

10th Maths Unit Exercise Chapter – 1

1. Given : $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal

$$(x^2 - 3x, y^2 + 4y) = (-2, 5)$$

$$\text{i.e. } x^2 - 3x = -2$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$x = 1, 2$$

$$y^2 + 4y = 5$$

$$y^2 + 4y - 5 = 0$$

$$(y + 5)(y - 1) = 0$$

$$y = -5, 1$$

2. Given : $n(A \times A) = 9$ Also two ordered pairs $= (-1, 0)$ and $(0, 1)$

$$n(A) \times n(A) = 9 \therefore n(A) = 3$$

From the given two ordered pairs $= (-1, 0)$ and $(0, 1)$

$$A = \{-1, 0, 1\}; \therefore A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$$

$$= \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$$

3. Given : $f(x) = \begin{cases} \sqrt{x-1}, & x \geq 1 \\ 4, & x < 1 \end{cases}$

$$(i) f(0) = 4 \quad [\because 0 < 1, \text{ It is satisfy 2nd condition}]$$

$$(ii) f(3) = \sqrt{3-1} = \sqrt{2} \quad [\because 3 \geq 1 \text{ It is satisfy 1st condition}]$$

$$(iii) f(a+1) = \sqrt{(a+1)-1} = \sqrt{a} \quad [\because 0+1 \geq 1 \text{ It is satisfy 1st condition}]$$

4. Given : $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$

$f(n) = \text{the highest prime factor}$ (Note : 1 is neither a prime nor a composite)

$f(9) = (9 = 3 \times 3)$, \therefore the highest prime = 3

$f(10) = (10 = 2 \times 5)$, \therefore the highest prime = 5

$f(11) = \text{It's prime number, } (11 = 1 \times 11) \therefore \text{the highest prime} = 11$

$f(12) = (12 = 2 \times 2 \times 3)$, \therefore the highest prime = 3

$f(13) = \text{It's prime number, } (13 = 1 \times 13) \therefore \text{the highest prime} = 13$

$f(14) = (14 = 2 \times 7)$, \therefore the highest prime = 7

$f(15) = (15 = 3 \times 5)$, \therefore the highest prime = 5

$f(16) = (16 = 2 \times 2 \times 2 \times 2)$, \therefore the highest prime = 2

$f(17) = \text{It's prime number, } (17 = 1 \times 17) \therefore \text{the highest prime} = 17$

$$f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$$

$$\text{Range } f = \{2, 3, 5, 7, 11, 13, 17\}$$

5. Given : $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$

$$\text{When } x = 0; f(0) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 0}}} = 1$$

$$\text{When } x = 1; f(1) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 1}}} = \sqrt{2}$$

$$\text{When } x = -1; f(-1) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 1}}} = 1$$

$$\text{When } x = 2; f(2) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 4}}} = \sqrt{1 + \sqrt{1 - \sqrt{-3}}} \text{ is an imaginary}$$

$$\text{When } x = -2; f(-2) = \sqrt{1 + \sqrt{1 - \sqrt{1 - 4}}} = \sqrt{1 + \sqrt{1 - \sqrt{-3}}} \text{ is an imaginary}$$

From the above, except $(-1, 0, 1)$ the result for the other value of x become an imaginary one.

$$\therefore \text{The domain} = \{-1, 0, 1\}$$

6. Given : $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$; To prove : $(f \circ g) \circ h = f \circ (g \circ h)$

$$\begin{aligned} (f \circ g) &= f(g(x)) \\ &= f(3x) \\ &= (3x)^2 = 9x^2 \end{aligned}$$

$$\begin{aligned} (f \circ g) \circ h &= f \circ g(h(x)) \\ &= f \circ g(x - 2) \\ &= 9(x - 2)^2 \quad \text{----- (1)} \end{aligned}$$

$$\begin{aligned} (g \circ h) &= g(h(x)) \\ &= g(x - 2) \\ &= 3(x - 2) \\ f \circ (g \circ h) &= f(goh) \\ &= (3(x - 2))^2 \\ &= 9(x - 2)^2 \quad \text{----- (2)} \end{aligned}$$

since (1) = (2), $(f \circ g) \circ h = f \circ (g \circ h)$ (Proved)

7. This question is also given as multiple choice no. (3)

Given : $A = \{1, 2\}$; $B = \{1, 2, 3, 4\}$; $C = \{5, 6\}$; $D = \{5, 6, 7, 8\}$; To show : $A \times C \subset B \times D$

$$A \times C = \{1, 2\} \times \{5, 6\} = \{(1, 5), (1, 6), (2, 5), (2, 6)\} \quad \text{----- (1)}$$

$$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\} = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$\begin{aligned} &= \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), \\ &\quad (4, 5), (4, 6), (4, 7), (4, 8)\} \quad \text{----- (2)} \end{aligned}$$

Comparing (1) & (2) : $A \times C \subset B \times D$ (Proved)

8. Given : If $f(x) = \frac{x-1}{x+1}$ show that $f(f(x)) = -\frac{1}{x}$

$$\begin{aligned} f(x) &= \frac{x-1}{x+1} \\ f(f(x)) &= \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} \\ &= \frac{\frac{x-1-x-1}{x+1}}{\frac{x-1+x+1}{x+1}} = \frac{-2}{x+1} \times \frac{x+1}{2x} \\ &= \frac{-2}{2x} = -\frac{1}{x} \quad \text{(Proved)} \end{aligned}$$

9. Given : $f(x) = 6x + 8$; $g(x) = \frac{x-2}{3}$

$$\begin{aligned} \text{(i)} \quad gg(x) &= g(g(x)) = \frac{\frac{x-2}{3}-2}{3} \\ &= \frac{x-2-6}{3 \times 3} = \frac{x-8}{9} \\ gg(x) &= \frac{x-8}{9} \\ gg\left(\frac{1}{2}\right) &= \frac{\frac{1}{2}-8}{9} \\ &= \frac{1-16}{2 \times 9} \\ &= -\frac{15}{18} = -\frac{5}{6} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad gf(x) &= g(f(x)) \\ &= g(6x + 8) \\ &= \frac{6x+8-2}{3} \\ &= \frac{6x+6}{3} \\ &= \frac{6(x+1)}{3} \\ &= 2(x+1) \end{aligned}$$

10. (i). $f(x) = \frac{2x+1}{x-9}$ By seeing the denominator except $x = 9$, the other values x are defined

\therefore Domain of $f = \mathbb{R} - \{9\}$

(Note : \mathbb{R} means Real number)

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(ii). $p(x) = \frac{-5}{4x^2+1}$ By seeing the denominator, all values x is defined

\therefore Domain of p = \mathbb{R}

(iii). $g(x) = \sqrt{x-2}$ By seeing the Square root , when $x < 2$, it will become an imaginary.

\therefore Domain of g = $[2, \infty)$

(ii). $h(x) = x + 6$ $h(x)$ is defined all values x

\therefore Domain of h = \mathbb{R}

Unit Exercise Chapter – 2

1. To Prove : $n^2 - n$ divisible by 2 for every positive integer n.

$$n^2 - n = n(n - 1)$$

Here, when n = Odd, n - 1 becomes even

when n = Even, n - 1 becomes odd

The product of one odd and one even is always an even number which is divisible by 2

$\therefore n^2 - n$ divisible by 2 for every positive integer n.

2. Cow's milk = 175 litres ; Buffalow's milk = 105 litres

The milkman wants to them separately with equal sizes of can

\therefore The size can is the HCF 175, 105

$$175 = 5 \times 5 \times 7 ; 105 = 3 \times 5 \times 7 \therefore \text{The HCF} = 5 \times 7 = 35$$

(i) Capacity of a can = 35 litre

$$(ii) \text{Number of cans of cow's milk} : \frac{175}{35} = 5$$

$$(iii) \text{Number of cans of buffalow's milk} : \frac{105}{35} = 3$$

3. As per given condition,

When a is divisible by 13, the remainder is 9

$$\therefore a \equiv 9 \pmod{13} \quad \dots \quad (1)$$

$$\text{Similarly } b \equiv 7 \pmod{13} \quad \dots \quad (2)$$

$$\text{Similarly } c \equiv 10 \pmod{13} \quad \dots \quad (3)$$

$$(2) \times 2 \rightarrow 2b \equiv 14 \pmod{13} \quad (\text{Multiplication of Modulo arithmetic})$$

$$2b \equiv 1 \pmod{13}$$

$$(3) \times 3 \rightarrow 3c \equiv 30 \pmod{13}$$

$$3c \equiv 4 \pmod{13}$$

$$a + 2b + 3c \equiv (9 + 1 + 4) \pmod{13} \quad (\text{Addition of Modulo arithmetic})$$

$$a + 2b + 3c \equiv 14 \pmod{13}$$

$$a + 2b + 3c \equiv 1 \pmod{13}$$

\therefore When (a + b + c) is divisible by 13 , the remainder is 1.

4. Let $107 = 4q + 3$

$$107 - 3 = 4q$$

$$104 = 4q$$

\because 104 is divisible by 4 for any integer q, 107 is of the form $4q + 3$.

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5. Let a and d be the 1st term and common difference of an AP

$$\text{It's } n^{\text{th}} \text{ term} \quad t_n = a + (n - 1)d$$

$$\begin{aligned} (\text{m+1})^{\text{th}} \text{ term} \quad t_{m+1} &= a + (m + 1 - 1)d \\ &= a + md \quad \dots \quad (1) \end{aligned}$$

$$\begin{aligned} (\text{n+1})^{\text{th}} \text{ term} \quad t_{n+1} &= a + (n + 1 - 1)d \\ &= a + nd \quad \dots \quad (2) \end{aligned}$$

$$(2) \times 2 \rightarrow 2(t_{n+1}) = 2[a + nd] \quad \dots \quad (3)$$

From the condition $(1) = (3)$

$$a + md = 2[a + nd] \quad \dots \quad (4)$$

$$\begin{aligned} t_{3m+1} &= a + (3m + 1 - 1)d \\ &= a + 3md \\ &= a + md + 2md \\ &= 2(a + nd) + 2md \quad [\text{As per (4)}] \\ &= 2(a + md + nd) \\ &= 2[a + (m + n)d] \\ &= 2[a + (m + n + 1 - 1)d] \\ &= 2t_{m+n+1} \end{aligned}$$

$$\therefore (3m+1)^{\text{th}} \text{ term} = 2 \times (\text{m+n+1})^{\text{th}} \text{ term}$$

6. Given A.P = -2, -4, -6, ... -100 ; It's 1st term $a = -2$; $d = -2$

By reversing the A.P = -100, -98, -96, ..., -2 ; Now $a = -100$, $d = 2$

$$t_n = a + (n - 1)d$$

$$12^{\text{th}} \text{ term} \quad t_{12} = -100 + (12 - 1)2$$

$$t_{12} = -100 + 22 = -78$$

7. Given : $\underline{\text{AP}_1}$ $\underline{\text{AP}_2}$
 1st term 2 7

Common difference is same for both AP's

Difference of 1st terms of two AP's = $2 - 7 = -5$

Since the common difference is same for both, then

The Difference of any corresponding terms two AP's = -5

$$\therefore t_{10} \text{ of AP}_1 - t_{10} \text{ of AP}_2 = -5$$

$$t_{21} \text{ of AP}_1 - t_{21} \text{ of AP}_2 = -5$$

$$\therefore t_n \text{ of AP}_1 - t_n \text{ of AP}_2 = -5$$

8. Given : $S_{10} = 16500$,

Let the 1st year savings = a

The 2nd year savings = $a + 100$

The 3rd year savings = $a + 100 + 100 = a + 200$

It forms an AP with a common difference $d = 100$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2} [2a + (10 - 1)100] = 16500$$

$$5[2a + 900] = 16500$$

$$2a + 900 = \frac{16500}{5} = 3300$$

$$2a = 3300 - 900 = 2400$$

$$\therefore a = \frac{2400}{2} = 1200$$

1st year he saved Rs 1200

9. Given : 2nd term of a GP i.e. $ar = \sqrt{6}$

6th term of a GP i.e. $ar^5 = 9\sqrt{6}$

$$\frac{ar^5}{ar} = \frac{9\sqrt{6}}{\sqrt{6}}$$

$$r^4 = 9$$

$$r^2 = 3$$

$$r = \sqrt{3}, -\sqrt{3}$$

$$\textcircled{1} \text{ When } r = \sqrt{3}, ar = \sqrt{6}; a = \frac{\sqrt{6}}{r} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{3}} = \sqrt{2}$$

$$\textcircled{2} \text{ When } r = -\sqrt{3}, ar = \sqrt{6}; a = \frac{\sqrt{6}}{r} = \frac{\sqrt{3} \times \sqrt{2}}{-\sqrt{3}} = -\sqrt{2}$$

