N K MATHS ACADEMY
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UNIT TEST-2022-23
MATHEMATICS
UNIT TEST -7

MARKS: 40   **TIME: 1.00 HR**

I. CHOOSE THE BEST ANSWER: 8X1=8

1. The volume of a sphere is increasing in volume at the rate of $3\pi cm^3 / \sec$ .The rate of change of its radius when radius is $\frac{1}{2}cm$

   (1) 3 cm/s   (2) 2 cm/s   (3) 1 cm/s   (4) $\frac{1}{2}cm/s$

2. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

   (1) 2   (2) 2.5   (3) 3   (4) 3.5

3. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

   (1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

4. What is the value of the limit $\lim\limits_{x \to 0}\left(\cot x - \frac{1}{x}\right)$?

   (1) 0   (2) 1   (3) 2   (4) ∞

5. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

   (1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

6. The minimum value of the function $|3 - x| + 9$ is

   (1) 0   (2) 3   (3) 6   (4) 9

7. The maximum slope of the tangent to the curve $y = e^x \sin x, x \in [0, 2\pi]$ is at

   (1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$

8. The point of inflection of the curve $y = (x-1)^3$ is

   (1) (0, 0)   (2) (0, 1)   (3) (1, 0)   (4) (1, 1)

II. ANSWER ANY 4 QUESTIONS: 4X2=8

9. For what value of x the tangent of the curve $y = x^3 - 3x^2 + x - 2$ is parallel to the line $y = x$

10. State Rolle's theorem

11. Explain why Lagrange's mean value theorem is not applicable $f(x) = |3x+1|, x \in [-1, 3]$

12. Evaluate: $\lim\limits_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

13. Find the asymptotes of the function $f(x) = \dfrac{1}{x}$

III. ANSWER ANY 3 QUESTIONS: 3X3=9

14. A particle moves so that the distance moved is according to the law $s(t) = \dfrac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively?

15. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$

16. Show that the value in the conclusion of the mean value theorem for $f(x) = Ax^2 + Bx + C$ on any interval $[a,b]$ is $\dfrac{a+b}{2}$

17. Write the Maclaurin series expansion of $\cos x$

18. Find the absolute extrema of the function $f(x) = 3\cos x$ on the closed interval $[0, 2\pi]$

IV. ANSWER ANY 3 QUESTIONS: 3X5=15

19. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

20. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0, 0), (1, 1)

21. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

22. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margin at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum

23. Prove that among all the rectangles of the given perimeter, the square has the maximum area