N K MATHS ACADEMY

TIRUPUR-98434 34491

UNIT TEST-2022-23

LN-3, 4

MARKS: 50 **TIME: 1.30 hrs**

I. CHOOSE THE BEST ANSWER: **10X1=10**

1. A polynomials equation in x of degree n always has

(1)n distinct roots (2) n real roots

(3) n complex roots (4)at most one root

2. The polynomial $x^3 - kx^3 + 9x$ has three real roots if and only if, k Satisfies

(1)$|k| \le 6$ (2)k=0 (3)$|k| \succ 6$ (4)$|k| \ge 6$

3. The number of positive roots of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(1)0 (2)n (3)<n (4)r

4. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

(1)$\frac{2\pi}{3}$ (2)$\frac{\pi}{3}$ (3)$\frac{\pi}{6}$ (4)π

5. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is vaild for

(1)$-\pi \le x \le 0$ (2)$0 \le x \le \pi$ (3)$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (4)$-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

6. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(1)$\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2)$\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3)$\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4)$\tan^{-1}\left(\frac{1}{2}\right)$

7. If $\cot^{-1}\left(\sqrt{\sin\alpha}\right) + \tan^{-1}\left(\sqrt{\sin\alpha}\right) = \mu$, then $\cos 2\mu$ is equal to

(1)$\tan^2\alpha$ (2)0 (3)-1 (4)$\tan 2\alpha$

8. The quadratic equation whose roots are $\pm i\sqrt{7}$ is

(1) $x^2 + 7 = 0$ (2) $x^2 - 7 = 0$ (3) $x^2 + x + 7 = 0$ (4) $x^2 - x - 7 = 0$

9. $\tan(\cos^{-1} x)$ is equal to

(1) $\frac{\sqrt{1-x^2}}{x}$ (2) $\frac{x}{1+x^2}$ (3) $\frac{\sqrt{1+x^2}}{x}$ (4) $\sqrt{1-x^2}$

10. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(1) $\frac{5\pi}{3}$ (2) $-\frac{5\pi}{3}$ (3) $-\frac{\pi}{3}$ (4) $\frac{4\pi}{3}$

II. ANSWER ANY 5 QUESTIONS: 5X2=10

11. If $x^2+2(k+2)x+9k=0$ has equal roots, find *k*.

12. Determine the number of positive and negative roots of the equation $x^9-5x^8-14x^7=0$

13. Find the period of amplitude of $y=\sin\left(\frac{1}{3}x\right)$.

14. State the reason for $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)\neq-\frac{\pi}{6}$

15. Find the domain of $f(x)=\sin^{-1}(x)+\cos^{-1}(x)$

16. Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

III. ANSWER ANY 5 QUESTIONS: 5X3=15

17. Find the value of $\cot^{-1}(-1)-\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\sec^{-1}\left(-\sqrt{2}\right)$

18. Find the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$

19. Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

20. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

21. Prove that $2\cos^{-1}x=\tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, if $x\geq1$

22. Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

IV. ANSWER THE FOLLOWING: 3X5=15

23. Solve the equation $(2x-3)(3x-2)(6x-1)(x-12)-7=0$

24. Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

25. Solve $\cos\left[\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right]=\sin\left[\cot^{-1}\left(\frac{3}{4}\right)\right]$

26. Solve $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{2}, x>0$

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4: INVERSE TRIGONOMETRICAL FUNCTIONS

MARKS: 50 **TIME: 1.30 Hrs**

PART – A: CHOOSE THE BEST ANSWER (10 x 1=10)

1. The value of $\sin^{-1}(\cos x),\ 0\le x\le\pi$ is

   (1) $\pi - x$ (2) $x-\dfrac{\pi}{2}$ (3) $\dfrac{\pi}{2}-x$ (4) $\pi - x$

2. If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution , then

   (1) $|\alpha|\le\dfrac{1}{\sqrt{2}}$ (2) $|\alpha|\ge\dfrac{1}{\sqrt{2}}$ (3) $|\alpha|<\dfrac{1}{\sqrt{2}}$ (4) $|\alpha|>\dfrac{1}{\sqrt{2}}$