GP : a, ar, ar^2, \dots

GP as per $\textcircled{1}$: $\sqrt{2}, \sqrt{2} \times \sqrt{3}, \sqrt{2} \times \sqrt{3}^2, \dots$

$$\sqrt{2}, \sqrt{6}, 3\sqrt{2}, \dots$$

GP as per $\textcircled{2}$: $-\sqrt{2}, (-\sqrt{2}) \times (-\sqrt{3}), (-\sqrt{2}) \times (-\sqrt{3})^2, \dots$

$$-\sqrt{2}, \sqrt{6}, -3\sqrt{2}, \dots$$

10. Given : Value motor cycle (a) = ₹ 45000

Depreciation = 15%

To find : Value of the motor cycle after 3 years : n = 3

Depreciated value after 1 year = $45000 \times (100 - 15)\% = 45000 \times 85\%$

$$\text{After 1 year} = 45000 \times \frac{85}{100}$$

$$\text{After 2 year} = 45000 \times \frac{85}{100} \times \frac{85}{100}$$

$$\begin{aligned} \text{After 3 year} &= 45000 \times \frac{85}{100} \times \frac{85}{100} \times \frac{85}{100} \\ &= 27635.625 \end{aligned}$$

Value of the motor cycle after 3 years = ₹ 27635

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Unit Exercise Chapter – 3

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$$\frac{1}{3}(x+y-5) = y-2$$

$$x+y-5 = 3y-3z$$

$$x+y-3y+3z = 5$$

$$x-2y+3z = 5 \quad \text{--- } ①$$

Taking middle two

$$y-2 = 2x-11$$

$$y = 2x+2-11 \quad \text{--- } ②$$

Put ② in ①

$$x-2(2x+2-11)-3z = 5$$

$$x-4x-2z+22-3z = 5$$

$$-3x+z = -17 \quad \text{--- } ③$$

Taking last two

$$2x-11 = 9-(x+2z)$$

$$2x+x+2z = 9+11$$

$$3x+2z = 20 \quad \text{--- } ④$$

② Let x, y, z be the students in A, B, C. According to conditions.

$$x+y+z = 150 \quad \text{--- } ①$$

$$x-6 = z+6$$

$$x = z+12 \quad \text{--- } ②$$

$$4z-x = y$$

$$-x-y+4z = 0 \quad \text{--- } ③$$

Putting ② in ①

$$z+12+y+z = 150$$

$$y+2z = 138 \quad \text{--- } ④$$

Putting ② in ③

$$-(z+12)-y+4z = 0$$

$$-z-12-y+4z = 0$$

$$-y+3z = 12 \quad \text{--- } ⑤$$

$$\text{Adding ④ & ⑤ } 5z = 150, \therefore z = 30$$

$$x = z+12 = 30+12 = 42$$

$$y = 150 - (30+42) = 78$$

Students in A = 42, B = 78, C = 30

③ Let x, y, z be the $100^{\text{th}}, 10^{\text{th}}$ & unit place.

\therefore The number = $100x + 10y + z$.

If 100^{th} & 10^{th} changed, then it is 54 more than twice of original.

$$100y + 10x + z = 3(100x + 10y + z) + 54$$

$$100y + 10x + z - 300x - 30y - 3z = 54$$

$$-290x + 70y - 2z = 54$$

$$\div \text{ by 2} \Rightarrow -145x + 35y - z = 27 \quad \text{--- } ①$$

If the digits are reversed then, it is 198 of the original.

$$100z + 10y + x = 100x + 10y + z + 198$$

$$100z + 10y + x - 100x - 10y - z = 198$$

$$-99x + 99z = 198$$

$$\div \text{ by 99} \quad -x + z = 2 \quad \text{--- } ②$$

$$z = x+2$$

As per 3rd condition.

$$y = 14-2 = 12$$

$$y-z-2y+2z = 0$$

$$-3x+z = -17$$

$$3x+2z = 20 \quad \text{--- } ④$$

$$\text{Adding } 0+3z = 3$$

$$2 = \frac{3}{3} = 1$$

$$\text{From ④ } 3x+2 \times 1 = 20$$

$$3x = 18$$

$$x = \frac{18}{3} = 6$$

$$\text{Put } x = 6, z = 1 \text{ in } ②$$

$$y = 2 \times 6 + 1 - 11$$

$$= 12 - 11 = 1$$

$$x = 6, y = 1, z = 1$$

$$(4) xy(k^2z) + k(z^2+y^2)$$

$$= k^2xy + xy + kz^2 + ky^2$$

$$= k^2xy + kz^2 + ky^2 + xy$$

$$= kx(ky+x) + y(ky+x)$$

$$= (kx+y)(ky+x)$$

$$xy(k^2z) + k(z^2-y^2)$$

$$= k^2xy - xy + kz^2 - ky^2$$

$$= k^2xy + kz^2 - ky^2 - xy$$

$$= kx(ky+x) - y(ky+x)$$

$$= (kx-y)(ky+x)$$

$$\text{L.C.M of Both} = (ky+x)(kx+y)(kx-y)$$

$$= (ky+x)(k^2x^2-y^2)$$

$$\text{L.C.M} = (ky+x)(k^2x^2-y^2)$$

$$-x-y+2z = 0 \quad \text{--- } ③$$

Putting ② in ①

$$-145x + 35y - (x+2) = 27$$

$$-145x + 35y - x - 2 = 27$$

$$-146x + 35y = 29 \quad \text{--- } ④$$

Putting ② in ③

$$-x-y+2x+4 = 0$$

$$x-y = -4$$

$$x = y+4 \quad \text{--- } ⑤$$

$$-146x + 35(x+4) = 29$$

$$-146x + 35x + 140 = 29$$

$$-111x = -111$$

$$x = 1$$

$$\text{From ② } z = 1+2 = 3$$

$$\text{From ⑤ } y = 1+4 = 5$$

$$\text{The Number} = 100x1 + 10x5 + 3 = 153$$

$$(5) \text{ Let } f(x) = 2x^4 + 13x^3 + 27x^2 + 23x + 7 \\ g(x) = x^3 + 3x^2 + 3x + 1 \\ h(x) = x^2 + 2x + 1.$$

$$(i) f(x) \div g(x) \quad \begin{array}{r} 2x + 7 \\ x^3 + 3x^2 + 3x + 1 \end{array}$$

$$\begin{array}{r} 2x^4 + 13x^3 + 27x^2 + 23x + 7 \\ 2x^4 + 6x^3 + 6x^2 + 2x \\ \hline 7x^3 + 21x^2 + 21x + 7 \\ 7x^3 + 21x^2 + 21x + 7 \\ \hline 0 \end{array}$$

Since the remainder is zero,
g(x) is GCD of f(x) and g(x).

$$(ii) g(x) \div h(x) \quad \begin{array}{r} x + 1 \\ x^2 + 2x + 1 \end{array}$$

$$\begin{array}{r} x^3 + 3x^2 + 3x + 1 \\ x^3 + 2x^2 + x \\ \hline x^2 + 2x + 1 \\ x^2 + 2x + 1 \\ \hline 0 \end{array}$$

Here also remainder = 0
∴ The GCD = $x^2 + 2x + 1$

$$(6) (i) \frac{x^3+8}{x^2+2x+4} = \frac{(x^2+2)(x+2)}{(x^2+2x+4)^2} = \frac{(x^2+2)[(x^2+2x+4)^2]}{(x^2+2x+4)^2} = \frac{x^2+2}{x^2+2x+4} = \frac{10x^3-25x^2+4x-10}{-4-10x^2} = \frac{5x^2(2x-5)+2(2x-5)}{-2(5x^2+2)} = \frac{(5x^2+2)(2x-5)}{-2(5x^2+2)} = \frac{2x-5}{-2} = -x+\frac{5}{2}$$

8. Arul, Rani, Ram together } 6 hr.

Complete the work in }

Their workmanship per hour = $\frac{1}{6}$

Let Arul complete it alone in x hr.

∴ Rani " " " " $2x$ hr.
Ram " " " " $3x$ hr.

Their individual } = $\frac{1}{x}, \frac{1}{2x}, \frac{1}{3x}$
workmanship }

$$\therefore \frac{1}{x} + \frac{1}{2x} + \frac{1}{3x} = \frac{1}{6}$$

$$\frac{6+3+2}{6x} = \frac{1}{6}$$

$$\frac{11}{6x} = \frac{1}{6} \therefore x = \frac{6 \times 11}{6} = 11 \text{ hrs.}$$

∴ Arul complete it in 11 hrs

Rani " " " 22 hrs

Ram " " " 33 hrs

$$9 \quad \begin{array}{r} 17x^2 - 18x + 19 \\ 17x^4 - 612x^3 + 970x^2 + 684x + 361 \\ 2.89x^4 \\ 34x^2 - 18x \\ 34x^2 - 36x \\ 19 \end{array}$$

$$\begin{array}{r} -612x^3 + 970x^2 \\ -612x^3 + 324x^2 \\ 646x^2 + 684x + 361 \\ 646x^2 + 684x + 361 \\ 0 \end{array}$$

∴ The Square root: $17x^2 + 18x + 19$

$$(10) \sqrt{y+1} + \sqrt{2y-5} = 3$$

Squaring both sides,

$$(\sqrt{y+1} + \sqrt{2y-5})^2 = 3^2$$

$$(\sqrt{y+1})^2 + (\sqrt{2y-5})^2 + 2\sqrt{y+1} \times \sqrt{2y-5} = 9$$

$$y+1 + 2y-5 + 2\sqrt{(y+1)(2y-5)} = 9$$

$$3y-4 + 2\sqrt{2y^2-5y+2y-5} = 9$$

$$2\sqrt{2y^2-3y-5} = 13-3y$$

Again squaring on both sides.

$$(2\sqrt{2y^2-3y-5})^2 = (13-3y)^2$$

$$4(2y^2-3y-5) = 169 + 9y^2 - 78y$$

$$8y^2 - 12y - 20 = 169 + 9y^2 - 78y$$

$$8y^2 - 12y - 20 - 9y^2 + 78y - 169 = 0$$

$$-y^2 + 66y - 189 = 0$$

$$y^2 - 66y + 189 = 0$$

$$(y-63)(y-3) = 0$$

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$$(11) \text{ Speed} = \frac{\text{Dist}}{\text{Time}} ; \text{ Time} = \frac{\text{Dist}}{\text{Speed}}$$

$$\text{Distance} = 36 \text{ km.}$$

$$\text{Speed of water current} = 4 \text{ km/hr}$$

$$\text{Let the speed of the boat} = x \text{ km/hr}$$

$$\therefore \text{Net speed along the current} = x+4$$

$$\text{Net speed opposite to the current} = x-4$$

$$\text{Difference time} = 1.6 \text{ hr} = \frac{16}{10} = \frac{8}{5}$$

$$t_2 - t_1 = \frac{8}{5}$$

$$\frac{36}{x-4} - \frac{36}{x+4} = \frac{8}{5}$$

$$36 \left[\frac{1}{x-4} - \frac{1}{x+4} \right] = \frac{8}{5}$$

$$36 \left[\frac{x+4 - x+4}{(x-4)(x+4)} \right] = \frac{8}{5}$$

$$\frac{36 \times 8}{x^2-16} = \frac{8}{5} ; \therefore x^2 - 16 = 36 \times 5$$

$$x^2 = 180 + 16 = 196$$

$$x = \pm 14$$

Speed can not be negative.

(12) Rectangular Park.

Perimeter = 320 m : Area = 4800 m²

Let l, b be the length and breadth

$$\therefore 2(l+b) = 320$$

$$l+b = 160 \quad \text{--- ①}$$

$$l \times b = 4800 ; l = \frac{4800}{b} \quad \text{--- ②}$$

Placing ② in ①

$$\frac{4800}{b} + b = 160$$

$$\times \text{ by } b, 4800 + b^2 = 160b$$

$$b^2 - 160b + 4800 = 0$$

$$(b-120)(b-40) = 0 \quad (-120) + (-40) = -160$$

$$b = 120 \text{ or } 40$$

\therefore Taking Breadth = 40 m
Length = 120 m.

