Sun Tuition center -Vpm

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STANDARD TEN 2 – Marks

Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution : Largest value L = 67; Smallest value S = 18

Range R = L − S = 67 − 18 = 49

Coefficient of range $=\frac{L-S}{L+S}=\frac{67-18}{67+18}=\frac{49}{85}=0.576$

Life is a good circle, you choose the best radius...

Find the range of the following distribution.

Age (in years)	16- 18	18- 20	20- 22	22- 24	24- 26	26- 28
Number of students	0	4	6	8	2	2

Solution : Here Largest value L = 28 Smallest value S = 18

Range R = L − S = 28 − 18 = 10Years.

The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution : Range R = 13.67 Largest value L = 70.08 Range R = L − S

13.67 = 70.08 − S S = 70.08 − 13.67 = 56.41

Find the range and coefficient of range of the following data. 63, 89, 98, 125, 79, 108, 117, 68

Solution: Range = L − S = 125 − 63 = 62

Coefficient of range $=\frac{L-S}{L+S}=\frac{125-63}{125+63}=\frac{62}{185}=0.33$

Find the range and coefficient of range of the following data. 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution: Range = L − S = 61.4 − 13.6 = 47.8

Coefficient of range $=\frac{L-S}{L+S}=\frac{61.4-13.6}{61.4+13.6}=\frac{47.8}{75}=0.64$

If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution : Given, S.D of a data = 4.5 each value is decreased by 5, then the new SD = 4.5

If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution : Given, S.D of a data = 3.6 each value is divided by 3 then the new S.D $=\frac{3.6}{3}=1.2$

New Variance = $(S.D)^2 = (1.2)^2 = 1.44$

Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution : Here, Largest value = L = 650 Smallest value = S = 400

∴ Range = L − S = 650 − 400 = 250

If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution : Given ; range = 36.8 Smallest value = 13.4 ∴ R = L − S 36.8 = L − 13.4

Find the standard deviation of first 21 natural numbers.

Solution : SD of first 21 natural numbers $=\sqrt{\frac{n^2-1}{12}}=\sqrt{\frac{441-1}{12}}=\sqrt{\frac{440}{12}}=6.05$

The standard deviation of some temperature data in degree celsius (°C) is 5. If the data were converted into degree Farenheit (°F) then what is the variance?

Solution : Given $\sigma_c = 5$ $F = \frac{9c}{5} + 32 \Rightarrow \sigma_F = \frac{9}{5}\sigma_c = \frac{9}{5} \times 5 = 9$ $\therefore \sigma_F{}^2 = 9^2 = 81.$

A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution : n(S) = 5 + 4 = 9 (i) Let A blue ball. n(A) = 5 , $P(A) = \frac{5}{9}$

(ii) B will be the event of not getting a blue ball. n(B) = 4 , $P(B) = \frac{4}{9}$

Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution : S = {HH,HT,TH,TT} Let A be the different faces on the coins.

n(S) = 4 A = {HT,TH}; n(A) = 2 , $P(A) = \frac{2}{4} = \frac{1}{2}$

What is the probability that a leap year selected at random will contain 53 saturdays. (Hint: 366 = 52×7 +2)

Solution : A leap year has 366 days. 52 weeks and 2 days.

S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)} ; n(S) = 7

Let A 53rd Saturday. A = {Fri-Sat, Sat-Sun}; n(A) = 2 , $P(A) = \frac{2}{7}$

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution : S = {1H,1T,2H,2T,3H,3T,4H,4T,5H,5T,6H,6T}; n (S) = 12

Let A odd number and a head. A = {1H, 3H, 5H}; n (A)= 3 , $P(A) = \frac{3}{12} = \frac{1}{4}$

A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution : Number of green balls is n (G) = 6 Let number of red balls is n (R) = x

Therefore, number of black balls is n (B) = $2x$ Total number of balls n(S) = $6 + x + 2x = 6 + 3x$

It is given that, P(G) = 3 × P(R) $\Rightarrow \frac{6}{6+3x} = 3 \times \frac{x}{6+3x} \Rightarrow 3x = 6 \Rightarrow x = \frac{6}{3} = 2 \Rightarrow x = 2.$

(i) Number of black balls = 2×2 = 4 (ii) Total number of balls = 6 +(3×2) = 12

If A is an event of a random experiment such that P(A) : P $(\overline{A})$=17:15 and n(S) = 640 then find (i) P $(\overline{A})$ (ii) n(A).

Solution : Given P(A) : P$(\overline{A})$ = 17 : 15 Total event = 17+15 = 32

(i) P $(\overline{A})$ $= \frac{15}{32}$ **(ii) n(A)**$= \frac{17}{32} \times 640 = 340$

A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution : **A coin is tossed thrice.** S = {(HHH), (HHT), (HTH), (HTT), (THH),(THT), (TTH), (TTT)} , n(S) = 8

Let A **two consecutive tails** A = {(HTT), (TTH), (TTT)} , n(A) = 3 , $P(A) = \frac{3}{8}$

At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?

i) Let A perfect squares between 500 and 1000 $A = \{23^2, 24^2, 25^2, 26^2 \ldots\ldots 31^2\}$, n(A) = 9 , $P(A) = \frac{9}{1000}$

(ii) Let A **the second player wins a prize, if the first has won** n(S) = 999, n(B) = 8 , $P(B) = \frac{8}{999}$

Find the diameter of a sphere whose surface area is 154 m^2.

Solution : surface area of sphere = 154 m^2 $\Rightarrow$ $4\pi r^2 = 154$

$$4\times\frac{22}{7}\times r^2 = 154 \Rightarrow r^2 = 154\times\frac{1}{4}\times\frac{7}{22} \Rightarrow r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2}$$

The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

Solution : C.S.A. of the cylinder = 88 sq. cm diameter = $2r$

Given that, $2\pi rh = 88 \Rightarrow 2\times\frac{22}{7}\times r\times 14 = 88 \Rightarrow 2r = \frac{88\times 7}{22\times 14} = 2$

Therefore, diameter = 2 cm

If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution : r = 7 cm T.S.A. of cone $= \pi r(l + r)$ sq. units

T.S.A. = 704 cm^2 $\Rightarrow$ $704 = \frac{22}{7}\times 7(l+7)$

$$(l+7) = \frac{\overset{32}{\cancel{704}}\times\cancel{7}}{\cancel{7}\times\cancel{22}} \Rightarrow l + 7 = 32 \Rightarrow l = 32 - 7 = 25 \text{ cm}$$

slant height of the cone is 25 cm.

The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Solution : Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Now, ratio of C.S.A. of balloons $= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{3^2}{4^2} = \frac{9}{16}$ Therefore, ratio of C.S.A. of balloons is 9:16.

The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

Solution : r = 7 m and h = 24 m $\Rightarrow$ $l = \sqrt{r^2 + h^2} = \sqrt{49+576} = \sqrt{625} = 25$m

C.S.A. of the conical tent $= \pi r l$ sq. units

Area of the canvas $= \frac{22}{7}\times 7\times 25 = 550\ m^2$

length of the canvas $= \frac{\text{Area of canvas}}{\text{Width}} = \frac{550}{4} = 137.5\ m$

A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Solution : d = 2.8 m
r = 1.4 m and height = 3 m

Area covered in one revolution = curved surface area of the cylinder $= 2\pi rh$ sq. units

$$= 2\times\frac{22}{7}\times 1.4\times 3 = 26.4$$

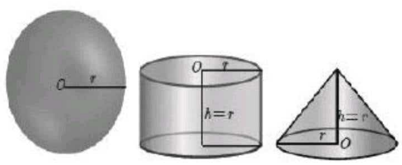
Area covered in 1 revolution = 26.4 m^2 Area covered in 8 revolutions $= 8\times 26.4 = 211.2$

A sphere, a cylinder and a cone are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.

Solution : Required Ratio = C.S.A. of the sphere: C.S.A. of the cylinder : C.S.A. of the cone

$= 4\pi r^2 : 2\pi rh : \pi rl$

$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$



The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution : $l = 5$ cm, R = 4 cm, r = 1 cm

C.S.A. of the frustum $= \pi(R + r)\,l = \frac{22}{7} \times (4+1) \times 5 = \frac{550}{7} = 78.57 \text{ cm}^2$

4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm² of the floor area, then find the height of the tent.

Solution : Given slant height of the cone $l = 19$ cm

Total floor area of 4 persons = 88 cm² $\Rightarrow \pi r^2 = 88 \Rightarrow \frac{22}{7} \times r^2 = 88 \Rightarrow r^2 = 28$

$$\therefore h = \sqrt{l^2 - r^2} = \sqrt{19^2 - 28} = \sqrt{361 - 28} = \sqrt{333} \simeq 18.25 \text{ cm.}$$

From a solid Cylinder **whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm³.**

Solution: Volume of the remaining solid = Vol. of Cylinder – Vol. of Cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4 = 2.46 \text{ cm}^3$$

The volume of a solid right circular cone is 11088 cm³. If its height is 24 cm then find the radius of the cone.

Solution : Volume of the cone = 11088 cm³ $\Rightarrow \frac{1}{3}\pi r^2 h = 11088 \Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

$$r^2 = 441$$

Therefore, radius of the cone r = 21 cm

The ratio of the volumes of two cones is 2 : 3. Find the ratio of their radii if the height of second cone is double the height of the first..

Solution : Given $h_2 = 2h_1$ and $\frac{\text{Volume of the cone I}}{\text{Volume of the cone II}} = \frac{2}{3}$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3} \Rightarrow \frac{r_1^2}{r_2^2} \times \frac{h_1}{2h_1} = \frac{2}{3} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{4}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

ratio of their radii = $2 : \sqrt{3}$

The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights.

Solution : Given volumes of 2 cones = 3600 cm³ & 5040 cm³ & base radius are equal

$$\therefore \text{Ratio of volumes} = \frac{V_1}{V_2} = \frac{3600}{5040} \Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3600}{5040} \Rightarrow \frac{h_1}{h_2} = \frac{40}{56} = \frac{5}{7}$$

$$\therefore h_1 : h_2 = 5 : 7$$

A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$: 4.

Solution : Given TSA of a solid sphere = TSA of a solid hemisphere

$$\Rightarrow 4\pi R^2 = 3\pi r^2 \Rightarrow \therefore \frac{R^2}{r^2} = \frac{3}{4} \qquad \therefore \frac{R}{r} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{Ratio of their volumes} = \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{2R^3}{r^3} = 2\left[\frac{R}{r}\right]^3 = 2\left(\frac{\sqrt{3}}{2}\right)^3 = 2 \times \frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}$$

$$\therefore \text{Ratio of the volumes} = 3\sqrt{3} : 4$$

Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of 10224 and 9648

Solution : HCF of 10224 and 9648

$10224 = 9648 \times 1 + 576$

$9648 = 576 \times 16 + 432$

$576 = 432 \times 1 + 144$ $\quad\therefore$ The last divisior "144" is the HCF.

$432 = (144) \times 3 + 0$

Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Solution : HCF of 1230 - 12 and 1926 - 12

i.e., HCF of 1218 and 1914

$1914 = 1218 \times 1 + 696$

$1218 = 696 \times 1 + 522$

$696 = 522 \times 1 + 174$

$522 = (174) \times 3 + 0$

$\therefore$ HCF = 174 $\quad\therefore$ The required largest number = 174.

When the positive integers a, b and c are divided by 13, the respective remainders are 9,7 and 10. Show that $a+b+c$ is divisible by 13.

Solution : When a is divided by 13, remainder is 9 $\quad$ i.e., $a = 13q + 9$ .... (1)

When b is divided by 13, remainder is 7 $\quad$ i.e., $b = 13q + 7$ .... (2)

When c is divided by 13, remainder is 11 $\quad$ i.e., $c = 13q + 11$ .... (3)

Adding (1), (2) & (3) $\quad a + b + c = 39q + 26 = 13(2q + 2)$

$a + b + c$ is divisible by 13

Find the HCF of 252525 and 363636.

Solution :

5	252525
5	50505
3	10101
7	3367
	481

2	363636
2	181818
3	90909
3	30303
3	10101
7	3367
	481

$\therefore 252525 = 5 \times 5 \times \underline{3} \times \underline{7} \times \underline{481}$

$363636 = 2 \times 2 \times \underline{3} \times 3 \times 3 \times \underline{7} \times \underline{481}$

$\therefore$ HCF $= 3 \times 7 \times 481 = 10{,}101$

Find the least number that is divisible by the first ten natural numbers.

Solution : The required number is the LCM of (1, 2, 3, ... 10)

$2 = \underline{2} \times 1 \quad 4 = \underline{2} \times 2 \quad 6 = 3 \times \underline{2} \quad 8 = 2 \times 2 \times \underline{2} \quad 9 = 3 \times 3$

$10 = 5 \times \underline{2}$ and 1, 3, 5, 7

L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

If $13824 = 2^a \times 3^b$ then find a and b.

Solution : Given $\quad 2^a \times 3^b = 13824$

$2^a \times 3^b = 2^9 \times 3^2$

$\therefore a = 9, b = 2$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
	3

Find the standard deviation of first 10 natural numbers.

Solution : SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{100-1}{12}} = \sqrt{\frac{99}{12}} = 2.87$

Find the standard deviation of first 13 natural numbers.

Solution : SD of first 21 natural numbers $= \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{169-1}{12}} = \sqrt{\frac{168}{12}} = 3.74$

A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Solution : A clock strikes bell at 1 o' clock once twice at 2 o' clock, 3 times at 3 o' clock ....

$\therefore$ Number of times it strikes in a particular day $= 2\,(1+2+3+ \ldots\ldots\ldots 12) = 2\left(\frac{12\times 13}{2}\right) = 156$ times

S.D of 2 (1, 2, 3, ...12) $= 2\left[\sqrt{\frac{n^2-1}{12}}\right] = 2\left[\sqrt{\frac{144-1}{12}}\right] = 2\sqrt{\frac{143}{12}} = 6.9$

The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution : Mean $\bar{x} = 25.6$, C.V. $= 18.75$ C.V. $= \frac{\sigma}{\bar{x}} \times 100$

$$18.75 = \frac{\sigma}{25.6}\times 100 \quad = \frac{18.75 \times 25.6}{100} = 4.8$$

The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution : Given $\sigma = 6.5, \bar{x} = 12.5$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}}\times 100 = \frac{6.5}{12.5}\times 100 = 52$

The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution : Given $\sigma = 1.2$, C.V $= 25.6$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}}\times 100$

$$25.6 = \frac{1.2}{\bar{x}}\times 100 \Rightarrow \bar{x} = \frac{1.2\times 100}{25.6} \Rightarrow \bar{x} = \frac{120}{25.6} = 4.69$$

If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution : Given $\bar{x} = 15$, CV $= 48$, $\sigma = ?$ $\therefore$ C.V $= \frac{\sigma}{\bar{x}}\times 100$

$$48 = \frac{\sigma}{15}\times 100 \quad \Rightarrow \sigma = \frac{15\times 48}{100} = \frac{720}{100} = 7.2$$

If $n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.

Solution : $n = 5$, $\bar{x} = 6$, $\Sigma x^2 = 765$, CV $= ?$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2} = \sqrt{117} = 10.82$$

$$\therefore \text{C.V} = \frac{\sigma}{\bar{x}}\times 100 = \frac{10.82}{6}\times 100 = 180.33$$

If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution : Given range $= 20$, Co.eff. of range $= 0.2$

$\Rightarrow$ L $-$ S $= 20$ ...(1) $\frac{L-S}{L+S} = 0.2 \Rightarrow \frac{20}{L+S} = 0.2 \quad \Rightarrow L+S = 100$ ...(2)

Solviong (1) and (2) L $= 60$, S $= 40$

Find the 12th term from the last term of the A.P – 2, – 4, – 6, ... –100.

Solution : Given A.P is – 2, – 4, – 6, ... – 100

12th term from the last term $a = -100$, d = 2

$t_{12} = a + 11d = -100 + 11(2) = -100 + 22 = -78$

If $1 + 2 + 3 + \ldots\ldots + k = 325$, then find $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + k^3$

Solution : $1 + 2 + 3 + \ldots\ldots + k = 325 \Rightarrow \frac{k(k+1)}{2} = 325$

$\therefore 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2 = (325)^2 = 105625$

Find the G.P. in which the 2nd term is $\sqrt{6}$ and the 6th term is $9\sqrt{6}$.

Solution : Given $t_2 = \sqrt{6},\ t_6 = 9\sqrt{6}$ in G.P.

$a.r = \sqrt{6}$ .... (1)

$a.r^5 = 9\sqrt{6}$ .... (2)

(2) divide (1) $\Rightarrow \frac{\cancel{a}.r^{\cancel{5}\,4}}{\cancel{a}.\cancel{r}} = \frac{9\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}}$ $\therefore r^4 = 9$ $\Rightarrow r = \sqrt{3}$

$\therefore a \times \sqrt{3} = \sqrt{6}$ $\therefore a = \sqrt{2}$

$\therefore$ The G.P is $\sqrt{2}, \sqrt{6}, \sqrt{18}, \ldots..$

When the positive integers *a, b* and *c* are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when $a + 2b + 3c$ is divided by 13.

Solution : Let $a = 13q + 9$ $b = 13q + 7 \Rightarrow 2b = 26q + 14$ $c = 13q + 10 \Rightarrow 3c = 39q + 30$

$a + 2b + c = (13q + 9) + (26q + 14) + (39q + 30) = 78q + 53 = 13(6q) + 13(4) + 1$

$\therefore$ When $a + 2b + 3c$ is divided by 13, the remainder is 1.

The value of a motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45,000 ?

Solution : P = ₹ 45000, n = 3, r = 15% (depreciation)

$A = P\left(1 - \frac{r}{100}\right)^n = 45,000\left(1 - \frac{15}{100}\right)^3 = 27636$

Show that the square of an odd integer is of the form $4q + 1$, for some integer q.

Solution : Let $x = 2k + 1$ be any odd integer.

