2$

$2^2 = (2x+1)^2 + adj^2$

$4 = 4x^2 + 1 + 4x + adj^2$

$adj^2 = -4x^2 - 1 - 4x + 4$

$= -4x^2 - 4x + 3$

$\boxed{adj^2 = 3-4x-4x^2}$

③ Find the value of (i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

Solution

$$\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)=\sin^{-1}\left(\cos\left(\pi/3\right)\right)$$

$$=\sin^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\text{Ans} = \pi/6}$$

(ii) $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$

Solution:

$$\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$$

$$=\cot\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{3}{5}\right)$$

W.k.T

$$\boxed{\cos^{-1}x+\sin^{-1}x=\pi/2}$$

R.w

$\sin^{-1}\frac{4}{5}$ $\left(\frac{op}{hy}\right)$

$hy^2 = op^2 + adj^2$

$5^2 = 4^2 + adj^2$

$25 = 16 + adj^2$

$25 - 16 = adj^2$

$9 = adj^2$

$\boxed{3 = adj}$

$= \cot(\pi/2)$

$\boxed{\text{Ans} = 0}$

(iii) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

Solution:-

$\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

$\tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

w.k.T

$\boxed{\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)}$

$\therefore \tan\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{\frac{9+8}{12}}{1-\frac{6}{12}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{17/12}{12-6/12}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{17/\cancel{12}}{6/\cancel{12}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{17}{6}\right)\right)$

$\boxed{\text{Ans} = \frac{17}{6}}$

R.W

$\sin^{-1}\frac{3}{5}$ — $\frac{op}{hy}$

$hy^2 = op^2 + adj^2$

$5^2 = 3^2 + adj^2$

$25 = 9 + adj^2$

$25 - 9 = adj^2$

$16 = adj^2$

$\boxed{adj = 4}$

$\tan^{-1}\left(\frac{3}{4}\right)$

(4) prove that

(i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Proof

L.H.S

$\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$= \tan^{-1}\left(\dfrac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right)$

$= \tan^{-1}\left[\dfrac{\frac{48+77}{264}}{1-\frac{14}{264}}\right]$

$= \tan^{-1}\left[\dfrac{125/264}{264-14/264}\right]$

$= \tan^{-1}\left[\dfrac{125}{250}\right]$

$= \tan^{-1}\left[\frac{1}{2}\right]$

$=$ R.H.S

$\therefore$ L.H.S = R.H.S

Hence proved.

R.W

W.K.T

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(ii) $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$

Proof

L.H.S $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13}$

$= \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{5}{13}$

W.K.T

$\boxed{\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)}$

R.W

$\cos^{-1}\frac{12}{13}$ — adj / — hy

$hy^2 = op^2 + adj^2$

$13^2 = 12^2 + op$

$169 = 144 + op^2$

$169 - 144 = op^2$

$25 = op^2$

$\boxed{op = 5}$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} - \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\frac{25}{169}} - \frac{5}{13}\sqrt{1-\frac{9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{\frac{169-25}{169}} - \frac{5}{13}\sqrt{\frac{25-9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{\frac{144}{169}} - \frac{5}{13}\sqrt{\frac{16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\left(\frac{12}{13}\right) - \frac{5}{13}\left(\frac{4}{5}\right)\right]$$

$$= \sin^{-1}\left[\frac{36}{65} - \frac{20}{65}\right]$$

$$= \sin^{-1}\left[\frac{16}{65}\right]$$

$= R.H.S$

Hence proved.

(5) prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

Proof:

$L.H.S = \tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$$

$$= \tan^{-1}\left[\frac{\frac{x+y}{1-xy} + z}{1-\left(\frac{x+y}{1-xy}\right)z}\right]$$

R.W

$\tan^{-1}x + \tan^{-1}y$

$= \tan^{-1}\left[\frac{x+y}{1-xy}\right]$

$$= \tan^{-1}\left[\frac{\dfrac{x+y+z(1-xy)}{\cancel{1-xy}}}{\dfrac{1-xy-z(x+y)}{\cancel{1-xy}}}\right]$$

$$= \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-zx-zy}\right]$$

L.H.S = R.H.S

Hence proved.

6) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, Show that $x+y+z = xyz$

Proof:

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$$

$$\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-zx-zy}\right) = \pi \quad (\because \text{from sum } 5)$$

$$\frac{x+y+z-xyz}{1-xy-zx-zy} = \tan\pi$$

$$\frac{x+y+z-xyz}{1-xy-zx-zy} = 0$$

$$x+y+z-xyz = 0$$

$$\boxed{x+y+z = xyz}$$

Hence proved.