(13) The time past after 2 min and the time need to 3 pm is www.Padasalai.Net

$$t + \frac{t^2}{4} - 3 = 60 \text{ mins.}$$

$$\times \text{ by } 4 \Rightarrow 4t + t^2 - 12 = 240$$

$$t^2 + 4t - 12 = 240 = 0$$

$$t^2 + 4t - 252 = 0 \quad -252$$

$$\frac{4\sqrt{10}}{4} = 14$$

$$(t+18)(t-14) = 0$$

$$18 + (-14) = 4$$

$$18 \times (-14) = -252$$

$$t = -18 \text{ or } 14$$

\therefore Time can not be negative,

$$t = 14 \text{ mins.}$$

(14) Let the number of rows = x

As per condition, No. of seats in a row also = x

\therefore Total seats in the hall = $x \times x = x^2$

when row doubled = $2x$

Seats reduced to 5 = $x-5$

As per the IInd condition,

$$2x(x-5) = x^2 + 375$$

$$2x^2 - 10x = x^2 + 375$$

$$2x^2 - x^2 - 10x - 375 = 0$$

$$x^2 - 10x - 375 = 0 \quad -375$$

$$(x-25)(x+15) = 0 \quad -25 \mid +15$$

$$x = 25 \text{ or } -15$$

$$(-25) + 15 = -10$$

$$(-25) \times 15 = -375$$

$\therefore x$ can not be a negative.

\therefore The number of rows = 25

seats = 25

(15) let $f(x) = x^2 + Px - 4$

It has a root $6b \in 4$

$$f(4) = (-4)^2 + P(-4) - 4 = 0$$

$$16 - 4P - 4 = 0$$

$$-4P = -12$$

$$P = \frac{-12}{-4} = 3.$$

$$g(x) = x^2 + bx + 9 = 0$$

It has equal roots.

$$\text{For equal roots, } b^2 - 4ac = 0$$

Here $a = 1, b = P, c = 9$

$$P^2 - 4 \times 1 \times 9 = 0$$

$$(-3)^2 - 4 \times 9 = 0$$

$$9 - 4 \times 9 = 0$$

$$4 \times 9 = 9$$

$$9 = \frac{9}{4}$$

(15) $f(x) = x^2 - 2x + 3$

Here $a = 1, b = -2, c = 3$

$$\text{Sum of roots } \alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{1} = 2$$

$$\text{Product of the roots } \alpha \cdot \beta = \frac{c}{a} = \frac{3}{1} = 3.$$

(i) $\alpha + 2, \beta + 2$

$$\begin{aligned} \text{Sum of the roots} &= \alpha + 2 + \beta + 2 \\ &= (\alpha + \beta) + 4 \\ &= 2 + 4 = 6 \end{aligned}$$

$$\begin{aligned} \text{Product of the roots} &= (\alpha + 2)(\beta + 2) \\ &= \alpha\beta + 2\alpha + 2\beta + 4 \\ &= 2\alpha + 2(\alpha + \beta) + 4 \\ &= 3 + 2 \times 2 + 4 = 11 \end{aligned}$$

$$\text{The required equation, } x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$

$$x^2 - 6x + 11 = 0.$$

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$$(i) \frac{x-1}{\alpha+1}, \frac{B-1}{B+1}$$

$$\text{Sum of roots} = \frac{x-1}{\alpha+1} + \frac{B-1}{B+1}$$

$$= \frac{(\alpha-1)(B+1) + (B-1)(\alpha+1)}{(\alpha+1)(B+1)}$$

$$= \frac{\alpha B + \alpha - B - 1 + \alpha B + B - \alpha - 1}{\alpha B + \alpha + B + 1}$$

$$= \frac{2\alpha B - 2}{\alpha B + (\alpha + B) + 1} = \frac{2 \times 3 - 2}{3 + 2 + 1} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Product of roots} = \frac{(\alpha-1)(B-1)}{(\alpha+1)(B+1)} = \frac{\alpha B - \alpha - B + 1}{\alpha B + \alpha + B + 1}$$

$$= \frac{\alpha B - (\alpha + B) + 1}{\alpha B + (\alpha + B) + 1} = \frac{3 - 2 + 1}{3 + 2 + 1} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \text{The eqn. : } x^2 - \left(\frac{2}{3}\right)x + \frac{1}{3} = 0$$

$$(ii) \text{ by 3, } 3x^2 - 2x + 1 = 0$$

$$(17) \text{ April Sales } A = \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix}$$

It is doubled in May.

$$\therefore \text{May Sales : } 2A = 2 \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix} = \begin{bmatrix} 1000 & 2000 & 3000 \\ 5000 & 3000 & 500 \end{bmatrix}$$

$$\text{Average Sales} = \frac{A + 2A}{2} = \frac{3 \times A}{2} = \frac{3}{2} \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix} = \begin{bmatrix} 750 & 1500 & 2250 \\ 3750 & 2250 & 750 \end{bmatrix}$$

$$\text{April} = A, \text{May} = 2A, \text{June} = 4A, \text{July} = 8A$$

$$\therefore \text{August} = 16A = 16 \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix} = \begin{bmatrix} 8000 & 16000 & 21000 \\ 40000 & 21000 & 8000 \end{bmatrix}$$

$$(18) LHS = \cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} -\sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} + \begin{bmatrix} -\sin \theta \cos \theta & -\sin \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix} = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = I_2.$$

$$\begin{bmatrix} \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & \cos^2 \theta + \sin^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos \theta = 1 - \cos^2 \theta$$

$$\sin \theta = \sqrt{\cos^2 \theta}$$

$$\cos \theta = \frac{\sin \theta}{\sqrt{\cos^2 \theta}}$$

$$\cos \theta = \sin \theta$$

$$(19) A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix}$$

$$C^2 = C \times C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + (-2) \times 2 & 2 \times (-2) + (-2) \times 2 \\ 2 \times 2 + 2 \times 2 & 2 \times (-2) + 2 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$BA = C^2 ; \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$p = 8, -2q = -8$$

$$q = \frac{-8}{-2} = 4$$

$$p = 8, q = 4$$

$$(20) A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}, C = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 \times 6 + 0 \times 8 & 3 \times 3 + 0 \times 5 \\ 4 \times 6 + 5 \times 8 & 4 \times 3 + 5 \times 5 \end{bmatrix} = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$$\text{Let } D = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \therefore CD = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$CD = \begin{bmatrix} 3a + 6c & 3b + 6d \\ a + c & b + c \end{bmatrix}$$

$$AB - CD = 0, \therefore CD = AB$$

$$\begin{bmatrix} 3a + 6c & 3b + 6d \\ a + c & b + d \end{bmatrix} = \begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$$\therefore 3a + 6c = 18 \Rightarrow a + 2c = 6 \quad \text{--- (1)}$$

$$3a + 6c = 18 \Rightarrow a + 2c = 6 \quad \text{--- (1)}$$

$$3b + 6d = 9 \Rightarrow b + 2d = 3 \quad \text{--- (2)}$$

$$a + c = 64 \quad \text{--- (3)}$$

$$b + d = 37 \quad \text{--- (4)}$$

cont....

(20) Contd.

$$a + 2c = 6 \quad \text{--- (1)}$$

$$a + c = 64 \quad \text{--- (3)}$$

$$\text{--- (2)} \rightarrow c = -58$$

$$\therefore a = 64 - c = 64 - (-58) = 122$$

$$b + 2d = 3 \quad \text{--- (2)}$$

$$b + d = 37 \quad \text{--- (4)}$$

$$\text{--- (2)} \rightarrow d = -34$$

$$\therefore b = 37 - d = 37 - (-34)$$

$$= 71$$

$$D = \begin{bmatrix} 122 & 71 \\ -58 & -34 \end{bmatrix}$$

$$(7) \frac{\frac{1}{P} + \frac{1}{Q+r}}{\frac{1}{P} - \frac{1}{Q+r}} = \frac{\frac{Q+r+P}{P(Q+r+P)}}{\frac{Q+r-P}{P(Q+r)}} = \frac{Q+r+P}{Q+r-P} \quad \text{--- (1)}$$

$$14 \frac{Q+r^2 - P^2}{2Qr} = \frac{2Qr + Q^2 + r^2 - P^2}{2Qr} = \frac{2Qr + Q^2 + r^2 - P^2}{2Qr}$$

$$= \frac{(Q+r)^2 - P^2}{2Qr}$$

$$= \frac{(Q+r+P)(Q+r-P)}{2Qr} \quad \text{--- (2)}$$

$$\text{--- (1)} \times \text{--- (2)} = \frac{(Q+r+P)}{(Q+r-P)} \times \frac{(Q+r+P)}{(Q+r-P)} \times \frac{(Q+r-P)}{(Q+r-P)} = \frac{(Q+r+P)^2}{2Qr} \quad \text{--- (3)}$$

$$= \frac{(b+Q+r)^2}{2Qr}$$

1. (i) Given : $BD \perp AC$, $CE \perp AB$

In $\triangle AEC$ and $\triangle ADB$,

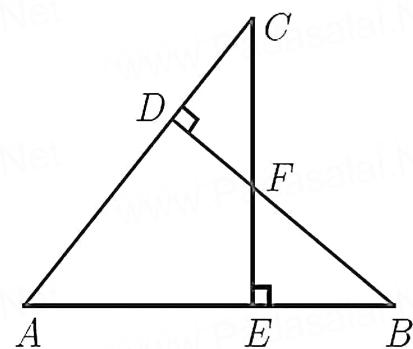
$$\angle AEC = \angle ADB = 90^\circ \text{ (Given)},$$

$\angle A$ is common for both the $\triangle AEC$ and $\triangle ADB$

When two angles are equal, the 3rd angles are also equal.

Due to AA symmetry, $\triangle AEC \sim \triangle ADB$ (Hence proved)

$$(ii) \text{ since } \triangle AEC \sim \triangle ADB, \frac{CA}{AB} = \frac{CE}{DB} \text{ (Hence proved)}$$



2. Given : $AB \parallel CD \parallel EF$

In $\triangle DAB$ and $\triangle DFE$,

$$\angle ADB = \angle FDE \text{ (}\because\text{Vertically opposite angle)}$$

$\angle DAB = \angle DFE$ (Alternate angles are equal $\because AB \parallel EF$)

Due to AA symmetry, $\triangle ADB \sim \triangle FDE$

$$\therefore \frac{DE}{DB} = \frac{FE}{AB} = \frac{DF}{DA};$$

$$\frac{y}{5} = \frac{4}{6} = \frac{DF}{AD} \quad \dots \dots \dots \text{①}$$

$$y = \frac{4 \times 5}{6} = \frac{10}{3} = 3.33 \text{ cm}$$

In $\triangle ADC$ and $\triangle AFE$, $CD \parallel EF$

By angular bisector theorem ,

$$\frac{AD}{AF} = \frac{CD}{EF}$$

$$\frac{AD}{AF} = \frac{x}{4} \quad \dots \dots \dots \text{②}$$

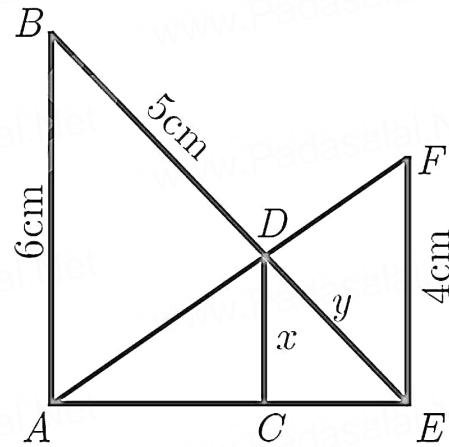
$$\text{From ①, } \frac{DF}{AD} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{DF}{AD} + 1 = \frac{2}{3} + 1$$

$$\frac{AD+DF}{AD} = \frac{2+3}{3}$$

$$\frac{AF}{AD} = \frac{5}{3} \text{ (or) } \frac{AD}{AF} = \frac{3}{5} \quad \dots \dots \dots \text{③}$$

$$\text{From ② and ③ } \frac{x}{4} = \frac{3}{5} \text{ or } x = \frac{4 \times 3}{5} = \frac{12}{5} = 2.4 \text{ cm}$$



3. In the $\triangle ABC$, mark a point O inside to it at anywhere.

Joint OA, OB, OC.