The square of an odd integer $x^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1 = 4q + 1$

A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over

Solution : No. of flower pots = 532 each row to contain 21 flower pots.

$\Rightarrow 532 = 21 \times 25 + 7$

$\therefore$ Number of completed rows = 25

Number of flower pots left out = 7

$$\begin{array}{r|r} & 25 \\ \hline 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & 7 \end{array}$$

'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution : $800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$

Hence, $a^b \times b^a = 2^5 \times 5^2$ This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$.

Prove that two consecutive positive integers are always coprime.

Solution : Let $x,\ x + 1$ be two consecutive integers.

G.C.D. of $(x, x + 1) = 1$ $\Rightarrow$ x & $x + 1$ are Co-prime.

Find the number of terms in the A.P. 3, 6, 9, 12, ... 111.

Solution : $a = 3$; $d = 6 - 3 = 3$; $l = 111$ $\quad n = \left(\frac{l-a}{d}\right) + 1 = \left(\frac{111-3}{3}\right) + 1 = \left(\frac{108}{3}\right) + 1 = 37$

the number of terms in the A.P. 37

Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n.

Solution : When n = 1, $2n + 6 \times 9n = 2 + (6 \times 9) = 56$, divisible by 7.

What is the time 100 hours after 7 a.m.?

Solution : Formula : $t + n = f \pmod{24}$ $\quad 100 + 7 = f \pmod{24}$

$\Rightarrow 107 - f$ is divisible by 24

$\therefore f = 11$ so that $107 - 11 = 96$ is divisible by 24.

$\therefore$ The time is 11 A.M.

Kala and Vani are friends, Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

Solution :

Let 0, 1, 2, 3, 4, 5, 6 to represent the weekdays from Sunday to Saturday respecitvely.

$-74 \pmod 7 \equiv -4 \pmod 7 \equiv 7 - 4 \pmod 7 \equiv 3 \pmod 7$

The day for the number 3 is Wednesday. Vani's birthday must be on Wednesday.

Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

Solution : Today is Tuesday $\quad$ Day after 45 days = ?

When we divide 45 by 7, remainder is 3. $\therefore$ The 3rd day from Tuesday is Friday

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A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi ?

Solution : Starting time 22.30, Travelling time 32 hours. Here we use modulo 24.

The reaching time is $22.30 + 32 \pmod{24} \equiv 54.30 \pmod{24} \equiv 6.30 \pmod{24}$

What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?

Solution : The required number is the LCM of (35, 56, 91) + remainder 7

$35 = 7 \times 5$

$56 = 7 \times 2 \times 2 \times 2$

$91 = 7 \times 13$

$\therefore$ L.C.M $= 7 \times 5 \times 13 \times 8 = 3640$

$\therefore$ The required number is $3640 + 7 = 3647$

Find the first five terms of the following sequence. $a_1 = 1,\ a_2 = 1,\ a_n = \frac{a_{n-1}}{a_{n-2} + 3};\ n \geq 3,\ n \in N$

Solution :

$$a_3 = \frac{a_{3-1}}{a_{3-2}+3} = \frac{a_2}{a_1+3} = \frac{1}{1+3} = \frac{1}{4}$$

$$a_4 = \frac{a_{4-1}}{a_{4-2}+3} = \frac{a_3}{a_2+3} = \frac{\frac{1}{4}}{1+3} = \frac{\frac{1}{4}}{4} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$a_5 = \frac{a_{5-1}}{a_{5-2}+3} = \frac{a_4}{a_3+3} = \frac{\frac{1}{16}}{\frac{1}{4}+3} = \frac{1}{16} \times \frac{4}{13} = \frac{1}{52}$$

The first five terms of the sequence are $1, 1, \frac{1}{4}, \frac{1}{16}, \frac{1}{25}$

The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.

Solution : R = 16 cm h = 13 cm thickness = 4 cm $\therefore$ r = R – w = 16 – 4 = 12

$\therefore$ TSA of hollow cylinder $= 2\pi(R+r)(R-r+h) = 2\times\frac{22}{7}(28)(4+13) = 44\times4\times17 = 2992\text{ cm}^2$

Find the volume of a cylinder whose height is 2 m and whose base area is 250 m².

Solution : height h = 2 m, base area = 250 m²

volume of a cylinder $= \pi r^2 h$ cu. units

$= \text{base area}\times h = 250\times2 = 500\text{ m}^3$

Therefore, volume of the cylinder = 500 m³

The ratio of the radii of two right circular cones of same height is 1 : 3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Solution : Given $r_1 : r_2 = 1 : 3$

$r_1 = 1$, $r_2 = 3$ | $h_1 = 3r_1 = 3$, $h_2 = 3r_1 = 3$

$l_1 = \sqrt{h_1^2 + r_1^2} = \sqrt{9+1} = \sqrt{10}$ | $l_2 = \sqrt{h_2^2 + r_2^2} = \sqrt{9+9} = 3\sqrt{2}$

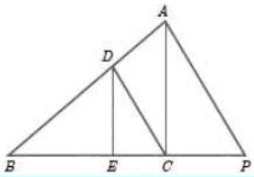
$\therefore$ Ratio of their CSA $= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{1}{3}\times\frac{\sqrt{10}}{3\sqrt{2}} = \frac{\sqrt{5}}{9} = \sqrt{5}:9$

Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

Solution : Given that, r = 21cm, R = 28 cm, h = 9 cm

volume of hollow cylinder $= \pi(R^2 - r^2)h = \frac{22}{7}(28^2 - 21^2)\times9 = \frac{22}{7}(784-441)\times9 = 9702\text{ cm}^3$.

In the Fig. DE | | AC and DC | | AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$



Solution : In ΔBPA, DC | | AP. In ΔBCA, DE | | AC. By Basic Proportionality Theorem,

$\frac{BC}{CP} = \frac{BD}{DA}$ ......(1) $\frac{BE}{EC} = \frac{BD}{DA}$ ......(2) From (1) and (2) we get, $\frac{BE}{EC} = \frac{BC}{CP}$

Hence proved.

If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

Solution : A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} Thus A = {3, 5} and B = {2, 4}.

Find A × B, A × A and B × A If A = B = {*p*, *q*}

Solution: A × B = {(*p*, *p*), (*p*, *q*), (*q*, *p*), (*q*, *q*)} ,

A × A = {(*p*, *p*), (*p*, *q*), (*q*, *p*), (*q*, *q*)} , B × A = {(*p*, *p*), (*p*, *q*), (*q*, *p*), (*q*, *q*)}

Find A × B, A × A and B × A If A = {*m*, *n*} ; B = ϕ

Solution: A = {m, n}, B = ϕ

If A = ϕ (or) B = ϕ, then A × B = ϕ. and B × A = ϕ A × B = ϕ and B × A = ϕ

A × A = {(*m*, *m*), (*m*, *n*), (*n*, *m*), (*n*, *n*)}

Let A = {1, 2, 3} and B = {*x* | *x* is a prime number less than 10}. Find A × B and B × A.

Solution : A = {1, 2, 3}, B = {*x* | *x* is a prime number less than 10}. $\therefore$ B = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

If B × A = {(– 2, 3), (– 2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.

Solution : B × A = {(–2, 3), (– 2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} $\therefore$ A = {3, 4}, B = {– 2, 0, 3}

Find the 19th term of an A.P. −11, −15, −19,...

Solution : A.P is −11, −15, −19, ... $a = -11$, $d = -15 - (-11) = -15 + 11 = -4$

$$t_n = a + (n-1)\,d$$

$$t_{19} = a + 18\,d = (-11) + 18\,(-4) = -11 - 72 = -83$$

Which term of an A.P. 16, 11, 6, 1.... is −54?

Solution : A.P. is 16, 11, 6, 1, ... − 54

$a = 16,\ d = -5,\ t_n = -54$

$\Rightarrow\quad a + (n-1)\,d = -54 \quad \Rightarrow 16 + (n-1)(-5) = -54 \quad \Rightarrow \quad 16 - 5n + 5 = -54$

$\Rightarrow \quad -5n + 21 = -54$

$\Rightarrow \quad -5n + 21 = -54$

$\Rightarrow \quad -5n = -54 - 21$

$\Rightarrow \quad -5n = -75$

$\therefore n = 15$

$\therefore$ 15th term of A.P. is − 54

If $a_1 = 1, a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$, $n \ge 3$, $n \in N$, then find the first six terms of the sequence.

Solution : Given $a_1 = 1$, $a_2 = 1$ $a_3 = 2a_2 + a_1 = 2\,(1) + 1 = 3$ $a_4 = 2a_3 + a_2 = 2\,(3) + 1 = 7$

$a_5 = 2a_4 + a_3 = 2\,(7) + 3 = 17$ $a_6 = 2a_5 + a_4 = 2\,(17) + 7 = 41$

$\therefore$ The first 6 terms are 1, 1, 3, 7, 17, 41

Find the middle term(s) of an A.P. 9, 15, 21, 27,…,183.

Solution : Given A.P is 9, 15, 21, 27, ... 183 $a = 9,\ d = 6,\ l = 183$

$$n = \frac{l-a}{d} + 1 = \frac{183-9}{6} + 1 = \frac{174}{6} + 1 = 29 + 1 = 30$$

$\therefore$ Middle terms are $30/2,\ 30/2 + 1 = 15^{th},\ 16^{th}$

$t_{15} = a + 14d = 9 + 14(6) = 9 + 84 = 93$

$t_{16} = a + 15d = 9 + 15(6) = 9 + 90 = 99$

A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Claculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.

Solution : Cow's milk = 175 lrs. Buffalow's milk = 105 lrs.

i) Capacity of a can = HCF of 175 and 105 = 35 litres

ii) Number of cans of Cow's milk = $\frac{175}{35} = 5$ iii) Number of cans of buffalow's milk = $\frac{105}{35} = 3$

If $3 + k$, $18 - k$, $5k + 1$ are in A.P. then find k.

Solution : a, b, c are in A.P. $\Rightarrow\ 2b = a + c$

$\Rightarrow\ 2\,(18 - k) = (3 + k) + (5k + 1)$

$36 - 2k = 6k + 4$

$8k = 32 \Rightarrow k = 4$

Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

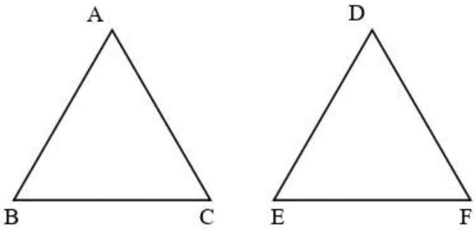
Solution : Given that x, 10, y, 24, z are in A.P. $\therefore$ y is the arithmetic mean of 10 & 24

$2y = 10 + 24 \Rightarrow y = \frac{10+24}{2} = \frac{34}{2} = 17$ Clearly $d = 7$

$\therefore$ x, 10, y, 24, z are in A.P. $\therefore x = 10 - 7 = 3$ & $z = 24 + 7 = 31$

$\therefore x = 3, y = 17, z = 31$

If ΔABC ~ ΔDEF such that area of ΔABC is $9cm^2$ and the area of ΔDEF is $16cm^2$ and BC = 2.1 cm. Find the length of EF.



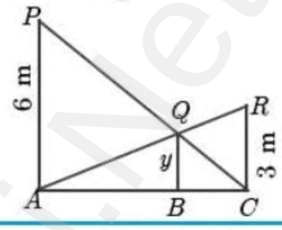
Solution : Given ΔABC ~ ΔDEF

$$\therefore \frac{\text{Area of } \Delta\, ABC}{\text{Area of } \Delta\, DEF} = \frac{BC^2}{EF^2} \Rightarrow \frac{9}{16} = \frac{(2.1)^2}{EF^2}$$

$$\Rightarrow EF^2 = \frac{16\times(2.1)^2}{9}$$

$$\therefore \quad EF = \frac{4\times 2.1}{3} = 2.8\ cm$$

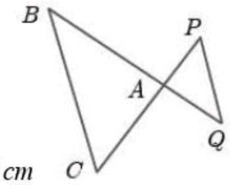
Two vertical poles of heights 6 m and 3m are erected above a horizontal ground AC. Find the value of y.



Solution : Using formula $y = \dfrac{ab}{a+b}$

$$= \frac{6\times 3}{6+3} = \frac{18}{9} = 2m$$

In the adjacent figure, ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.



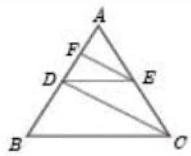
Solution : Given Δ ACB ~ Δ APQ

$$\therefore \frac{BC}{PQ} = \frac{AC}{AP} = \frac{AB}{AQ} \Rightarrow \frac{8}{4} = \frac{AC}{2.8} = \frac{6.5}{AQ}$$

$$\therefore \frac{AC}{2.8} = 2 \qquad \frac{6.5}{AQ} = 2$$

$$\Rightarrow AC = 5.6\ cm \qquad \Rightarrow AQ = \frac{6.5}{2} = 3.25\ cm$$

In figure DE | | BC and CD | | EF. Prove that $AD^2 = AB \times AF$

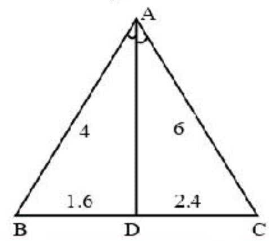


Solution : In figure DE | | BC and CD | | EF

In $\Delta\ ACD$, by BPT, $\dfrac{AF}{AD} = \dfrac{AE}{AC}$ ..... (1) In $\Delta\ ABC$, by BPT, $\dfrac{AD}{AB} = \dfrac{AE}{AC}$ ..... (2)

$\therefore$ From (1) & (2) $\dfrac{AF}{AD} = \dfrac{AD}{AB} \Rightarrow$ $\mathbf{AD^2 = AB \times AF}$

Check whether AD is bisector of ∠A of ΔABC, AB = 4 cm, AC = 6 cm, BD = 1.6. cm and CD = 2.4 cm.

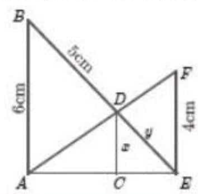


Solution : $\dfrac{AB}{AC} = \dfrac{4}{6} = \dfrac{2}{3}$, $\dfrac{BD}{DC} = \dfrac{1.6}{2.4} = \dfrac{2}{3}$

$\therefore$ By Converse of ABT, $\therefore \dfrac{AB}{AC} = \dfrac{BD}{DC}$

AD is the bisector of ∠A.

In the given figure AB | | CD | | EF . If AB = 6 cm, CD = x cm, EF = 4 cm, BD = 5 cm and DE = y cm. Find x and y.



Solution :

$$x = \frac{ab}{a+b}$$

$$= \frac{6\times 4}{6+4} = \frac{24}{10} = \frac{12}{5}$$

Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Solution : $\sin A \times \sin A = (1+\cos A)\times(1-\cos A) \Rightarrow \sin^2 A = 1-\cos^2 A \Rightarrow \sin^2 A = \sin^2 A$

Prove that $1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = \operatorname{cosec}\theta$

Solution : $\frac{\cot^2\theta}{1+\operatorname{cosec}\theta} = (\operatorname{cosec}\theta-1) \Rightarrow \frac{\cancel{(\operatorname{cosec}\theta+1)}(\operatorname{cosec}\theta-1)}{\cancel{\operatorname{cosec}\theta+1}} = (\operatorname{cosec}\theta-1) \Rightarrow (\operatorname{cosec}\theta-1) = (\operatorname{cosec}\theta-1)$

Prove that sec θ – cos θ = tan θ sin θ

Solution : $\sec\theta-\cos\theta = \frac{1}{\cos\theta}-\cos\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}\times\sin\theta = \tan\theta\sin\theta$

Prove that $\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta} = \cot\theta$

Solution : $\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta} = \frac{\sec\theta\cos\theta-\sin\theta\sin\theta}{\cos\theta\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\cancel{\cos\theta}\cos\theta}{\sin\theta\cancel{\cos\theta}} = \cot\theta$

Show that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2$

Solution : $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 \Rightarrow \left(\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}\right) = \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2 \Rightarrow \tan^2 A = (-\tan A)^2 \Rightarrow \tan^2 A = \tan^2 A$

Prove that (cosecθ – sinθ) (secθ – cosθ) (tanθ + cotθ) = 1

Solution :

$(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta) = \left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$

$= \frac{1-\sin^2\theta}{\sin\theta}\times\frac{1-\cos^2\theta}{\cos\theta}\times\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{\cancel{\cos^2\theta}\cancel{\sin^2\theta}\times 1}{\cancel{\sin^2\theta}\cancel{\cos^2\theta}} = 1$

Prove that $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A} = 2\operatorname{cosec} A$

Solution : $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A} = \frac{2\sin A}{(1-\cos A)\times(1+\cos A)} = \frac{2\sin A}{1-\cos^2 A} = \frac{2\sin A}{\sin^2 A} = \frac{2\cancel{\sin A}}{\sin A\cancel{\sin A}} = 2\operatorname{cosec} A$

Prove that $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B+\sin^2 A\sin^2 B = 1$

Solution : $\sin^2 A\cos^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B+\sin^2 A\sin^2 B$

$= \sin^2 A\cos^2 B+\sin^2 A\sin^2 B+\cos^2 A\sin^2 B+\cos^2 A\cos^2 B$

$= \sin^2 A(\cos^2 B+\sin^2 B)+\cos^2 A(\sin^2 B+\cos^2 B)$

$= \sin^2 A(1)+\cos^2 A(1) = \sin^2 A+\cos^2 A = 1$

Prove $\sec^6\theta = \tan^6\theta+3\tan^2\theta\sec^2\theta+1$

Solution Take $a = 1$ $b = \tan^2\theta$ $(\because a+b)^3 = a^3+b^3+3ab(a+b))$

$(1+\tan^2\theta)^3 = 1+\tan^6\theta+3(1)\tan^2\theta)(1+\tan^2\theta)$

$\sec^6\theta = 1+\tan^6\theta+3\tan^2\theta\,.\,\sec^2\theta$

Prove $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2 = 1+(\sec\theta+\operatorname{cosec}\theta)^2$

Solution : $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2 = \sin^2\theta+\sec^2\theta+2\sin\theta\,.\,\sec\theta+\cos^2\theta+\operatorname{cosec}^2\theta+2\cos\theta\,.\,\operatorname{cosec}\theta$

$= 1+(\sec\theta\operatorname{cosec}\theta)^2$

Find the sum of $0.40 + 0.43 + 0.46 + \ldots + 1$.