3. If $\cot^{-1}x=\dfrac{2\pi}{5}$ for some $x\in R$, the value of $\tan^{-1}$ x is

   (1) $-\dfrac{\pi}{10}$ (2) $\dfrac{\pi}{5}$ (3) $\dfrac{\pi}{10}$ (4) $-\dfrac{\pi}{5}$

4. If $x=\dfrac{1}{5}$, the value of $\cos\left(\cos^{-1}x+2\sin^{-1}x\right)$ is

   (1) $-\sqrt{\dfrac{24}{25}}$ (2) $\sqrt{\dfrac{24}{25}}$ (3) $\dfrac{1}{5}$ (4) $-\dfrac{1}{5}$

5. $\tan^{-1}\left(\dfrac{1}{4}\right)+\tan^{-1}\left(\dfrac{2}{9}\right)$ is equal to

   (1) $\dfrac{1}{2}\cos^{-1}\left(\dfrac{3}{5}\right)$ (2) $\dfrac{1}{2}\sin^{-1}\left(\dfrac{3}{5}\right)$ (3) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{3}{5}\right)$ (4) $\tan^{-1}\left(\dfrac{1}{2}\right)$

6. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle , then the third angle is

   (1) $\dfrac{\pi}{4}$ (2) $\dfrac{3\pi}{4}$ (3) $\dfrac{\pi}{6}$ (4) $\dfrac{\pi}{3}$

7. $\sin^{-1}\left(2\cos^2 x-1\right)+\cos^{-1}\left(1-2\sin^2 x\right)=$

   (1) $\dfrac{\pi}{2}$ (2) $\dfrac{\pi}{3}$ (3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{6}$

8. If $|x|\le 1$ then $2\tan^{-1}x-\sin^{-1}\dfrac{2x}{1+x^2}$ is equal to

   (1) $\tan^{-1}x$ (2) $\sin^{-1}x$ (3)0 (4) π

9. If $\sin^{-1}x+\cot^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{2}$, then x is equal to

   (1) $\dfrac{1}{2}$ (2) $\dfrac{1}{\sqrt{5}}$ (3) $\dfrac{2}{\sqrt{5}}$ (4) $\dfrac{\sqrt{3}}{2}$

10. $\sin\left(\tan^{-1}x\right),|x|<1$ is equal to

   (1) $\dfrac{x}{\sqrt{1-x^2}}$ (2) $\dfrac{1}{\sqrt{1-x^2}}$ (3) $\dfrac{1}{\sqrt{1+x^2}}$ (4) $\dfrac{x}{\sqrt{1+x^2}}$

PART-B: ANSWER ANY 5 QUESTIONS: (5X2=10)

11. Find the principal value of $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

12. Find the period of amplitude of $y=\sin 7x$.

13. Find $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

14. Find the domain of $f(x)=\sin^{-1}(x)+\cos^{-1}(x)$

15. Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

16. If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$

PART-C: ANSWER ANY 5 QUESTIONS: (5X3=15)

17. Sketch the graph of $y=\sin\left(\frac{1}{3}x\right)$ for $0\le x<6\pi$

18. Find value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}+\cos\frac{\pi}{7}\sin\frac{5\pi}{17}\right)$

19. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x,\ |x|>1$

20. Find the value of $\cos\left[\frac{1}{2}-\cos^{-1}\left(\frac{1}{8}\right)\right]$

21. Show that $\cot(\sin^{-1}x)=\frac{\sqrt{1-x^2}}{x}, -1\le x\le 1$ and $x\ne 0$

22. Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

PART-D: ANSWER ANY 3 QUESTIONS: (3X5=15)

23. Find the domain of $f(x)=\sin^{-1}\left(\frac{x^2+1}{2x}\right)$.

24. If $a_1,a_2,a_3,......a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+......+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$$

25. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

26. Simplify: $\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{x-y}{x+y}\right)$

27. Solve $2\tan^{-1}(x)=\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right), a<0, b>0$

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UNIT TEST-2022-23

5. TWO DIMENSIONAL ANALYTICAL GEOMETRY-5.1, 5.2

MARKS: 50 **TIME: 1.30 hrs**

I. CHOOSE THE BEST ANSWER: **10X1=10**

1. The equation of the circle passing through (1,5) and (4,1) and touching y-axis $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$ where λ is equal to