(7) prove that $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-3x^2}$, $|x| < \frac{1}{\sqrt{3}}$

Proof:

$$L.H.S = \tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$$

R.W

$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$$= \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)}\right)$$

$$= \tan^{-1}\left[\frac{\frac{x(1-x^2)+2x}{\cancel{1-x^2}}}{\frac{1-x^2-2x^2}{\cancel{1-x^2}}}\right]$$

$$= \tan^{-1}\left[\frac{x-x^3+2x}{1-3x^2}\right]$$

$$= \tan^{-1}\left[\frac{-x^3+3x}{1-3x^2}\right]$$

$$= \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$$

$$= R.H.S$$

Hence proved

(8) Simplify $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$

Solution

$$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$$

R.W

$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

$$= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \frac{x}{y}\left(\frac{x-y}{x+y}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{y(\cancel{x}+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$$

$$= \tan^{-1}\left[\frac{x^2+\cancel{xy}-\cancel{yx}+y^2}{\cancel{xy}+y^2+x^2-\cancel{xy}}\right]$$

$$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)$$

$$= \tan^{-1}(1)$$

$$\boxed{\text{Ans} = \pi/4}$$

9) Solve :

(i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

Solution.

$$\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$$

$$\sin^{-1}\frac{5}{x} = \frac{\pi}{2} - \sin^{-1}\frac{12}{x}$$

$$\sin^{-1}\frac{5}{x} = \cos^{-1}\frac{12}{x} = \alpha$$

R.w

$\sin^{-1}x + \cos^{-1}x = \pi/2$

$\cos^{-1}x = \pi/2 - \sin^{-1}x$

Let $\sin^{-1}\frac{5}{x} = \alpha$

$\frac{5}{x} = \sin\alpha$

$\cos^{-1}\frac{12}{x} = \alpha$

$\frac{12}{x} = \cos\alpha$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{12}{x}\right)^2 + \left(\frac{5}{x}\right)^2 = 1$

$\frac{144}{x^2} + \frac{25}{x^2} = 1$

$\frac{169}{x^2} = 1$

$169 = x^2$

$x = \pm 13$

$x = -13$ not satisfied

$\Rightarrow \boxed{x = 13}$

(ii) Solve $2\tan^{-1}x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$, $a>0$, $b>0$

Solution

~~Solutio~~

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$

$2\tan^{-1}x = 2\tan^{-1}a - 2\tan^{-1}b$

$\div 2$

$\tan^{-1}x = \tan^{-1}a - \tan^{-1}b$

$\tan^{-1}x = \tan^{-1}\left(\frac{a-b}{1+ab}\right)$

$\boxed{x = \frac{a-b}{1+ab}}$

RW

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

iii) Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

Solution:-

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\frac{\cancel{2}\cos x}{1-\cos^2 x} = \cancel{2}\operatorname{cosec} x$$

$$\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\cos x \sin x = \sin^2 x$$

$$\cos x \sin x - \sin^2 x = 0$$

$$\sin x(\cos x - \sin x) = 0$$

$\sin x = 0$

$\boxed{x = n\pi,\ n\in Z}$

$\cos x - \sin x = 0$

$\cos x = \sin x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1 \Rightarrow x = \pi/4$

$\boxed{x = n\pi + \pi/4,\ n\in Z}$

R.W

$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1-\cos^2\theta$

$\sin x \Rightarrow$ $n\in Z$

$x = n\pi + (-1)^n$

$\cos x \Rightarrow$

$x = 2n\pi \pm \theta$

$\tan x \Rightarrow$

$x = n\pi + \theta$

$\therefore$ The Solution:-

$$\boxed{\begin{array}{l} x = n\pi,\ n\in Z \quad \& \\ x = n\pi + \pi/4,\ n\in Z \end{array}}$$

(iv) solve : $\cot^{-1} x - \cot^{-1}(x+2) = \pi/12$

Solution :-

$$\cot^{-1} x - \cot^{-1}(x+2) = \pi/12$$

$$\cot^{-1}\left(\frac{(x+2)x+1}{x+2-x}\right) = \pi/12$$

$$\cot^{-1}\left(\frac{x^2+2x+1}{2}\right) = \pi/12$$

$$\cot^{-1}\frac{(x+1)^2}{2} = \frac{180}{12}$$

$$\cot^{-1}\left(\frac{(x+1)^2}{2}\right) = 15^\circ$$

$$\frac{(x+1)^2}{2} = \cot 15^\circ$$

$$\frac{(x+1)^2}{2} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\frac{(x+1)^2}{2} = \frac{(\sqrt{3}+1)^2}{\sqrt{3}^2-1^2}$$

$$\frac{(x+1)^2}{2} = \frac{(\sqrt{3}+1)^2}{3-1}$$

$$\frac{(x+1)^{\cancel{2}}}{\cancel{2}} = \frac{(\sqrt{3}+1)^{\cancel{2}}}{\cancel{2}}$$

$$x+\cancel{1} = \sqrt{3}+\cancel{1}$$

$$\boxed{x = \sqrt{3}}$$

R-W

$$\cot^{-1}A - \cot^{-1}B = \cot^{-1}\left(\frac{AB+1}{B-A}\right)$$

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow \cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

(10) Find the number of Solution of the equation

$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

Solution

$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$\tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right)=\tan^{-1}\left(\frac{3x-x}{1+3x(x)}\right)$

$\tan^{-1}\left(\frac{2x}{1-(x^2-1^2)}\right)=\tan^{-1}\left(\frac{2x}{1+3x^2}\right)$

$\frac{2x}{1-x^2+1}=\frac{2x}{1+3x^2}$

$\frac{2x}{2-x^2}=\frac{2x}{1+3x^2}$

$2x(1+3x^2)=2x(2-x^2)$

$2x+6x^3=4x-2x^3$

$6x^3+2x-4x+2x^3=0$

$\div 2$

$3x^3+x-2x+x^3=0$

$3x^3+x^3-x=0$

$\boxed{4x^3-x=0}$ which is a cubic equation

$\therefore$ The given equation has 3 solution.