Solution : $a = 0.40$ and $l = 1$, $d = 0.43 - 0.40 = 0.03$. $n = \left(\frac{l-a}{d}\right) + 1 = \left(\frac{1-0.40}{0.03}\right) + 1 = 21$

$$S_n = \frac{n}{2}[a+l] \qquad S_{21} = \frac{21}{2}[0.40+1] = 14.7$$

Find the sum of first 15 terms of the A.P. $8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, \ldots$

Solution : $a = 8$, $d = 7\frac{1}{4} - 8 = -\frac{3}{4}$, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{15} = \frac{15}{2}\left[2\times 8 + (15-1)(-\frac{3}{4})\right] \qquad S_{15} = \frac{15}{2}\left[16 - \frac{21}{2}\right] = \frac{165}{4}$$

In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution : $a = 20$, $d = 2$, $n = 30$ $\qquad t_{30} = a + 29d = 20 + 29(2) = 20 + 58 = 78$

$t_n = a + (n-1)\,d$ $\qquad \therefore$ The no. of seats in 30^{th} row $= 78$

Find the sum of all odd positive integers less than 450.

Solution : $1 + 3 + 5 + 7 + \ldots\ldots + 449 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{449+1}{2}\right]^2 = \left[\frac{450}{2}\right]^2 = [225]^2 = 50,625$

In a G.P. 729, 243, 81,… find t_7.

Solution : 729, 243, 21, …….. $\qquad a = 729$, $r = 8/243 = 1/3$

$\therefore t_n = a \cdot r^{n-1}$

$$\Rightarrow t_7 = a \cdot r^6 = 729 \times \left(1/3\right)^6 = 729 \times \left(1/729\right) = 1$$

Find x so that $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution : Given $x + 6$, $x + 12$, $x + 15$ are consecutive terms of a G.P.

a, b, c are in G.P. $\Rightarrow b^2 = ac$

$$\Rightarrow (x+12)^2 = (x+15)(x+6) \Rightarrow x^2 + 24x + 144 = x^2 + 21x + 90 \Rightarrow 3x = -54 \Rightarrow x = -18$$

Find the 10th term of a G.P. whose 8th term is 768 and the common ratio is 2.

Solution : $t_8 = 768$, $\qquad r = 2$

$\Rightarrow a \cdot r^7 = 768 \Rightarrow a \times 2^7 = 768 \Rightarrow a \times 128 = 768 \Rightarrow a = 6$

$\therefore \qquad t_{10} = a \cdot r^9 = 6 \times 2^9 = 6 \times 512 = 3072$

If a, b, c are in A.P. then show that $3^a, 3^b, 3^c$ are in G.P.

Solution : Given a, b, c are in A.P. $\Rightarrow 2b = a + c$ …..(1)

To Prove : $3^a, 3^b, 3^c$ are in G.P. i.e. TP : $(3^b)^2 = 3^a \cdot 3^c$

LHS : $(3^b)^2 = 3^{2b} = 3^{a+c} = 3^a \cdot 3^c = \text{RHS}$ (from (1))

$\therefore 3^a, 3^b, 3^c$ are in G.P.

Find the sum of $2 + 4 + 6 + \ldots + 80$

Solution : $2 + 4 + 6 + \ldots + 80 = 2\,(1 + 2 + 3 + \ldots + 40) = 2 \times \frac{40\times(40+1)}{2} = 1640$

Find the sum of $1 + 3 + 5 + \ldots + 55$

Solution : $1 + 3 + 5 + \ldots + 55 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{(55+1)}{2}\right]^2 = \left[\frac{56}{2}\right]^2 = (28)^2 = 784.$

In a box there are 20 non-defective and some defective bulbs. If the probability that a bul selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

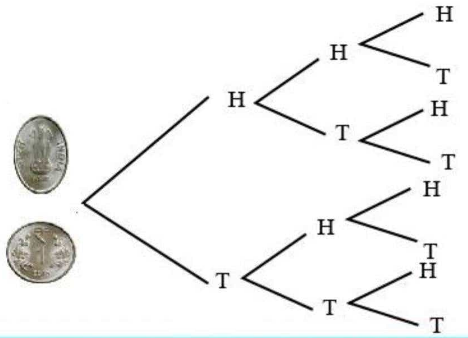
Solution : Let x be the number of defective bulbs. $\therefore$ n(S) $= x + 20$

Let A defective balls $\therefore$ n(A) $= x$ $\quad P(A) = \frac{x}{x+20}$

Given $\frac{x}{x+20} = \frac{3}{8} \Rightarrow 8x = 3x + 60 \Rightarrow 5x = 60 \Rightarrow x = 12$

Write the sample space for tossing three coins using tree diagram

Solution :

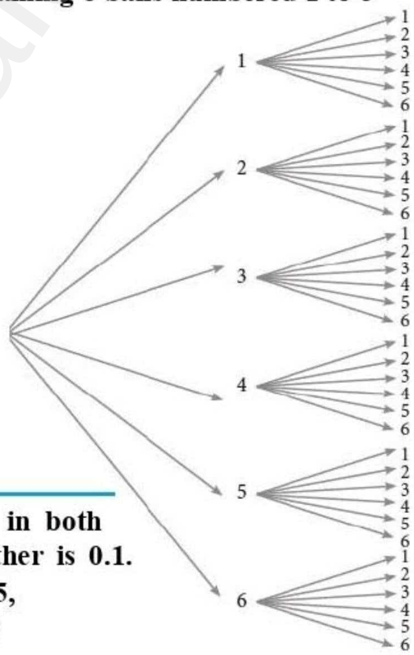


Sample space = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Express the sample space for rolling two dice using tree diagram.



The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?

Solution : $P(E\cap T) = 0.5$; $\quad P(\overline{E}\cap\overline{T}) = 0.1$ & $P(E) = 0.75 \Rightarrow P(E\cup T) = 1 - 01 = 0.9$

$P(E\cup T) = P(E) + P(T) - P(E\cap T) \Rightarrow 0.9 = 0.75 + P(T) - 0.5$

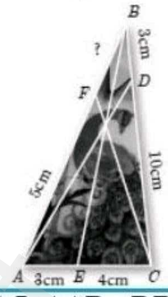
$P(T) = 0.9 - 0.25 = 0.65 = \frac{65}{100} = \frac{13}{20}$

Reduce to lowest form $\frac{x^{3a}-8}{x^2a+2xa+4}$ ***Solution :*** $\frac{x^{3a}-8}{x^2a+2xa+4} = \frac{(x^a-2)\cancel{(x^{2a}+2x^a+4)}}{\cancel{x^{2a}+2x^a+4}} = x^a - 2$

If x = $\frac{a^2+3a-4}{3a^2-3}$ **and y =** $\frac{a^2+2a-8}{2a^2-2a-4}$ **find the value of $x^2\,y^{-2}$.**

Solution : $x = \frac{a+4}{3(a+1)}$ | $y = \frac{a+4}{2(a+1)}$ $\quad \therefore x^2y^{-2} = \frac{x^2}{y^2} = \frac{\cancel{(a+4)^2}}{9\cancel{(a+1)^2}} \times \frac{4\cancel{(a+1)^2}}{\cancel{(a+4)^2}} = \frac{4}{9}$

An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.



Solution: By applying Ceva's theorem,

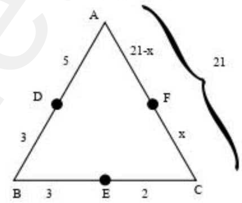
$$BD \times CE \times AF = DC \times EA \times FB$$

$$\Rightarrow \quad 3 \times 4 \times 5 = 10 \times 3 \times FB$$

$$\Rightarrow \quad 60 = 30 \times FB$$

$$\therefore \quad FB = 2 \text{ cm}$$

Let ABC be a triangle and D,E,F are points on the respective sides AB, BC, AC Let AD : DB = 5 : 3 BE : EC = 3 : 2 and AC = 21 . Find the length of the line segment CF.



Solution : By Ceva's theorem, $\dfrac{AD}{DB} \times \dfrac{BE}{EC} \times \dfrac{CF}{FA} = 1 \quad \Rightarrow \dfrac{5}{3} \times \dfrac{3}{2} \times \dfrac{x}{21-x} = 1$

$$\Rightarrow \frac{x}{21-x} = \frac{2}{5}$$

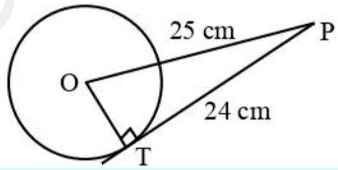
$$\Rightarrow 5x = 42 - 2x \quad \Rightarrow 7x = 42 \quad \therefore x = 6 \quad \therefore CF = 6$$

Ceva's Theorem : Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\dfrac{BD}{DC} \times \dfrac{CE}{EA} \times \dfrac{AF}{FB} = 1$

Menelaus Theorem : A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB of a triangle ABC to be collinear is that $\dfrac{BP}{PC} \times \dfrac{CQ}{QA} \times \dfrac{AR}{RB} = 1$

The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

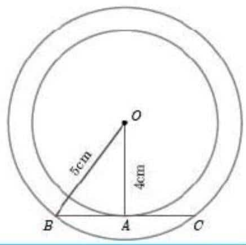
Solution: $\therefore \quad OT = \sqrt{25^2 - 24^2} = \sqrt{625 - 576} = \sqrt{49} = 7$ cm



$\therefore$ Radius = 7 cm

If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

Solution : $OB^2 = OA^2 + AB^2$

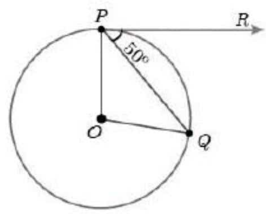


$5^2 = 4^2 + AB^2$ gives $AB^2 = 9$

Therefore AB = 3 cm

BC = 2AB hence BC = 2 × 3 = 6 cm

In Figure O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$



Solution : $\angle OPQ = 90^0 - 50^0 = 40^0$

OP = OQ (Radii of a circle are equal)

$\angle OPQ = \angle OQP = 40^0$ (ΔOPQ is isosceles)

$\angle POQ = 180^0 - \angle OPQ - \angle OQP$

$\angle POQ = 180^0 - 40^0 - 40^0 = 100^0$

Find the sum of first six terms of the G.P. 5, 15, 45, …

Solution : Given G.P is 5, 15, 45, ... $a = 5, r = 3 > 1$ $S_n = a \cdot \frac{r^n - 1}{r - 1}$

$$\therefore S_6 = 5 \cdot \frac{3^6 - 1}{3 - 1} = \frac{5}{2} \times 728 = 5 \times 364 = 1820$$

Find the sum $3 + 1 + \frac{}{} + \ldots \infty$

Solution : Here $a = 3,\ r = \frac{t_2}{t_1} = \frac{1}{3}$ Sum of infinite terms $= \frac{a}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{9}{2}$

Find the sum to infinity of $21 + 14 + \frac{28}{3} + \ldots$

Solution : $a = 21,\ r = \frac{14}{21} = \frac{2}{3} < 1$ $\therefore S_\infty = \frac{a}{1-r} = \frac{21}{1-\frac{2}{3}} = \frac{21}{1-\frac{2}{3}} = \frac{21}{\frac{1}{3}} = 63$

If the first term of an infinite G.P. is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.

Solution : $a = 8,\ S_\infty = \frac{32}{3},\ r = ?$

$$\Rightarrow \frac{a}{1-r} = \frac{32}{3} \Rightarrow \frac{\cancel{8}^{1}}{1-r} = \frac{\cancel{32}^{4}}{3} \Rightarrow 3 = 4 - 4r \Rightarrow 4r = 1$$

$$\therefore r = \frac{1}{4}$$

Find the first term of G.P. in which $S_6 = 4095$ and $r = 4$.

Solution : $S_n = \frac{a(r^n - 1)}{r - 1} = 4095$ $r = 4,\ \frac{a(4^6 - 1)}{4 - 1} = 4095$ gives $a \times \frac{4095}{3} = 4095$ $a = 3.$

Find the rational form of the number $0.\overline{123}$.

Solution : Let $x = 0.\overline{123}$ $x = 0.123123123$

$$\Rightarrow 1000x = 123.123123$$

$$\Rightarrow 1000x = 123 + 0.123123123$$

$$\Rightarrow 1000x = 123 + x$$

$$1000x - x = 123 \Rightarrow 999x = 123 \Rightarrow x = \frac{123}{999} \quad \therefore x = \frac{41}{333}$$

Find the 8^{th} term of the G.P. 9, 3, 1, ...

Solution : First term $a = 9$, common ratio $r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$ $\Rightarrow$ $t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^7 = \frac{1}{243}$

If $1^3 + 2^3 + 3^3 + \ldots\ldots + k^3 = 44100$, then find $1 + 2 + 3 + \ldots\ldots\ldots + k$.

Solution : Given $1^3 + 2^3 + 3^3 + \ldots\ldots + k^3 = 44100$

$$\Rightarrow \left(\frac{k(k+1)}{2}\right)^2 = 44100 \quad \Rightarrow \frac{k(k+1)}{2} = 210 \quad \Rightarrow 1 + 2 + 3 + \ldots\ldots + k = 210$$

Find the sum of the following series $3 + 6 + 9 + \ldots\ldots + 96$

Solution : $3 + 6 + 9 + \ldots\ldots + 96$

$$= 3\,(1 + 2 + 3 + \ldots\ldots\ldots + 32) = 3\left(\frac{32 \times 33}{2}\right) = 3 \times 16 \times 33 = 1584$$

Find the sum of the following series $1 + 4 + 9 + 16 + \ldots\ldots + 225$

Solution : $1 + 4 + 9 + 16 + \ldots\ldots + 225 = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 15^2 = \frac{15 \times 16 \times 31}{6} = 1240$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Find the sum of the following series $1 + 3 + 5 + \ldots\ldots + 71$

Solution : $1 + 3 + 5 + \ldots\ldots + 71$ $(\because 1 + 3 + 5 + \ldots\ldots\ldots + n \text{ terms} = n^2)$

$$\therefore 1 + 3 + 5 + \ldots\ldots + 71 = (36)^2 = 1296$$

Find the excluded values of $\frac{x^2+6x+8}{x^2+x-2}$ **Solution :** $\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+4)(x+2)}{(x+2)(x-1)} = \frac{x+4}{x-1}$

$\therefore$ The excluded values is 1

Find the excluded values of $\frac{t}{t^2-5t+6}$

Solution : $\frac{t}{t^2-5t+6} = \frac{t}{(t-3)(t-2)}$ $\therefore$ The excluded values are 3, 2

Find the excluded values of $\frac{x^3-27}{x^3+x^2-6x}$

Solution :

$\frac{x^3-27}{x^3+x^2-6x} = \frac{x^3-3^3}{x(x^2+x-6)} = \frac{(x-3)(x^2+3x+9)}{x(x+3)(x-2)}$ $\therefore$ The excluded values are 0, −3, 2

Simplify $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$

Solution : $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2} = \frac{x+2}{4y} \times \frac{12y^2}{x^2-x-6} = \frac{\cancel{x+2}}{\cancel{4y}} \times \frac{\cancel{12}3\cancel{y^2}y}{(x-3)\cancel{(x+2)}} = \frac{3y}{x-3}$

Multiply $\frac{x^4b^2}{x-1}$ by $\frac{x^2-1}{a^4b^3}$

$\frac{x^4b^2}{x-1} \times \frac{x^2-1}{a^4b^3} = \frac{x^4 \times \cancel{b^2}}{\cancel{x-1}} \times \frac{(x+1)\cancel{(x-1)}}{a^4 \times \cancel{b^3}_{b}} = \frac{x^4(x+1)}{a^4b}$

Simplify $\frac{14x^4}{y} \div \frac{7x}{3y^4}$ **Solution :** $\frac{14x^4}{y} \div \frac{7x}{3y^4} = \frac{14x^4}{y} \times \frac{3y^4}{7x} = 6x^3y^3$

Simplify $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$ **Solution :** $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4} = \frac{\cancel{(x+4)}\cancel{(x-4)}}{\cancel{(x+4)}} \times \left(\frac{x+4}{\cancel{x-4}}\right) = x+4$

Simplify $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$ **Solution :** $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20} = \frac{\cancel{x+4}}{\cancel{3x+4y}} \times \frac{\cancel{(3x+4y)}(3x-4y)}{\cancel{(x+4)}(2x-5)}$

$= \frac{3x-4y}{2x-5}$

Simplify $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$ **Solution :** $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$

$= \frac{\cancel{(x-y)}(x^2+xy+y^2)}{3(x^2+3xy+2y^2)} \times \frac{\cancel{(x+y)}\,\cancel{(x+y)}}{\cancel{(x+y)}\,\cancel{(x-y)}} = \frac{(x^2+xy+y^2)\cancel{(x+y)}}{3(x+2y)\cancel{(x+y)}} = \frac{x^2+xy+y^2}{3(x+2y)}$

If a polynomial p(x) = x² − 5x − 14 is divided by another polynomial q(x) we get $\frac{x-7}{x+2}$ find q(x).

