11

Time : 3.00 hrs.

Quarterly Examination - 2022
MATHEMATICS

Reg. No. | | | | | | | |

Max. Marks : 90

PART – A

i) Answer all the questions. **20 x 1 = 20**

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. Let A and B be subsets of the universal set N, the set of natural numbers. Then $A' \cup [(A \cap B) \cup B']$ is
   a) A b) A' c) B d) N
2. Let $f : R \to R$ be defined by $f(x) = 1 - |x|$. Then the range of f is
   a) R b) $(1, \infty)$ c) $(-1, \infty)$ d) $(-\infty, 1]$
3. The number of constant functions from a set containing m elements to a set containing n elements is
   a) mn b) m c) n d) m + n
4. The relation 'less than' in the set of natural number is
   a) only symmetric b) only transitive c) only reflexive d) Equivalence
5. Given that x, y and b are real numbers $x < y$, $b > 0$, then
   a) $xb < yb$ b) $xb > yb$ c) $xb \leq yb$ d) $\frac{x}{b} \geq \frac{y}{b}$
6. The value of $\log_7 343$ is a) 5 b) 3 c) 6 d) 9
7. If 8 and 2 are the roots of $x^2 + ax + c = 0$ and 3, 3 are the roots of $x^2 + dx + b = 0$, then the roots of the equation $x^2 + ax + b = 0$ are a) 1, 2 b) –1, 1 c) 9, 1 d) –1, 2
8. The number of roots of $(x + 3)^4 + (x + 5)^4 = 16$ is a) 4 b) 2 c) 3 d) 0
9. $(1 + \cos \pi/8)(1 + \cos 3\pi/8)(1 + \cos 5\pi/8)(1 + \cos 7\pi/8) =$ a) 1/8 b) 1/2 c) $1/\sqrt{3}$ d) $1/\sqrt{2}$
10. Which of the following is not true? a) $\sin\theta = -3/4$ b) $\cos\theta = -1$ c) $\tan\theta = 25$ d) $\sec\theta = 1/4$
11. A wheel is spinning at 2 radians / second - How many seconds will it take to make 10 complete rotations?
    a) 10π seconds b) 20π seconds c) 5π seconds d) 15π seconds
12. $\cos 2A =$ a) $\cos^2 A - \sin^2 A$ b) $\frac{1 - \tan^2 A}{1 + \tan^2 A}$ c) $2\sin A \cos A$ d) both 1 & 2
13. In 3 fingers, the number of ways four rings can be worn in.....ways a) $4^3 - 1$ b) 3^4 c) 68 d) 64
14. In $2nC_3 : nC_3 = 11 : 1$ then n is a) 5 b) 6 c) 11 d) 7
15. Number of sides of a polygon having 44 diagonals is..... a) 4 b) 4! c) 11 d) 22
16. $1 + 3 + 5 + 7 + \ldots + 17$ is equal to a) 101 b) 81 c) 71 d) 61
17. The coefficient of x^8y^{12} in the expansion of $(2x + 3y)^{20}$ is a) 0 b) $2^8 3^{12}$ c) $2^8 3^{12} + 2^{12} 3^8$ d) $20C_8\ 2^8 3^{12}$
18. The HM of two positive numbers whose AM and GM are 16, 8 respectively is a) 10 b) 6 c) 5 d) 4
19. The n^{th} term of the sequence $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots\ldots$ .....is a) $2^n - n - 1$ b) $1 - 2^{-n}$ c) $2^{-n} + n - 1$ d) 2^{n-1}
20. The value of $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots$ is a) $\frac{e^2 + 1}{2e}$ b) $\frac{(e+1)^2}{2e}$ c) $\frac{(e-1)^2}{2e}$ d) $\frac{e^2 - 1}{2e}$

PART - B

II. Answer any seven questions. Question number 20 is compulsory. **7 x 2 = 14**

21. Find the number of subsets of A if $A = \{x : x = 4n + 1, 2 \leq n \leq 5, n \in N\}$
22. Show that the relation $xy = -2$ is a function for a suitable domain. Find the domain and the range of the function.
23. Find a positive number smaller than $\frac{1}{2^{1000}}$. Justify.
24. Solve $|2x - 17| = 3$ for x.
25. Eliminate θ from $a\cos\theta = b$ and $c\sin\theta = d$, where a, b, c, d are constants.
26. Find the principal solution of i) $\sin\theta = 1/2$ ii) $\cos\theta = 1/2$
27. Find the value of $\frac{8!}{5! \times 2!}$
28. Find the number of strings of length 4, which can be formed using the letters of the word BIRD, without repetition of the letters.
29. Find the middle term in the expansion of $(x + y)^6$.
30. Express the equation $\sqrt{3}x - y + 4 = 0$ in the slope - intercept form.

PART - C

III. Answer any seven questions. Question number 40 is compulsory. **7 x 3 = 21**

31. The formula for converting from Fahrenheit to celsius temperatures is $y = \frac{5x}{9} = \frac{160}{9}$. Find the inverse of this function and determine whether the inverse is also a function.

32. Find the range of the function $f(x) = \frac{1}{1-3\cos x}$

33. Evaluate $\left(((256)^{-1/2})^{-1/4}\right)^3$

34. Shade the region given by the linear inequality $x + 2y > 3$.

35. Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

36. Find the area of the triangle whose sides are 13cm, 14cm and 15cm.

37. If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, then find the rank of the word (i) DANGER.

38. If $15C_{2r-1} = 15\ C_{2r+4}$, find r.

39. Write the first 6 terms of the sequences nth term an is given below

$$an = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$$

40. Prove that $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

PART - D

IV. Answer all the questions. **7 x 5 = 35**

41. a) Write the values of f at –3, 5, 2, –1, 0 if $f(x) \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$ **(OR)**

b) In a survey of 5000 persons in a town, it was found that 45% of the persons know language A, 25% know language 8, 10% know language C, 5% know languages A and B, 4% know languages B and C, and 4% know languages A and C. If 3% of the persons know all the three languages, find the number of persons who knows only language A.

42. a) Let A = {a, b, c} and R = { (a, a), (b, b), (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it. i) reflexive ii) symmetric iii) transitive iv) equivalence **(OR)**

b) Resolve into partial fractions $\frac{x^2 + x + 1}{x^2 - 5x + 6}$.

43. a) Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$. **(OR)**

b) Prove $\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243} = \log 2$

44. a) Prove that $\frac{\cot(180^0 + \theta)\sin(90^0 - \theta)\cos(-\theta)}{\sin(270^0 + \theta)\tan(-\theta)\operatorname{cosec}(360^0 + \theta)} = \cos^2\theta \cot\theta$

b) State and prove Napier's Formula.

45. a) If $\sin x = \frac{4}{5}$ (in I quadrant) and $\cos y = \frac{-12}{13}$ (in II quadrant), then find $\sin(x - y)$ **(OR)**

b) Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.

46. a) By the principles of mathematical induction, prove that, for $n \geq 1$. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ **(OR)**

b) An exam paper contains 8 questions, 4 in Part A and 4 in Part B. Examiners are required to answer 5 questions. In how many ways can this be done if i) There are no restrictions of choosing a number of questions in either parts. ii) Atleast two questions from Part A must be answered.

47. a) If the product of the 4th, 5th and 6th terms of a geometric progression in 4096 and if the product of the 5th, 6th and 7th terms of it is 32768, find the sum of first 8 terms of the geometric progression. **(OR)**

b) Prove that $\sqrt[3]{x^3 + 6} - \sqrt[3]{x^3 + 3}$ in approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.