Join OD as the angular bisector of $\angle AOB$ in $\triangle AOB$

Join OE as the angular bisector of $\angle BOC$ in $\triangle BOC$

Join OF as the angular bisector of $\angle COA$ in $\triangle COA$

As per Angular bisector theorem,

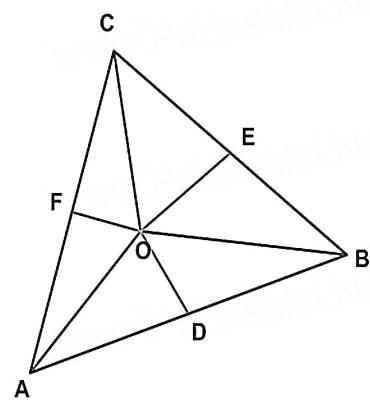
$$\because OD \text{ is the bisector of the } \angle AOB \text{ in the } \triangle AOB, \frac{AD}{DB} = \frac{AO}{BO} \quad \dots \dots \dots \text{①}$$

$$\because OE \text{ is the bisector of the } \angle AOB \text{ in the } \triangle BOC, \frac{BE}{EC} = \frac{BO}{CO} \quad \dots \dots \dots \text{②}$$

$$\because OD \text{ is the bisector to the } \angle AOB \text{ in the } \triangle COA, \frac{CF}{FA} = \frac{CO}{AO} \quad \dots \dots \dots \text{③}$$

$$\text{①} \times \text{②} \times \text{③} \rightarrow \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA} = \frac{AO}{BO} \times \frac{BO}{CO} \times \frac{CO}{AO}$$

$$\frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA} = 1 \quad \therefore AD \times BE \times CF = DB \times EC \times FA \text{ (Proved)}$$



4. In the given fig. $\therefore AB = AC$, The $\triangle AOB$ is an equilateral \triangle
 $\therefore \angle ABC \text{ or } \angle DBC = \angle ACB \text{ or } \angle ECB \quad \dots \text{--- } ①$

In the quadrilateral BCED, $DE \parallel BC$ ($\because AD = AE$)

$\therefore BD$ is transversal of BC and DE, $\angle EDB + \angle DBC = 180^\circ \quad \dots \text{--- } ②$

$\therefore CE$ is transversal of BC and DE, $\angle DEC + \angle ECB = 180^\circ \quad \dots \text{--- } ③$

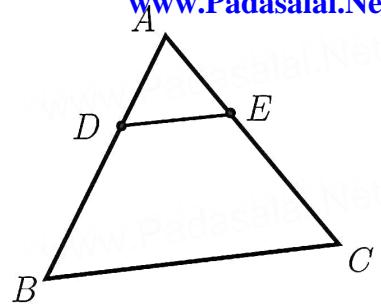
From ① and ② $\angle EDB + \angle ECB = 180^\circ$

From ① and ③ $\angle DEC + \angle DBC = 180^\circ$

From the above two ,

\therefore The sum of the opposite angles are 180° , the quadrilateral BCED lies on a same circle.

i.e. BCED is a cyclic quadrilateral. (Proved)



5. Let O be Rly Station. From it,

Train A departs towards (due) west at a speed of 20 km/hr

After 2 hour, Train A is at $20 \times 2 = 40$ km from O.

Train B departs towards (due) north at a speed of 30 km/hr

After 2 hour, Train B is at $30 \times 2 = 60$ km from O.

Now the points A, O, and B are form a Right Triangle.

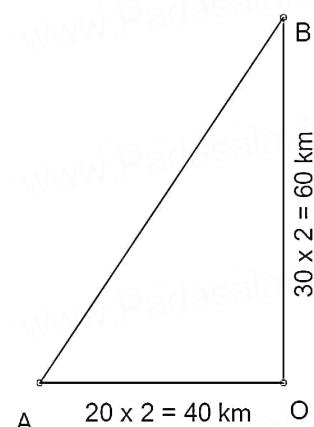
$$AB^2 = AO^2 + BO^2$$

$$AB = \sqrt{AO^2 + BO^2}$$

$$AB = \sqrt{40^2 + 60^2}$$

$$= \sqrt{1600 + 3600}$$

$$= \sqrt{5200} = \sqrt{400 \times 13} = 20\sqrt{13} \text{ km}$$



6. (In the question the word "In a $\triangle ABC$ " is missing. Anyhow)

In the fig. $AB = c$, $AC = b$, $BC = a$, $AD = p$, $AE = h$;

$\therefore D$ is the midpoint of BC, $BD = DC = \frac{a}{2}$, $ED = x$, $\therefore BE = \frac{a}{2} - x$

$\therefore AE \perp BC$, the $\triangle AED$ is right triangle.

$$h^2 = p^2 - x^2 \quad \dots \text{--- } ①$$

(i) In right triangle $\triangle AEC$,

$$AC^2 = AE^2 + EC^2$$

$$b^2 = h^2 + \left[x + \frac{a}{2}\right]^2$$

$$\text{From } ① \quad b^2 = p^2 - x^2 + x^2 + ax + \frac{a^2}{4}$$

$$b^2 = p^2 + ax + \frac{a^2}{4} \quad (\text{Proved}) \quad \dots \text{--- } ②$$

(ii) In right triangle $\triangle AEB$,

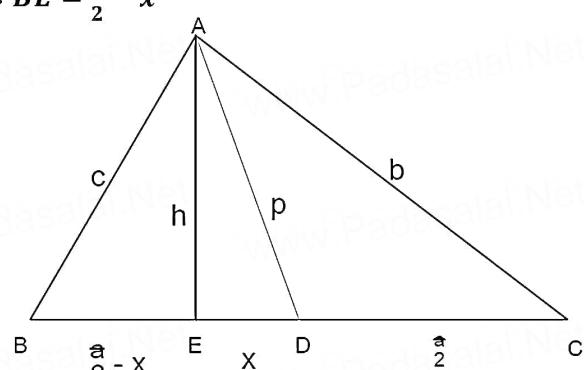
$$AB^2 = AE^2 + EB^2$$

$$c^2 = h^2 + \left[x - \frac{a}{2}\right]^2$$

$$\text{From } ① \quad c^2 = p^2 - x^2 + x^2 - ax + \frac{a^2}{4}$$

$$c^2 = p^2 - ax + \frac{a^2}{4} \quad (\text{Proved}) \quad \dots \text{--- } ③$$

$$\text{(iii) Adding } ② \text{ and } ③ \quad b^2 + c^2 = 2p^2 + \frac{a^2}{2} \quad (\text{Proved})$$



7. From the fig.

Man's eye level $CE = 2$ m

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Let Tree's height $AD = x \text{ m}$

B is the mirror point

Now DB is incident ray, BE is reflected ray

Always *The angle of incident ray = The angle of reflected ray*

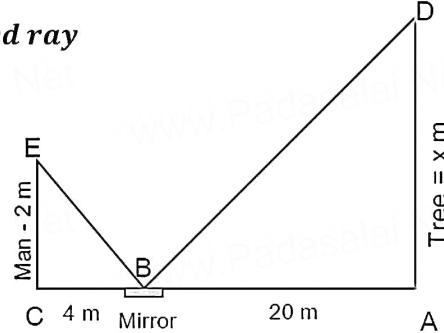
$$\therefore \angle ABD = \angle CBE$$

Also $\angle BAD = \angle BCE = 90^\circ$ ($\because \perp$ to the ground)

\therefore Due to AA similarity, $\triangle BAD \sim \triangle BCE$,

$$\frac{AD}{CE} = \frac{BA}{BC} \rightarrow \frac{x}{2} = \frac{20}{4}$$

$$x = \frac{20 \times 2}{4} = 10 \text{ m}$$



8. (One data is missing i.e. "there is a light at the top of the pillar".)

$AB = 30 \text{ ft}$ is the pillar with a light at top.

If the emu ($CD = 8 \text{ ft}$) is walking away from the foot of the pillar,

It's shadow is in front of it's.

The length of the shadow of the emu is based on it's distance from the light pillar.

Let the distance x, y is as marked in the fig.

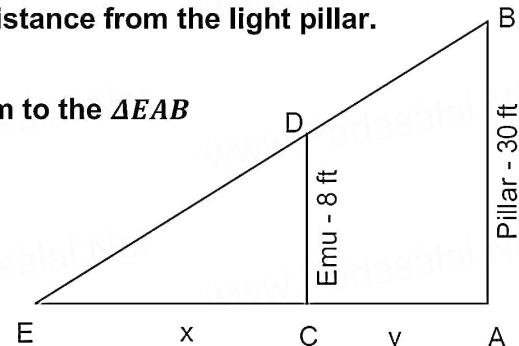
$\because AB \parallel CD$, Applying the basic proportionality theorem to the $\triangle EAB$

$$\frac{EC}{EA} = \frac{CD}{AB} \rightarrow \frac{x}{x+y} = \frac{8}{30}$$

$$30x = 8x + 8y$$

$$22x = 8y$$

$$x = \frac{8y}{22} = \frac{4}{11} \times y$$



Length of the shadow = $\frac{4}{11} \times \text{Distance of the emu from the pillar.}$

9. Draw two circles C_1 and C_2 with are intersecting at A and B.

Marking a point P on the circle C_1 and drawn a tangent XY to it.

Join PA, PB which intersect circle C_2 at C, D

Since the quadrilateral ABCD is a cyclic on circle C_1

The sum of the opposite angles = 180°

Also The exterior angle = The interior angle

$$\therefore \angle PDC = \angle PBA \quad \text{--- (1)}$$

$$\angle PCD = \angle PAB \quad \text{--- (2)}$$

Also XY is a tangent at P for circle C_1

(Alternate segment angles are equal)

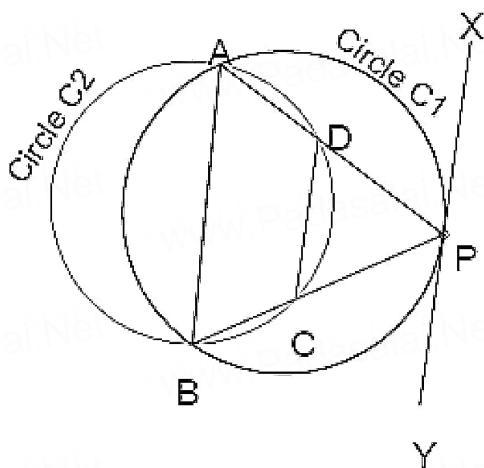
$$\therefore \angle XPA = \angle PBA \quad \text{--- (3)}$$

$$\angle YPB = \angle PAB \quad \text{--- (4)}$$

From (1) and (4), $\angle PDC = \angle YPB$

\therefore According to the Alternate Segment Theorem,

CD is parallel to the tangent at P (Proved)



10. Given : $AD:DB = 5:3$; $BE:EC = 3:2$; $AC = 21$

$$\therefore \frac{AD}{DB} = \frac{5}{3}; \quad \frac{BE}{EC} = \frac{3}{2}$$

In $\triangle ABC$, D, E and F on AB, BC and CA (or their extension)

According to Menelaus theorem, for the collinearity of D, E and F

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$$\frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA} = -1 \quad (\text{The line segments are with direction})$$

(Or) $\frac{AD}{DB} \times \frac{BE}{EC} \times \frac{FC}{FA} = 1 \quad (\text{If any one of the line segments is changed with direction})$

$$\frac{5}{3} \times \frac{3}{2} \times \frac{FC}{FA} = 1$$

$$\frac{FC}{FA} = \frac{2}{5}$$

$$\frac{FC}{FC+CA} = \frac{2}{5}$$

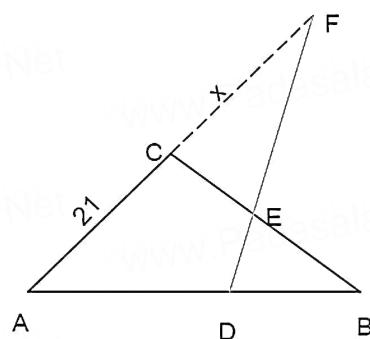
$$\frac{FC}{FC+21} = \frac{2}{5}$$

$$5FC = 2FC + 42$$

$$5FC - 2FC = 42$$

$$3FC = 42$$

$$FC = \frac{42}{3} = 14$$



10th Maths Unit Exercise Chapter – 5 Coordinate Geometry

1. Given : PQRS is a rectangle,

Their points are $P(-1, -1)$, $Q(-1, 4)$, $R(5, 4)$ and $S(5, -1)$

A, B, C and D are the mid-points of PQ, QR, RS and SP respectively.

(Hints to this problem :

1. For square, diagonals are equal and bisects perpendicular.
2. For rectangle, diagonals are equal and not bisects perpendicular.
3. For rhombus, diagonals are unequal but bisects perpendicular.

