mathematics

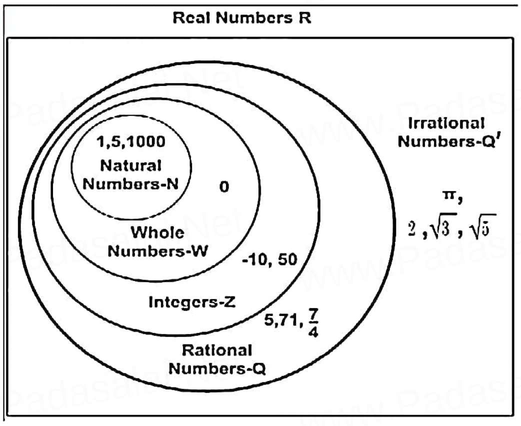
Std- X

Based Excercise Formula

1. RELATIONS AND FUNCTIONS

Exercise 1.1

1. Leibniz is hailed as "The Father of Applied Sciences".
2. A set is a collection of well-defined distinguishable objects.
   a) The concept of sets in two forms called Relations and Functions.
   b) A number pair is called an ordered pair of numbers.
3. If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$, $b \in B$ is called the Cartesian Product of A and B, and is denoted by $A \times B$.
   Thus, $A \times B = \{(a,b) \mid a \in A, b \in B\}$.
   a) If a = b then (a, b) = (b, a)
   b) The Cartesian product is also referred as cross product
   c) $A \times B \neq B \times A$, but $n(A \times B) = n(B \times A)$.
   d) $A \times B = \phi$ if and only if $A = \phi$ or $B = \phi$.
4. If n(A) = p & n(B) = q, n(A × B) = pq. Three sets $A \times B \times C = \{(a, b, c)/a \in A, b \in B, c \in C\}$



5. Natural Numbers N = {1, 2, 3, 4, ...}
   Whole Numbers W = {0, 1, 2, 3, ...}
   Integers Z = { ..... -2, -1, 0, 1, 2, ...}
   Rational Numbers $Q = \left\{\frac{p}{q} / p, q \in Z, q \neq 0\right\}$
   Real Numbers $R = