COMMON QUARTERLY EXAM-2022

Standard - XII

MATHS

Time : 3.00 hrs Marks : 90

Section - A

20×1=20

I. Answer all the questions:-

1) If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I_2 - A =$

a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

2) If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$

a) $(\cos^2\frac{\theta}{2})A$ b) $(\cos^2\frac{\theta}{2})A^T$ c) $(\cos^2\theta)I$ d) $(\sin^2\frac{\theta}{2})A$

3) If $P(A) = P(A|B)$ then the system $Ax=B$ of linear equations is

a) consistent and has a unique solution b) consistent

c) consistent and has infinitely many solution d) inconsistent

4) Which of the following is the solution of the system $x+2y=0$, $2x+4y=0$

a) (2, –1) b) (4, –2) c) (8, –4) d) All

5) The value of $i^{1001}+i^{1002}+i^{1003} =$

a) 1 b) –1 c) 0 d) i

6) If $\frac{z-1}{z+1}$ is purely imaginary then $|z|$ is

a) ½ b) 1 c) 2 d) 3

7) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

a) cis $\frac{2\pi}{3}$ b) cis $\frac{4\pi}{3}$ c) –cis $\frac{2\pi}{3}$ d) –cis $\frac{4\pi}{3}$

8) If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

a) –2 b) –1 c) 1 d) 2

9) If f and g are polynomiale of degrees m and n respectively and if h(x)=(fog)(x), then the degree of h is

a) mn b) m+n c) m^n d) n^m

10) If α, β and γ are the zero x^3+px^2+qx+r, then $\sum \frac{1}{\alpha}$ is

a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

11) The polynomial x^3+2x+3 has

a) one negative and two imaginary zeros

b) one positive and two imaginary zeros

c) Three real roots d) no zeros

12) $sin^{-1}\frac{3}{5} - cos^{-1}\frac{12}{13} + sec^{-1}\frac{5}{13} - cosec^{-1}\frac{13}{12}$ is equal to

a) 2π b) π c) 0 d) $tan^{-1}\frac{12}{65}$

13) If $Cot^{-1}2$ and $cot^{1}3$ are two angles of a triangle then the third angle is

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$

14) If $\sin^{-1}x+\cot^{1}(\frac{1}{2})= \frac{\pi}{2}$ then x is equal to

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

15) The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

a) $\frac{4}{3}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{3}{2}$

16) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

17) If $x+y=k$ is a normal to the parabola $y^2=12x$ then the value of k is

a) 3 b) –1 c) 1 d) 9

18) If the two tangents drawm from a point P to the parabola $y^2=4x$ are at right angles then the locus of P is

a) $2x+1=0$ b) $x=-1$ c $2x-1=0$ d) $x=1$

19) If $\vec{a}=\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$ then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

20) If the direction cosiness of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

a) $c=\pm3$ b) $c=\pm\sqrt{3}$ c) $c>0$ d) $0<c<1$

Section - B

Answer any seven questions. Question number 30 is compulsory: 7×2=14

21) Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is an orthogonal

22) Find the rank of the matrix $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

23) Find $|\overline{(1+i)}(2+3i)(4i-3)|$

24) Find z^{-1}, if $z=(2+3i)(1-i)$

25) Find a polynomial equation of minimum degree with rational coefficient, having $2+\sqrt{3}i$ as a root.

26) Discuss the nature of the roots of the polynomial equation $x^{2018}+1947x^{1950}+15x^8+26x^6+2019=0$

27) Find the principal value of $\text{Sin}^{-1}\left(\text{Sin}\left(\frac{-\pi}{3}\right)\right)$

28) Draw the graph of $\sin^{-1}x$.

29) Find the length of the latus rectum of the parabola $y^2-4y-8x+12=0$

30) Find the angle between the planes $\vec{r}.(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$

Section - C

Answer any seven questions. Question number 40 is compulsory: 7×3=21

31) Solve $2x+5y=-2$, $x+2y=-3$ by matrix inverse method.

32) If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

33) Show that $(2+i\sqrt{3})^n + (2-i\sqrt{3})^n$, $n\in N$ is real.

34) Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular form.

35) If $x^2+2(k+2)x+9k=0$ has equal roots, find k.

36) Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$

37) Find the equation of the tangent at t=2 to the parabola $y^2=8x$.

38) Prove that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

39) Show that the points (2,3,4), (-1, 4,5) and (8,1,2) are collinear.

40) Find the equation of the ellipse with foci (±2, 0) and vertices (±3,0)

Section - D

Answer all the questions:- **7×5=35**

41) a) Solve: $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$, $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$, $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$ by Cramer's rule.

[or]

b) Solve: $6x^4-35x^3+62x^2-35x+6=0$.

42) a) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$ show that $x^2+y^2=1$ [or]

b) Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

43) a) If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ find all roots. [or]

b) Find the number of solutions of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$

44) a) For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices and the foci. Also prove that the length of latus rectum is 2. Draw a rough. [or]

b) Prove by vector method $\sin(A+B) = \sin A \cos B + \cos A \sin B$

45) a) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8+O_2 \rightarrow CO_2+H_2O$ [or]

b) Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

46) a) If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$ show that $xy-\frac{1}{xy} = 2i\sin(\alpha+\beta)$,

ii) $x^m y^n+\frac{1}{x^m y^n} = 2\cos(m\alpha+n\beta)$ [or]

b) A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides.

47) a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$ [or]

b) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one alone and that of one women alone to finish the same work by using matrix inversion method.

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