SECOND MID TERM TEST - 2019

X - Standard

Reg No. | | | | | | |

Mathematics

Time : 1.30 hrs *Marks : 50*

I **Answer all the questions:** [10x1=10]

1) If A is a 2 x 3 matrix and B is a 3 x 4 matrix, how many columns does AB have
   a) 1 b) 4 c) 2 d) 5

2) If number of columns and rows are not equal in a matrix then it is said to be a
   a) diagonal matrix b) rectangular matrix c) square matrix d) identity matrix

3) Find is the order of the matrix $\begin{pmatrix} \sin\theta \\ \cos\theta \\ \tan\theta \end{pmatrix}$
   a) 3x1 b) 1x3 c) 3x3 d) 1x1

4) A tangent is perpendicular to the radius at the
   a) centre b) point of contact c) infinity d) chord

5) A tower is 60m height. Its shadow is x metres shorter when the sun's attitude is $45°$ than when it has been $30°$, then x is equal to
   a) 41.92m b) 43.92m c) 43m d) 45.6m

6) If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}$: 1, then the angle of elevation of the sun has measure
   a) $45°$ b) $30°$ c) $90°$ d) $60°$

7) $1-\cos^2\theta$ = _____
   a) $\sin^2\theta$ b) $\cos^2\theta$ c) $\tan^2\theta$ d) $\cot^2\theta$

8) The difference between the T.SA and C.SA of a right circular cylinder is _____
   a) $2\pi r^2$ b) $3\pi r^2$ c) πr^2 d) $4\pi r^2$

9) The total surface area of a hemi sphere is how much times the square of its radius.
   a) π b) 4π c) 3π d) 2π

10) The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is
   a) 1 : 2 : 3 b) 2 : 1 : 3 c) 1 : 3 : 2 d) 3 : 1 : 2

II. **Answer Any Five Questions :-** [5x2=10]

11) Construct a 3 x 3 matrix whose elements are given by $a_{ij} = \frac{(i+j)^3}{3}$

12) If A = $\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$; B = $\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ Find the value of 3A - 9B.

13) Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

14) Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

15) If the base area of a hemispherical solid is 1386 sq.metres, then find its total surface area?

16) Find the volume of a cylinder whose height is 2 m and whose base area is 250 m^2.

17) The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

III. Answer Any Five Questions :- **[5x5=25]**

18) A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.

19) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^T . A^T$

20) State and prove pythagoras theorem

21) A pole 5 m high is fined on the top of a tower. The angle of elevation of the top of pole observed from a point 'A' on the ground is $60°$ and the angle of depression to the point 'A' from the top of the tower is $45°$. Find the height of the tower.

22) A man is watching a boat speeding away from the top of a tower. The boat makes an angles of depression of $60°$ with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes $45°$. What is the approximate speed of the boat (in km/hr), assuming that is sailing in still water? ($\sqrt{3}$ =1.732)

23) An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

24) A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.

IV. Answer Any One : **[1x5=5]**

25) Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

(or)

Draw the Graph of $y=x^2-4x+3$ and use it to solve $x^2-6x+9=0$.

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SECOND MIDTERM TEST
X STD (2019-2020)
MATHEMATICS ANSWER KEY
15.11.2019

NALANDHA INNOVATION SCHOOL
a synonym of education

PREPARED BY : A.NAZIYA BEGUM M.Sc.,B.Ed.

I.

1. b) 4
2. b) Rectangular matrix
3. a) 3×1
4. b) Point of Contact
5. b) 43.92m
6. d) 60°
7. a) $\sin^2\theta$
8. a) $2\pi r^2$
9. c) 3π
10. d) 3 : 1 : 2.

II.

