2MS

SECOND MID TERM TEST - 2022

CLASS : **10**

MATHEMATICS

TIME : 1.30

MARKS :50

I Choose the correct answer. 4 X 1 = 4

1. The two tangents from an external point P to a circle with centre at O are PA and PB. If $\angle APB = 70^\circ$ then the value of $\angle AOB$ is

   a) 100° b) 110° c) 120° d) 130°

2. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$ then the angle of elevation of the sun has measure.

   a) 45° b) 30° c) 90° d) 60°

3. The height of a right circular cone whose radius is 5cm and slant height is 13cm will be

   a) 12cm b) 10cm c) 13 cm d) 5 cm

4. If $A=\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$, and $B=\begin{pmatrix} Sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$ then AB =

   a) $\begin{pmatrix} \sin^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix}$ b) $\begin{pmatrix} \cos^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

II Answer any 5 of the following questions. Question number 12 is compulsory.

5. If $A=\begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$ then find the transpose of -A. 5 X 2 = 10

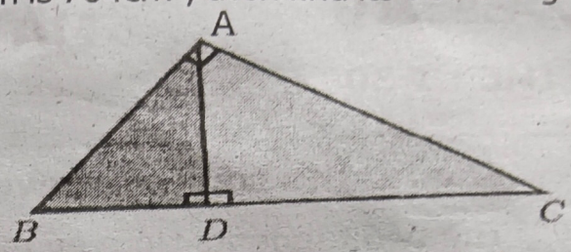
6. Verify that $A^2 = I$ when $A=\begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}$.

7. If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

8. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.

9. From the top of a rock $50\sqrt{3}$ high, the angle of depression of a car on the ground observed to be 30° . Find the distance of the car from the rock.

10. Find the diameter of a sphere whose surface area is $154m^2$.
11. If the total surface area of a cone of radius 7 cm is $704cm^2$, then find its slant height.
12. In the given figure $\angle A = 90^\circ$, BD = 4 cm, DC = 9cm and AD $\perp$ BC, then find the length of AD.



III Answer any 4 of the following questions. 5 X 4 = 20

Question number 19 is compulosry.

13. Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ and $X - Y \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$.
14. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$ show that $(AB)^T = B^T A^T$.
15. State and prove Pythagoras Theorem.
16. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ship is $4h/\sqrt{3}$ m.
17. From the top of 12m high building, but angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.
18. A container open at the top is in form of a frustum of a cone of height 16cm with radii of its lower and upper ends are 8cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of Rs.40 per litre.
19. Two circles with centres O and O of radii 3cm and 4 cm, respectively intersects at two points P and Q, such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

IV Answer the following 2 questions. 2 X 8 = 16

20. a) Draw Circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment. ***(OR)***
    b) Draw the tangents from a point which is 10cm away from the centre of a circle of radius 5 cm. Also measure the lengths of the tangents.
21. a) Discuss the nature of solutions of the quadratic equation $x^2 - 8x + 16 = 0$. ***(OR)*** b) Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$.

VINAYAGA MATRIC HIGHER SECONDARY SCHOOL
SUNDAKKAL KANNANDHERI
SALEM DISTRICT
SECOND MIDTERM MATHEMATICS KEYANSWER- NOV-2022

I. 1 MARKS

1. b) 110^0

2. d) 60^0

3. a) 12 cm

4. d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

II. 2 MARKS

5. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of $-A$.

$$-A = -\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix} = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$\therefore (-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

6. Verify that $A^2 = I$ when $A = \begin{pmatrix} 5 & 4 \\ 6 & -5 \end{pmatrix}$

$$A^2 = I \Longrightarrow A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix} \times \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \;\; -4 & 5 \;\; -4 \\ \hline 5 & -4 \\ 6 & -5 \\ \hline 6 \;\; -5 & 6 \;\; -5 \\ \hline 5 & -4 \\ 6 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 25-24 & -20+20 \\ 30-30 & -24+25 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$A^2 = I \Longrightarrow \therefore A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$$