So finding the length of the diagonals and it's slopes will give the answer)

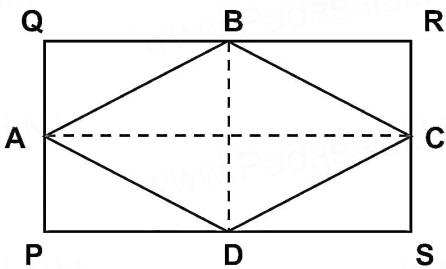
Using mid-point formula of $A = \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right]$

$$A = \left[\frac{(-1)+(-1)}{2}, \frac{(-1)+(4)}{2} \right] = \left(-1, \frac{3}{2} \right)$$

$$B = \left[\frac{(-1)+(5)}{2}, \frac{(4)+(4)}{2} \right] = (2, 4)$$

$$C = \left[\frac{(5)+(5)}{2}, \frac{(4)+(-1)}{2} \right] = \left(5, \frac{3}{2} \right)$$

$$D = \left[\frac{(5)+(-1)}{2}, \frac{(-1)+(-1)}{2} \right] = (2, -1)$$



For the quadrilateral ABCD, AC and BD are the two diagonals.

The length of the diagonals

The distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\text{Distance of } AC = \sqrt{(5 - (-1))^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = \sqrt{(5 + 1)^2 + (0)^2} = 6$$

$$\text{Distance of } BD = \sqrt{(2 - 2)^2 + ((-1) - 4)^2} = \sqrt{(0)^2 + (-5)^2} = 5$$

The Slopes of the diagonals

$$\text{Slope of } AC = \left[\frac{y_2-y_1}{x_2-x_1} \right] = \left[\frac{\frac{3}{2}-\frac{3}{2}}{5-(-1)} \right] = \frac{0}{6} = 0 \text{ i.e. } \tan\theta = 0, \theta = 0^\circ$$

$$\text{Slope of } BD = \left[\frac{y_2-y_1}{x_2-x_1} \right] = \left[\frac{-1-(-4)}{2-2} \right] = \frac{3}{0} = \infty \text{ i.e. } \tan\theta = \infty, \theta = 90^\circ$$

Since the diagonals are unequal but bisecting perpendicular,

The quadrilateral ABCD is a Rhombus one.

2. Given : The area of the triangle = 5 sq.units, Two of its vertices are A (2, 1) and B (3, -2).

Let the 3rd vertex be : C (x, y), where $y = x + 3$ ----- ①

$$\text{Area of a triangle} = \frac{1}{2} \left\{ x_1 \times y_2 - x_2 \times y_1 \right\}$$

$$\frac{1}{2} \left\{ 2 \times 3 - (-2) \times 1 \right\} = 5$$

$$(-4 + 3y + x) - (3 - 2x + 2y) = 5 \times 2$$

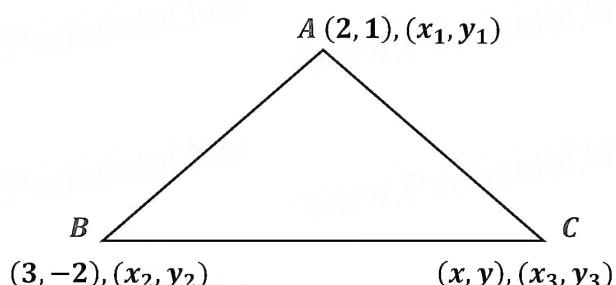
$$-4 + 3y + x - 3 + 2x - 2y = 10$$

$$3x + y - 7 = 10$$

$$3x + y = 17$$

$$\text{From ① } 3x + x + 3 = 17$$

$$4x = 14 \text{ (or) } x = \frac{14}{4} = \frac{7}{2}, \quad y = \frac{7}{2} + 3 = \frac{13}{2}$$



3. Given : Three straight lines

$$3x + y - 2 = 0 \text{ ---- ①}, \quad 5x + 2y - 3 = 0 \text{ ---- ②}, \quad 2x - y - 3 = 0 \text{ ---- ③}$$

(Solving these three equations we can get the three vertices of a triangle.)

Taking ① and ②

$$3x + y = 2 \rightarrow ①$$

$$5x + 2y = 3 \rightarrow ②$$

$$① \times 2 \rightarrow 6x + 2y = 4 \rightarrow ④$$

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(or) $x = 1, \therefore y = -1$

Taking (2) and (3)

$$2x - y = 3 \rightarrow (3)$$

$$5x + 2y = 3 \rightarrow (2)$$

$$(3) \times 2 \rightarrow 4x - 2y = 6 \rightarrow (5)$$

$$(2) + (5) \rightarrow 9x = 9$$

(or) $x = 1, \therefore y = -1$

Taking (3) and (1)

$$2x - y = 3 \rightarrow (3)$$

$$3x + y = 2 \rightarrow (1)$$

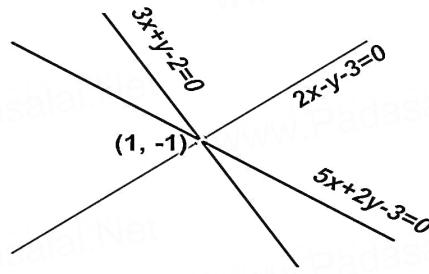
$$(1) + (3) \rightarrow 5x = 5$$

(or) $x = 1, \therefore y = -1$

Here the three points found out are the same. \therefore They are concurrent.

Since the three lines are concurrent, there will be no triangle formed

\therefore The area of the triangle = 0



4. Given : Area of a quadrilateral = 72 sq.units

The vertices of the quadrilateral are $A(-5, 7), B(-4, k), C(-1, -6), D(4, 5)$

$$\text{The area of a quadrilateral} = \frac{1}{2} \left\{ \frac{x_1}{y_1} \otimes \frac{x_2}{y_2} \otimes \frac{x_3}{y_3} \otimes \frac{x_4}{y_4} \otimes \frac{x_1}{y_1} \right\}$$

$$\frac{1}{2} \left\{ \frac{-5}{7} \otimes \frac{-4}{k} \otimes \frac{-1}{-6} \otimes \frac{4}{5} \otimes \frac{-5}{7} \right\} = 72$$

$$\{(-5k + 24 - 5 + 28) - (-28 - k - 24 - 25)\} = 72 \times 2$$

$$-5k + 47 + 77 + k = 144$$

$$-4k + 124 = 144$$

$$-4k = 144 - 124$$

$$k = \frac{20}{-4} = -5$$

5. Given : Four vertices $A(-2, -1), B(4, 0), C(3, 3)$ and $D(-3, 2)$

(Hint : If the slopes of the opposite sides are equal, they are parallel.)

(If so it's a parallelogram.)

$$\text{Slope of a line (Two points)} = \left[\frac{y_2 - y_1}{x_2 - x_1} \right]$$

$$\text{Slope of line } AB = \left[\frac{0 - (-1)}{4 - (-2)} \right] = \left[\frac{1}{4+2} \right] = \frac{1}{6}$$

$$\text{Slope of line } BC = \left[\frac{3 - 0}{3 - 4} \right] = \left[\frac{3}{-1} \right] = -3$$

$$\text{Slope of line } CD = \left[\frac{2 - 3}{(-3) - 3} \right] = \left[\frac{-1}{-6} \right] = \frac{-1}{-6} = \frac{1}{6}$$

$$\text{Slope of line } DA = \left[\frac{(-1) - 2}{(-2) - (-3)} \right] = \left[\frac{-3}{1} \right] = -3$$

$$\text{Slope of line } AB = \text{Slope of line } CD$$

$$\text{Slope of line } BC = \text{Slope of line } DA$$

Here the slopes of the opposite sides are equal, so they are parallel.

\therefore The given points are the vertices of parallelogram.

6. Given : Sum of the intercepts = 1, Product of the intercepts = -6

Let a, b be the intercepts.

$$a + b = 1, ab = -6$$

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$$a - a^2 = -6$$

$$a^2 - a - 6 = 0$$

$$(a - 3)(a + 2) = 0 \text{ (or) } a = 3, -2$$

When $a = 3$, $b = 1 - a \rightarrow 1 - 3 = -2$

When $a = -2$, $b = 1 - a \rightarrow 1 - (-2) = 3$

Eqn. of the st. line for two intercept form $\frac{x}{a} + \frac{y}{b} = 1$

Eqn. of the st. line for the intercepts $= 3, b = -2$:

$$\frac{x}{3} + \frac{y}{-2} = 1$$

Multiplying with 6 (LCM of 3 and 2) on both sides,

$$\frac{6x}{3} + \frac{6y}{-2} = 1 \times 6$$

$$2x - 3y = 6 \text{ (or) } 2x - 3y - 6 = 0$$

Eqn. of the st. line for the intercepts $= -2, b = 3$:

$$\frac{x}{-2} + \frac{y}{3} = 1$$

Multiplying with 6 (LCM of 3 and 2) on both sides,

$$\frac{6x}{-2} + \frac{6y}{3} = 1 \times 6$$

$$-3x + 2y = 6 \text{ (or) } 3x - 2y + 6 = 0$$

The required eqns are : $2x - 3y - 6 = 0$

$$3x - 2y + 6 = 0$$

7. Given : 1st week price and quantity (A) = (Rs. 14, 980 litre) (x_1, y_1)
 2nd week price and quantity (B) = (Rs. 16, 1220 litre) (x_2, y_2)
 3rd week price and quantity (C) = (Rs. 17, x litre) (x_3, y_3)
 Relationship is linear. It means A, B and C are lie on the same line.

Slope of a line BC = Slope of a line AB

$$\left[\frac{y_3 - y_2}{x_3 - x_2} \right] = \left[\frac{y_2 - y_1}{x_2 - x_1} \right]$$

$$\left[\frac{x - 1220}{17 - 16} \right] = \left[\frac{1220 - 980}{16 - 14} \right]$$

$$x - 1220 = \left[\frac{240}{2} \right]$$

$$x = 120 + 1220 = 1340$$

The milk vendor sold 1340 lit milk at the rate of Rs.17 per lit on that week.

8. Given : The mirror of the line : $x + 3y = 7$, Object Point : A (3, 8)

To be found : Image point of the object.

(Hints : Object and it's image is always equidistant from the mirror perpendicularly.)

Mirror line : $x + 3y = 7$ ----- ①

A (3, 8) Object

The perpendicular line of the mirror : $3x - y + k = 0$ ----- ②

It passes through the object point : A (3, 8)

Placing the value of A (3, 8) in ②

$$3 \times 3 - 8 + k = 0, \therefore k = -1$$

\therefore The perpendicular eqn. is $3x - y - 1 = 0$

$$(\text{Or}) 3x - y = 1 \text{ ----- ③}$$

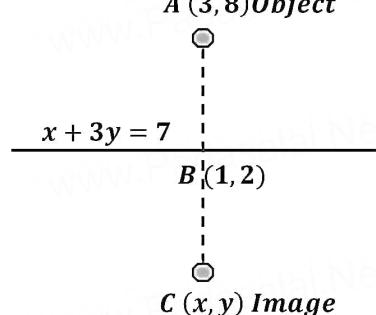
$$\textcircled{1} \times 3 \rightarrow 3x + 9y = 21 \text{ ----- ④}$$

$$\textcircled{2} - \textcircled{3} \rightarrow -10y = -20$$

$$\therefore y = 2, x = 1 \text{ (This is midpoint of A and C)}$$

Midpoint of A(3, 8) and C(x, y) is B(1, 2)

$$\text{i.e. } \left(\frac{3+x}{2}, \frac{8+y}{2} \right) = (1, 2)$$



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$$\frac{3+x}{2} = 1 \quad \frac{8+y}{2} = 2$$

$$x = -1, \quad y = -4$$

9. Given : $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$

$$4x + 7y = 3 \quad \dots \textcircled{1}$$

$$2x - 3y = -1 \quad \dots \textcircled{2}$$

$$\textcircled{2} \times 2 \rightarrow 4x - 6y = -2 \quad \dots \textcircled{3}$$

$$\textcircled{1} - \textcircled{3} \quad 13y = 5 \quad \dots \textcircled{3} \quad \text{or} \quad y = \frac{5}{13}$$

Put $y = -\frac{5}{13}$ in $\textcircled{2} \rightarrow 2x - 3\left(\frac{5}{13}\right) = -1$

$$2x - \frac{15}{13} = -1$$

$$2x = \frac{15}{13} - 1 = \frac{2}{13} \quad (\text{Or}) \quad x = \frac{1}{13}$$

The point of intersection is $\left(\frac{1}{13}, \frac{5}{13}\right)$

The required eqn. has equal intercepts. And let it be a , a

$$\therefore \frac{x}{a} + \frac{y}{a} = 1 \quad (\text{or}) \quad x + y = a \quad \text{And this passes through the point of intersection } \left(\frac{1}{13}, \frac{5}{13}\right)$$

$$\frac{1}{13} + \frac{5}{13} = a \quad (\text{or}) \quad a = \frac{6}{13}$$

$$\text{The required eqn. } x + y = a$$

$$x + y = \frac{6}{13} \quad (\text{or}) \quad 13x + 13y = 6$$

10. Given : $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$; 3rd eqn. $6x - 7y + 8 = 0$

$$2x - 3y = -4 \quad \dots \textcircled{1}$$

$$6x - 7y + 8 = 0$$

$$3x + 4y = 5 \quad \dots \textcircled{2}$$

$$\textcircled{1} \times 3 \rightarrow 6x - 9y = -12 \quad \dots \textcircled{3}$$

$$\textcircled{2} \times 2 \rightarrow 6x + 8y = 10 \quad \dots \textcircled{4}$$

$$\textcircled{3} - \textcircled{4} \quad -17y = -22 \quad \text{or} \quad y = \frac{22}{17}$$

Put $y = \frac{22}{17}$ in $\textcircled{1} \rightarrow 2x - 3\left(\frac{22}{17}\right) = -4$

$$2x - \frac{66}{17} = -4$$

$$2x = \frac{66}{17} - 4 = \frac{66-68}{17} = -\frac{2}{17} \quad \text{Or} \quad x = -\frac{1}{17}$$

The point of intersection is $\left(-\frac{1}{17}, \frac{22}{17}\right)$

3rd eqn. $6x - 7y + 8 = 0$

The perpendicular of this eqn. which is passing through the point of intersection gives the shortest distance.