11. $a_{ij} = \frac{(i+j)^3}{3}$

$a_{11} = \frac{(1+1)^3}{3} = \frac{2^3}{3} = \frac{8}{3}$

$a_{12} = \frac{(1+2)^3}{3} = \frac{27}{3} = 9$

$a_{13} = \frac{(1+3)^3}{3} = \frac{64}{3} = \frac{64}{3}$

$a_{21} = \frac{(2+1)^3}{3} = \frac{27}{3} = 9$

$a_{22} = \frac{(2+2)^3}{3} = \frac{64}{3} = \frac{64}{3}$

$$a_{23} = \frac{(2+3)^3}{3} = \frac{125}{3} = \frac{125}{3}$$

$$a_{31} = \frac{(3+1)^3}{3} = \frac{64}{3} = \frac{64}{3}$$

$$a_{32} = \frac{(3+2)^3}{3} = \frac{125}{3} = \frac{125}{3}$$

$$a_{33} = \frac{(3+3)^3}{3} = \frac{216}{3} = 72$$

The required Matrix $A = \begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$.

12. $A = \begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$; $B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$

$$3A = 3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix}$$

$$9B = 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} = \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}$$

$$3A - 9B = \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} - \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}$$

$$3A - 9B = \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

3. $\boxed{\text{Radius} = 3 \text{ Cm}}$.

Distance from the centre = 5 Cm.

Length of tangent $= \sqrt{(\text{distance})^2 - (\text{radius})^2}$

$= \sqrt{5^2 - 3^2}$

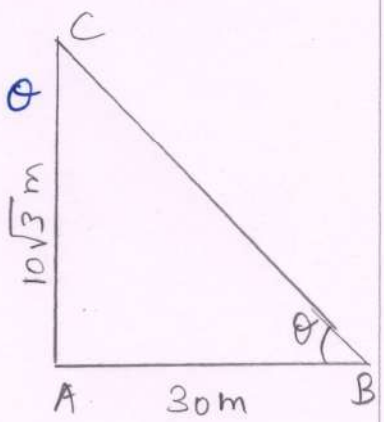
$= \sqrt{25 - 9} = \sqrt{16}$.

Length of tangent = $\boxed{4 \text{ Cm}}$

4. Height of the tower (AC) = $10\sqrt{3}$ m

Distance between the base of the tower and Point of observation (AB) = 30m

Let the angle of elevation $\angle ABC$ be θ



In the right ΔABC, $\tan\theta = \frac{AC}{AB}$

$= \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3}$

$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\therefore$ Angle of Inclination is $\boxed{30^\circ}$

15. Base area of the Hemisphere $= (\pi r^2) = 1386.$

Total surface area of a hemisphere $= 3\pi r^2$

$= 3 \times 1386$

Total Surface area of a hemisphere $= \boxed{4158\ m^2}$

16. Area of the base $\pi r^2 = 250 m^2$

Height $= 2m$

Volume of the cylinder $= \pi r^2 h$

$= 250 \times 2$

Volume of the cylinder $= 500 m^3.$

17. Given:

Volume of the right Circular cone $= 11088\ cm^3$

Height $= 24\ cm$

Radius $= ?$

Volume of the right Circular Cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24$

$= 11088.$

$r^2 = 11088 \times \frac{3}{1} \times \frac{7}{22} \times 24$

$r = 21^2$

$\boxed{r = 21}$

III.

18. Let the age of the sister be "x"

$\therefore$ The age of the girl $= 2x$

Five years hence

Age of the sister $= x+5$

Age of the girl $= 2x+5$

By the given Condition

$(x+5)(2x+5) = 375$

$2x^2 + 5x + 10x + 25 = 375$

$2x^2 + 15x - 350 = 0$

$a = 2, \quad b = 15, \quad c = -350$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-15 \pm \sqrt{225 - 4(2)(-350)}}{4}$$

$$= \frac{-15 \pm \sqrt{225 + 2800}}{4}$$

$$= \frac{-15 \pm \sqrt{3025}}{4}$$

$$= \frac{-15+55}{4} \text{ or } \frac{-15-55}{4}$$

$$= \frac{40}{4} \text{ or } \frac{-70}{4}$$

$$x = 10$$

Age will not be Negative

Age of the girl = 10 years.

Age of the sister = 20 years (2×10)

19.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix} \qquad (AB)^T = B^T A^T$$

$$AB = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2+0 & -1+8+2 \\ 4 & +1+0 & -2-4+2 \end{bmatrix}$$

$$AB = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

Hence, Proved.