Solution : $\frac{p(x)}{q(x)} = \frac{x-7}{x+2} \Rightarrow \frac{x^2-5x-14}{q(x)} = \frac{x-7}{x+2} \Rightarrow \frac{(x-7)(x+2)}{q(x)} = \frac{x-7}{x+2} \Rightarrow q(x) = (x+2)^2$

$\Rightarrow q(x) = x^2+4x+4$ is another polynomial

Simplify $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$ **Solution :** $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2} = \frac{x^2+x+x-x^2}{x-2} = \frac{2x}{x-2}$

Subtract $\frac{1}{x^2+2}$ **from** $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Solution :

$$\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}=\frac{(2x^3+x^2+3)-(x^2+2)}{(x^2+2)^2}=\frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2}=\frac{2x^3+1}{(x^2+2)^2}$$

Find the square root of $256(x-a)^8(x-b)^4(x-c)(x-d)^{20}$

Solution : $\sqrt{256\,(x-a)^8\,(x-b)^4\,(x-c)^{16}\,(x-d)^{20}}=16\left|\,(x-a)^4\,(x-b)^2\,(x-c)^8\,(x-d)^{10}\,\right|$

Find the square root of $\frac{144\,a^8\,b^{12}\,c^{16}}{81\,f^{12}\,g^4\,h^{14}}$ ***Solution :*** $\sqrt{\frac{144\,a^8\,b^{12}\,c^{16}}{81\,f^{12}\,g^4\,h^{14}}}=\frac{4}{3}\left|\frac{a^4\,b^6\,c^8}{f^6\,g^2\,h^7}\right|$

Find the square root of $16x^2+9y^2-24xy+24x-18y+9$

Solution : $\sqrt{16x^2+9y^2-24xy+24x-18y+9}=\sqrt{(4x-3y+3)^2}=|\,4x-3y+3\,|$

Find the square root $\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$ ***Solution :*** $\sqrt{\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}}=\sqrt{\frac{(\sqrt{7}x+\sqrt{2})^2}{\left(x-\frac{1}{4}\right)^2}}=4\left|\frac{\sqrt{7}x+\sqrt{2}}{(4x-1)}\right|$

Find the square root of $4x^2+20x+25$ ***Solution :*** $\sqrt{4x^2+20x+25}=\sqrt{(2x+5)^2}=|\,2x+5\,|$

Find the square root of $9x^2-24xy+30xz-40yz+25z^2+16y^2$

Solution : $\sqrt{9x^2-24xy+30xz-40yz+25z^2+16y^2}=\sqrt{(3x-4y+5z)^2}=|\,3x-4y+5z\,|$

Find the square root of $1+\frac{1}{x^6}+\frac{2}{x^3}$ ***Solution :*** $\sqrt{1+\frac{1}{x^6}+\frac{2}{x^3}}=\sqrt{\left(1+\frac{1}{x^3}\right)^2}=\left|1+\frac{1}{x^3}\right|$

Find the zeroes of the quadratic expression $x^2+8x+12$.

Solution : Let $p(x)=x^2+8x+12=(x+2)(x+6)$ Therefore -2 and -6 are zeros of $p(x)$

Write down the quadratic equation in general form for which sum and product of the roots are $-\frac{3}{5}, -\frac{1}{2}$

Solution : $x^2-(\text{SOR})\,x+\text{POR}=0$ $\quad x^2-\left(-\frac{3}{5}\right)x+\left(-\frac{1}{2}\right)=0$ Therefore $10x^2+6x-5=0$

Determine quadratic equations, whose sum and product **of roots are** $-(2-a)^2,(a+5)^2$

Solution : Given SOR $=-(2-a)^2$, POR $=(a+5)^2$

∴ The required equation is $\Rightarrow x^2+(2-a)^2x+(a+5)^2=0$

Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ **to get** $\frac{3}{x^2-2x+4}$

Solution : $\frac{x^2+6x+8}{x^3+8}-\frac{3}{x^2-2x+4}=\frac{(x+4)\cancel{(x+2)}}{\cancel{(x+2)}(x^2-2x+4)}-\frac{3}{x^2-2x+4}=\frac{x+4-3}{x^2-2x+4}=\frac{x+1}{x^2-2x+4}$

Solve $\sqrt{a(a-7)}=3\sqrt{2}$ **by factorization method**

Solution : Given $\sqrt{a(a-7)}=3\sqrt{2}$ Squaring on both sides $a^2-7a=18 \Rightarrow a^2-7a-18=0$

$\Rightarrow (a-9)(a+2)=0 \Rightarrow$ Roots are $9, -2$

Solve $\sqrt{2}x^2+7x+5\sqrt{2}=0$ **by factorization method**

Solution : $\Rightarrow \sqrt{2}x^2+2x+5x+5\sqrt{2}=0 \Rightarrow (x+\sqrt{2})(\sqrt{2}x+5)=0 \quad \therefore$ Roots are $-\frac{5}{\sqrt{2}}, -\sqrt{2}$

Solve $2x^2-x+\frac{1}{8}=0$ **by factorization method**

Solution : $\Rightarrow 16x^2-8x+1=0 \Rightarrow (4x-1)(4x-1)=0 \quad \therefore$ Roots are $\frac{1}{4}, \frac{1}{4}$

Find the sum and product of the roots for the quadratic equation $x^2+3x=0$

Solution : $x^2+3x=0$ $a=1, b=3, c=0$ $\therefore \alpha+\beta=\frac{-b}{a}=\frac{-3}{1}=-3$ $\alpha\beta=\frac{c}{a}=\frac{0}{1}=0$

Find the sum and product of the roots for the quadratic equation $3+\frac{1}{a}=\frac{10}{a^2}$

Solution : Given $3+\frac{1}{a}=\frac{10}{a^2} \Rightarrow \frac{3a+1}{a}=\frac{10}{a^2} \Rightarrow 3a+1=\frac{10}{a} \Rightarrow 3a^2+a-10=0$

$A=3, B=1, C=-10$ $\therefore \alpha+\beta=\frac{-B}{A}=\frac{-1}{3}$ $\therefore \alpha\beta=\frac{C}{A}=\frac{-10}{3}$

Solve $2x^2-2\sqrt{6}x+3=0$

Solution : $2x^2-2\sqrt{6}x+3=0 \Rightarrow (\sqrt{2}x-\sqrt{3})(\sqrt{2}x-\sqrt{3})=0$ $\Rightarrow$ The solution is $x=\frac{\sqrt{3}}{\sqrt{2}}$

Solve $x^4-13x^2+42=0$

Solution : $(x^2)^2-13x^2+42=0 \Rightarrow (x^2-7)(x^2-6)=0 \Rightarrow x=\pm\sqrt{7}$ or $x=\pm\sqrt{6}$

Solve $3(p^2-6)=p(p+5)$ by factorization method

Solution : Given $3(p^2-6)=p(p+5) \Rightarrow 3p^2-18=p^2+5p \Rightarrow 2p^2-5p-18=0 \Rightarrow p=9/2, -2$

The number of volleyball games that must be scheduled in a league with n teams is given by $G(n)=\frac{n^2-n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

Solution : $G(n)=\frac{n^2-n}{2}=15 \Rightarrow n^2-n=30 \Rightarrow n^2-n-30=0 \Rightarrow (n-6)(n+5)=0 \Rightarrow n=6,-5$

$\therefore$ Number of terms in the league = 6

A ball rolls down a slope and travels a distance $dt=t^2-0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.

Solution : By data given, $t^2-0.75t-11.25=0$

$\Rightarrow (t-3.75)(t+3)=0$ $\therefore t=3.75$ sec

Solve $2x^2-3x-3=0$ by formula method

Solution :

$a=2, b=-3, c=-3$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x=\frac{-(-3)\pm\sqrt{(-3)\;\;-4(2)(-3)}}{2(2)}=\frac{3\pm\sqrt{33}}{4}$

Solve $x^2+2x-2=0$ by formula method

Solution : $a=1, b=2, c=-2$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}=\frac{-2\pm\sqrt{12}}{2}=-1\pm\sqrt{3}$

Solve $3p^2+2\sqrt{5}p-5=0$ by formula method

Solution : $a=3, b=2\sqrt{5}, c=-5$ $p=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow p=-2\sqrt{5}\pm\sqrt{\frac{(2\sqrt{5})^2-4(3)(-5)}{2(3)}}$

$\Rightarrow p=\frac{-2\sqrt{5}\pm\sqrt{80}}{6}=\frac{-\sqrt{5}\pm2\sqrt{5}}{3}=\frac{\sqrt{5}}{3}, -\sqrt{5}$

If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

Solution :

Let x be the required number $\frac{1}{x}$ be its reciprocal Given $x-\frac{1}{x}=\frac{24}{5}$

$\Rightarrow \frac{x^2-1}{x}=\frac{24}{5} \Rightarrow 5x^2-24x-5=0 \Rightarrow \therefore$ The required numbers are $5, -1/5$

Determine the nature of the roots $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0\ b \neq 0$

Solution : $A = 9a^2b^2, B = -24abcd, C = 16c^2d^2$

$\therefore \quad \Delta = B^2 - 4AC = 576\, a^2b^2c^2d^2 - 576\, a^2b^2c^2d^2 = 0$

$\therefore$ The roots are real & equal.

Determine the nature of the roots $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

Solution : $a = 9, b = -6\sqrt{2}, c = 2$

$\therefore \quad \Delta = b^2 - 4ac = (-6\sqrt{2})^2 - 4\,(9)\,(2) = 72 - 72 = 0$

$\therefore$ The roots are real & equal.

Determine the nature of the roots $9y^2 - 6\sqrt{2}\,y + 2 = 0$

Solution : $a = \sqrt{2}, b = -3, c = 3\sqrt{2}$

$\therefore \quad \Delta = b^2 - 4ac = 9 - 4\,(\sqrt{2})\,(3\sqrt{2}) = 9 - 24 = -15 < 0$

$\therefore$ The roots are unreal.

Discuss the nature of solutions of the equations $\frac{y+z}{4} = \frac{z+x}{3} = \frac{x+y}{2};\ x+y+z = 27$

Solution :

$\Rightarrow \frac{y+z}{4} = \frac{z+x}{3}$ & $\frac{z+x}{3} = \frac{x+y}{2} \Rightarrow 3y + 3z = 4z + 4x$ & $2z + 2x = 3x + 3y$

$\Rightarrow 4x - 3y + z = 0$ & $x + 3y - 2z = 0$

$\therefore 4x - 3y + z = 0$ ...... (1) $\quad x + 3y - 2z = 0$ ..... (2) $\quad x + y + z = 27$ .... (3)

From (1) & (2) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ *ie* $\frac{4}{1} \neq \frac{-3}{3} \neq \frac{-1}{2}$ From (2) & (3), $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ie $1 \neq 3 \neq -2$

$\therefore$ The system of equations has a unique solutions.

If $A = \begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$ find A+B.

Solution : A is of order 3×3 B is of order 3×2 It is not possible to add A and B because different orders.

Given $A = \begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$ **and if BA = C², find p and q.**

Solution : Given $BA = C^2$ $\Rightarrow \begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & -2q \\ p & 0 \end{pmatrix} = \begin{pmatrix} 0 & -8 \\ 8 & 0 \end{pmatrix}$

$\therefore p = 8, \quad -2q = -8, \quad q = 4$

If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ **prove that $AA^T = I$**

Solution : $A.A^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ Hence proved.

Verify that $A^2 = I$ when $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$

Solution : $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$ $\quad A^2 = A.A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix} = \begin{pmatrix} 25-24 & -20+20 \\ 30-30 & -24+25 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

Queen of science is mathematics

If $\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta\\ \cos\theta & x\end{pmatrix}=I_2$ **find x.**

Solution :

$$\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta\\ \cos\theta & x\end{pmatrix}=I_2 \Rightarrow \begin{pmatrix}\cos^2\theta+x\sin\theta & 0\\ 0 & \cos^2\theta+x\sin\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\Rightarrow \cos^2\theta + x\sin\theta = 1 \Rightarrow x\sin\theta = 1-\cos^2\theta \Rightarrow x\sin\theta=\sin^2\theta \Rightarrow x=\sin\theta$$

Let $A=\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$, $C=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ **Show that** $(A-B)^T=A^T-B^T$

Solution : $A=\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$, $C=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ $(A-B)^T=\begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}^T=\begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix}$ .... (1)

$$A^T-B^T=\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}-\begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}=\begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix} \text{.......(2)}$$

From (1) & (2) $(A-B)^T=A^T-B^T$

Show that the matrices $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ **satisfy commutative property** $AB=BA$

Solution :

$$AB=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}=\begin{pmatrix}1-6 & -2+2\\ 3-3 & -6+1\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$$BA=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}=\begin{pmatrix}1-6 & 2-2\\ -3+3 & -6+1\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$\therefore AB=BA$

$\therefore$ Commutative property is true.

If $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$ **then show that** $A^2+B^2=I$.

Solution : $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$

$$A^2=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta & 0\\ 0 & \cos^2\theta\end{pmatrix}$$

$$B^2=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}=\begin{pmatrix}\sin^2\theta & 0\\ 0 & \sin^2\theta\end{pmatrix}$$

$$\therefore A^2+B^2=\begin{pmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$ **$A^2+B^2=I$.**

If A is of order $p\times q$ and B is of order $q\times r$ what is the order of AB and BA?

Solution : A is of order $p\times q$ B is of order $q\times r$

$\therefore$ Order of AB $=p\times r$ Order of BA is not defined

Construct a 3×3 matrix whose elements are given by $a_{ij} = \frac{(i+j)^3}{3}$

$a_{11} = \frac{8}{3}$, $a_{12} = \frac{27}{3} = 9$, $a_{13} = \frac{64}{3}$, $a_{21} = \frac{27}{3} = 9$, $a_{22} = \frac{64}{3}$, $a_{23} = \frac{125}{3}$, $a_{31} = \frac{64}{3}$, $a_{32} = \frac{125}{3}$, $a_{33} = \frac{216}{3} = 72$

$$\therefore A = \begin{pmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{pmatrix}$$

If $\begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ **then find the transpose of A.** ***Solution :*** $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ $\therefore A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$

If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution : Given, a matrix has 18 elements. The possible orders $18 \times 1, 1 \times 18, 9 \times 2, 2 \times 9, 6 \times 3, 3 \times 6$

The matrix has 6 elements. The order are $1 \times 6, 6 \times 1, 3 \times 2, 2 \times 3$

If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ **then find the transpose of-A.**

Solution : $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ $\quad -A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$ $\quad \therefore$ Transpose of $-A = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$

Construct a 3×3 matrix whose elements are given by $a_{ij} = |i - 2j|$

Solution : Given $a_{ij} = |i - 2j|$, 3×3 $\quad A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

$a_{11} = |1-2| = |-1| = 1 \quad a_{12} = |1-4| = |-3| = 3$

$a_{13} = |1-6| = |-5| = 5 \quad a_{21} = |2-2| = 0$

$a_{22} = |2-4| = |-2| = 2 \quad a_{23} = |2-6| = |-4| = 4 \qquad \therefore A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$

$a_{31} = |3-2| = |1| = 1 \quad a_{32} = |3-4| = |-1| = 1$

$a_{33} = |3-6| = |-3| = 3$

If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ **then verify** $(A^T)^T = A$

Solution : $A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$ $\quad (A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix} = A$

Find the values of x, y and z from the following equations $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

Solution : $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix} \Rightarrow x = 3,\ y = 12,\ z = 3$

Find the values of x, y and z from the following equations $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution :

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix} \quad \begin{array}{l|l|l} \Rightarrow\ x+y+z=0 & x+z=5 & y+z=7 \\ \Rightarrow\ 5+y=9 & \Rightarrow\ x+3=5 & \Rightarrow\ 4+z=7 \\ \Rightarrow\ y=4 & \Rightarrow\ x=2 & \Rightarrow\ z=3 \end{array}$$

If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then Find $2A + B$

Solution : $2A+B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$

Find the values of x, y and z from the following equations $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

Solution : $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$ $\Rightarrow x+y=6,\quad xy=8,\quad 5+z=5 \quad \Rightarrow z=0$

$x = 2$ (or) 4, $\quad y = 4$ (or) 2

If $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$ then Find $4A - 3B$

Solution :

$$4A - 3B = 4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix} - 3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix} = \begin{pmatrix} 20 & 6 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} + \begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix} = \begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find $A + B$

Solution :

$$A+B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{pmatrix} = \begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

If $A = \begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$ compute $3A + 2B - C$

Solution :

$$3A + 2B - C = 3\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} + 2\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{pmatrix} + \begin{pmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{pmatrix} + \begin{pmatrix} -5 & -3 & 0 \\ 1 & 7 & -2 \\ -1 & -4 & -3 \end{pmatrix} = \begin{pmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{pmatrix}$$

If $A=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}, B=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}$ **then verify that $A+B=B+A$**

Solution :

$$\therefore A+B=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}+\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}=\begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix} \quad B+A=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}+\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}=\begin{pmatrix}6 & 16\\ 6 & 7\\ 9 & -3\end{pmatrix}$$

$$\therefore A+B=B+A$$

Find the value of a, b, c, d, x, y from the following matrix equation. $\begin{pmatrix}d & 8\\ 3b & a\end{pmatrix}+\begin{pmatrix}3 & a\\ -2 & -4\end{pmatrix}=\begin{pmatrix}2 & 2a\\ b & 4c\end{pmatrix}+\begin{pmatrix}0 & 1\\ -5 & 0\end{pmatrix}$

Solution : $\begin{pmatrix}d+3 & 8+a\\ 3b-2 & a-4\end{pmatrix}=\begin{pmatrix}2 & 2a+1\\ b-5 & 4c\end{pmatrix}$ $d+3=2 \Rightarrow d=2-3 \Rightarrow d=-1$

$8+a=2a+1 \Rightarrow 8-1=2a-a \Rightarrow a=7$

$3b-2=b-5 \Rightarrow 3b-b=-5+2 \Rightarrow 2b=-3 \Rightarrow b=\frac{-3}{2}$

Substituting a = 7 in $a-4=4c \Rightarrow 7-4=4c \Rightarrow 3=4c \Rightarrow c=\frac{3}{4}$

If $A=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}, B=\begin{pmatrix}5 & 7\\ 3 & 3\\ 1 & 0\end{pmatrix}$ **then verify that $A+(-A)=(-A)+A=O$**