\therefore The perpendicular eqn. of 3rd eqn. is $-7x - 6y + k = 0$

It passes through the point $\left(-\frac{1}{17}, \frac{22}{17}\right)$

$$-7\left(-\frac{1}{17}\right) - 6\left(\frac{22}{17}\right) + k = 0 \quad \dots \textcircled{1}$$

$$\textcircled{1} \times 17 \rightarrow 7 - 132 + 17k = 0$$

$$17k = 125 \quad \text{or} \quad k = \frac{125}{17}$$

The required eqn. is $-7x - 6y + \frac{125}{17} = 0$

$$\text{Multiplying by 17, } -7x \times 17 - 6y \times 17 + \frac{125}{17} \times 17 = 0 \times 17$$

$$-119x - 102y + 125 = 0 \quad (\text{Or}) \quad 119x + 102y - 125 = 0$$

$$\text{The required eqn. } 119x + 102y - 125 = 0$$

(Note : In the above all problems, linear equations with two variables are solved by elimination method. Students can follow their own method if feel easy.)

10th Maths Unit Exercise Chapter – 6 Trigonometry www.Padasalai.Net

1. (i)
$$\begin{aligned} LHS &= \cot^2 A \left[\frac{\sec A - 1}{1 + \sin A} \right] + \sec^2 A \left[\frac{\sin A - 1}{1 + \sec A} \right] \\ &= \frac{\cot^2 A (\sec A - 1)(1 + \sec A) + \sec^2 A (\sin A - 1)(1 + \sin A)}{(1 + \sin A)(1 + \sec A)} \\ &= \frac{\cot^2 A (\sec A - 1)(\sec A + 1) + \sec^2 A (\sin A - 1)(\sin A + 1)}{(1 + \sin A)(1 + \sec A)} \\ &= \frac{\cot^2 A (\sec^2 A - 1) + \sec^2 A (\sin^2 A - 1)}{(1 + \sin A)(1 + \sec A)} \\ &= \frac{\cot^2 A \times \tan^2 A + \sec^2 A \times (-\cos^2 A)}{(1 + \sin A)(1 + \sec A)} \\ &= \frac{\frac{1}{\tan^2 A} \times \tan^2 A + \frac{1}{\cos^2 A} \times (-\cos^2 A)}{(1 + \sin A)(1 + \sec A)} \\ &= \frac{1 - 1}{(1 + \sin A)(1 + \sec A)} \\ &= \frac{0}{(1 + \sin A)(1 + \sec A)} \\ &= 0 \\ &= RHS \end{aligned}$$

(ii)
$$\begin{aligned} LHS &= \frac{\tan^2 \theta - 1}{\tan^2 \theta + 1} \\ &= \frac{\frac{\sin^2 \theta}{\cos^2 \theta} - 1}{\frac{\sin^2 \theta}{\cos^2 \theta} + 1} \\ &= \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta}} \\ &= \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta}} \\ &= \frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{1} \\ &= \sin^2 \theta - \cos^2 \theta \\ &= 1 - \cos^2 \theta - \cos^2 \theta \\ &= 1 - 2\cos^2 \theta \\ &= RHS \end{aligned}$$

2.
$$\begin{aligned} LHS &= \left[\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} \right]^2 \\ &= \frac{(1 + \sin \theta - \cos \theta)^2}{(1 + \sin \theta + \cos \theta)^2} \\ &= \frac{1 + \sin^2 \theta + \cos^2 \theta + 2\sin \theta + 2\sin \theta(-\cos \theta) + 2(-\cos \theta)}{1 + \sin^2 \theta + \cos^2 \theta + 2\sin \theta + 2\sin \theta \cos \theta + 2\cos \theta} \quad [(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca] \\ &= \frac{1 + 1 + 2\sin \theta - 2\sin \theta \cos \theta - 2\cos \theta}{1 + 1 + 2\sin \theta + 2\sin \theta \cos \theta + 2\cos \theta} \\ &= \frac{2 - 2\cos \theta + 2\sin \theta - 2\sin \theta \cos \theta}{2 + 2\cos \theta + 2\sin \theta + 2\sin \theta \cos \theta} \\ &= \frac{2(1 - \cos \theta) + 2\sin \theta(1 - \cos \theta)}{2(1 + \cos \theta) + 2\sin \theta(1 + \cos \theta)} \\ &= \frac{(1 - \cos \theta)(2 + 2\sin \theta)}{(1 + \cos \theta)(2 + 2\sin \theta)} \\ &= \frac{(1 - \cos \theta)}{(1 + \cos \theta)} \\ &= RHS \end{aligned}$$

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3. Given : $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ ----- ①

$$x \sin \theta = y \cos \theta \quad \text{----- ②}$$

$$\text{From ②} \rightarrow x = \frac{y \cos \theta}{\sin \theta} \quad \text{----- ③}$$

$$\text{Put ③ in ①} \rightarrow \frac{y \cos \theta}{\sin \theta} \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$$

$$y \cos \theta \sin^2 \theta + y \cos^3 \theta = \sin \theta \cos \theta$$

$$y \cos \theta (\sin^2 \theta + \cos^2 \theta) = \sin \theta \cos \theta$$

$$y \cos \theta (1) = \sin \theta \cos \theta$$

$$y = \frac{\sin \theta \cos \theta}{\cos \theta} = \sin \theta$$

$$\therefore \text{From ③ } x = \frac{\sin \theta \cos \theta}{\sin \theta} = \cos \theta$$

$$x^2 + y^2 = \sin^2 \theta + \cos^2 \theta$$

$$\therefore x^2 + y^2 = 1 \quad (\text{Proved})$$

4. Given : $a \cos \theta - b \sin \theta = c$

$$\text{Squaring on both sides} \rightarrow (a \cos \theta - b \sin \theta)^2 = c^2$$

$$a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cos \theta = c^2$$

$$a^2 (1 - \sin^2 \theta) + b^2 (1 - \cos^2 \theta) - 2ab \sin \theta \cos \theta = c^2$$

$$a^2 - a^2 \sin^2 \theta + b^2 - b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = c^2$$

$$-a^2 \sin^2 \theta - b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = -a^2 - b^2 + c^2$$

$$a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = a^2 + b^2 - c^2$$

$$(a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$$

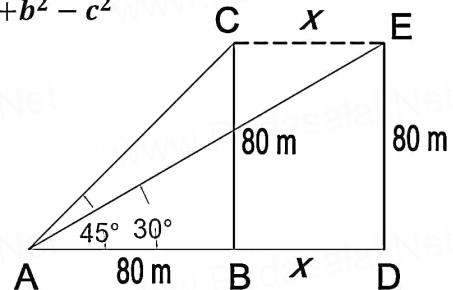
Taking square root on both sides, $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$

5. Let the bird is sitting initially at C which is 80 m high.

The angle of elevation of C, i.e. $\angle BAC = 45^\circ$

$$\tan 45^\circ = \frac{BC}{AB}$$

$$1 = \frac{80}{AB}; \quad \therefore AB = 80 \text{ m}$$



Then the bird is flying horizontally x m from C to the point E for 2 seconds.

$$CE = BD = x \text{ m}$$

Now the angle of elevation of E i.e. $\angle DAE = 30^\circ$

$$\tan 30^\circ = \frac{DE}{AD} = \frac{DE}{AB+BD}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{80+x}$$

$$80 + x = \sqrt{3} \times 80$$

$$x = \sqrt{3} \times 80 - 80$$

$$x = 80(\sqrt{3} - 1) = 80(1.732 - 1)$$

$$x = 80 \times 0.732 = 58.56 \text{ m}$$

Distance travelled by the bird $x = 58.56 \text{ m}$; Time taken for it = 2 seconds

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{58.56}{2} = 29.28 \text{ m/second.}$$

6. Let the plane be at C initially which is 600 m high.

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The angle of elevation $\angle BAC = 37^\circ$

$$\tan 37^\circ = \frac{BC}{AB}$$

$$0.7536 = \frac{600}{x+y} ;$$

$$\therefore x + y = \frac{600}{0.7536} = 796.18 \text{ m}$$

Then the plane is flying horizontally $x \text{ m}$ to the point E.

$$CE = BD = x \text{ m}$$

Now the angle of elevation $\angle DAE = 53^\circ$; $DE = BC = 600 \text{ m}$

$$\tan 53^\circ = \frac{DE}{AD}$$

$$1.3270 = \frac{600}{y}$$

$$y = \frac{600}{1.327} = 452.15 \text{ m}$$

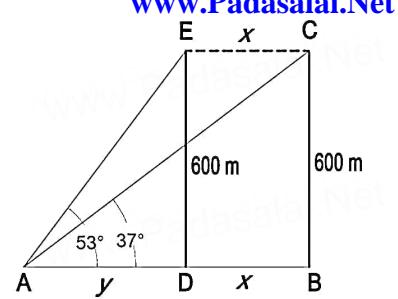
$$x + y = 796.18 \text{ m}$$

$$x = 796.18 - y$$

$$x = 796.18 - 452.15 = 344.03 \text{ m}$$

Distance travelled by the plane $x = 344.03 \text{ m}$; Speed of the plane = 175 m/seconds

$$\text{Time taken} = \frac{\text{Distance travelled}}{\text{Speed}} = \frac{344.03}{175} = 1.97 \text{ second.}$$



7. The bird is flying from A to B at an angle of 35° with respect to north.

$$AB = 30 \text{ m}; \angle LAB = 90^\circ - 35^\circ = 55^\circ$$

In the right angled triangle ALB

$$\sin 55^\circ = \frac{LB}{AB};$$

$$0.8192 = \frac{LB}{30};$$

$$LB = 0.8192 \times 30;$$

$$LB = 24.58 \text{ m};$$

$$\cos 55^\circ = \frac{AL}{AB}$$

$$0.5736 = \frac{AL}{30}$$

$$AL = 0.5736 \times 30$$

$$AL = 17.21 \text{ m}$$

Now the bird is flying from B to C at an angle of 48°

with respect to north. $BC = 32 \text{ m}; \angle MAC = 90^\circ - 48^\circ = 42^\circ$

In the right angled triangle BMC

$$\sin 42^\circ = \frac{MC}{BC}$$

$$0.6691 = \frac{MC}{32}$$

$$MC = 0.6691 \times 32$$

$$MC = 21.41 \text{ m}$$

$$\cos 42^\circ = \frac{BM}{BC}$$

$$0.7431 = \frac{BM}{32}$$

$$BM = 0.7431 \times 32$$

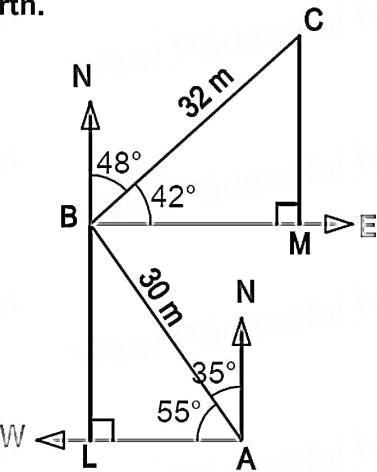
$$BM = 23.78 \text{ m}$$

(i) B is 24.58 m from North of A (i.e. LB)