20. Pythagoras theoram:-

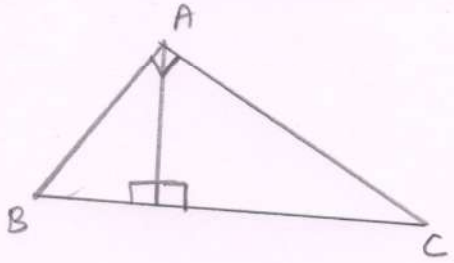
Statement: In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof:-

Given : In ΔABC, $\angle A = 90^\circ$

To Prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$



No	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is Common $\angle BAC = \angle BDA = 90^\circ$ Therefore, $\Delta ABC \sim \Delta ABD$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$	Given $\angle BAC = 90^\circ$ and by Construction $\angle BDA = 90^\circ$ By AA Similarity
2.	Compare ΔABC and ΔADC $\angle C$ is Common $\angle BAC = \angle ADC = 90^\circ$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$	Given $\angle BAC = 90^\circ$ and by Construction $\angle CDA = 90^\circ$ By AA Similarity

Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$

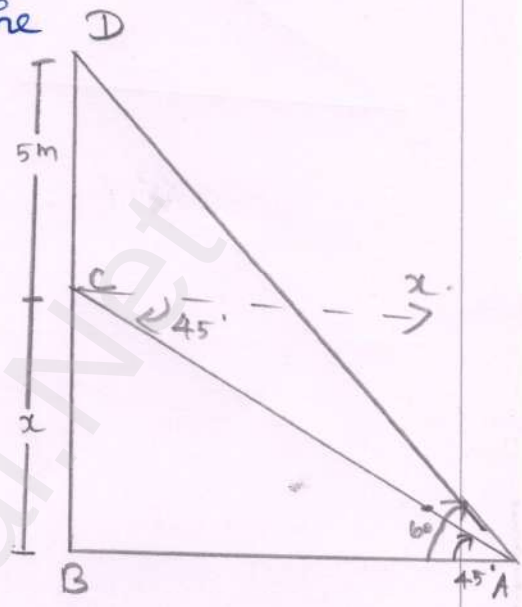
$$= BC(BD + DC) = BC \times BC$$

$$AB^2 + AC^2 = BC^2$$

Hence, the theorem is proved.

21. Solution:

Let BC be the height of the tower and CD be the height of the Pole.



Let 'A' be the point of observation.

Let $BC = x$ and $AB = y$

From the diagram,

$\angle BAD = 60^\circ$ and $\angle xCA = 45^\circ = \angle BAC$

In right triangle ABC, $\tan 45^\circ = \frac{BC}{AB}$

gives $1 = \frac{x}{y}$ So, $x = y$ ...(1)

In right triangle ABD, $\tan 60^\circ = \frac{BD}{AB} = \frac{BC+CD}{AB}$

gives $\sqrt{3} = \frac{x+5}{y}$ So, $\sqrt{3}y = x+5$

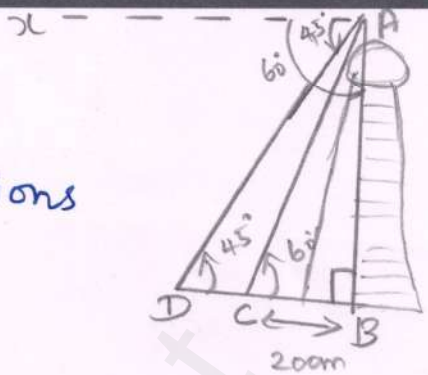
we get, $\sqrt{3}x = x+5$ [From (1)]

So, $x = \frac{5}{\sqrt{3}-1} = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5(1.732+1)}{2}$

$= 6.83.$

Hence, the height of the tower is 6.83 m.

22 Solution:



Let AB be the tower.

Let C and D be the positions of the boat.

