Quarterly Examination - sep.'22

XI - Mathematics key (CBE-Dist.)

Part - A

1. d) N
2. d) $(-\infty, 1]$
3. c) n
4. b) only transitive
5. a) $xb < yb$
6. b) 3
7. c) 9, 1
8. a) 4
9. a) $1/8$
10. d) $\sec\theta = \frac{1}{4}$
11. a) 10π seconds
12. d) both 1 & 2
13. b) 3^4
14. b) 6
15. c) 11
16. b) 81
17. d) $20C_8\, 2^8 3^{12}$
18. d) 4
19. b) $1-2^{-n}$
20. c) $\frac{(e-1)^2}{2e}$

Part - B

21) $A = \{19, 13, 17, 21\}$

$P(A) = 2^4 = 16.$

22) $y = -\frac{2}{x}$

Domain : $\mathbb{R} - \{0\}$

Range : $\mathbb{R} - \{0\}$

23) $\frac{1}{2^{1001}} < \frac{1}{2^{1000}}$

24) $2x - 17 = \pm 3$

$x = 7$ & $x = 10$

25) $a\cos\theta = b$ & $c\sin\theta = d$

$ac\cos\theta = bc$ — ① $ac\sin\theta = ad$ — ②

①^2+②^2: $\boxed{a^2c^2 = b^2c^2 + a^2d^2}$

26) (i) $\theta = \pi/6$ (ii) $\theta = \pi/3$

27) $\frac{8\times7\times6\times5!}{5!\times2} = 168$

28) BIRD : $4\times3\times2\times1 = 24$

29) $T_{\frac{n}{2}+1} = T_4 = T_{3+1} = 6C_3\, x^3 y^3$

$= 20\, x^3 y^3$

30) $y = \sqrt{3}x + 4$

BANANA

Total no of Ways $= \frac{6!}{3!2!} = 60$

Part – C

31) $y = \frac{5x}{9} - \frac{160}{9} = \frac{5x-160}{9}$

$= f(x)$

$\Rightarrow x = \frac{9y+160}{5} = g(y)$

now,

$(g \circ f)(x) = g[f(x)]$

$= \frac{9\left(\frac{5x-160}{9}\right)+160}{5}$

$= x$

& $(f \circ g)(y) = f[g(y)]$

$= y$

$\Rightarrow$ g & f are bijections & Inverses to each other

$\therefore f^{-1}(y) = \frac{9y+160}{5}$

$f^{-1}(x) = \frac{9x+160}{5}$

32) $f(x) = \frac{1}{1-3\cos x}$

$-1 \le \cos x \le 1$

$\Rightarrow -2 \le 1-3\cos x \le 4$

$\Rightarrow \frac{1}{1-3\cos x} \ge \frac{1}{4}$ & $\frac{1}{1-3\cos x} \le \frac{1}{2}$

$\therefore$ Range : $(-\infty, \frac{1}{2}] \cup [\frac{1}{4}, \infty)$

33) $\left(\left((256)^{-1/2}\right)^{-1/4}\right)^3$

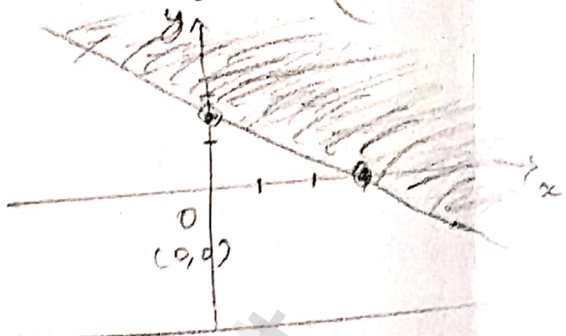
$= \left((2^8)^{1/8}\right)^3$

$= 2^3 = 8$

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

34) $x + 2y > 3$ (0,0) X (not satisfies the inequality)

$x + 2y = 3$



35). $n > 10$ & $(n+2) > 10$

From the data,

$n + (n+2) < 40$

$\boxed{n < 19}$

(11, 13), (13, 15), (15, 17), (17, 19)

36) $s = \frac{a+b+c}{2} = \frac{13+14+15}{2} = 21$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= 84$ sq.cm.

37)

2	1	5	4	3	6
D	A	N	G	E	R
1	0	2	1	0	0
5!	4!	3!	2!	1!	0!

Rank $= 120 + 12 + 2 + 1$

$= 135$

38) ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$

$2r - 1 + 2r + 4 = 15$

$\boxed{r = 3}$

39) $a_1 = 1$, $a_2 = 2$, $a_3 = 3$

$a_4 = 5$, $a_5 = 8$, $a_6 = 13$

40) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

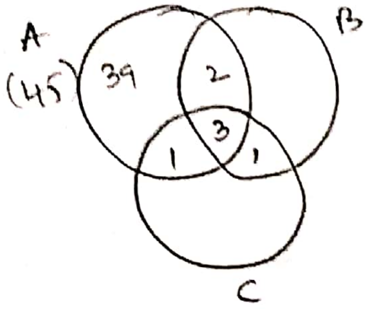
$\Rightarrow \cot(A+B) = \frac{1 - \tan A \tan B}{\tan A + \tan B}$

Part - D

41) a) $f(x) = \begin{cases} x^2+x-5 & (-\infty, 0) \\ x^2+3x-2 & (3, \infty) \\ x^2 & (0,2) \\ x^2-3 & \text{Otherwise} \end{cases}$

f at	-3	5	2	-1	0
	1	38	1	-5	-3

b)