   (1) $0,-\frac{40}{9}$ (2) 0 (3) $\frac{40}{9}$ (4) $\frac{-40}{9}$

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

   (1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

3. The length of the diameter of the circle which touches the x-axis at point (1,0) and passes through the point (2,3).

   (1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

4. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

   (1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

5. The equation the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x+4y=3$ is

   (1) $x+2y=3$ (2) $x+2y+3=0$ (3) $2x+4y+3=0$ (4) $x-2y+3=0$

6. If $P(x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2 is

   (1) 8 (2) 6 (3) 10 (4) 12

7. The area of quadrilateral formed with foci of the hyberbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

   (1) $4(a^2+b^2)$ (2) $2(a^2+b^2)$ (3) a^2+b^2 (4) $\frac{1}{2}(a^2+b^2)$

8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

   (1) 2ab (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

9. The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

   (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

10. The circle passing through (1,-2) and touching the axis of x at (3, 0) passing through the point

    (1) (-5, 2) (2) (2,-5) (3) (5,-2) (4) (-2,5)

II. ANSWER ANY 5 QUESTIONS: **5X2=10**

11. Find the general equation of a circle with center $(-3,-4)$ and radius 3 units.

12. Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.

13. If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c.

14. Determine whether the point $(-2,1)$ lies outside, on or inside the circle $x^2+y^2-5x+2y-5=0$.

15. Find the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix $x=\sqrt{2}$.

16. Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y-axis and passing through $(3,6)$.

III. ANSWER ANY 5 QUESTIONS: 5X3=15

17. A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with center of the circle $(2,1)$ find the equation of the circle in general form.

18. Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

19. Find the equation of the circle with centre $(2,-1)$ and passing through the point $(3,6)$ in standard form.

20. Find the length of latus rectum of the parabola $y^2=4ax$.

21. Find the equation of the hyperbola with vertices $(0,\pm4)$ and foci $(0,\pm6)$.

22. Find the vertices, foci for the hyperbola $9x^2-16y^2=144$

23. Identify the type of conic and find center, foci, vertices and directrices of $\frac{x^2}{3}+\frac{y^2}{10}=1$.

IV. ANSWER ANY 5 QUESTIONS: 3X5=15

24. Find the equation of the circle passing through the points $(1,1),(2,-1),(3,2)$.

25. Find the vertex, focus, directrix and length of the lotus rectum of the parabola .

26. Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$.

27. Find the center, foci and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$.

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11. PROBABILITY DISTRIBUTIONS

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MARKS: 50 **TIME: 1.30 hrs**

I. CHOOSE THE BEST ANSWER: **10X1=10**

1. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is

(a)1 (b)2 (c)3 (d)4

2. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

(a) $i+2n, i=0,1,2..n$ (b) $21-n, i=0,1,2..n$

(c) $n-i, i=0,1,2..n$ (d) $2i+2n, i=0,1,2...n$

3. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is

(a) 0.11 (b) 1.1 (c) 11 (d) 1

4. If $p\{X=0\}=1-p\{X=1\}$. If $E[X]=3\operatorname{var}(X)$, then $p\{X=0\}$

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

5. If X is a binomial random variable with expected value 6 and variance 2.4 , then $p\{x=5\}$ is

(a) $\binom{10}{5}\left(\frac{3}{6}\right)^{6}\left(\frac{2}{5}\right)^{4}$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^{5}$

(c) $\binom{10}{5}\left(\frac{3}{5}\right)^{4}\left(\frac{2}{5}\right)^{6}$ (d) $\binom{10}{5}\left(\frac{3}{5}\right)^{5}\left(\frac{2}{5}\right)^{5}$

6. Which of the following is a discrete random variable?
   I. The number of cars crossing a particular signal in a day.
   II. The number of customers in a queue to buy train tickets at a moment.
   III. The time taken to complete a telephone call.