Solution :

$$A+(-A)=\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}+\begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}=O$$

$$(-A)+A=\begin{pmatrix}-1 & -9\\ -3 & -4\\ -8 & 3\end{pmatrix}+\begin{pmatrix}1 & 9\\ 3 & 4\\ 8 & -3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}=O$$

If $A=\begin{pmatrix}1 & 8 & 3\\ 3 & 5 & 0\\ 8 & 7 & 6\end{pmatrix}, B=\begin{pmatrix}8 & -6 & -4\\ 2 & 11 & -3\\ 0 & 1 & 5\end{pmatrix}, C=\begin{pmatrix}5 & 3 & 0\\ -1 & -7 & 2\\ 1 & 4 & 3\end{pmatrix}$ compute $\frac{1}{2}A-\frac{3}{2}B$

Solution :

$$\frac{1}{2}A-\frac{3}{2}B=\frac{1}{2}(A-3B)=\frac{1}{2}\left(\begin{pmatrix}1 & 8 & 3\\ 3 & 5 & 0\\ 8 & 7 & 6\end{pmatrix}-3\begin{pmatrix}8 & -6 & -4\\ 2 & 11 & -3\\ 0 & 1 & 5\end{pmatrix}\right)=\begin{pmatrix}-\frac{23}{2} & 13 & \frac{15}{2}\\ -\frac{3}{2} & -14 & \frac{9}{2}\\ 4 & 2 & -\frac{9}{2}\end{pmatrix}$$

If $A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}, B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$ **find the value of $B-5A$**

Solution :

$$B-5A=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}-\begin{pmatrix}0 & 20 & 45\\ 40 & 15 & 35\end{pmatrix}=\begin{pmatrix}7 & -17 & -37\\ -39 & -11 & -26\end{pmatrix}$$

If $A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}, B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$ **find the value of $3A-9B$**

Solution :

$$3A-9B=\begin{pmatrix}0 & 12 & 27\\ 24 & 9 & 21\end{pmatrix}-\begin{pmatrix}63 & 27 & 72\\ 9 & 36 & 81\end{pmatrix}=\begin{pmatrix}-63 & -65 & -45\\ 15 & -27 & -60\end{pmatrix}$$

Find the non-zero values of x satisfying the matrix equation $x\begin{pmatrix}2x & 2\\3 & x\end{pmatrix}+2\begin{pmatrix}8 & 5x\\4 & 4x\end{pmatrix}=2\begin{pmatrix}x^2+8 & 24\\10 & 6x\end{pmatrix}$

Solution :

$$\Rightarrow \begin{pmatrix}2x^2 & 2x\\3x & x^2\end{pmatrix}+\begin{pmatrix}16 & 10x\\8 & 8x\end{pmatrix}=\begin{pmatrix}2x^2+16 & 48\\20 & 12x\end{pmatrix} \Rightarrow \begin{pmatrix}2x^2+16 & 12x\\3x+8 & x^2+8x\end{pmatrix}=\begin{pmatrix}2x^2+16 & 48\\20 & 12x\end{pmatrix}$$

$\therefore 12x = 48 \Rightarrow x = 4$

Construct a 3 × 3 matrix whose elements are $a_{ij} = i^2j^2$

Solution : The general 3 × 3 matrix is given by $A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{pmatrix}$ $\quad a_{ij}=i^2j^2$

$a_{11} = 1^2\times 1^2 = 1\times 1 = 1$; $a_{12} = 1^2\times 2^2 = 1\times 4 = 4$; $a_{13} = 1^2\times 3^2 = 1\times 9 = 9$

$a_{21} = 2^2\times 1^2 = 4\times 1 = 4$; $a_{22} = 2^2\times 2^2 = 4\times 4 = 16$; $a_{23} = 2^2\times 3^2 = 4\times 9 = 36$

$a_{31} = 3^2\times 1^2 = 9\times 1 = 9$; $a_{32} = 3^2\times 2^2 = 9\times 4 = 36$; $a_{33} = 3^2\times 3^2 = 9\times 9 = 81$

$A=\begin{pmatrix}1 & 4 & 9\\4 & 16 & 36\\9 & 36 & 81\end{pmatrix}$

If one root of the equation $2y^2 - ay + 64=0$ is twice the other then find the values of *a*.

Solution : Let α, β be the roots of $2y^2 - ay + 64 = 0$ $\quad \alpha+\beta=a/2 \quad \alpha\beta=32$

Given $\alpha = 2\beta$ $\quad \alpha\beta = 32 \Rightarrow 2\beta\,.\,\beta = 32$ $\quad \therefore \alpha+\beta=a/2 \Rightarrow 3\beta=a/2 \Rightarrow \beta=a/6$

$\Rightarrow \beta^2 = 16 \Rightarrow \beta = \pm 4$

$\therefore a/6=\pm 4 \Rightarrow a=\pm 24$

The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Solution : Let the present age of Kumaran ⇒x years.

Two years ago, (x − 2) years. Four years from now, (x + 4) years.

Given, $(x-2)(x+4) = 1 + 2x \Rightarrow x^2 + 2x - 8 = 1 + 2x$ gives $(x-3)(x+3) = 0$

Kumaran's present age is 3 years.

If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17 find k.

Solution : $x^2 - 13x + k = 0$ here, a = 1, b = −13, c = k

$\alpha+\beta=\frac{-b}{a}=\frac{-(-13)}{1}=13$ ...... (1) $\qquad \alpha-\beta = 17$ .... (2)

(1) + (2) we get, $2\alpha = 30$ gives $\alpha = 15$ Therefore, $15 + \beta = 13$ (from (1)) gives $\beta = -2$

But $\alpha\beta=\frac{c}{a}=\frac{k}{1}$ gives $15\times(-2)=k$ $\quad$ we get, $k=-30$

A has 'a' rows and 'a + 3' columns. B has 'b' rows and '17–b' columns, and if both products AB and BA exist, find a, b?

Solution : Given Order of A is $a\times(a+3)$ Order of B is $b\times(17-b)$

$\Rightarrow a+3=b \Rightarrow a-b=-3$........ (1) $\qquad$ Solving (1) & (2) $2a = 14$ $\quad a = 7$

$\Rightarrow 17-b=a \Rightarrow a+b=17$ .... (2) $\qquad$ Sub a = 7 in (1) $7-b=-3$ $\quad b = 10$

In the matrix A = $\begin{pmatrix}8 & 9 & 4 & 3\\-1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5\\1 & 4 & 3 & 0\\6 & 8 & -11 & 1\end{pmatrix}$ **write** , **(i) The number of elements (ii) The order of the matrix (iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$**

Solution : i) A has 4 rows and 4 columns $\quad$ Number of elements = 16

ii) Order of the matrix = 4 × 4

iii) $a_{22} = \sqrt{7}$, $a_{23} = \sqrt{3}/2$ $\quad a_{24} = 5$ $a_{34} = 0$, $a_{43} = -11$, $a_{44} = 1$

Simplify $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$

Solution : $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50} = \frac{2a^2+5a+3}{2a^2+7a+6} \times \frac{-5a^2-35a-50}{a^2+6a+5} = \frac{(2a+3)(a+1)}{(2a+3)(a+2)} \times \frac{-5(a^2+7a+10)}{(a+5)(a+1)}$

$= \frac{\cancel{(2a+3)}\ \cancel{(a+1)}}{\cancel{(2a+3)}\ \cancel{(a+2)}} \times \frac{-5\cancel{(a+5)}\ \cancel{(a+2)}}{\cancel{(a+5)}\ \cancel{(a+1)}} = -5$

Find the excluded values of $\frac{x+10}{8x}$ expressions

Solution : $\frac{x+10}{8x}$ is undefined when $8x = 0$ or $x = 0$. Hence the excluded value is 0.

Find the excluded values of $\frac{x}{x^2+1}$ expressions

Solution : $x^2 + 1 \neq 0$ for any x. Therefore, no real excluded values

Reduce to lowest form. $\frac{x^2-1}{x^2+x}$ ***Solution :*** $\frac{x^2-1}{x^2+x} = \frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}} = \frac{x-1}{x}$

Find the LCM and GCD for (x^2y+xy^2), (x^2+xy) and verify that $f(x) \times g(x) = \text{LCM} \times \text{GCD}$

Solution : $f(x) = x^2y + xy^2 = xy(x+y)$ $g(x) = x^2 + xy = x(x+y)$

$\therefore$ GCD $= x(x+y)$ $\therefore$ LCM $= xy(x+y)$

$\because f(x) \times g(x) = xy(x+y) \times x(x+y) = x^2y(x+y)^2 = \text{LCM} \times \text{GCD}$

Find the LCM of $a^2+4a-12$, a^2-5a+6 whose GCD is $a-2$

Solution : GCD $= a-2$ Let $f(x) = a^2+4a-12 = (a+6)(a-2)$ $g(x) = a^2-5a+6 = (a-3)(a-2)$

$\therefore \text{LCM} = \frac{f(x)\times g(x)}{GCD} = \frac{(a+6)(a-2)\times(a-3)\cancel{(a-2)}}{\cancel{a-2}} = (a+6)(a-3)(a-2)$

The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

Solution : Let the present age of father be x years **and** the present age of son be y years

Given, $x = 6y$ — (1) $x+6 = 4(y+6)$ — (2)

Substituting (1) in (2), $6y + 6 = 4(y+6)$

$6y + 6 = 4y + 24$ gives, $y = 9$

son's age = 9 years and father's age = 54 years.

Solve $2x - 3y = 6$, $x + y = 1$

Solution : $2x - 3y = 6$ — (1)

$x + y = 1$ — (2)

(1) × 1 gives, $2x - 3y = 6$

(2) × 2 gives, $2x + 2y = 2$

(−) (−) (−)

$-5y = 4$ gives, $y = \frac{-4}{5}$

$y = \frac{-4}{5}$ in (2),

$x - \frac{4}{5} = 1 = 1 + \frac{4}{5} = \frac{9}{5}$

Find the LCM of $x^4 - 1$, $x^2 - 2x + 1$

Solution : $x^4 - 1 = (x^2+1)(x+1)(x-1)$

$x^2 - 2x + 1 = (x-1)(x-1) = (x-1)^2$

LCM $= (x^2+1)(x+1)(x-1)^2$

Find the LCM of x^3-27, $(x-3)^2$, x^2-9

Solution : $x^3 - 27 = (x-3)(x^2+3x+9)$ | $(x-3)^2 = (x-3)^2$ | $(x^2-9) = (x+3)(x-3)$

LCM $= (x-3)^2(x+3)(x^2+3x+9)$

Find the LCM of $p^2 - 3p + 2$, $p^2 - 4$

Solution : $p^2 - 3p + 2 = (p-2)(p-1)$ | $p^2 - 4 = (p-2)(p+2)$

$\therefore$ LCM $= (p-2)(p-1)(p+2)$

Find the LCM and GCD for $(x^3-1)(x+1)$, (x^3+1) and verify that $f(x) \times g(x) = \text{LCM} \times \text{GCD}$

Solution : $f(x) = (x^3-1)(x+1) = (x-1)(x^2+x+1)\underline{(x+1)}$ $g(x) = x^3+1 = \underline{(x+1)}(x^2-x+1)$

$\therefore$ GCD $= x+1$ $\therefore$ LCM $= (x^3+1)(x^3-1) = x^6 - 1$

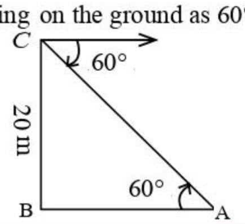
$f(x) \times g(x) = (x-1)(x^2+x+1)(x+1)(x+1)(x^2-x+1) = (x+1)(x^6-1) = \text{LCM} \times \text{GCD}$

Simplify $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

Solution :

$\frac{x^3}{x-y} + \frac{y^3}{y-x} = \frac{x^3}{x-y} - \frac{y^3}{x-y} = \frac{x^3-y^3}{x-y} = \frac{\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{x-y}} = x^2+xy+y^2$

A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3}=1.732$)

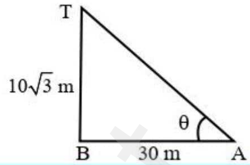


Solution : $\tan 60^0=\dfrac{BC}{AB} \Rightarrow \sqrt{3}=\dfrac{20}{\text{AB}} \Rightarrow \text{AB}=\dfrac{20}{\sqrt{3}} \Rightarrow \text{AB}=\dfrac{20\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\dfrac{20\times 1.732}{3}$

$=11.54\text{m}$

distance between the foot of the tower and the ball is 11.54 m.

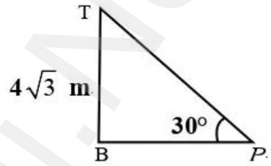
Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.



Solution :

$\tan\theta=\dfrac{10\sqrt{3}}{30} \Rightarrow \tan\theta=\dfrac{\sqrt{3}}{3} \Rightarrow \tan\theta=\dfrac{1}{\sqrt{3}} \quad \therefore \theta=30^0$

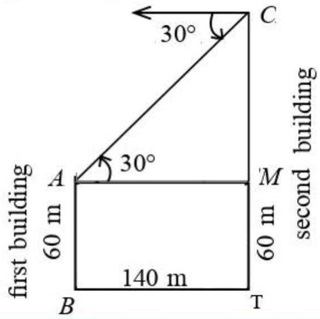
A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}$ m with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road.



Solution :
$\tan 30^0=\dfrac{4\sqrt{3}}{PB} \Rightarrow \dfrac{1}{\sqrt{3}}=\dfrac{4\sqrt{3}}{PB} \Rightarrow PB=12\text{ m}$

$\therefore$ Width of the road $=2\ PB=2\,(12)=24$ m

The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30°. If the height of the first building is 60 m, find the height of the second building. ($\sqrt{3}=1.732$)



Solution :

$\tan 30^0=\dfrac{CM}{140}$

$\dfrac{1}{\sqrt{3}}=\dfrac{CM}{140}$

$CM=\dfrac{140}{\sqrt{3}}=\dfrac{140\sqrt{3}}{3}=\dfrac{140\times 1.732}{3}$

$CM=80.78$

height of the second building $=60+80.78=140.78$ m

Prove $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

Solution : $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=(\sec\theta+\cancel{\tan\theta})+(\sec\theta-\cancel{\tan\theta})=\sec\theta+\sec\theta=2\sec\theta$

Prove that $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}=\operatorname{cosec}\theta+\cot\theta$

Solution : $\sqrt{\dfrac{1+\cos\theta}{1\ \ \cos\theta}}=\sqrt{\dfrac{(1+\cos\theta)\times(1+\cos\theta)}{(1-\cos\theta)\times(1+\cos\theta)}}=\sqrt{\dfrac{(1+\cos\theta)^2}{\sin^2\theta}}=\dfrac{1+\cos\theta}{\sin\theta}=\operatorname{cosec}\theta+\cot\theta$

Prove $\tan^4\theta+\tan^2\theta=\sec^4\theta-\sec^2\theta$

Solution : $\tan^2\theta\,(\tan^2\theta+1)=\sec^2\theta\,.\,(\sec^2\theta-1) \Rightarrow \tan^2\theta\sec^2\theta=\tan^2\theta\sec^2\theta$

Prove that $\left(\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\dfrac{1-\cos\theta}{1+\cos\theta}$

Solution : Take $1+\sin\theta=a \quad \cos\theta=b \quad \therefore \dfrac{(a-b)^2}{(a+b)^2}=\dfrac{a^2+b^2-2ab}{a^2+b^2+2ab}=\dfrac{2\cancel{(1+\sin\theta)}\,[1-\cos\theta]}{2\cancel{(1+\sin\theta)}\,[1+\cos\theta]}=\dfrac{1-\cos\theta}{1+\cos\theta}$

Prove $\dfrac{1-\tan^2\theta}{\cot^2\theta-1}=\tan^2\theta$

Solution : $1-\tan^2\theta=\tan^2\theta\,(\cot^2\theta-1) \Rightarrow 1-\tan^2\theta=\tan^2\theta\cot^2\theta-\tan^2\theta \Rightarrow 1-\tan^2\theta=1-\tan^2\theta$

Prove that $\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$

Solution : $\tan^2\theta-\sin^2\theta=\dfrac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta=\sin^2\theta\left(\dfrac{1}{\cos^2\theta}-1\right)=\sin^2\theta(\sec^2\theta-1)=\tan^2\theta\sin^2\theta$

If cotθ + tanθ = x and secθ – cosθ = y, then prove that $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

Solution : $\therefore\ x^2y = \sec^3\theta$ and $xy^2 = \tan^3\theta \Rightarrow (x^2y)^{2/3} - (xy^2)^{2/3} = (\sec^3\theta)^{2/3} - (\tan^3\theta)^{2/3} = \sec^2\theta - \tan^2\theta = 1$

Prove $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

Solution : $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}\times\dfrac{1+\sin\theta}{1+\sin\theta}} = \sqrt{\dfrac{(1+\sin\theta)^2}{\cos^2\theta}} = \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

Prove $\dfrac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

Solution : $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{\cos\theta}{1+\sin\theta}\times\dfrac{1-\sin\theta}{1-\sin\theta} = \dfrac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \dfrac{\cancel{\cos\theta}(1-\sin\theta)}{\cancel{\cos\theta}\cos\theta} = \dfrac{1-\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta$

Prove that **cot θ + tan θ = sec θ cosec θ**

Solution: $\cot\theta + \tan\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos\theta\,\cos\theta + \sin\theta\,\sin\theta}{\sin\theta.\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta} = \dfrac{1}{\sin\theta.\cos\theta} = \sec\theta.\operatorname{cosec}\theta$

Prove that $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

Solution :

$\tan^2 A - \tan^2 B = \dfrac{\sin^2 A}{\cos^2 A} - \dfrac{\sin^2 B}{\cos^2 B} = \dfrac{\sin^2 A\cos^2 B - \sin^2 B\cos^2 A}{\cos^2 A\cos^2 B} = \dfrac{\sin^2 A(1-\sin^2 B) - \sin^2 B(1-\sin^2 A)}{\cos^2 A\cos^2 B}$

$= \dfrac{\sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B}{\cos^2 A\cos^2 B} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

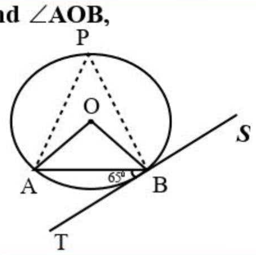
Prove that $\left(\dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$

Solution : LHS $= \left(\dfrac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \cos A\sin A)}{\cos A - \sin A}\right) - \left(\dfrac{(\cos A + \sin A)(\cos^2 A + \sin^2 A - \cos A\sin A)}{\cos A + \sin A}\right)$

$= (1 + \cos A\sin A) - (1 - \cos A\sin A) = 2\cos A\sin A$

A tangent ST to a circle touches it at B. AB is a chord such that ∠ABT = 65⁰. Find ∠AOB, where "O" is the centre of the circle.