From the diagram,

$\angle xAC = 60^\circ = \angle ACB$ and

$\angle xAD = 45^\circ = \angle ADB$, $BC = 200m$

In right triangle ABC, $\tan 60^\circ = \frac{AB}{BC}$

gives $\sqrt{3} = \frac{AB}{200}$

we get $AB = 200\sqrt{3}$ ...(1)

In right triangle ABD, $\tan 45^\circ = \frac{AB}{BD}$

gives $1 = \frac{200\sqrt{3}}{BD}$ [by (1)]

we get, $BD = 200\sqrt{3}$

Now, $CD = BD - BC$

$CD = 200\sqrt{3} - 200$

$= 200(\sqrt{3}-1)$

$= 146.4$

It is given that the distance CD is covered in 10 seconds.

That is, the distance of 146.4m is covered in 10 seconds.

Therefore, speed of the boat $= \frac{\text{distance}}{\text{time}}$

$= \frac{146.4}{10} = 14.64$ m/s

gives $14.64 \times \frac{3600}{1000}$ km/hr $= 52.704$ km/hr.

23. Solution:

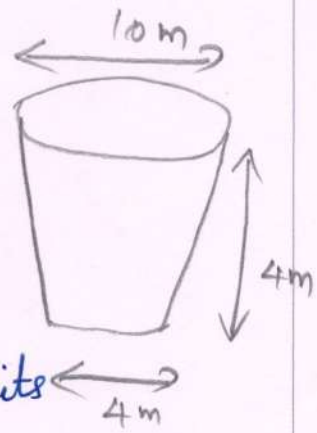
Given that, Diameter of the top = 10 m;
radius of the top $R = 5$ m.

Diameter of the bottom = 4m;
radius of the bottom $r = 2$ m, height $h = 4$ m.

Now, $l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{4^2 + (5-2)^2}$

$l = \sqrt{16+9} = \sqrt{25} = 5$ m



Here, C.S.A $= \pi (R+r) l$ sq.units

$= \frac{22}{7}(5+2) \times 5 = 110 m^2$

T.S.A $= \pi(R+r) l + \pi R^2 + \pi r^2$ sq.units

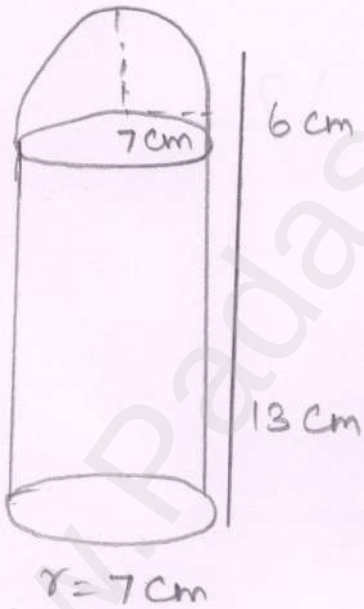
$$= \frac{22}{7}\left[(5+2)\times 5 + 25+4\right]$$

$$= \frac{1408}{7} = 201.14$$

Therefore C.S.A $= 110 m^2$ and T.S.A $= 201.14 m^2$

24. Sol:-

Radius of a Hemisphere = Radius of the cylinder



Height of the cylinder

$(h) = 13 - 7 = 6$ Cm.

Capacity of the vessel = Volume of the cylinder + Volume of the hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left[h + \frac{2r}{3} \right]$$

$$= \frac{22}{7} \times 7 \times 7 \left[6 + \frac{2 \times 7}{3} \right]$$

$$= 22 \times 7 \times \left[\frac{18+14}{3} \right]$$

$$= 22 \times 7 \times \frac{32}{3} \text{ cm}^3$$

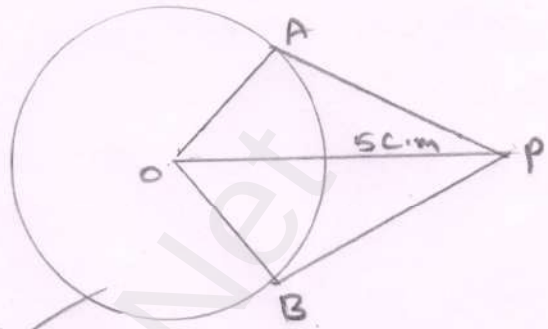
Capacity of the vessel = 1642.67 cm^3.