(a)I and II (b)II only (c)III only (d)II and III

7. The probability function of a random variable is defined as:

x	-2	-1	0	1	2
f(x)	k	2k	3k	4k	5k

Then E(X) is equal to:

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

8. A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

9. Variance of the random variable X is 4. Its mean is 2. Then $E(X^2)$ is

(a)2 (b)4 (c)6 (d)8

10. Var (4X+ 3) is (a)7 (b)16 Var (X) (c)19 (d)0.

II. ANSWER ANY 5 QUESTIONS: 5X2=10

11.Suppose *X* is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable *X* and number of points in its inverse images

12. The probability density function of *X* is given by $f(x) = \begin{cases} ke^{-2x} & , x > 0 \\ 0 & , x \le 0 \end{cases}$ Find the value of *k*

13. Find the mean and variance of a random variable *X*, whose probability mass function is

$$f(x) = \begin{cases} \frac{4-x}{6} & , x = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

14. Find the binomial distribution function for a fair die is rolled 10 times and *X* denotes the number of times 4 appeared

15. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where $n = 10$, $p = \frac{1}{5}$, $k = 4$

16. The probability that a certain kind of component will survive an electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive

III. ANSWER ANY 5 QUESTIONS: 5X3=15

17. In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images

18. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If *X* denotes the total score in two throws, find the values of the random variable and number of points in its inverse images

19. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours

20. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred

21. Find the mean and variance of a random variable *X* , whose probability density function is

$$f(x) = \begin{cases} 2(x-1) & , 1 < x \le 2 \\ 0 & , \textit{otherwise} \end{cases}$$

22. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

IV. ANSWER ANY 3 QUESTIONS: 3X5=15

23. If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
f(x)	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

Find (i) its cumulative distribution function; hence find (ii) $P(X \le 3)$ (iii) $P(X \ge 2)$

24. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 liters and a maximum of 600 liters with probability density function $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & otherewise \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 liters and 500 liters?

25. Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	2c

Find (i) the value of c (ii) Mean and variance

26. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective

27. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights

(i) Exactly 10 will have a useful life of at least 600 hours;

(ii) At least 11 will have a useful life of at least 600 hours;

(iii) At least 2 will *not* have a useful life of at least 600 hours

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APPLICATIONS OF MATRICES AND DETERMINANTS

MARKS: 50 **TIME: 1.30 hrs**

I. CHOOSE THE BEST ANSWER: 10X1=10

1. If $|adj(adjA)| = |A|^9$, then the order of the square matrix A is

(1)3 (2)4 (3)2 (4)5

2. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adjA$ and $C = 3A$, then $\frac{|adjB|}{|C|} =$

(1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4)1

3. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I - A =$ (1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $m+n$

4. If $p = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1)15 (2)12 (3)14 (4)11

5. If A, B and C are invertible matrices of some order, then which one of the following is not true?

(1) $adjA = |A|A^{-1}$ (2) $adj(AB) = (adjA)(adjB)$ (3) $\det A^{-1} = (\det A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

6. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $\left(A^T\right)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

7. If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then B =

(1) $\left(\cos^2\frac{\theta}{2}\right)A$ (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (3) $\left(\cos^2\theta\right)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

8. The rank of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 4 & -4 & 8 \end{pmatrix}$ is, (1) 1 (2) 2 (3) 3 (4) 4

9. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

(1) $\frac{1}{k^2}$ I (2) $\frac{1}{k^3}$ I (3) $\frac{1}{k}$ I (4) k I

10. If A = $\begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$, then A^{12} is,

(1) $\begin{pmatrix} 0 & 0 \\ 0 & 60 \end{pmatrix}$ (2) $\begin{pmatrix} 0 & 0 \\ 0 & 5^{12} \end{pmatrix}$ (3) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

II. ANSWER ANY 5 QUESTIONS: **5X2=10**

11. If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

12. If A is non-Singular, prove that $\left(A^{-1}\right)^T=\left(A^T\right)^{-1}$

13. Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.