Solution:



(angles in altemate segment).

$\angle TBA = 65^0 \Rightarrow \angle APB = 65^0$

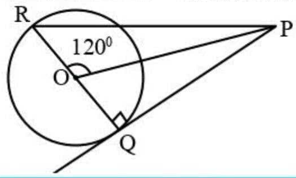
$\therefore \angle AOB = 2\angle APB = 2(65^0) = 130^0$

PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120⁰. Find ∠OPQ .

Solution: Given $\angle POR = 120^0 \Rightarrow \angle POQ = 60^0$ (linear pair)

$\angle OQP = 90^0$ (Radius ⊥ tangent)

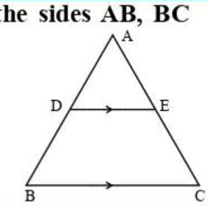
$\therefore \angle OPQ = 90^0 - 60^0 = 30^0$



O is any point inside a triangle ABC. The bisector of ∠AOB , ∠BOC and ∠COA meet the sides AB, BC and CA in point D, E and F respectively. Show that AD × BE × CF = DB × EC × FC

Solution : By using Ceva's Theorem, $\dfrac{AD}{DB}\times\dfrac{BE}{EC}\times\dfrac{AF}{FC} = 1$

$\Rightarrow$ AD × BE × AF = DB × EC × FC Hence proved.



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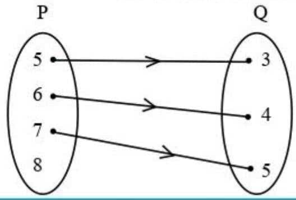
life is a good circle,
you choose the best radius...

The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



Solution : (i) Set builder form of $R = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain of $R = \{5, 6, 7\}$ and range of $R = \{3, 4, 5\}$

Let A = {1, 2, 3, 4 ....., 45} and R be the relation defined as "is square of" on A. Write R as a subset of A × A. Also, find the domain and range of R.

Solution : $A = \{1, 2, 3, 4, \ldots\ldots.. 45\}$ R : "is square of" $R = \{1, 4, 9, 16, 25, 36\}$ Clearly R is a subset of A.

$\therefore$ Domain $= \{1, 2, 3, 4, 5, 6\}$ $\therefore$ Range $= \{1, 4, 9, 16, 25, 36\}$

A Relation R is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.

Solution : Given $R = \{(x, y) / y = x + 3,$

$x \in \{0, 1, 2, 3, 4, 5\}\}$

$x = 0 \Rightarrow y = 3$ $x = 1 \Rightarrow y = 4$

$x = 2 \Rightarrow y = 5$ $x = 3 \Rightarrow y = 6$

$x = 4 \Rightarrow y = 7$ $x = 5 \Rightarrow y = 8$

$\therefore R = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$

$\therefore$ Domain : $\{0, 1, 2, 3, 4, 5\}$

Range : $\{3, 4, 5, 6, 7, 8\}$

Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible. $\{(x, y) \mid y = x + 3, x, y$ are natural numbers $< 10\}$

Solution : x, y **are natural numbers < 10**

$y = x + 3$

$x = 1 \Rightarrow y = 4$

$x = 2 \Rightarrow y = 5$

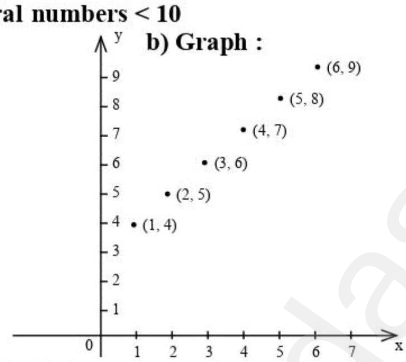
$x = 3 \Rightarrow y = 6$

$x = 4 \Rightarrow y = 7$

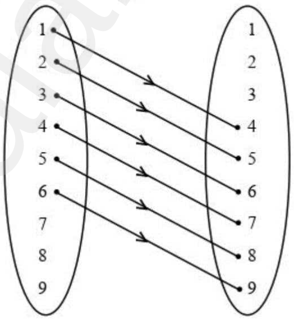
$x = 5 \Rightarrow y = 8$

$x = 6 \Rightarrow y = 9$

b) Graph :



a) Arrow Diagram :



c) a set in roster : $\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible. $\{(x, y) \mid x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}$

Solution :

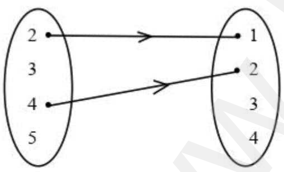
$y = 1 \Rightarrow x = 2$

$y = 2 \Rightarrow x = 4$

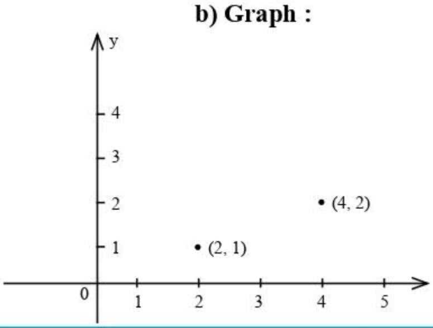
$y = 3 \Rightarrow x = 6$

$y = 4 \Rightarrow x = 8$

a) Arrow diagram :



b) Graph :

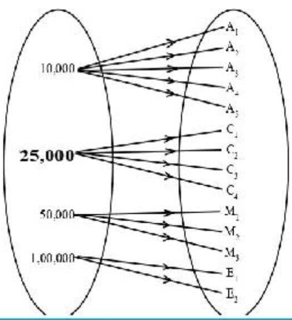


c) a set in roster : $\{(2, 1), (4, 2)\}$

A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants ; C_1, C_2, C_3, C_4 were Clerks ; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.

Solution : **a) Ordered Pair :** $\{$(₹10,000, A_1), (₹10,000, A_2), (₹10,000, A_3), (₹10,000, A_4), (₹10,000, A_5), (₹25,000, C_1), (₹25,000, C_2), (₹25,000, C_3), (₹25,000, C_4), (₹50,000, M_1), (₹50,000, M_2), (₹50,000, M_3), (₹1,00,000, E_1), (₹1,00,000, E_2)$\}$

b) arrow diagram.



Find the values of x, y, z if $(x \quad y-z \quad z+3)+(y \quad 4 \quad 3)=(4 \quad 8 \quad 16)$

Solution : $(x \quad y-z \quad z+3)+(y \quad 4 \quad 3)=(4 \quad 8 \quad 16)$

$\Rightarrow x+y=4$	$y-z+4=8$	$z+6=16$
$\Rightarrow x+14=4$	$y-z=4$	$z=10$
$\Rightarrow x=-10$	$y-10=4$	
	$\therefore y=14$	

$\therefore x=-10, y=14, z=10$

Find x and y if $x\begin{pmatrix}4\\-3\end{pmatrix}+y\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}4\\6\end{pmatrix}$

Solution :

Given $x\begin{pmatrix}4\\-3\end{pmatrix}+y\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}4\\6\end{pmatrix}$ $\quad 4x-2y=4 \;\ldots\ldots\; (1)$ $\quad -3x+3y=6 \;\ldots\ldots\; (2)$

$\Rightarrow 2x-y=2 \;\ldots\ldots\; (1)$

$\Rightarrow -x+y=2 \;\ldots\ldots\; (2)$

(1) $\Rightarrow 2x-y=2$

(2) $\Rightarrow -x+y=2$

Adding, $x=4$

Sub $x=4$ in (2)

$-4+y=2 \Rightarrow y=6$

$\therefore x=4, y=6$

Find the values of x, y, z if $\begin{pmatrix}x-3 & 3x-z\\ x+y+7 & x+y+z\end{pmatrix}=\begin{pmatrix}1 & 0\\ 1 & 6\end{pmatrix}$

Solution : $\Rightarrow x-3=1$, $\therefore x=4$ | $3x-z=0$, $12-z=0 \Rightarrow z=12$ | $x+y+7=1$ $\Rightarrow x+y=-6 \Rightarrow 4+y=-6 \Rightarrow y=-10$

At t minutes past 2 pm, the time needed to 3 pm is 3 minutes less than $\dfrac{t^2}{4}$ **Find t.**

Solution :

$\therefore$ As per the data given, $\dfrac{t^2}{4}-3=60-t \Rightarrow t^2-12=240-4t \Rightarrow t^2+4t-252=0$

$\Rightarrow (t+18)(t-14)=0 \quad \therefore t=14$ min.

If a matrix has 16 elements, what are the possible orders it can have?

Solution : The matrix has 16 elements . Hence possible orders are $1\times16,\ 16\times1,\ 4\times4,\ 2\times8,\ 8\times2$.

If $A=\begin{pmatrix}1 & 2 & 0\\ 3 & 1 & 5\end{pmatrix}, B=\begin{pmatrix}8 & 3 & 1\\ 2 & 4 & 1\\ 5 & 3 & 1\end{pmatrix}$ find AB.

Solution :

$$AB=\begin{pmatrix}1 & 2 & 0\\ 3 & 1 & 5\end{pmatrix}\times\begin{pmatrix}8 & 3 & 1\\ 2 & 4 & 1\\ 5 & 3 & 1\end{pmatrix}=\begin{pmatrix}8+4+0 & 3+8+0 & 1+2+0\\ 24+2+25 & 9+4+15 & 3+1+5\end{pmatrix}=\begin{pmatrix}12 & 11 & 3\\ 51 & 28 & 9\end{pmatrix}$$

If $A=\begin{pmatrix}2 & -2\sqrt{2}\\ \sqrt{2} & 2\end{pmatrix}$ and $B=\begin{pmatrix}2 & 2\sqrt{2}\\ -\sqrt{2} & 2\end{pmatrix}$ Show that A and B satisfy commutative property with respect to matrix multiplication.

Solution :

$$AB=\begin{pmatrix}2 & -2\sqrt{2}\\ \sqrt{2} & 2\end{pmatrix}\times\begin{pmatrix}2 & 2\sqrt{2}\\ -\sqrt{2} & 2\end{pmatrix}=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix} \quad BA=\begin{pmatrix}2 & 2\sqrt{2}\\ -\sqrt{2} & 2\end{pmatrix}\times\begin{pmatrix}2 & -2\sqrt{2}\\ \sqrt{2} & 2\end{pmatrix}=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}$$

A and B satisfy commutative property

Solve $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Solution :

By matrix multiplication $\begin{pmatrix} 2x+y \\ x+2y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $2x + y = 4$ .... (1)

$x + 2y = 5$ .... (2)

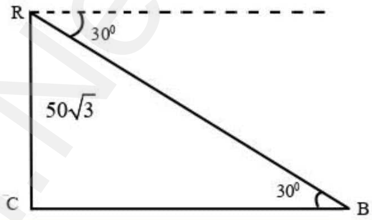
$(1) - 2 \times (2)$ gives $2x + y = 4$

$2x + 4y = 10$ $(-)$

$-3y = -6$ gives $y = 2$

Substituting $y = 2$ in (1), $2x + 2 = 4$ gives $x = 1$ Therefore, $x = 1, y = 2$.

From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.



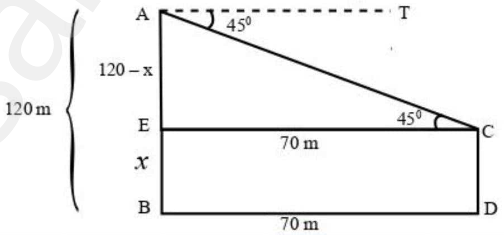
Solution:

$$\tan 30^0 = \frac{RC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{CB}$$

$$\Rightarrow CB = 50\sqrt{3}\sqrt{3} \Rightarrow CB = 150m$$

$\therefore$ Dist. of the car from the rock = 150m

The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.



Solution: $\tan 45^0 = \frac{AE}{EC} \Rightarrow 1 = \frac{120-x}{70}$

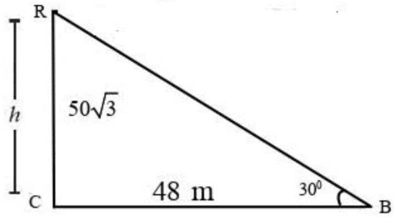
$$\Rightarrow 70 = 120 - x$$

$$\Rightarrow x = 120 - 70$$

$$\Rightarrow x = 50$$

$\therefore$ Height of 1st building = 50 m

A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is30°. Find the height of the tower.

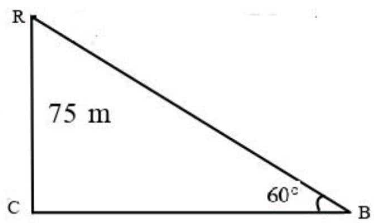


Solution : $\tan 30^0 = \frac{h}{48}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{48} \Rightarrow h = 16\sqrt{3}$$

The height of the tower is $16\sqrt{3}$ m

A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.



Solution : $\sin 60^0 = \frac{75}{RB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{RB}$

$$\Rightarrow RB = \frac{150}{\sqrt{3}} = 50\sqrt{3}$$

length of the string is $50\sqrt{3}$ m

If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Solution : base area = πr^2 = 1386 sq. m

T.S.A. = $3\pi r^2$ sq.m $= 3 \times 1386 = 4158$ m^2.

Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$.

Solution : $f(x) = 2x + 1, g(x) = x^2 - 2$

$f \circ g = (2x + 1)(x^2 - 2) = 2(x^2 - 2) + 1 = 2x^2 - 3$

$g \circ f = (x^2 - 2)(2x + 1) = (2x + 1)^2 - 2 = 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1 \quad \therefore f \circ g \neq g \circ f$

Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Solution : $f_2(x) = 2x^2 - 5x + 3$ and $f_1(x) = \sqrt{x}$

$f(x) = \sqrt{2x^2 - 5x + 3} = \sqrt{f_2(x)} = f_1 f_2(x)$

Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Solution : $f \circ f(k) = (2k - 1)(2k - 1) = 2(2k - 1) - 1 = 4k - 3$. But, $f \circ f(k) = 5$

Therefore $4k - 3 = 5 \Rightarrow 4k = 5 + 3 \Rightarrow 4k = 8 \Rightarrow k = 2$.

If $f(x) = x - 6, g(x) = x^2$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = x - 6, g(x) = x^2$

$(f \circ g) = (x - 6)(x^2) = x^2 - 6$

$(g \circ f) = (x^2)(x - 6) = (x - 6)^2 = x^2 - 12x + 36$

$\therefore f \circ g \neq g \circ f$ $\quad$ **, $g(x) = 1 + x$**

If $f(x) = 4x^2 - 1, g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = 4x^2 - 1, g(x) = 1 + x$

$(f \circ g) = (4x^2 - 1)(1 + x) = 4(1 + x^2) - 1 = 4(1 + x^2 + 2x) - 1 = 4x^2 + 8x + 3$

$(g \circ f) = (1 + x)(4x^2 - 1) = 1 + 4x^2 - 1 = 4x^2 \quad \therefore f \circ g \neq g \circ f$

If $f(x) = 4x^2 - 1, g(x) = 1 + x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

$(f \circ g) = \left(\frac{2}{x}\right)(2x^2 - 1) = \frac{2}{2x^2 - 1}$

$(g \circ f) = \left(\frac{2}{x}\right)(2x^2 - 1) = 2\left(\frac{2}{x}\right)^2 - 1 = \frac{8}{x^2} - 1 \quad \therefore f \circ g \neq g \circ f$

If $f(x) = \frac{x+6}{3}, g(x) = 3 - x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution : $f(x) = \frac{x+6}{3}, g(x) = 3 - x$

$(f \circ g) = \left(\frac{x+6}{3}\right)(3 - x) = \frac{(3 - x) + 6}{3} = \frac{9 - x}{3}$

$(g \circ f)(x) = \left(\frac{x+6}{3}\right)(3 - x) = 3 - \frac{x+6}{3} = \frac{9 - x - 3}{3} = \frac{6 - x}{3} \quad \therefore f \circ g \neq g \circ f$

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If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a, if $g \circ f(a) = 1$.

Solution : $f(x) = x^2 - 1$, $g(x) = x - 2$

Given $g \circ f(a) = 1 \Rightarrow (x-2)(x^2-1) = 1 \Rightarrow (x-2)(a^2-1) = 1$

$\Rightarrow a^2 - 1 - 2 = 1 \Rightarrow a^2 - 3 = 1 \Rightarrow a^2 = 4 \quad \therefore a = \pm 2$

Find k, if $f(k) = 2k - 1$ and $f \circ f(k) = 5$.

Solution : $f(k) = 2k - 1 \Rightarrow f \circ f(k) = 5 \Rightarrow (2k-1)(2k-1) = 5 \Rightarrow 2(2k-1) - 1 = 5$

$\Rightarrow 4k - 2 = 6 \Rightarrow 4k = 8 \quad \therefore k = 2$

If $f(x) = 2x - k$, $g(x) = 4x + 5$ Find k, $f \circ g = g \circ f$

$\Rightarrow (f \circ g) = (g \circ f) \Rightarrow (2x-k)(4x+5) = (4x+5)(2x-k)$

$\Rightarrow 2(4x+5) - k = 4(2x-k) + 5$

$\Rightarrow 8x + 10 - k = 8x - 4k + 5 \Rightarrow 10 - k = -4k + 5 \Rightarrow -k + 4k = 5 - 10 \Rightarrow 3k = -5$

Let A, B, C $\subseteq$ N and a function $f : A \to B$ be defined by $f(x) = 2x + 1$ and $g : B \to C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$.