7. If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

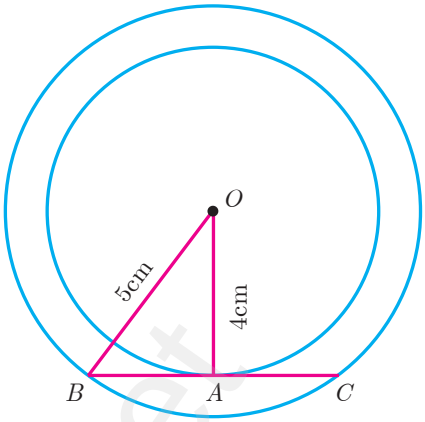
Solution $OA = 4$ cm, OB $= 5$ cm; also $OA \perp BC$.

$$OB^2 = OA^2 + AB^2$$

$$5^2 = 4^2 + AB^2 \text{ gives } AB^2 = 9$$

Therefore $AB = 3$ cm

$BC = 2$AB hence $BC = 2 \times 3 = 6$ cm



8. A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60°$. Find the length of the string, assuming that there is no slack in the string.

Solution Let AB be the height of the kite above the ground. Then, $AB = 75$.

Let AC be the length of the string.

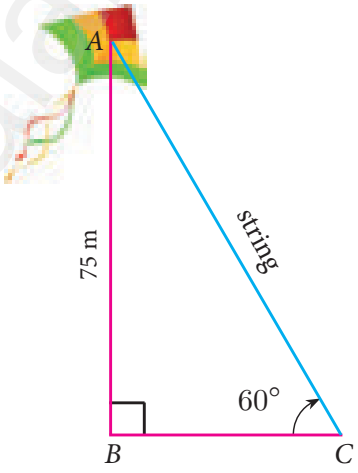
In the right angled ΔABC, $\angle ACB = 60°$

$$\sin\theta = \frac{AB}{AC}$$

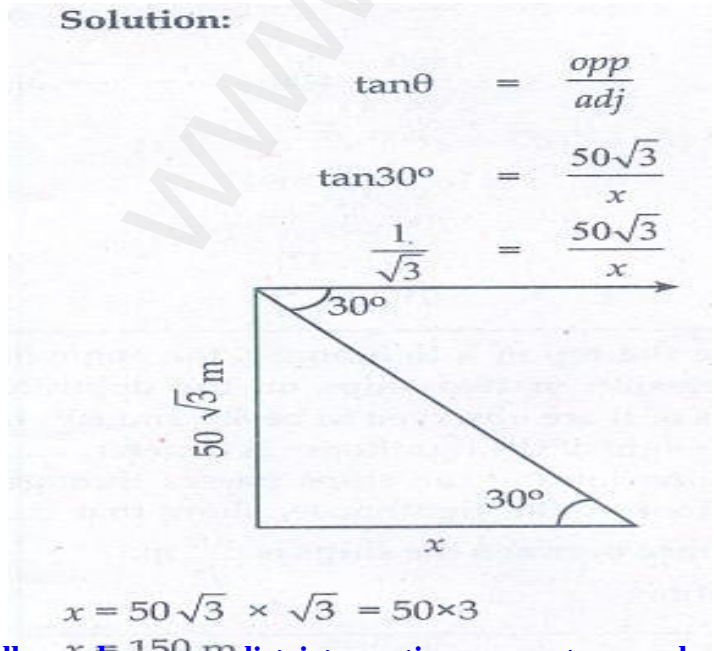
$$\sin 60° = \frac{75}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{75}{AC} \Rightarrow AC = \frac{150}{\sqrt{3}} = 50\sqrt{3}$$

Hence, the length of the string is $50\sqrt{3}$ m.



9. From the top of a rock 50 3 m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.



10. Find the diameter of a sphere whose surface area is 154 m^2.

Solution Let r be the radius of the sphere.

Given that, surface area of sphere $= 154\ m^2$

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow \quad r^2 = 154 \times \frac{1}{4} \times \frac{7}{22}$$

$$r^2 = \frac{49}{4} \text{ we get } r = \frac{7}{2}$$

Therefore, diameter is 7 m

11. If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution Given that, radius $r = 7$ cm

Now, total surface area of the cone $= \pi r(l + r)$ sq. units

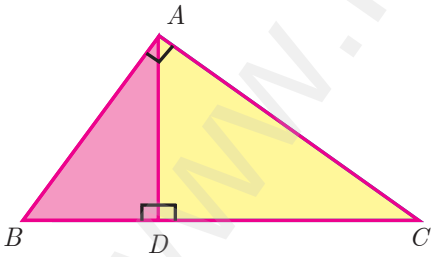
$$\text{T.S.A.} = 704\ cm^2$$

$$704 = \frac{22}{7} \times 7(l + 7)$$

$$32 = l + 7 \Rightarrow l = 25 \text{ cm}$$

Therefore, slant height of the cone is 25 cm.