14. Find the rank of matrix $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ by minor method.

15. If A is non-singular, then prove that A^{-1} is also non-singular and $(A^{-1})^{-1}=A$.

16. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

III. ANSWER ANY 5 QUESTIONS: **5X3=15**

17. Find the inverse of $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ (if it exists).

18. If $A=\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2-3A-7I_2=O_2$, hence find A^{-1}.

19. If $A=\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

20. Find the rank of $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ by row reduction methods.

21. Solve the system of linear equation $5x+2y=3, 3x+2y=5$ using matrix inversion methods.

22. Solve $5x-2y+16=0, x+3y-7=0$ by Cramer's rule.

IV. ANSWER THE FOLLOWING: **3X5=15**

23. If $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(adjA)=(adjA)A=|A|I_3$

24. Solve $2x_1+3x_2+3x_3=5,\ x_1-2x_2+x_3=-4,\ 3x_1-x_2-2x_3=3$, by matrix inversion method.

25. If $A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and Hence solve

$x-y+z=4, x-2y-2z=9, 2x+y+3z=1$

26. Solve $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$ by Cramer's rule.

N K MATHS ACADEMY

TIRUPUR-98434 34491

UNIT TEST-2022-23

12

APPLICATIONS OF MATRICES AND DETERMINANTS

MARKS: 50 **TIME: 1.30 hrs**

I. CHOOSE THE BEST ANSWER: **10X1=10**

1. If A is 3×3 Non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$
   (1)A (2)B (3)I (4) B^T

2. If $A=\begin{bmatrix}7 & 3\\4 & 2\end{bmatrix}$,then $9I - A$ =
   (1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

3. If $A^T A^{-1}$ is symmetric , then $A^2 =$
   (1) A^{-1} (2) $\left(A^T\right)^2$ (3) A^T (4) $\left(A^{-1}\right)^2$

4. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$ and $A(adjA)=\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$,then K=
   (1)0 (2) $\sin\theta$ (3) $\cos\theta$ (4)1

5. The rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\2 & 4 & 6 & 8\\-1 & -2 & -3 & -4\end{bmatrix}$ is
   (1)1 (2)2 (3)4 (4)3

6. If $\rho(A)=\rho([A\,|\,B])$,then the system $AX = B$ of linear equations is
   (1)consistent and has unique solution (2)consistent
   (3)Consistent and has infinitely many solution (4)inconsistent.

7. Let $A=\begin{bmatrix}2 & -1 & 1\\-1 & 2 & -1\\1 & -1 & 2\end{bmatrix}$ and $AB=\begin{bmatrix}3 & 1 & -1\\1 & 3 & x\\-1 & 1 & 3\end{bmatrix}$. If B is the inverse of A , then the value of x is
   (1)2 (2)4 (3)3 (4)1

8. If $A=\begin{pmatrix}1\\2\\3\end{pmatrix}$, then the rank of AA^T is,
   (1) 3 (2) 0 (3) 1 (4) 2

9. If A is a square matrix of order n then $|\text{adj } A|$ is
   (1) $|A|^2$ (2) $|A|^n$ (3) $|A|^{n-1}$ (4) $|A|$

10. If the equations –2x+y+z=l ; x-2y+z=m ; x+y-2z=n such that l+m+n = 0, then the system has
    (1) non-zero unique solution (2) trivial solution
    (3) infinitely many solutions (4) no solution

II. ANSWER ANY 5 QUESTIONS: **5X2=10**

11. If A is non-singular matrix of odd order, prove that $|adj\ A|$ is positive.

12. Find the inverse (if it exists) of $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

13. Find the rank of matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$ by minor method.

14. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

15. Find the rank of $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

16. Solve $x+2y+3z=0,\ 3x+4y+4z=0,\ 7x+10y+12z=0$

III. ANSWER ANY 5 QUESTIONS: **5X3=15**

17. If $A=\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2-3A-7I_2=O_2$, hence find A^{-1}.

18. If $A=\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

19. Find the rank of $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

20. Solve $5x-2y+16=0, x+3y-7=0$ by Cramer's rule

21. A chemist has one solution which is 50% acid and another solution which is25% acid .How much each should be mixed to make 10 liters of a 40% acid solution? (Use Cramer's rule).