Solution : $f : A \to B$, $g : B \to C$ where A, B, C $\subseteq$ N. $f(x) = 2x + 1$, $g(x) = x^2$

Range of $f \circ g$ $= (2x+1)(x^2) = 2x^2 + 1$ $\therefore$ Range of $f \circ g = \{y / y = 2x^2 + 1, x \in N\}$.

Range of $g \circ f$ $= (x^2)(2x+1) = (2x+1)^2$ $\therefore$ Range of $g \circ f = \{y/y = (2x+1)^2, x \in N\}$.

Let $f(x) = x^2 - 1$. Find $f \circ f$

Solution :

Given $f(x) = x^2 - 1$

a) $f \circ f = ?$

$(f \circ f) = (x^2-1)(x^2-1) = (x^2-1)^2 - 1 = x^4 - 2x^2 + 1 - 1 = x^4 - 2x^2$

Let $f(x) = x^2 - 1$. Find $f \circ f \circ f$

Solution :

$(f \circ f \circ f) = (x^2-1)(x^2-1)(x^2-1) = (x^4 - 2x^2)(x^2-1) = (x^2-1)^4 - 2(x^2-1)^2 = (x^4-2x^2)^2 - 1$

If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one ?

Solution : Let A be the domain. B be the co-domain.

For every element $\in$ A, there is a unique image in B. Since f is an odd function $\therefore f$ is $1 - 1$.

But $g(x)$ is an even function.

$\therefore$ Two elements of domain will have the since image in co-domain. $\therefore g$ is not $1 - 1$.

Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from Z into Z. Find $f(x)$.

Solution : Given $f = \{(-1, 3), (0, -1), (2, -9)\}$ is a linear function from Z into Z.

Let $y = ax + b$ When $x = -1, y = 3 \Rightarrow 3 = -a + b$ — (1)

When $x = 0$, $y = -1 \Rightarrow -1 = 0 + b \quad \therefore b = -1 \quad \therefore (1) \Rightarrow 3 = -a - 1 \Rightarrow a = -4$

$\therefore a = -4, b = -1$

$\therefore y = -4x - 1$ is the required linear function.

In electrical circuit theory, a circuit C(t) is called a linear circuit if it satisfies the superposition principle given by C $(at_1 + bt_2) = a$C$(t_1) + b$C(t_2), where a, b are constants. Show that the circuit C(t) = 3t is linear.

Solution : Given C(t) = 3t To Prove : C(t) is linear.

$C(at_1) = 3at_1$, $C(bt_2) = 3bt^2$ Adding,

$C(at_1) + C(bt_2) = 3at_1 + 3bt_2 = 3(at_1 + bt_2)$ $\therefore$ C(t) = 3t is a linear function.

In ΔABC, with $B = 90°$, $BC = 6$ cm and $AB = 8$ cm, D is a point on AC such that $AD = 2$ cm and E is the midpoint of AB. Join D to E and extend it to meet at F. Find BF.

Solution: Given In ΔABC, $AB = 8$ cm, $BC = 6$ cm

$$\therefore AC = \sqrt{64+36} = \sqrt{100} = 10$$

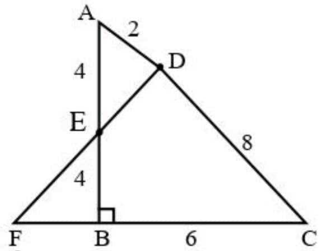
$AD = 2 \Rightarrow CD = 8$ cm

E is the mid point of AB $\Rightarrow$ $AE = EB = 4$ cm

By Menelaus Theorem, $\frac{AE}{EB} \times \frac{BF}{FC} \times \frac{CD}{DA} = 1 \Rightarrow \frac{\cancel{4}}{\cancel{4}} \times \frac{BF}{BF+6} \times \frac{8}{2} = 1$

$\Rightarrow 4BF = BF + 6 \Rightarrow 3BF = 6$

$\therefore BF = 2$ cm

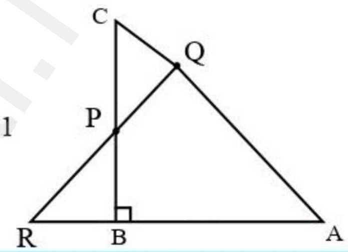


In a garden containing several trees, three particular trees P, Q, R are located in the following way, $BP = 2$ m, $CQ = 3$ m, $RA = 10$ m, $PC = 6$ m, $QA = 5$ m, $RB = 2$ m, where A, B, C are points such that P lies on BC, Q lies on AC and R lies on AB. Check whether the trees P, Q, R lie on a same straight line.

Solution : By Meanlau's theorem, $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{RA}{RB} = 1 \quad \ldots(1)$

$$\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{RA}{RB} = \frac{2}{6} \times \frac{3}{5} \times \frac{10}{2} = \frac{60}{60} = 1$$

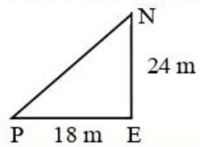
Hence The trees P, Q, R lie on a same straight line.



A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?

Solution: $PN = \sqrt{18^2 + 24^2} = \sqrt{324 + 576} = \sqrt{900} = 30\ m$

$\therefore$ Distance of his current position from the starting point = 30 m



What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

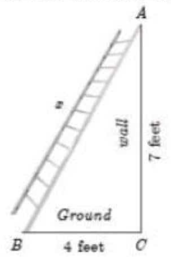
Solution : Let x be the length of the ladder.

By Pythagoras theorem, $x^2 = 7^2 + 4^2$

$x^2 = 49 + 16$

$x = \sqrt{65}$

Therefore, length of the ladder is approximately 8.1 ft.



If ΔABC is similar to ΔDEF such that $BC = 3$ cm, $EF = 4$ cm and area of $\Delta ABC = 54$ cm^2. Find the area of ΔDEF.

Solution : $\frac{Area\ (\Delta ABC)}{Area\ (\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{Area\ (\Delta DEF)} = \frac{3^2}{4^2} \Rightarrow \frac{54}{Area\ (\Delta DEF)} = \frac{9}{16}$

$\Rightarrow Area\ (\Delta DEF) = \frac{16 \times 54}{9} = 96\ cm^2$

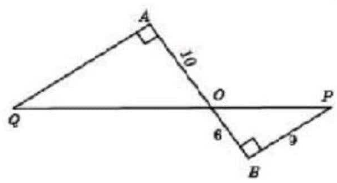
In Fig. QA and PB are perpendiculars to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ.

Solution : In ΔAOQ and ΔBOP, $\angle OAQ = \angle OBP = 90^0$

$\angle AOQ = \angle BOP$ (Vertically opposite angles)

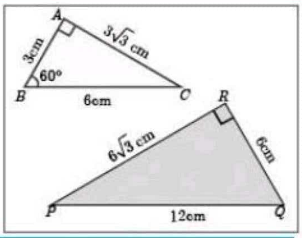
by AA Criterion $\Delta AOQ \sim \Delta BOP$

$$\frac{AO}{BO} = \frac{AQ}{BP} = \frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = \frac{10 \times 9}{6} = 15cm$$



Observe Fig. and find $\angle P$.

Solution : In ΔBAC and ΔPRQ, $\frac{AB}{RQ}=\frac{3}{6}=\frac{1}{2}$; $\frac{BC}{QP}=\frac{6}{12}=\frac{1}{2}$; $\frac{CA}{PR}=\frac{3\sqrt{3}}{6\sqrt{3}}=\frac{1}{2}$



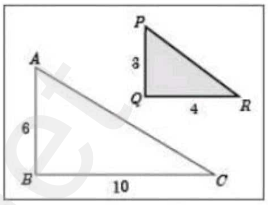
By SSS similarity, $\Delta BAC \sim \Delta QRP$

$$\frac{AB}{RQ}=\frac{BC}{QP}=\frac{CA}{PR}$$

$$\angle P = 180^0 - 150^0 = 30^0$$

Is $\Delta ABC \sim \Delta PQR$?

Solution : In ΔABC and ΔPQR, $\frac{PQ}{AB}=\frac{3}{6}=\frac{1}{2}$,

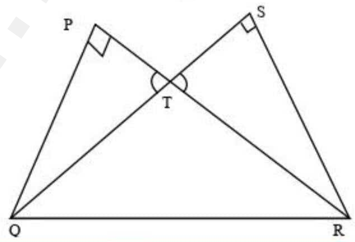


$$\frac{QR}{BC}=\frac{4}{10}=\frac{2}{5} \quad \Rightarrow \frac{PQ}{AB}\neq\frac{QR}{BC}$$

ΔABC is not similar to ΔPQR.

Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that PT × TR = ST × TQ.

Solution : ΔPQT and ΔSRT $\angle P = \angle S = 90^0$



$\angle PTQ = \angle STR$ (Vertically Opp.angle)

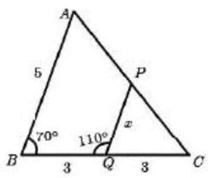
$\therefore$ By AA similarity,

$\Delta PQT \sim \Delta SRT$

$$\therefore \frac{QT}{TR}=\frac{PT}{ST} \Rightarrow PT\times TR = ST\times TQ$$

Check whether the triangles are similar and find the value of x.

Solution: Given $\angle PQB=110^0$



$\angle QBA = \angle PQC = 70^0$

$$\therefore \frac{CQ}{QB}=\frac{PQ}{AB} \Rightarrow \frac{3}{3}=\frac{x}{5}$$

$$\therefore x = 5$$

Converse of Basic Proportionality Theorem

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Converse of Angle Bisector

If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex.

Basic Proportionality Theorem (BPT) or Thales theorem

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

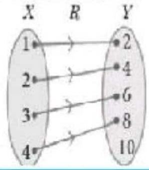
Angle Bisector Theorem

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Alternate Segment theorem

The angles between the tangent and the chord equal to the angles in the corresponding alternate segments.

Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and R = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that R is a function and find its domain, co-domain and range ?



Solution : Thus all elements in X have only one image in Y. Therefore R is a function.

Domain X = {1, 2, 3, 4}; Co-domain Y = {2, 3, 6, 8, 10}; Range of f = {2, 4, 6, 8}.

A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ (i) List the elements of f (ii) If f a function ?

Solution : $f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3\}$ (i) $f(-2) = (-2)^2 - 2 = 2$;

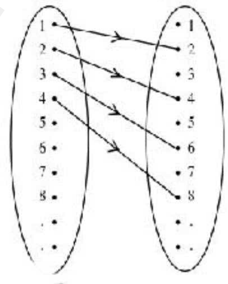
$f(-1) = (-1)^2 - 2 = -1$

$f(0) = (0)^2 - 2 = -2$; $f(3) = (3)^2 - 2 = 7$

$f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) each element in the domain of f has a unique image. Therefore f is a function.

Let $f = \{(x, y) \mid x, y \in N$ and $y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function ?



Solution : Given $f = \{(x, y) \mid x, y \in N$ and $y = 2x\}$

Domain = {1, 2, 3, 4,....} Co-domain = {1, 2, 3, 4, ...} Range = {2, 4, 6, 8, ...}

Since all the elements has unique element Yes, f is a function.

Let f(x) = 2x + 5. If $x \neq 0$ then find $\dfrac{f(x+2)-f(2)}{x}$.

Solution : $f(x) = 2x + 5$

$f(x + 2) = 2(x + 2) + 5 = 2x + 9$

$f(2) = 2(2) + 5 = 9$

$$\therefore \frac{f(x+2)-f(x)}{x} = \frac{2x+9-9}{x} = 2$$

Let X = {3, 4, 6, 8}. Determine whether the relation R = {(x, f(x)) | $x \in$ X, f(x) = x^2 + 1} is a function from X to N ?

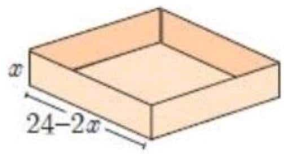
Solution : X = {3, 4, 6, 8} Given R = $\{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$

$x = 3 \Rightarrow f(x) = f(3) = 9 + 1 = 10$ $x = 6 \Rightarrow f(x) = f(6) = 36 + 1 = 37$

$x = 4 \Rightarrow f(x) = f(4) = 16 + 1 = 17$ $x = 8 \Rightarrow f(x) = f(8) = 64 + 1 = 65$

R = {(3, 10), (4, 17), (6, 37), (8, 65)} $\therefore$ The relation R : X $\rightarrow$ N is a function.

An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in the figure. Express the volume V of the box as a function of x.



Solution : $l = b = 24 - 2x$ cm, height = x cm.

$\therefore$ Volume of the box, $V = lbh = (24 - 2x)(24 - 2x)x = (24 - 2x)^2 x$

$= (576 + 4x^2 - 96x)x$

$= 4x^3 - 96x^2 + 576x$

$\therefore$ Volume is expressed as a function of x.

A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution : $f(x) = 3 - 2x$ and $f(x^2) = (f(x))^2$

$\Rightarrow 3 - 2x^2 = (3 - 2x)^2 \Rightarrow 3 - 2x^2 = 9 + 4x^2 - 12x \Rightarrow 6x^2 - 12x + 6 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$ (twice)

A plane is flying at a speed of 500km per hour. Express the distance d travelled by the plane as function of time t in hours.

Solution : Speed of the plane = 500 km / h $\therefore$ Distance = Time × Speed = 500t

Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f.

Solution : $3a - 5 = 4 \Rightarrow a = 3$

$3(1) - 5 = b \Rightarrow b = -2$

If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Solution : $A = \{-2, -1, 0, 1, 2\}$ and $f(x) = x^2 + x + 1$

$f(-2) = (-2)^2 + (-2) + 1 = 3$;

$f(-1) = (-1)^2 + (-1) + 1 = 1$;

$f(0) = 0^2 + 0 + 1 = 1$;

$f(1) = 1^2 + 1 + 1 = 3$

$f(2) = 2^2 + 2 + 1 = 7$

f is an onto function, range of $f = B =$ co-domain of f. Therefore, $B = \{1, 3, 7\}$.

The Cartesian product A × A has 9 elements among which (–1, 0) and (0, 1) are found. Find the set A and the remaining elements of A × A.

Solution : $n(A \times A) = 9$ and $(-1, 0), (0, 1) \in A \times A$ $A = \{-1, 0, 1\}$

set A and the remaining elements of A × A $= \{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$

Find the domain of the function $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$.

Solution : If $x > 1$ and $x < -1$, $f(x)$ leads to unreal $\therefore$ The domain of $f(x) = \{-1, 0, 1\}$

Write the domain $f(x) = \dfrac{2x + 1}{x - 9}$ **ng real**

Solution : If $x = 9, f(x) \to \infty$ The domain is $R - \{9\}$

Write the domain $g(x) = \sqrt{x - 2}$

Solution : The function exists only if $x \geq 2$ $\therefore$ The domain is $[2, \infty)$.

Let A = {–1, 1} and B = {0, 2}. If the function $f : A \to B$ defined by $f(x) = ax + b$ is an onto function ? Find a and b.

Solution : $\therefore f(-1) = 0 \Rightarrow -a + b = 0$ — (1)

$f(1) = 2 \Rightarrow a + b = 2$ — (2)

Solving (1) and (2) $2b = 2 \Rightarrow b = 1$

$\Rightarrow a = 1$ $\therefore a = 1, b = 1$

If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y.

Solution : Given $(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$\therefore x^2 - 3x = -2$	$y^2 + 4y = 5$
$\Rightarrow x^2 - 3x + 2 = 0$	$y^2 + 4y - 5 = 0$
$\Rightarrow (x - 2)(x - 1) = 0$	$(y + 5)(y - 1) = 0$
$\therefore x = 2, 1$	$y = -5, 1$

Let A = {1, 2} and B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify whether A × C is a subset of B × D =?

Solution : $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$, $D = \{5, 6, 7, 8\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{$**(1, 5), (1, 6)**, (1, 7), (1, 8), **(2, 5), (2, 6)**, (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8).$\}$

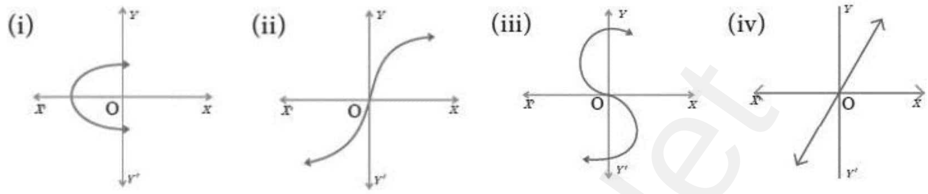
Clearly $A \times C$ is a subset of $B \times D$.

Let A = {9, 10, 11, 12, 13, 14, 15, 16, 17} and let f : A → N be defined by f (n) = the highest prime factor of $n \in$ A. Write f as a set of ordered pairs and find the range of f.

Solution : f = {(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)}

Range of f = {2, 3, 5, 7, 11, 13, 17}

Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



Solution :

(i) The curve do not represent a function since it meets y-axis at 2 points.
(ii) The curve represents a function as it meets x-axis or y-axis at only one point.
(iii) The curve do not represent a function since it meets y-axis at 2 points.
(iv) The line represents a function as it meets axes at origin.

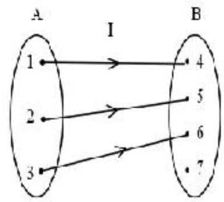
Let A = {1, 2, 3, 4} and B = N. Let f : A → B be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function

Solution : A = {1, 2, 3, 4}, B = N $f(x) = x^3$

$x = 1 \Rightarrow f(1) = 1$ $x = 3 \Rightarrow f(3) = 27$ (diff. elements have diff. images)

$x = 2 \Rightarrow f(2) = 8$ $x = 4 \Rightarrow f(4) = 64$ (Range ≠ co-domain)

(i) Range of f = {1, 8, 27, 64} (ii) f is one-one and f is into



Let A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one but not onto function.