12. In the given figure $\angle A = 90^0$, BD = 4 cm, DC = 9 cm , and AD $\perp$ BC, then find the length of AD.



Given : In ΔABC, $\angle A = 90°$ BD = 4 cm , DC = 9 cm

$\angle ADB = \angle ADC = 90^0$ **Side AD is common**

$\Delta ADB \sim \Delta ADC$

$\frac{AD}{BD} = \frac{DC}{AD}$

$AD^2 = DC \times BD$

$AD^2 = 9 \times 4$

$AD = \sqrt{36}$

Length of $AD = 6$ cm

III. FIVE MARKS:

Find X and Y if $X+Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}$ and $X-Y=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}$

$X+Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}$ ----------①

$X-Y=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}$ ----------②

$①+②= X+Y+X-Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}+\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}$

$$2X=\begin{pmatrix}10 & 0\\ 3 & 9\end{pmatrix}$$

$$X=\frac{1}{2}\begin{pmatrix}10 & 0\\ 3 & 9\end{pmatrix}=\begin{pmatrix}\frac{10}{2} & \frac{0}{2}\\ \frac{3}{2} & \frac{9}{2}\end{pmatrix}$$

$$X=\begin{pmatrix}5 & 0\\ \frac{3}{2} & \frac{9}{2}\end{pmatrix}$$

$①-②= X+Y-(X-Y)=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}-\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}$

$$2Y=\begin{pmatrix}4 & 0\\ 3 & 1\end{pmatrix}$$

$$Y=\frac{1}{2}\begin{pmatrix}4 & 0\\ 3 & 1\end{pmatrix}=\begin{pmatrix}\frac{4}{2} & \frac{0}{2}\\ \frac{3}{2} & \frac{1}{2}\end{pmatrix}$$

$$Y=\begin{pmatrix}2 & 0\\ \frac{3}{2} & \frac{1}{2}\end{pmatrix}$$

14. If $A=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}$ show that $(AB)^T=B^TA^T$

Solution

LHS $=(AB)^T$

$$AB=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}_{2\times3}\times\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}_{3\times2}$$

$$=\begin{pmatrix}2-2+0 & -1+8+2\\ 4+1+0 & -2-4+2\end{pmatrix}=\begin{pmatrix}0 & 9\\ 5 & -4\end{pmatrix}$$

$$(AB)^T=\begin{pmatrix}0 & 9\\ 5 & -4\end{pmatrix}^T=\begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix} \quad \ldots(1)$$

RHS $=(B^TA^T)$

$$B^T=\begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix},\ A^T=\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix}_{2\times3}\times\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}_{3\times2}$$

$$=\begin{pmatrix}2-2+0 & 4+1+0\\ -1+8+2 & -2-4+2\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix} \quad \ldots(2)$$

From (1) and (2), $(AB)^T=B^TA^T$.

Hence proved.

15. State and prove Pythagoras theorem

Pythagoras Theorem

Statement

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

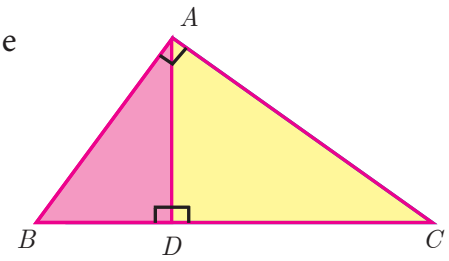


Fig. 4.46

Proof

Given : In ΔABC, $\angle A = 90^\circ$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common $\angle BAC = \angle BDA = 90^\circ$ Therefore, $\Delta ABC \sim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ … (1)	Given $\angle BAC = 90^\circ$ and by construction $\angle BDA = 90^\circ$ By AA similarity
2.	Compare ΔABC and ΔDAC $\angle C$ is common $\angle BAC = \angle ADC = 90^\circ$ Therefore, $\Delta ABC \sim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ …(2)	Given $\angle BAC = 90^\circ$ and by construction $\angle ADC = 90^\circ$ By AA similarity

