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Reg.No.: | | | | | | |

COMMON QUARTERLY EXAMINATION - 2022

Std - XII

MATHEMATICS

Time : 3.00 Hours Marks: 90

Part - I

Answer all the questions. Choose the correct answer : 20 x 1 = 20

1. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adjA) = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$ then K =

   a) 0 b) $\sin\theta$ c) $\cos\theta$ d) 1

2. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ then $9I_2 - A =$ a) A^{-1} b) $\frac{A^{-1}}{2}$ c) $3A^{-1}$ d) $2A^{-1}$

3. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$, $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, adj(AB) =

   a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

4. If $0 \leq 0 \leq p$ and the system of equations $x + (\sin\theta)y - (\cos\theta) = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$, has a non-trivial solution then θ is

   a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{4}$

5. If $\left|z - \frac{3}{2}\right| = 2$, then the least value of $|z|$ is a) 1 b) 2 c) 3 d) 5

6. If $\left|\frac{z-1}{z+1}\right|$ is purely imaginary, then $|z|$ is a) 1/2 b) 1 c) 3 d) 4

7. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

   a) -110° b) -70° c) 70° d) 110°

8. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{3/4}$ is a) -2 b) -1 c) 1 d) 2

9. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisifies

   a) $|k| \leq 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \geq 6$

10. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

    a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

11. A zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) -4

12. The principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is a) $\frac{\pi}{6}$ b) $\frac{-\pi}{3}$ c) $\frac{-\pi}{6}$ d) $\frac{\pi}{4}$

13. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then value of x is a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

14. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π

15. The radius of the circle is $3x^2 + 6y^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

16. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

17. If x+y = k is a normal to the parabola $y^2 = 12x$, then the value of k is

a) 3 b) -1 c) 1 d) 9

18. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

19. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

a) $C = \pm 3$ b) $C = \pm\sqrt{3}$ c) $C > 0$ d) $0 < C < 1$

20. If $[\vec{a}, \vec{b}, \vec{c}] = 3$, then the value of $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$ is

a) 9 b) 6 c) 4 d) 5

Part - II

7 x 2 = 14

Answer any seven Questions. Question No. 30 is compulsory.

21. If $A = \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$ find A^{-1}.

22. Find the rank of the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

23. Simplify $i^{1947} + i^{1950}$

24. If $z = (2 + 3i)(1 - i)$ find z^{-1}

25. Find a polynomial equation of minimum degree with rational cofficients having $2-\sqrt{3}$ as a root.

26. Solve $x^4 - 9x^2 + 20 = 0$

27. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

28. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$.

29. Find the general equation of the circle whose diameter is the line segment joining the points (-4, -2) and (1,1)

30. Find the volume of the parallelopiped whose coterminus edges are given by the vectors $2\vec{i}-3\vec{j}-4\vec{k}$, $\vec{i}+2\vec{j}-\vec{k}$ and $3\vec{i}-\vec{j}+2\vec{k}$

Part - III **7 x 3 = 21**

Answer any seven questions. Question No. 40 is compulsory.

31. If $\text{adj}A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ find A^{-1}

32. Find the rank of the matrix by rwo reduction method $A \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

33. Show that the equation $z^2 = \bar{z}$ has four solutions.

34. If p and q are the roots of the equation $lx^2 + mx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

35. Find the domain of $\sin^{-1}(2-3x^2)$

36. Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

37. If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$. Find the value of C.

38. Find the equation of the parabola whose vertex is (5, -2) and focus (2, -2)

39. Find the angle between the line $\vec{r} = (2\vec{i}+3\vec{j}+\vec{k}) + t(\vec{i}-\vec{j}+\vec{k})$ and the plane $2x - y + z = 5$.

40. A particle acted on by constant forces $8\vec{i}+2\vec{j}-6\vec{k}$ and $6\vec{i}+2\vec{j}-2\vec{k}$ is displaced from the point (1, 2, 3) to the point (5, 4, 1). Find the total work done by the forces.

7 x 5 = 35

Part - IV

Answer all the questions.

41. a) Solve by Cramer's rule $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}-\frac{2}{y}-\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$. **(OR)**

b) Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

42. a) Ivestigative for what values of λ and μ the system of linear equations.

$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has

i) no solution ii) a unique solution iii) an infinite number of solutions **(OR)**

b) If $z = x + iy$ is complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

43. a) Find all cube roots of $\sqrt{3}+i$ **(OR)**

b) Solve $(x - 5)(x - 7)(x + 6)(x + 4) = 504$

44. a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

(OR)

b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$ also. Find the cooridnates of the pont of contact.

45. a) Find the vertex, focus, directrix and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$ **(OR)**

b) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = p$ prove that $x + y + z = xyz$

46. a) If z_1, z_2, z_3 are three complex numbers such that $|z_1+z_2+z_3| = 1$

prove that $|9z_1z_2 + 4z_1z_3+z_2z_3| = 6$ **(OR)**

b) Prove by vector method $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

47. a) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$ **(OR)**

b) If $\vec{a} = \vec{i} - \vec{j}, \vec{b} = \vec{i} - \vec{j} - 4\vec{k}, \vec{c} = 3\vec{j} - \vec{k}, \vec{d} = 2\vec{i} + 5\vec{j} + \vec{k}$ prove that

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$