(ii) B is 17.21 m from West of A (i.e. AL)

(iii) C is 21.41 m from North of B (i.e. MC)

(iv) C is 23.78 m from East of B (i.e. BM)



8. Let A and B be the two ships on either side of the light house CD

$$\text{Distance between two ships } AB = 200 \left[\frac{\sqrt{3}+1}{\sqrt{3}} \right] \text{ m}$$

The angle depressions from the top light house are 60° and 45°

\therefore The angle of elevation from A is 60°

The angle of elevation from B is 45°

Let the height of light house CD be $h \text{ m}$

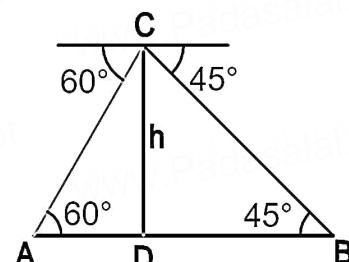
$$\text{From the rt. Triangle } ADC, \tan 60^\circ = \frac{h}{AD}$$

$$\sqrt{3} = \frac{h}{AD} \quad \therefore AD = \frac{h}{\sqrt{3}}$$

$$\text{From the rt. Triangle } BDC, \tan 45^\circ = \frac{h}{BD}$$

$$1 = \frac{h}{BD} \quad \therefore BD = h$$

$$AD + BD = \frac{h}{\sqrt{3}} + h$$



$$AB = h \left[\frac{1}{\sqrt{3}} + 1 \right]$$

$$200 \left[\frac{\sqrt{3}+1}{\sqrt{3}} \right] = h \left[\frac{\sqrt{3}+1}{\sqrt{3}} \right]$$

$$\therefore h = 200 \text{ m}$$

Hieght of the light house = 200 m

9. Width of the street = 35 m ; AD is the Buiding ; BC is the Statue.

From the top of the building ,

The angle of elevation of the top of the statue = 24°

The angle of depression of the top of the statue = 34°

\therefore The angle of elevation of the top of the building = 34°

In the right $\triangle BAD$, $\tan 34^\circ = \frac{AD}{AB}$

$$0.6745 = \frac{AD}{35}$$

$$AD = 35 \times 0.6745 = 23.61 \text{ m}$$

$$\therefore BE = AD = 23.61 \text{ m}$$

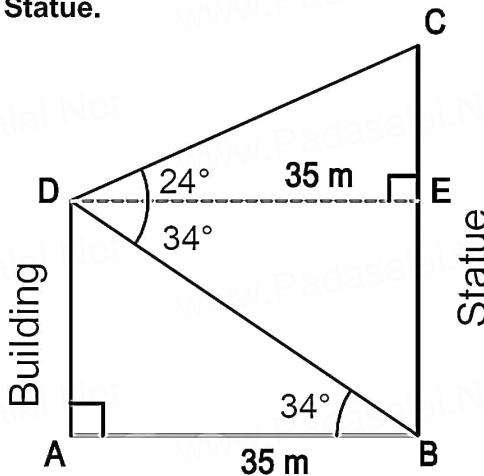
In the right $\triangle DEC$, $\tan 24^\circ = \frac{EC}{DE}$

$$0.4452 = \frac{EC}{35}$$

$$EC = 35 \times 0.4452 = 15.58 \text{ m}$$

$$\begin{aligned} \text{Height of the statue} &= BE + EC \\ &= 23.61 + 15.58 \\ &= 39.19 \text{ m.} \end{aligned}$$

Hieght of the Statue = 39.19 m



10th Maths Unit Exercise Chapter – 7 Mensurattion www.Padasalai.Net

1. Given : Pen's Cylindrical barrel length = 7 cm ; Dia. = 5 mm (or) 0.5 cm ; ∴ Radius = 0.25 cm

Volume of the ink boottle = $\frac{1}{5}$ th of 1 litre ; Number of words written in 1 barrel = 330

$$\text{Vol. of Cylindrical barrel} = \pi r^2 h = \frac{22}{7} \times 0.25 \times 0.25 \times 7 = 22 \times 0.25 \times 0.25 \text{ cm}^3$$

$$\text{Vol. of ink bottle} = \frac{1}{5} \text{ of 1 lit.} = \frac{1}{5} \times 1000 = 200 \text{ ml or } 200 \text{ cm}^3$$

$$\text{Number of barrels to be filled up} = \frac{\text{Vol. of ink bottle}}{\text{Vol. of 1 Cylindrical barrel}} = \frac{200}{22 \times 0.25 \times 0.25}$$

Number of words to be written = Number of barrels × Number of words written in 1 barrel

$$= \frac{200}{22 \times 0.25 \times 0.25} \times 330 = \frac{200 \times 100 \times 100}{22 \times 25 \times 25} \times 330 = 48000$$

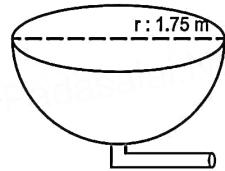
Number of words written using the ink in the bottle = 48000

2. Given : Radius of the hemispherical tank = 1.75 m ; Emptying speed of the pipe = 7 lit. per second

$$\text{Vol. of hemispherical tank} = \frac{2}{3} \pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 1.75^3 \text{ m}^3 \text{ (or) } \frac{2}{3} \times \frac{22}{7} \times 1.75^3 \times 1000 \text{ lit.}$$

$$\text{Time taken to empty the tank} = \frac{\text{Vol. of hemispherical tank}}{\text{Emptying speed of the pipe}} \\ = \frac{2}{3} \times \frac{22}{7} \times \frac{1.75 \times 1.75 \times 1.75 \times 1000}{7} = 1604 \text{ seconds}$$

$$\text{Time taken to empty the tank} = 1604 \text{ seconds } (\text{Or}) \frac{1604}{60} = 26 \text{ min } 44 \text{ sec.}$$

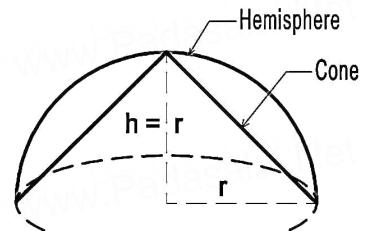


3. Given : Solid hemisphere of radius = r

$$\text{It's Volume} = \frac{2}{3} \pi r^3$$

Radius of the cone maximum carved from it = r unit

Height of the cone maximum carved from it also = r unit



$$\text{Maximum Vol. of the Cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^2 r = \frac{1}{3} \pi r^3 \text{ cubic units.}$$

4. Given : Cylinder portion dia = 8 cm ; ∴ r = 4 cm ; h = 10 cm

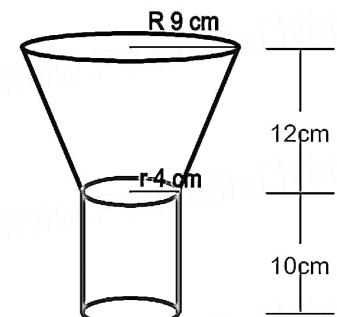
Frustum portion d = 8 cm ; ∴ r = 4 cm ;

D = 18 cm ; ∴ R = 9 cm ; h = 12 cm

$$\text{Frustum's Slant height} = \sqrt{(R - r)^2 + h^2} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

$$\begin{aligned} \text{CSA of funnel} &= 2\pi rh + \pi(R + r)l = \pi[2rh + (R + r)l] \\ &= \pi[2 \times 4 \times 10 + (9 + 4)13] \\ &= \pi[80 + 169] \\ &= \frac{22}{7} \times 249 = 782.57 \text{ cm}^2 \end{aligned}$$

Sheet required to funnel = 782.57 cm²



5. Given : Coin dia = 1.5 cm ; ∴ r = 0.75 cm ; Thickness (h) = 2 mm (or) 0.2 cm

Cylinder dia = 4.5 cm ; ∴ r = 2.25 cm ; Height (h) = 10 cm

$$\text{Volume of a coin} = \pi r^2 h = \pi \times 0.75^2 \times 0.2 \text{ cm}^3$$

$$\text{Volume of the cylinder} = \pi r^2 h = \pi \times 2.25^2 \times 10 \text{ cm}^3$$

$$\begin{aligned} \text{Number of coins required to the cylinder} &= \frac{\text{Volume of the cylinder}}{\text{Volume of a coin}} \\ &= \frac{\pi \times 2.25 \times 2.25 \times 10}{\pi \times 0.75 \times 0.75 \times 0.2} = \frac{\cancel{\pi} \times 225 \times 225 \times 100}{\cancel{\pi} \times 75 \times 75 \times 2} = 450 \end{aligned}$$

Number of coins required to the cylinder = 450

6. Given : Hollow Cylinder R = 4.3 cm ; r = 1.1 cm ; Height (h) = 4 cm

Solid Cylinder Height (h) = 12 cm ; r = ?

$$\text{Volume of the Hollow Cylinder} = \pi h(R + r)(R - r) = \pi \times 4 \times (4.3 + 1.1)(4.3 - 1.1)$$

$$\text{Volume of the Solid cylinder} = \pi r^2 h = \pi \times r^2 \times 10$$

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Volume of the Solid cylinder = Volume of the Hollow Cylinder

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$$\pi \times r^2 \times 12 = \pi \times 4 \times (4.3 + 1.1)(4.3 - 1.1)$$

$$\pi \times r^2 \times 12 = \pi \times 4 \times 5.4 \times 3.2$$

$$r^2 = \frac{4 \times 5.4 \times 3.2}{12} = \sqrt{1.8 \times 3.2}$$

$$r = \sqrt{5.76} = 2.4 \text{ cm}$$

$$\text{Diameter of the Solid cylinder} = 2r = 2 \times 2.4 = 4.8 \text{ cm}$$

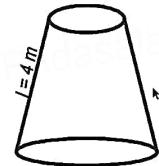
7. Given : Frustum cone Slant height = 4 m ; Perimeters = 16 m, 18 m

$$\therefore \text{It's radii } r = \frac{P}{2\pi} = \frac{16}{2\pi} = \frac{8}{\pi}; R = \frac{P}{2\pi} = \frac{18}{2\pi} = \frac{9}{\pi}$$

$$\text{CSA of funnel} = \pi(R + r)l = \pi \left(\frac{9}{\pi} + \frac{8}{\pi} \right) 4 = \pi \left(\frac{9+8}{\pi} \right) 4 = 17 \times 4 = 68 \text{ m}^2$$

$$\text{Rate of Painting} = \text{Rs. } 100 \text{ per sq.m}$$

$$\text{Cost of Painting} = 68 \times 100 = \text{Rs. } 6800$$



8. Given : Hemi-spherical hollow bowl : External dia. = 14 cm ; $\therefore R = 7 \text{ cm}$

$$\text{It's material volume} = \frac{436\pi}{3} \text{ cm}^3$$

$$\text{It's material volume} = \frac{2\pi(R^3 - r^3)}{3} = \frac{436\pi}{3}$$

$$7^3 - r^3 = \frac{436}{2} = 218$$

$$343 - r^3 = 218$$

$$-r^3 = 218 - 343$$

$$r^3 = -218 + 343 = 125$$

$$r = \sqrt[3]{125} = 5 \text{ cm}$$



$$\text{Thickness of the bowl} : (R - r) = 7 - 5 = 2 \text{ cm}$$

9. Given : Volume of a cone = $1005\frac{5}{7} \text{ cm}^3$; It's base area = $201\frac{1}{7} \text{ cm}^2$; Slant height of cone = ?

$$\text{Base Area} = \pi r^2 = 201\frac{1}{7}; \quad \frac{22}{7}r^2 = \frac{1402}{7}$$

$$r^2 = \frac{1408}{22} = 64$$

$$r = \sqrt{64} = 8 \text{ cm}$$

$$\text{It's volume} = \frac{1}{3}\pi r^2 h = 1005\frac{5}{7}; \quad \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times h = \frac{7040}{7}$$

$$h = \frac{7040 \times 3}{22 \times 8 \times 8} = 15 \text{ cm}$$

$$\text{Slant height of cone} = \sqrt{r^2 + h^2} = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \text{ cm}$$