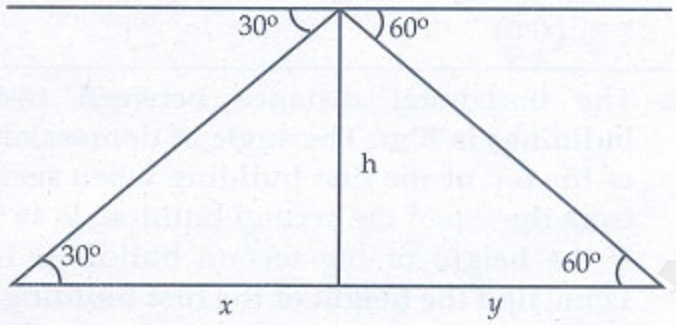
Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$
$$= BC(BD + DC) = BC \times BC$$
$$AB^2 + AC^2 = BC^2.$$

Hence the theorem is proved.

16. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be $30°$ and $60°$. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Solution:



$$\tan\theta = \frac{opp}{adj}$$

$$\tan 30° = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = \sqrt{3}\,h \quad \text{------(1)}$$

$$\tan 60° = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}}$$

$$\text{Distance} = x+y$$

$$= \frac{h}{\sqrt{3}} + h\sqrt{3}$$

$$= \frac{h+3h}{\sqrt{3}}$$

$$x+y = \frac{4h}{\sqrt{3}} \text{ m}$$

17. From the top of a 12 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Solution As shown in Fig.6.27, OA is the building, O is the point of observation on the top of the building OA. Then, $OA = 12$ m.

PP' is the cable tower with P as the top and P' as the bottom.

Then the angle of elevation of P, $\angle MOP = 60°$.

And the angle of depression of P', $\angle MOP' = 30°$.

Suppose, height of the cable tower $PP' = h$ metres.

Through O, draw $OM \perp PP'$

$MP = PP' - MP' = h - OA = h - 12$

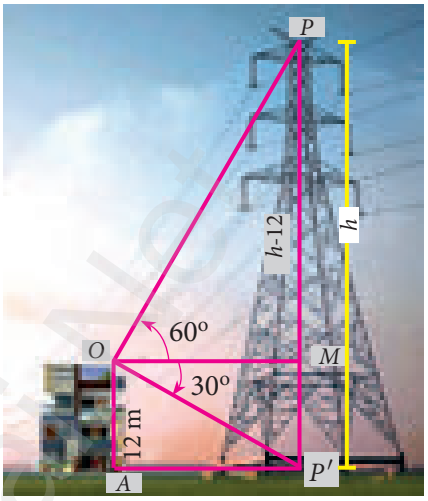


Fig. 6.27

In the right angled ΔOMP, $\frac{MP}{OM} = \tan 60°$

$$\Rightarrow \quad \frac{h-12}{OM} = \sqrt{3}$$

$$OM = \frac{h-12}{\sqrt{3}} \quad \ldots(1)$$

In the right angled $\Delta OMP'$, $\frac{MP'}{OM} = \tan 30°$

$$\Rightarrow \quad \frac{12}{OM} = \frac{1}{\sqrt{3}}$$

$$OM = 12\sqrt{3} \quad \ldots(2)$$

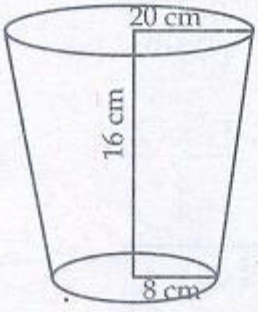
From (1) and (2) we have, $\frac{h-12}{\sqrt{3}} = 12\sqrt{3}$

$$\Rightarrow \quad h - 12 = 12\sqrt{3} \times \sqrt{3} \quad \text{we get, } h = 48$$

Hence, the required height of the cable tower is 48 m.

18. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹40 per litre.