Solution: A = {1, 2, 3}, B = {4, 5, 6, 7} ; f = {(1, 4), (2, 5), (3, 6)}

different elements in A are different images in B.
Hence f is one-one function. Note that the element 7 does not have any pre-image in A
Hence f is not onto.

Show that the function f : N → N defined by $f(m) = m^2 + m + 3$ is one-one function.
Solution :

Given f : N → N defined by $f(m) = m^2 + m + 3$

$m = 1 \Rightarrow f(1) = 1 + 1 + 3 = 5$ $m = 3 \Rightarrow f(3) = 9 + 3 + 3 = 15$

$m = 2 \Rightarrow f(2) = 4 + 2 + 3 = 9$ $m = 4 \Rightarrow f(4) = 16 + 4 + 3 = 23$ ....

different elements in **N** are different images in **N** $\therefore f$ is one-one function.

Show that the function f : N → N defined $f(x) = 2x - 1$ is one-one but not onto.

Solution : Given f : N → N defined by $f(x) = 2x - 1$.

$x = 1 \Rightarrow f(1) = 2 - 1 = 1$ $x = 3 \Rightarrow f(3) = 6 - 1 = 5$

$x = 2 \Rightarrow f(2) = 4 - 1 = 3$ $x = 4 \Rightarrow f(4) = 8 - 1 = 7$ ....

different elements in **N** are different images in **N** $\therefore f$ is one-one function.

$\therefore$ Range ≠ Co-domain. $\therefore f$ is not on-to.

The line p passes through the points (3, –2), (12, 4) and the line q passes through the points (6, –2) and (12, 2) . Is p parallel to q ?

Solution : The slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ The slope of line p is $m_1 = \dfrac{(4)-(-2)}{(12)-(3)} = \dfrac{4+2}{12-3} = \dfrac{6}{9} = \dfrac{2}{3}$

The slope of line q is $m_2 = \dfrac{(2)-(-2)}{(12)-(6)} = \dfrac{2+2}{12-6} = \dfrac{4}{6} = \dfrac{2}{3}$

Thus, slope of line p = slope of line q. Therefore, line p is parallel to the line q.

The line r passes through the points (–2, 2) and (5, 8) and the line s passes through the points (–8,7) and (–2, 0). Is the line r perpendicular to s ?

Solution : The slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

The slope of line r is $m_1 = \dfrac{(8)-(2)}{(5)-(-2)} = \dfrac{8-2}{5+2} = \dfrac{6}{7}$

The slope of line s is $m_2 = \dfrac{(0)-(7)}{(-2)-(-8)} = \dfrac{0-7}{-2+8} = \dfrac{-7}{6}$

The product of slopes $= \dfrac{6}{7} \times \dfrac{-7}{6} = -1$ That is, $m_1 m_2 = -1$

Without using Pythagoras theorem, show that the points (1,-4) , (2, –3) and (4, –7) form a right angled triangle.

Solution : Let the given points be A(1, – 4) , B(2, –3) and C(4, –7) .

The slope of AB $= \dfrac{-3+4}{2-1} = \dfrac{1}{1} = 1$ The slope of BC $= \dfrac{-7+3}{4-2} = \dfrac{-4}{2} = -2$

The slope of AC $= \dfrac{-7+4}{4-1} = \dfrac{-3}{3} = -1$ Slope of AB slope of AC = (1)(–1) = –1

AB is perpendicular to AC. $\angle A = 90°$ Therefore, ΔABC is a right angled triangle.

If the three points (3, –1), (*a*, 3) and (1, –3) are collinear, find the value of *a*.

Solution : $\therefore$ Slope of AB = Slope of BC

$$\Rightarrow \frac{4}{a-3} = \frac{-6}{1-a} \Rightarrow 4 - 4a = -6a + 18 \Rightarrow 2a = 14 \Rightarrow a = 7$$

Find the slope of a line joining the points (sin θ, – cos θ) and (– sin θ, cos θ)

Solution : $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\cos\theta + \cos\theta}{-\sin\theta - \sin\theta} = \frac{2\cos\theta}{-2\sin\theta} = -\cot\theta$$

Show that the given points are collinear (– 3, – 4), (7, 2) and (12, 5).

Solution : Given points are A (–3, –4), B (7, 2), C (12, 5)

Slope of AB $= \dfrac{2+4}{7+3} = \dfrac{6}{10} = \dfrac{3}{5}$ Slope of BC $= \dfrac{5-2}{12-7} = \dfrac{3}{5}$

$\therefore$ Slope of AB = Slope of BC $\therefore$ A, B, C are collinear.

What is the slope of a line perpendicular to the line joining A (5, 1) and P where P is the mid-point of the segment joining (4, 2) and (–6, 4).

Solution : P is the midpoint of (4, 2), (–6, 4) $\Rightarrow P = \left(\dfrac{4-6}{2}, \dfrac{2+4}{2}\right) = (-1, 3)$

$\therefore$ Slope of the line joining A (5, 1), P (–1, 3) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{-1-5} = \dfrac{2}{-6} = \dfrac{-1}{3}$

$\therefore$ Slope of the line perpendicular = 3

The line through the points (–2, a) and (9, 3) has slope $-\frac{1}{2}$. Find the value of a.

Solution :

Slope of the line joining (–2, a), (9, 3) = $-\frac{1}{2}$ $\Rightarrow \frac{3-a}{9+2}=\frac{-1}{2}$ $\Rightarrow \frac{3-a}{11}=\frac{-1}{2}$ $\Rightarrow 6-2a=-11$

$\Rightarrow 2a=17 \quad \therefore a=\frac{17}{2}$

The line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). FInd the value of x.

Solution : Slope of line joining (–2, 6), (4, 8) $m_1=\frac{8-6}{4+2}=\frac{2}{6}=\frac{1}{3}$

Slope of line joining (8, 12), (x, 24) $m_2=\frac{24-12}{x-8}=\frac{12}{x-8}$

Since two lines are perpendicular, $\Rightarrow \frac{1}{3}\times\frac{12}{x-8}=-1$ $\Rightarrow$ $\frac{4}{x-8}=-1 \Rightarrow -x+8=4 \Rightarrow x=4$

Find the intercepts made by the line $4x-9y+36=0$ on the coordinate axes.

Solution : **put** $x=0 \Rightarrow 4x=-36$ x intercept $a=-9$

put $y=0 \Rightarrow -9y+36=0.\ -9y=-36 \Rightarrow y$ intercept $b=4$

Show that the straight lines $2x+3y-8=0$ and $4x+6y+18=0$ are parallel.

Solution :

Slope of the straight line $2x+3y-8=0$ is $m_1=\frac{-\text{coefficient of }x}{\text{coefficient of }y}=\frac{-2}{3}$

Slope of the straight line $4x+6y+18=0$ is $m_2=\frac{-4}{6}=\frac{-2}{3}$ Here, $m_1=m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

Show that the straight lines $x-2y+3=0$ and $6x+3y+8=0$ are perpendicular.

Solution : Slope of the straight line $x-2y+3=0$ is $m_1=\frac{-1}{-2}=\frac{1}{2}$

Slope of the straight line $6x+3y+8=0$ is $m_2=\frac{-6}{3}=-2$

Now, $m_1\times m_2=\frac{1}{2}\times(-2)=-1$ Hence, the two straight lines are perpendicular.

Find the equation of a straight line which is parallel to the line $3x-7y=12$ and passing through the point (6, 4).

Solution : Equation of the straight line, parallel to $3x-7y-12=0$ is $3x-7y+k=0$

$3(6)-7(4)+k=0 \Rightarrow k=28-18=10$

the required straight line is $3x-7y+10=0$.

Find the equation of a straight line perpendicular to the line $y=\frac{4}{3}x-7$ and passing through the point (7, – 1).

Solution : The equation $y=\frac{4}{3}x-7$ can be written as $4x-3y-21=0$.

Equation of a straight line perpendicular to $4x-3y-21=0$ is $3x+4y+k=0$

Since it is passes through the point (7, –1), $21-4+k=0$ we get, $k=-17$

the required straight line is $3x+4y-17=0$.

Find the equation of a straight line passing through (5,7) and is (i) parallel to X axis (ii) parallel to Y axis.

Solution : (i) The equation of any straight line parallel to X axis is y=b

The required equation of the line is y=7.

(ii) The equation of any straight line parallel toY axis is x=c

The required equation of the line is $x = 5$.

Find the equation of a straight line whose Slope is 5 and x intercept is -9

Solution : Given, Slope = 5, x intercept, $d = -9$

The equation of a straight line is $y = m(x - d)$ $\quad y = 5(x+9)$ $\quad y = 5x + 45$

Find the equation of a line passing through the point $(3, -4)$ and having slope $\frac{-5}{7}$.

Solution : Given slope of the line is $-\frac{5}{7}$ and $(3, -4)$ is a point on the line.

$$y - y_1 = m(x - x_1) \qquad y + 4 = -\frac{5}{7}(x - 3) \qquad 5x + 7y + 13 = 0.$$

Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point $(-1,2)$.

Solution : slope of the line is $\frac{-5}{4}$ and $(-1, 2)$ is a point on the line. $\therefore$ its equation is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 2 = \frac{-5}{4}(x+1) \quad \Rightarrow 4y - 8 = -5x - 5 \quad \Rightarrow 5x + 4y - 3 = 0$$

Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6, 10) to (14, 12), find the equation of the rod joining the buildings ?

Solution : $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y-10}{12-10} = \frac{x-6}{11-6} \Rightarrow \frac{y-10}{2} = \frac{x-6}{8} \Rightarrow x - 4y + 34 = 0.$

Find the equation of a straight line passing through the mid-point of a line segment joining the points (1,-5), (4,2) and parallel to (i) X axis (ii) Y axis

Solution : Equation of a Straight line parallel to the Y axis is $x = c$.

Equation of a straight line parallel to X axis is $y = b$

Mid point of the line joining the points $(1,-5), (4, 2)$ is $= \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right] = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$

(i) Parallel to x-axis is $y = -3/2$ **(ii)** Parallel to y-axis is $x = 5/2$

Determine the sets of points are collinear ? $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$

Solution :

Area of triangle $= \frac{1}{2}\begin{bmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{bmatrix}$

$$= \frac{1}{2}[(a^2 + b^2 + c^2 + ab + bc + ca) - (a^2 + b^2 + c^2 + ab + bc + ca)] = \frac{1}{2}[0] = 0$$

$\therefore$ The 3 points are collinear.

If the straight lines $12y = -(p + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p'.

Solution : $12y = -(p + 3)x + 12$,

$\Rightarrow (p + 3)x + 12y = 12$ **and** $12x - 7y = 16$ are perpendicular

$$m_1 = \frac{-(p+3)}{12} \qquad m_2 = \frac{12}{7} \qquad m_1 \times m_2 = -1 \quad \Rightarrow \frac{-(p+3)}{12} \times \frac{12}{7} = -1 \quad \Rightarrow \quad p = 4$$

Determine the sets of points are collinear ? $\left\{-\frac{1}{2}, 3\right\}$**, (–5, 6) and (–8, 8)**

Solution : Area of triangle $= \frac{1}{2}\begin{bmatrix} -\frac{1}{2} & -5 & -8 & -\frac{1}{2} \\ 3 & 6 & 8 & 3 \end{bmatrix} = \frac{1}{2}[(-3-40-24)-(-15-48-4)]$

$= \frac{1}{2}[-67-(67)] = \frac{1}{2}(0) = 0$

$\therefore$ The 3 points are collinear.

Find the value of 'p'.

Vertices	Area (sq. units)
(p, p), (5, 6), (5, –2)	32

Solution : Area of triangle $= \frac{1}{2}\begin{bmatrix} p & 5 & 5 & p \\ p & 6 & -2 & p \end{bmatrix} = 32$

$\Rightarrow (6p - 10 + 5p) - (5p + 30 - 2p) = 64 \quad \Rightarrow (11p - 10) - (3p + 30) = 64$

$\Rightarrow 8p = 104 \Rightarrow p = \frac{104}{8} \Rightarrow p = 13$

If the points **(2, 3), (4, *a*) and (6, –3) are collinear**, then find **the value of '*a*'**

Solution : $\frac{1}{2}\begin{bmatrix} 2 & 4 & 6 & 2 \\ 3 & a & -3 & 3 \end{bmatrix} = 0$

$\Rightarrow (2a - 12 + 18) - (12 + 6a - 6) = 0$

$\Rightarrow (2a + 6) - (6a + 6) = 0 \Rightarrow -4a = 0 \Rightarrow a = 0$

If the points **(–a+1, 2*a*) and (– 4 – *a*, 6 – 2*a*) are collinear**, then find **the value of '*a*'**

Solution : $\frac{1}{2}\begin{bmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{bmatrix}$

$\Rightarrow 8a^2 + 4a - 4 = 0 \quad \Rightarrow 2a^2 + a - 1 = 0 \quad \Rightarrow a = -1, \frac{1}{2}$

Find the slope of a line joining the given points $\left(-\frac{1}{3}, \frac{1}{2}\right)$ and $\left(\frac{2}{7}, \frac{3}{7}\right)$

Solution : The slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(\frac{3}{7}\right) - \left(\frac{1}{2}\right)}{\left(\frac{2}{7}\right) - \left(-\frac{1}{3}\right)} = \frac{\frac{3}{7} - \frac{1}{2}}{\frac{2}{7} + \frac{1}{3}} = \frac{\frac{6-7}{14}}{\frac{6+7}{21}} = -\frac{1}{14} \times \frac{21}{13} = -\frac{3}{26}$.

Find the slope of a line joining the given points (14, 10) and (14, – 6)

Solution : The slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-6) - (10)}{(14) - (14)} = \frac{-6-10}{14-14} = \frac{-16}{0}$. The slope is undefined.

Show that the points (–2, 5), (6, –1) and (2, 2) are collinear.

Solution : The vertices are A (–2,5), B (6, –1) and C (2, 2). The slope $= \frac{y_2 - y_1}{x_2 - x_1}$

Slope of AB $= \frac{(-1) - (5)}{(6) - (-2)} = \frac{-1-5}{6+2} = \frac{-6}{8} = \frac{-3}{4}$

Slope of BC $= \frac{(2) - (-1)}{(2) - (6)} = \frac{2+1}{2-6} = \frac{3}{-4} = \frac{-3}{4}$

We get, Slope of AB = Slope of BC.

Hence the points A, B and C are collinear.

Find the slope and y intercept of $\sqrt{3}x + (1-\sqrt{3})y = 3$.

Solution : $a = \sqrt{3}$ $b = (1-\sqrt{3})$ $c = -3$

Slope of the line $= \dfrac{-a}{b} = \dfrac{-\sqrt{3}}{(1-\sqrt{3})} = \dfrac{3+\sqrt{3}}{2}$

Interecept on y-axis $= \dfrac{-c}{b} = \dfrac{-(-3)}{1-\sqrt{3}} = \dfrac{3+3\sqrt{3}}{-2}$

Find the equation of a line whose inclination is 30° and making an intercept -3 on the Y axis.

Solution : Given $\theta = 30^\circ \Rightarrow m = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$ and y - intercept $= -3$

The required equation of the line is $y = mx + c$ $\Rightarrow y = \dfrac{1}{\sqrt{3}}x - 3 \Rightarrow \sqrt{3}y = x - 3\sqrt{3}$
$\Rightarrow x - \sqrt{3}y - 3\sqrt{3} = 0$

Find the equation of a line through the given pair of points $\left(2, \frac{2}{3}\right)$ and $\left(\frac{-1}{2}, -2\right)$

Solution : Given points are $\left(2, \frac{2}{3}\right), \left(\frac{-1}{2}, -2\right)$ two-point form $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}} = \frac{x-2}{\frac{-1}{2}-2} \Rightarrow \frac{3y-2}{-8} = \frac{x-2}{\frac{-5}{2}} \Rightarrow \frac{3y-2}{-8} = \frac{2x-4}{-5} \Rightarrow 15y - 10 = 16x - 32 \Rightarrow 16x - 15y - 22 = 0$$

The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and intercept on the Y axis.

Solution : $2(x-y)+5=0 \Rightarrow 2x - 2y + 5 = 0$

i) Slope of the line $= \dfrac{-a}{b} = \dfrac{-2}{-2} = 1$

ii) The slope of the straight line is $m = \tan\theta$

Slope of the line $= 1$ $\therefore \tan\theta = 1$ $\therefore \theta = 45^\circ$.

iii) Interecept on y-axis $= \dfrac{-c}{b} = \dfrac{-5}{-2}$ $\therefore y$ - intercept $= 5/2$

The hill in the form of a right triangle has its foot at (19, 3). The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.

Solution : $\therefore$ Equation of slope $m = \tan 45^\circ = 1$ and passing through C(19, 3)

$\Rightarrow y - y_1 = m(x - x_1) \Rightarrow y - 3 = 1(x - 19) \Rightarrow x - y - 16 = 0.$

Find the value of 'a', if the line through (–2, 3) and (8, 5) is perpendicular to $y = ax + 2$.

Solution :

Slope of the line joining (– 2, 3), (8, 5). $\Rightarrow \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{5-3}{8+2} = \dfrac{2}{10} \Rightarrow m_1 = \dfrac{1}{5}$

Slope of the line $y = ax + 2$ $\Rightarrow ax - y + 2 = 0$ Slope of the line $= \dfrac{-a}{b} = \dfrac{-a}{-1} \Rightarrow m_2 = a.$

$$m_1 m_2 = -1 \Rightarrow \frac{1}{5} \times a = -1 \Rightarrow a = -5$$

Find the equation of a straight line passing through (5, –3) and (7, – 4).

Solution : $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1} \Rightarrow \dfrac{y+3}{-4+3} = \dfrac{x-5}{7-5} \Rightarrow 2y + 6 = -x + 5 \Rightarrow x + 2y + 1 = 0.$

The equation of the required straight line is $x + 2y + 1 = 0$.