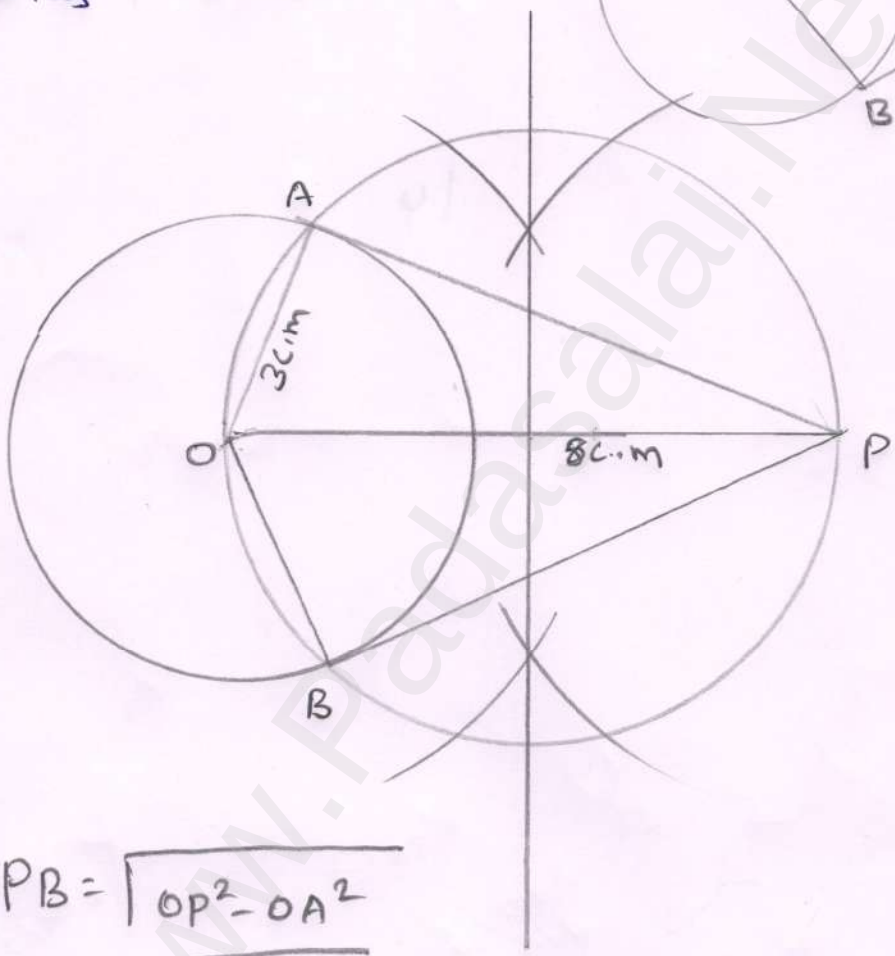
25.a

Given

diameter = 6 c.m

radius = 3 c.m





$PB = \sqrt{OP^2 - OA^2}$

$= \sqrt{8^2 - 3^2}$

$= \sqrt{64 - 9}$

$= \sqrt{55}$

$= 7.4$ c.m

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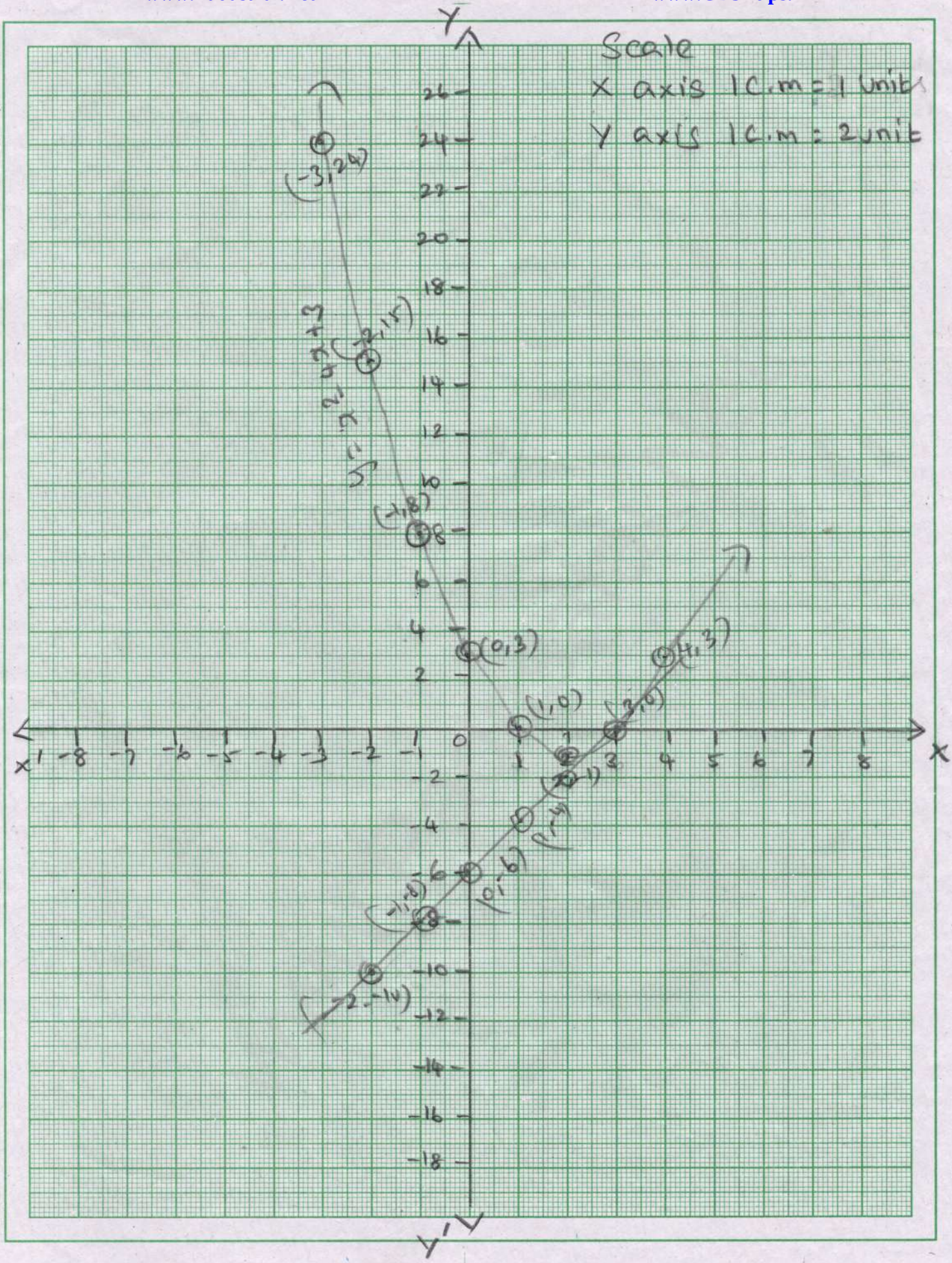
PREPARED BY : A.NAZIYA BEGUM M.Sc.,B.Ed.

5)
a.

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	-4	-8	-12	-16
3	3	3	3	3	3	3	3	3	3
$y=x^2-4x+3$	35	24	15	8	3	0	-1	0	3

$$y = x^2 - 4x + 3$$
$$0 = x^2 - 6x + 9$$
$$(-) \quad (+) \quad (-)$$
$$2x - 6$$

x	-2	-1	0	1	2
$2x$	-4	-2	0	2	4
-6	-6	-6	-6	-6	-6
$y=2x-6$	-10	-8	-6	-4	-2


Scale
X axis 1C.m = 1 Unit
Y axis 1C.m : 2 unit
Y
Y'
X
X'
(-3,24)
(-2,15)
(-1,8)
(0,3)
(1,0)
(3,0)
(4,3)
(1,-4)
(0,-6)
(-1,-8)
(-2,-10)