Solution:



Given: h = 16 cm, R = 20 cm, r = 8 cm

Volume of frustum

$= \frac{1}{3}\pi h\,(R^2+r+Rr)$ cu. unit

$= \frac{1}{3} \times \frac{22}{7} \times 16(20^2+8^2+20\times 8)$

$= \frac{1}{3} \times \frac{22}{7} \times 16(400+64+160)$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 624$

$= 10456.43\ cm^3$

$= \frac{10459.43}{1000}\ lt \quad [1000 cm^3 = 1l$

Volume = 10.45943 *lt*

The cost of milk per litre = Rs. 40

The cost of milk per 10.45943 *l*

= Rs. 1045943×40

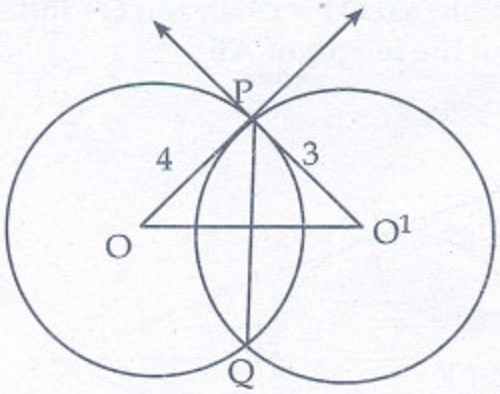
= Rs. 418.36

19. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ.

Solution :

Since the radius is perpendicular to tangent at point of contact,

$\angle OPO' = 90^0$



$\therefore PL = \sqrt{4^2 + (3.2)^2}$

$= \sqrt{16 - 10.04}$

$= \sqrt{5.76}$

$= 2.4$

$\therefore PQ = 2 \times 2.4$

$= 4.8$ cm

$O'O = \sqrt{4^2 + 3^2} = 5$

Let $O'L = x$, then $OL = 5 - x$

$PL^2 = 4^2 - x = 3^2 - (5 - x)^2$

$\Rightarrow 16 - x^2 = 9^2 - (25 + x^2 - 10x)$

$16 = -16 + 10x$

$10x = 32$

$x = \frac{32}{10}$

$\boxed{x = 3.2 \text{ cm}}$

IV. 8 MARKS

20. a) Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution

Given, radius=4 cm

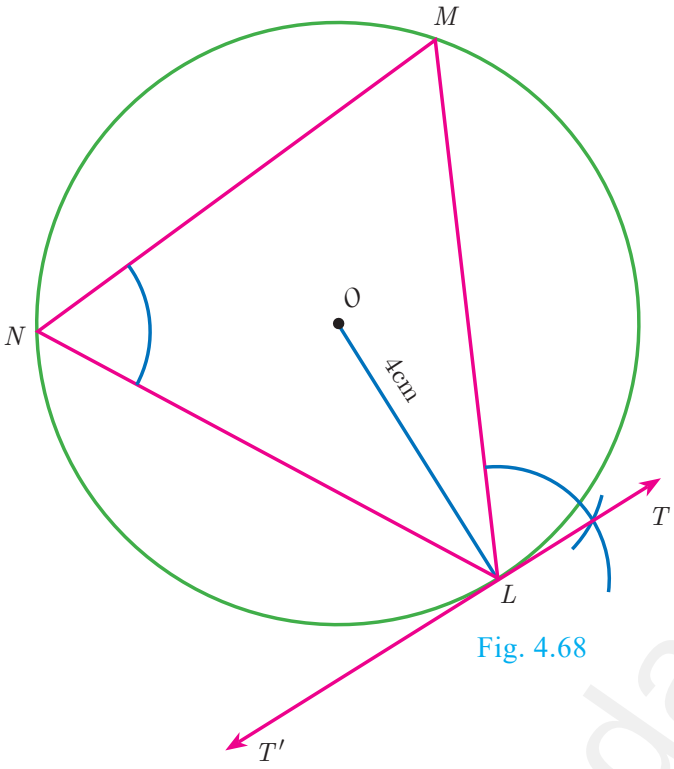
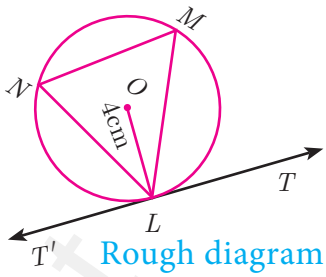