22. $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1}=\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

IV. ANSWER ANY 3 QUESTIONS: **3X5=15**

23. The prices of three commodities *A*, *B*, and *C* are Rs *x*, *y*, and *z* per units respectively. A person *P* purchases 4 units of *B* and sells two units of *A* and 5 units of *C*. Person *Q* purchases 2 units of *C* and sells 3 units of *A* and one unit of *B* . Person *R* purchases one unit of *A* and sells 3 unit of *B* and one unit of *C*. In the process *P*, *Q*, and *R* earn Rs 15,000, Rs 1,000 and Rs 4,000 respectively. Find the prices per unit of *A*, *B*, and *C* . (Use matrix inversion method to solve the problem.)

24. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadai is Rs150 .The cost of two dosai , two idlies and four vadais is Rs 200.The cost of five dosais, four idlies and two vadais is Rs 250.The family has Rs 350 in hand and they ate 3 dosai and six idlies and 6 vadais. Will they be able to manage to pay the bill within the amount they had?

25. If ax^2+bx+c is divided by $x+3, x-5$ and $x-1$, the remainders are 21, 61 and 9 respectively .Find a, b and c. (Use Gaussian elimination method)

26. Investigate for what values of λ and μ the system of linear equation

    $x+2y+z=7, x+y+\lambda z=\mu, x+3y-5z=5$ has (i) no solution (ii) a unique solution. (iii) infinitely many solutions.

12

N K MATHS ACADEMY

TIRUPUR-98434 34491

UNIT TEST-2022-23

THEORY OF EQUATIONS

MARKS: 50 **TIME: 1.30 hrs.**

I. CHOOSE THE BEST ANSWER: **10X1=10**

1. A zero of x^3+64 is

(1)0 (2)4 (3)4i (4)-4

2. A polynomials equation in x of degree n always has

(1)n distinct roots (2) n real roots (3) n complex roots (4)at most one root

3. According to the rational root theorem, which number is not possible rational root of $4x^2+2x^4-10x^3-5$?

(1)-1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4)5

4. The polynomial x^3-kx^3+9x has three real roots if and only if, k Satisfies

(1) $|k|\le 6$ (2) k=0 (3) $|k|\succ 6$ (4) $|k|\ge 6$

5. If $x^3+12x^3+10ax+1999$ definitely has a positive root, if and only if

(1) $a\ge 0$ (2) $a\succ 0$ (3) $a<0$ (4) $a\le 0$

6. The number of positive roots of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(1)0 (2)n (3)<n (4)r

7. If $-i+2$ is one is one root equation $ax^2-bx+c=0$, then the other root is

(1) $-i-2$ (2) $i-2$ (3) $2+i$ (4) $2i+i$

8. The equation having $4-3i$ and $4+3i$ as roots is

(1) $x^2+8x+25=0$ (2) $x^2+8x-25=0$ (3) $x^2-8x+25=0$ (4) $x^2-8x-25=0$

9. If $\frac{1-i}{1+i}$ is a root of $ax^2+bx+1=0$, where a, b are real then (a, b) is

(1) (1, 1) (2) (1, -1) (3) (0, 1) (4) (1, 0)

10. Polynomial equation P(x) = 0 admits conjugate pairs of roots only if the coefficients are

(1) imaginary (2) complex (3) real (4) either real or complex

II. ANSWER ANY 5 QUESTIONS: **5X2=10**

11. Construct a cubic equation with roots 1, 1, and −2

12. Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$ as a root.

13. Solve the equation: $x^4-14x^2+45=0$

14. Determine the number of positive and negative roots of the equation $x^9-5x^8-14x^7=0$

15. Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3=0$.

16. Find the roots of $2x^3+3x^2+2x+3=0$

III. ANSWER ANY 5 QUESTIONS: **5X3=15**

17. If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$

18. If k is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$, in terms of k.

19. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

20. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

21. Solve the equations $x^4+3x^3-3x-1=0$

22. Solve the equation $x^4-4x^2+8x+35=0$, if one of its roots is $2+\sqrt{3}\,i$

IV. ANSWER ANY 3 QUESTIONS: **3X5=15**

23. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

24. If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots.

25. Solve the equation $(x-2)(x-7)(x-3)(x+2)+19=0$

26. Solve the equations $6x^4-35x^3+62x^2-35x+6=0$