10. Given : Segment of a circular sheet Radius $r = 21 \text{ cm}$

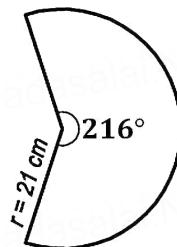
$$\text{It's central angle angle } x = 216^\circ$$

$$\text{It's arc length} = \frac{x}{360} \times 2\pi r = \frac{216}{360} \times 2\pi \times 21$$

$$\text{Perimeter of the cone formed} = \text{Arc length of the segment}$$

$$2\pi r = \frac{216}{360} \times 2\pi \times 21$$

$$r = \frac{63}{5} \text{ cm}$$



$$\text{Slant height of the cone} = \text{Radius of the segment}; \quad \therefore l = 21 \text{ cm}$$

$$\text{Height of the cone} = \sqrt{l^2 - r^2} = \sqrt{21^2 - \left(\frac{63}{5}\right)^2} = \sqrt{441 - \frac{3969}{25}} = \sqrt{\frac{441 \times 25 - 3969}{25}} = \sqrt{\frac{7056}{25}} = \frac{84}{5} \text{ cm}$$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{63}{5} \times \frac{63}{5} \times \frac{84}{5} = \frac{349272}{125} = 2794.18 \text{ cm}^3$$

10th Maths Unit Exercise Chapter – 8 Mensurattion www.Padasalai.Net

1. Given : The data and the frequency table:

Class Interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 - 120
Frequency	5	f_1	10	f_2	7	8

$$\text{Sum of all frequencies} = 5 + f_1 + 10 + f_2 + 7 + 8 = f_1 + f_2 + 30 = 50$$

$$f_1 + f_2 = 50 - 30 = 20 ; \therefore f_1 = 20 - f_2 \quad \text{--- (1)}$$

Class Mid value : 10 30 50 70 90 110

$$\text{Class mean} = \frac{10 \times 5 + 30 \times f_1 + 50 \times 10 + 70 \times f_2 + 90 \times 7 + 110 \times 8}{50} = 62.8$$

$$50 + 30f_1 + 500 + 70f_2 + 630 + 880 = 62.8 \times 50$$

$$\text{Dividing by 10 on both sides : } 5 + 3f_1 + 50 + 7f_2 + 63 + 88 = 62.8 \times 5$$

$$3f_1 + 7f_2 + 216 = 314$$

$$3f_1 + 7f_2 = 314 - 216 = 108$$

$$\text{From (1)} \rightarrow 3(20 - f_2) + 7f_2 = 108$$

$$4f_2 = 108 - 60 = 48 ;$$

$$f_2 = \frac{48}{4} = 12 ; \quad f_1 = 20 - 12 = 8$$

2. Given : The data and the frequency table:

Diameters	33 – 36	37 – 40	41 – 44	45 – 48	49 – 52
Number of circles	15	17	21	22	25

The continuous frequency = 32.5 – 36.5, 36.5 – 40.5, 40.5 – 44.5, 44.5 – 48.5, 48.5 – 52.5

The Midvalue = 34.5, 38.5, 42.5, 46.5, 50.5

Let the assumed mean A = 42.5 and C = 4

Diameters	Midvalue	f_i	$di = \frac{x_i - A}{C}$	d_i^2	$f_i d_i$	$f_i d_i^2$
32.5 – 36.5	34.5	15	-2	4	-30	60
36.5 – 40.5	38.5	17	-1	1	-17	17
40.5 – 44.5	42.5	21	0	0	0	0
44.5 – 48.5	46.5	22	1	1	22	22
48.5 – 52.5	50.5	25	2	4	50	100
		N=100	$\sum d_i = 0$		$\sum f_i d_i = 25$	$\sum f_i d_i^2 = 199$

$$\text{Standard Deviation } \sigma = C \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$= 4 \times \sqrt{\frac{199}{100} - \left(\frac{25}{100}\right)^2}$$

$$= 4 \times \sqrt{1.99 - \left(\frac{1}{4}\right)^2}$$

$$= 4 \times \sqrt{\frac{1.99 \times 16 - 1}{16}}$$

$$= 4 \times \frac{\sqrt{31.84 - 1}}{\sqrt{16}} = 4 \times \frac{\sqrt{30.84}}{4} = 5.55$$

$$\text{The SD } \sigma = 5.55$$

3. Given : Variance $\sigma^2 = 160$; The data and the frequency table: www.Padasalai.Net

x	k	k	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

Using direct method

x	f	$f_i x_i$	$(f_i x_i)^2$
k	2	$2k$	$4k^2$
$2k$	1	$2k$	$4k^2$
$3k$	1	$3k$	$9k^2$
$4k$	1	$4k$	$16k^2$
$5k$	1	$5k$	$25k^2$
$6k$	1	$6k$	$36k^2$
	$\sum f_i = 7$	$\sum (f_i x_i) = 22k$	$\sum (f_i x_i)^2 = 92k^2$

$$\text{Standard Deviation } \sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$$

$$\text{Variance } \sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$\frac{92k^2}{7} - \left(\frac{22k}{7}\right)^2 = 160$$

$$\frac{92k^2 \times 7 - (22k)^2}{7 \times 7} = 160$$

$$\frac{644k^2 - 484k^2}{7 \times 7} = 160$$

$$\frac{160k^2}{7 \times 7} = 160 ;$$

$$k^2 = 49 \quad (\text{Or}) \quad k = 7$$

4. Given : The SD of some temperature data in degree Celsius ($^{\circ}\text{C}$) = 5

$$\text{Celsius}({}^{\circ}\text{C}) \text{ to Fahrenheit } ({}^{\circ}\text{F}) \text{ conversion} = \frac{9}{5} \times {}^{\circ}\text{C} + 32$$

$$\therefore \text{The SD of that temperature data in Fahrenheit } ({}^{\circ}\text{F}) = \frac{9}{5} \times 5 = 9 \quad [\text{Leaving the addl. constant of 32}]$$

$$\text{It's variance } \sigma^2 = 9^2 = 81$$

5. Given : $\sum(x - 5) = 3$; i.e. $\sum d_i = 3$; Number of data $n = 18$

$$\sum(x - 5)^2 = 43 ; \text{ i.e. } \sum d_i^2 = 43$$

$$SD = \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{43}{18} - \left(\frac{3}{18}\right)^2}$$

$$= \sqrt{\frac{43 \times 18 - 9}{18^2}}$$

$$= \sqrt{\frac{765}{18^2}} = \frac{\sqrt{765}}{\sqrt{18^2}} = \frac{27.66}{18} = 1.54$$

6. Prices in city A : 20, 22, 19, 23, 16

- Prices in city B : 10, 20, 18, 12, 15

$$\text{Mean of } A = \frac{20+22+19+23+16}{5} = 20 ; \quad \text{Mean of } B = \frac{10+20+18+12+15}{5} = 15$$

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To find SD for city A

x_i	$d_i = x_i - \bar{x}$ $= x_i - 20$	d_i^2
20	0	0
22	2	4
19	-1	1
23	3	9
16	-4	16
		20

$$\text{SD of A } (\sigma) = \sqrt{\frac{\sum d_i^2}{n}} = \sqrt{\frac{30}{5}} = \sqrt{6} = 2.45$$

$$\text{SD of B } (\sigma) = \sqrt{\frac{\sum d_i^2}{n}} = \sqrt{\frac{68}{5}} = \sqrt{13.6} = 3.69$$

$$\text{C.V of A} = \frac{\sigma}{\bar{x}} \times 100 = \frac{2.45}{20} \times 100 = 12.25 \quad \text{--- (1)}$$

$$\text{C.V of B} = \frac{3.69}{15} \times 100 = 24.6 \quad \text{--- (2)}$$

Comparing (1) and (2) City A is more consistent.

7. Given : Range : $L - S = 20$; Coefficient : $\frac{L-S}{L+S} = 0.2$

$$\frac{20}{L+S} = 0.2 \quad (\text{or}) \quad L + S = \frac{20}{0.2} = \frac{200}{2} = 100$$

$$L - S = 20 \quad \text{--- (1)}$$

$$L + S = 100 \quad \text{--- (2)}$$

$$\text{Adding (1) and (2)} \rightarrow 2L = 120; \quad L = 60; \quad S = 100 - 60 = 40$$

8. Two dice are rolled.

Then it's Sample space = $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$
 $n(S) = 36$

(i). Let A be the event of getting the product value is 6

$$A = \{(2,3), (3,2), (1,6), (6,1)\}; \quad n(A) = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$$

(ii). Let B be the event of getting a product as a prime number

$$B = \{(1,6), (6,1)\}; \quad n(A) = 6$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$$

Also $(A \cap B) = \{(1,6), (6,1)\}; \quad n(A \cap B) = 2$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{4}{36} + \frac{2}{36} - \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$$

Probability of getting the product of face value 6 } or the difference of face values 5. } = $\frac{1}{9}$

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To find SD for city B

x_i	$d_i = x_i - \bar{x}$ $= x_i - 15$	d_i^2
10	-5	25
20	5	25
18	3	9
12	-3	9
15	0	0
		68

10 கணிதம் - பள்ளியல்

வர்க்குமூலம் நான் ஒத்து சூத்திரம்..

- Very useful
- Simple
- Accurate.

$$\sqrt{x \pm y} = \sqrt{x} \pm \frac{y}{2\sqrt{x}}$$

① எடுத்துக்காட்டி - 8.4:

$$\sqrt{8} = \sqrt{9-1}$$

$$\sqrt{x-y} = \sqrt{x} - \frac{y}{2\sqrt{x}}$$

$$\begin{aligned} x=9, y=1 \Rightarrow \sqrt{9-1} &= \sqrt{9} - \frac{1}{2\sqrt{9}} \\ &= 3 - \frac{1}{2\times 3} = 3 - \frac{1}{6} \\ &= 3 - 0.166 = 2.834 \end{aligned}$$

Book Answer: ≈ 2.83

② எ.நா. 8.5:

$$\begin{aligned} \sqrt{8.53} &= \sqrt{9-0.47} = \sqrt{9} - \frac{0.47}{2\sqrt{9}} \\ &= 3 - \frac{0.47}{6} \\ &= 3 - 0.078 = 2.92 \end{aligned}$$

Book ≈ 2.9

③ எ.நா: 8.6.

$$\begin{aligned} \sqrt{44.49} &= \sqrt{49-4.51} = 7 - \frac{4.51}{2\times 7} \\ &= 7 - 0.32 = 6.68 \end{aligned}$$

Book ≈ 6.67

④ எ.நா: 8.7

$$\begin{aligned} \sqrt{5.5-0.25} &= \sqrt{5.25} = \sqrt{4+1.25} \\ &= 2 + \frac{1.25}{2\times 2} = 2 + 0.31 \\ &\approx 2.31 \end{aligned}$$

Book ≈ 2.29

⑤ எ.நா.: 8.8.

$$\begin{aligned} \sqrt{6} &= \sqrt{4+2} \\ &= 2 + \frac{2}{2\times 2} = 2.5 \end{aligned}$$

Book ≈ 2.45

⑥ எ.நா. 8.9:

$$\begin{aligned} \sqrt{5.2} &= \sqrt{4+1.2} = 2 + \frac{1.2}{2\times 2} \\ &= 2 + 0.3 = 2.3 \end{aligned}$$

Book ≈ 2.28

⑦ எ.நா. 8.11:

$$\begin{aligned} \sqrt{2.58} &= \sqrt{4-1.42} \\ &= 2 - \frac{1.42}{2\times 2} = 2 - 0.355 \\ &= 1.645 \end{aligned}$$

Book ≈ 1.6

⑧ எ.நா: 8.13:

$$\begin{aligned} \sqrt{2.779} &= \sqrt{4-1.221} = 2 - \frac{1.221}{2\times 2} \\ &= 2 - 0.305 = 1.695 \end{aligned}$$

Book: 1.667

⑨ எ.நா: 8.14:

$$\begin{aligned} \sqrt{35} &= \sqrt{36-1} = 6 - \frac{1}{2\times 6} \\ &= 6 - 0.083 = 5.917 \end{aligned}$$

Book $\rightarrow 5.9$

⑩ எ.நா: 8.17

$$\begin{aligned} \sqrt{19.43-18.40} &= \sqrt{1.03} = \sqrt{1+0.03} \\ &= 1 + \frac{0.03}{2} = 1.015 \end{aligned}$$

1.01
Book

$$\begin{aligned} \sqrt{26.29-18.40} &= \sqrt{7.89} = \sqrt{9-1.11} \\ &= 3 - \frac{1.11}{6} = 3 - 0.183 \\ &= 2.817 \end{aligned}$$

Book: 2.8)

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