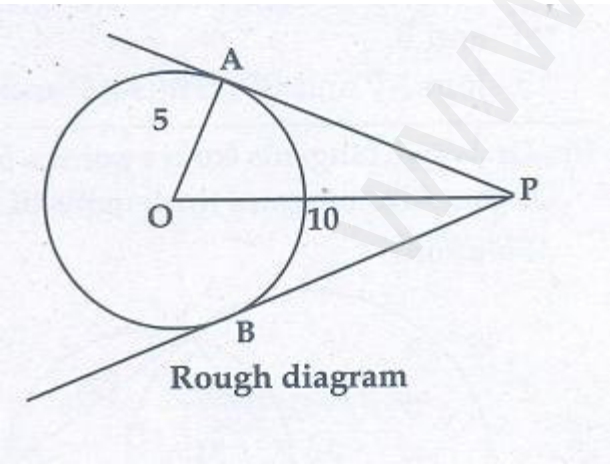
Fig. 4.68



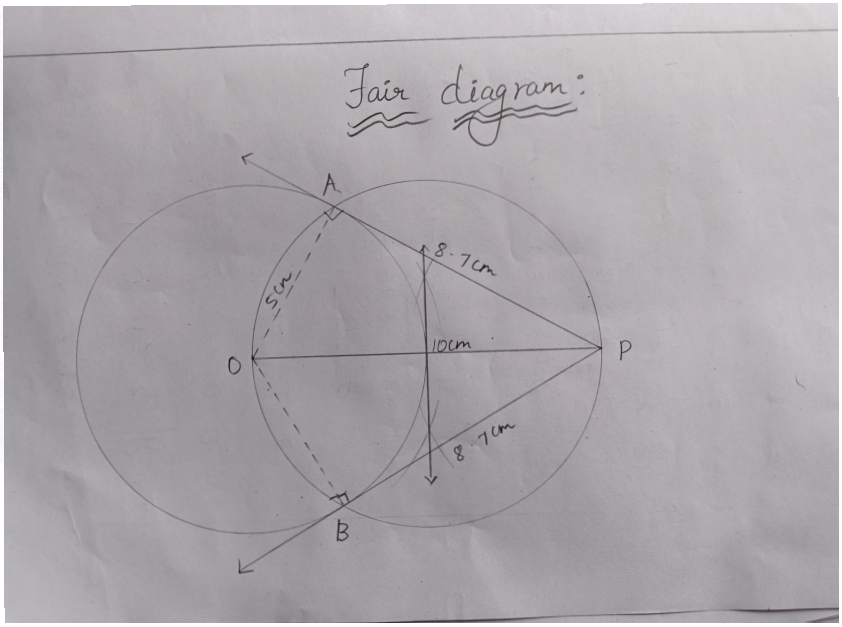
Rough diagram

b) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

Fair diagram



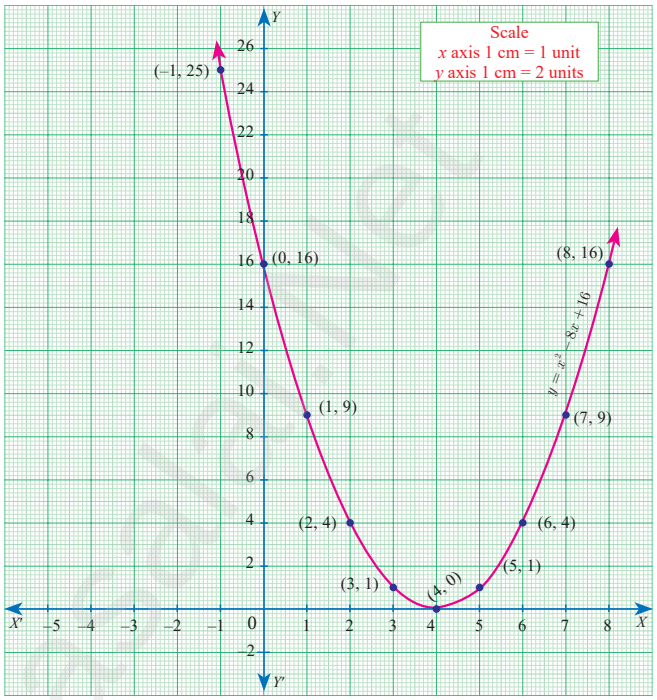
Rough diagram



21. a) Discuss the nature of solutions of the following quadratic equation $x^2 - 8x + 16 = 0$

