CENTUM ACHIEVERS' ACADEMY

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XII STD	APPLICATIONS OF VECTOR ALGEBRA	TIME : 2 ½Hrs MARKS : 90

PART-A

Choose the correct or the most suitable answer : **(20× 1 = 20)**

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a},\vec{c},\vec{b}]$ is equal to

   (1) 2 (2) -1 (3) 1 (4) 0

2. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

   (1) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=1$ (2) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=-1$ (3) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=0$ (4) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=2$

3. If $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is

   (1) $|\vec{a}||\vec{b}||\vec{c}|$ (2) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (3) 1 (4) -1

4. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

   (1) $\vec{a}$ (2) $\vec{b}$ (3) $\vec{c}$ (4) $\vec{0}$

5. If $[\vec{a},\vec{b},\vec{c}]=1$, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}}+\frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}}+\frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

   (1) 1 (2) -1 (3) 2 (4) 3

6. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath},\hat{\imath}+2\hat{\jmath},\hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is

   (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

7. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a},\vec{b},\vec{a}\times\vec{b}]=\frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

   (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

8. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k},\ \vec{b}=\hat{\imath}+\hat{\jmath},\ \vec{c}=\hat{\imath}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then the value of $\lambda+\mu$ is

   (1) 0 (2) 1 (3) 6 (4) 3

9. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a},\vec{b},\vec{c}]=3$, then $\{[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]\}^2$ is equal to

   (1) 81 (2) 9 (3) 27 (4) 18

10. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

    (1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) π

11. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}\cdot\vec{c}\neq0$ and $\vec{a}\cdot\vec{b}\neq0$, then $\vec{a}$ and $\vec{c}$ are

    (1) perpendicular (2) parallel (3) inclined at angle $\frac{\pi}{3}$ (4) inclined at angle $\frac{\pi}{6}$

12. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2},z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

    (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

13. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then (α,β) is

(1) $(-5,5)$ (2) $(-6,7)$ (3) $(5,-5)$ (4) $(6,-7)$

14. The angle between the line $\vec{r}=(\hat{\imath}+2\hat{\jmath}-3\hat{k})+t(2\hat{\imath}+\hat{\jmath}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{\imath}+\hat{\jmath})+4=0$ is

(1) $0°$ (2) $30°$ (3) $45°$ (4) $90°$

15. The coordinates of the point where the line $\vec{r}=(6\hat{\imath}-\hat{\jmath}-3\hat{k})+t(-\hat{\imath}+4\hat{k})$ meets the plane $\vec{r}\cdot(\hat{\imath}+\hat{\jmath}-\hat{k})=3$ are

(1) $(2,1,0)$ (2) $(7,-1,-7)$ (3) $(1,2,-6)$ (4) $(5,-1,1)$

16. Distance from the origin to the plane $3x-6y+2z+7=0$ is

(1) 0 (2) 1 (3) 2 (4) 3

17. The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

(1) $\frac{\sqrt{7}}{2\sqrt{2}}$ (2) $\frac{7}{2}$ (3) $\frac{\sqrt{7}}{2}$ (4) $\frac{7}{2\sqrt{2}}$

18. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then

(1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

19. If the distance of the point $(1,1,1)$ from the origin is half of its distance from the plane $x+y+z+k=0$, then the values of k are

(1) ± 3 (2) ± 6 (3) $-3,9$ (4) $3,-9$

20. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1, \lambda>0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

PART-B

Answer the following questions: **$(7\times 2=14)$**

21. A particle is acted upon by the forces $3\hat{\imath}-2\hat{\jmath}+2\hat{k}$ and $2\hat{\imath}+\hat{\jmath}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.

22. Prove by vector method that an angle in a semi-circle is a right angle.

23. The volume of the parallelepiped whose coterminous edges are $7\hat{\imath}+\lambda\hat{\jmath}-3\hat{k}, \hat{\imath}+2\hat{\jmath}-\hat{k}$, $-3\hat{\imath}+7\hat{\jmath}+5\hat{k}$ is 90 cubic units. Find the value of λ.

24. Prove that $[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

25. Find the intercepts cut off by the plane $\vec{r}\cdot(6\hat{\imath}+4\hat{\jmath}-3\hat{k})=12$ on the coordinate axes.

26. Find the angle between the straight line $\vec{r}=(2\hat{\imath}+3\hat{\jmath}+\hat{k})+t(\hat{\imath}-\hat{\jmath}+\hat{k})$ and the plane $2x-y+z=5$.

27. Find the equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also, find the distance between the two planes.

PART-C

Answer the following questions: **$(7\times 3=21)$**

28. Find the magnitude and direction cosines of the torque of a force represented by $3\hat{\imath}+4\hat{\jmath}-5\hat{k}$ about the point with position vector $2\hat{\imath}-3\hat{\jmath}+4\hat{k}$ acting through a point whose position vector is $4\hat{\imath}+2\hat{\jmath}-3\hat{k}$.

29. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2=\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.

30. For any vector $\vec{a}$, prove that $\hat{\imath}\times(\vec{a}\times\hat{\imath})+\hat{\jmath}\times(\vec{a}\times\hat{\jmath})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$.

31. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

32. Show that the straight lines $\vec{r} = (5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}) + s(4\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ and $\vec{r} = (8\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + t(7\hat{\imath} + \hat{\jmath} + 3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.
33. Find the image of the point whose position vector is $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) = 38$.
34. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\hat{\imath} - 7\hat{\jmath} + 4\hat{k}) = 3$ and $3x - 5y + 4z + 11 = 0$, and the point $(-2,1,3)$.

PART-D

Answer the following questions: **$(7\times 5 = 35)$**

35. Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$.
36. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent
37. If $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}, \vec{c} = 3\hat{\jmath} - \hat{k}$ and $\vec{d} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$
38. Show that the lines $\vec{r} = (6\hat{\imath} + \hat{\jmath} + 2\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}) + t(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ are skew lines and hence find the shortest distance between them.
39. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$
40. Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1, -2, 4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.
41. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4,3,2) to the plane $x + 2y + 3z = 2$.

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