The no of persons who knows only l-A $\Big\} = 5000 \times \frac{39}{100}$

$= 1950.$

42) a) $A = \{a,b,c\}$; $R = \{(a,a),(b,b),(a,c)\}$

i) ref. : (c,c)

ii) Symm : (c,a)

iii) Transi. : Nothing

iv) Equi : $(c,c),(c,a)$

b) $\frac{x^2+x+1}{x^2-5x+6} = 1 + \frac{6x-5}{x^2-5x+6}$

$= 1 + \frac{A}{(x-2)} + \frac{B}{(x-3)}$

$A=-7$
$B=13$

$= 1 + \frac{13}{x-3} - \frac{7}{x-2}$

43)(a) $f(x) = \frac{x^3(x-1)}{(x-2)} > 0$

Cri. pts : 0, 1, 2

$x=0,\ x=1\ \&\ x \neq 2$

(-) (+) (-) (+)

$-\infty$ 0 1 2 ∞

$x \in (0,1) \cup (2,\infty)$ $\{\because f(x) > 0$

(b) LHS :

$\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$

$= \log 75 - \log 16 - \log 25 + \log 81$
$+ \log 16 + \log 2 - \log 81 - \log 3$

$= \log 2.$ $\{\because \log 75 = \log 25 + \log 3$

44)a) LHS :

$= \frac{\cot(180+\theta) \cdot \sin(90-\theta) \cdot \cos(-\theta)}{\sin(270+\theta)\tan(-\theta)\,\text{cosec}(360+\theta)}$

$= \frac{\cot\theta \times \cos\theta \times \cos\theta}{(-\cos\theta)(-\tan\theta)(\text{cosec}\,\theta)}$

$= \cot\theta \cdot \cos^2\theta.$ $\because \cos\theta \times \tan\theta \times \text{cosec}\,\theta = 1$

b) Napier's formula : In $\triangle ABC$,

(i) $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

RHS :

$\frac{a-b}{a+b}\cot\frac{C}{2} = \frac{2R(\sin A - \sin B)}{2R(\sin A + \sin B)} \times \cot\frac{C}{2}$

$= \frac{2\cos\frac{A+B}{2} \cdot \sin\frac{A-B}{2}}{2\sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}} \times \cot\frac{C}{2}$

$= \cot\left(\frac{A+B}{2}\right) \cdot \tan\left(\frac{A-B}{2}\right) \cdot \cot\frac{C}{2}$

$= \tan\frac{C}{2} \cdot \tan\left(\frac{A-B}{2}\right) \cdot \cot\frac{C}{2}$

$= \tan\left(\frac{A-B}{2}\right) = \text{LHS}$

45) a) Gy: $\sin x = \frac{4}{5}$ & $\cos y = -\frac{12}{13}$ (II Q)

$\Rightarrow \cos x = \frac{3}{5}$ | $\sin y = \frac{5}{13}$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$= \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = -\frac{63}{65}$.

(b) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$

$a = 2,\ h = -1/2,\ b = -3$

$ab - h^2 = -6 - \frac{1}{4} \neq 0$

$\Rightarrow$ not pair of $\parallel^{el}$ St. lines

$\therefore$ It is a pair of $\times^{ing}$ lines

$\theta = \tan^{-1}\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

$= \tan^{-1}(5)$

46) a) $P(n): 1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

(i) $n=1$ $\quad 1=1$

$P(n)$ is true for $n=1$

(ii) $n=k$: Assume that

$P(k): 1^3+2^3+\cdots+k^3 = \left(\frac{k(k+1)}{2}\right)^2$

(iii) $n=k+1$

LHS

$P(k+1): 1^3+2^3+\cdots+k^3+(k+1)^3$

$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$

$= \frac{(k+1)^2 (k^2+4(k+1))}{4}$

$= \left(\frac{(k+1)(k+2)}{2}\right)^2 = RHS$.

b) (i) No restriction

$= 8C_5 = 8C_3 = 56$

(ii) At least 2:

A	B
2	3
3	2
4	1

$= 4C_2 \times 4C_3$

$+ 4C_3 \times 4C_2$

$+ 4C_4 \times 4C_1$

$= 24 + 24 + 4 = 52$

47) a)

From the data,

$a^3 r^{12} = 4096$

& $a^3 r^{15} = 32768$

$\Rightarrow r^3 = 8 \Rightarrow \boxed{r = 2}$

& $\boxed{a = 1}$

$S_8 = \frac{a(1-r^8)}{1-r}$

$= \frac{1-2^8}{1-2} = 255$

b) $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$

$= x(1+6/x^3)^{1/3} - x(1+3/x^3)^{1/3}$

$\simeq x\left(1+\frac{2}{x^3}\right) - x\left(1+\frac{1}{x^3}\right)$

$\simeq \frac{1}{x^2}$

45) b)

$A+B = 45°$

$\tan(A+B) = 1$

$\tan A + \tan B = 1 - \tan A \tan B$

$\tan A + \tan B + \tan A \tan B = 1$

$tA + tB + tA\,tB + 1 = 2$

$tA(1+tB) + (1+tB) = 2$

$(1+tB)(1+tA) = 2$

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Dear Trs, If there any correction, mail me: dharmu2020@gmail.com.