Prepare the table of values for the equation $y = x^2 - 8x + 16$

x	–1	0	1	2	3	4	5	6	7	8
y	25	16	9	4	1	0	1	4	9	16



Since there is only one point of intersection with X axis, the quadratic equation $x^2 - 8x + 16 = 0$ has real and equal roots.

b) Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$

Solution :

Prepare a table for $y = x^2 + 3x - 4$

x	– 5	– 4	– 3	– 2	– 1	0	1	2
y	6	0	– 4	– 6	– 6	– 4	0	6

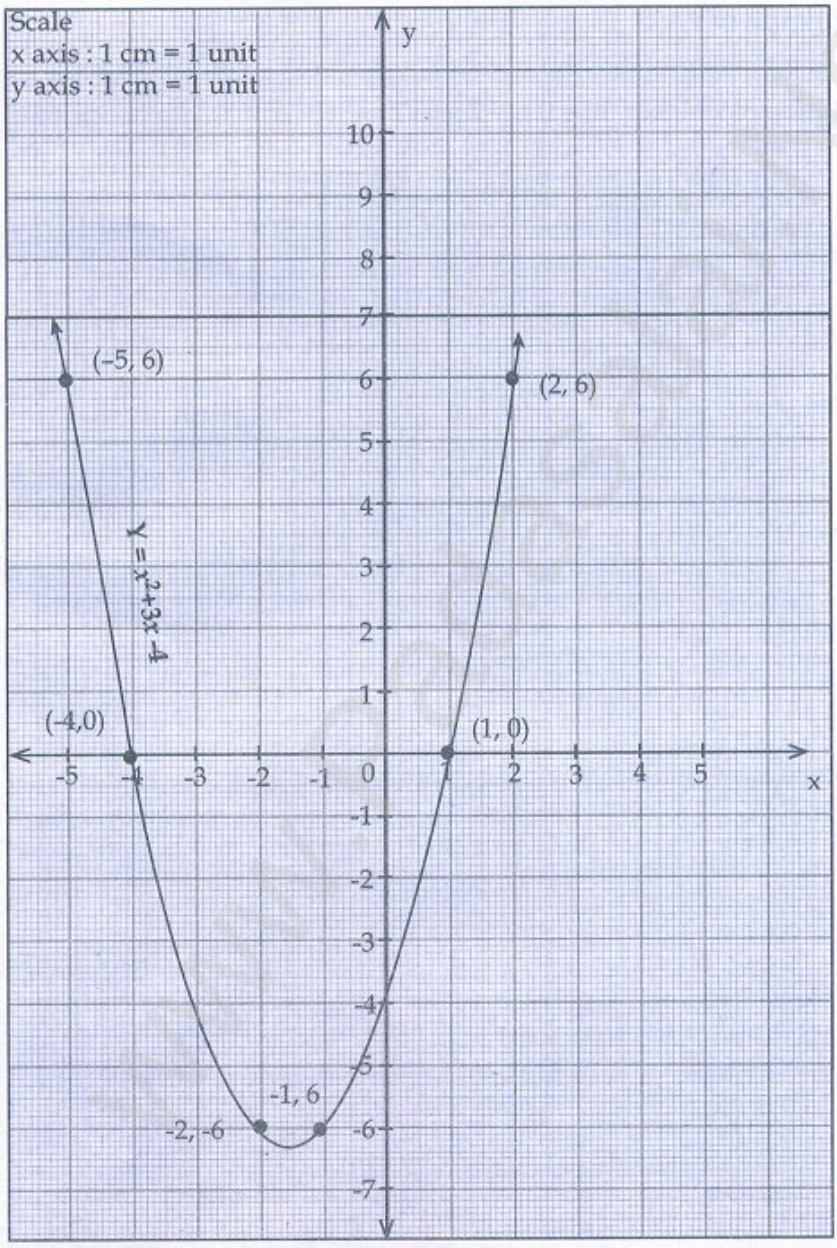
Now $y = x^2 + 3x - 4$

$0 = x^2 + 3x - 1$ (–)

$y = 0$

$y = 0$ is the straight line of x axis

There fore, the solution for $x^2 + 3x - 4 = 0$ is – 4 and 1



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