ANNE'S ACADEMY

Model Question Paper (Quarterly Exam)
CLASS – XII
MATHEMATICS

Time Allowed : 3 Hrs | Maximum Marks : 90

**PART – I**

**I. Answer ALL questions.** **20X1=20**

1) If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then $\lambda$ is
(1) 17 (2) 14 (3) 19 (4) 21

2) The rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 6 & 8\\ -1 & -2 & -3 & -4\end{bmatrix}$ is
(1) 1 (2) 2 (3) 4 (4) 3

3) If $\rho(A)=\rho([A\,|\,B])$, then the system $AX=B$ of linear equations is
(1) consistent and has a unique solution (2) consistent
(3) consistent and has infinitely many solution (4) inconsistent

4) If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is
(1) $z$ (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

5) If $|z_1|=1,\ |z_2|=2,\ |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is
(1) 1 (2) 2 (3) 3 (4) 4

6) The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
(1) –2 (2) –1 (3) 1 (4) 2

7) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?
(1) –1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

8) A zero of $x^3+64$ is
(1) 0 (2) 4 (3) $4i$ (4) -4

9) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if
(1) $a\geq 0$ (2) $a>0$ (3) $a<0$ (4) $a\leq 0$

10) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}+\sec^{-1}\frac{5}{3}-\text{cosec}^{-1}\frac{13}{12}$ is equal to

(1) $2\pi$ (2) $\pi$ (3) 0 (4) $\tan^{-1}\frac{12}{65}$

11) The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is

(1) $[1,\ 2]$ (2) $[-1,\ 1]$ (3) $[0,\ 1]$ (4) $[-1,\ 0]$

12) If the function $f(x)=\sin^{-1}\left(x^2-3\right)$, then $x$ belongs to

(1) $[-1,1]$ (2) $\left[\sqrt{2},\ 2\right]$

(3) $\left[-2,-\sqrt{2}\right]\cup\left[\sqrt{2},\ 2\right]$ (4) $\left[-2,-\sqrt{2}\right]$

13) The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

14) The length of the diameter of the circle which touches the $x$-axis at the point (1,0) and passes through the point $(2,3)$.

15) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

16) The values of $m$ for which the line $y=mx+2\sqrt{5}$ touches the hyperbola $16x^2-9y^2=144$ are the roots of $x^2-(a+b)x-4=0$, then the value of $(a+b)$ is

17) If $[\vec{a},\vec{b},\vec{c}]=1$, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}}+\frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(1) 1 (2) $-1$ (3) 2 (4) 3

18) The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j},\ \hat{i}+2\hat{j},\ \hat{i}+\hat{j}+\pi\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\pi$ (4) $\frac{\pi}{4}$

19) If $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}+\hat{j},\ \vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then the value of $\lambda+\mu$ is

(1) 0 (2) 1 (3) 6 (4) 3

20) The angle between the line $\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+t(2\hat{i}+\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j})+4=0$ is

(1) $0^\circ$ (2) $30^\circ$ (3) $45^\circ$ (4) $90^\circ$

## PART – II

**II. Answer 7 questions. Question No. 30 is compulsory.** **7x2 = 14**

21) If $A$ is symmetric, prove that adj $A$ is also symmetric.

22) Simplify: $i^{59}+\frac{1}{i^{59}}$

23) Solve the equation : $x^4-14x^2+45=0$

24) Find solution, if any, of the equation $2\cos^2 x-9\cos x+4=0$.

25) Find the period and amplitude of $y=-\sin\left(\frac{1}{3}x\right)$

26) Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

27) Determine whether $x+y-1=0$ is the equation of a diameter of the circle $x^2+y^2-6x+4y+c=0$ for all possible values of $c$.

28) Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to $y$-axis and passing through $(3,6)$.

29) Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i}+4\hat{j}-5\hat{k}$ about the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ acting through a point whose position vector is $4\hat{i}+2\hat{j}-3\hat{k}$.

30) Prove that $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

## PART – III

**III. Answer 7 questions. Question No. 40 is compulsory.** **7x3 = 21**

31) Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

32) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

33) Find the inverse of each of the following by Gauss-Jordan method: $\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$

34) Prove that the point of intersection of the tangents at '$t_1$' and '$t_2$'on the parabola $y^2 = 4ax$ is $\left[at_1t_2, a(t_1+t_2)\right]$.

35) Show that the equation $z^3 + 2\overline{z} = 0$ has five solutions.

36) If $z = x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of $z$ is $2x^2+2y^2+x-2y=0$.

37) Find the value of $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}\left(-\sqrt{2}\right)$

38) If $\alpha, \beta, \gamma$, and $\delta$ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find a quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.

39) With usual notations, in any triangle $ABC$, prove the following by vector method.
$b^2 = c^2 + a^2 - 2ca\cos B$

40) If $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{c} = 3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a}+m\vec{b}+n\vec{c}$, find the values of $l, m, n$.

## PART – IV

**IV. Answer ALL questions.** **7x5 = 35**

41) i) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

**OR**

ii) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference $d$, prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$

42) i) Identify the type of conic and find centre, foci, vertices, and directrices of the following :
$9x^2 - y^2 - 36x - 6y + 18 = 0$

**OR**

ii) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be?

43) i) Find the equations of the tangent and normal to hyperbola $12x^2-9y^2=108$ at $\theta=\frac{\pi}{3}$. (Hint use parametric form)

**OR**

ii) Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$,

44) i) Find the direction cosines of the straight line passing through the points $(5,6,7)$ and $(7,9,13)$. Also, find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points.

**OR**

ii) Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+t(\hat{i}-2\hat{j}+\hat{k})$ and hence find the shortest distance between the lines.

45) i) If the system of equations $px+by+cz=0$, $ax+qy+cz=0$, $ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.

**OR**

ii) If $ax^2+bx+c$ is divided by $x+3, x-5$, and $x-1$, the remainders are $21, 61$ and $9$ respectively. Find $a, b$ and $c$. (Use Gaussian elimination method.)

46) i) Let $z_1, z_2$, and $z_3$ be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

**OR**

ii) Solve the equation $z^3+8i=0$, where $z\in\mathbb{C}$.

47) i) If $p$ is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$, in terms of $p$.

**OR**

ii) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

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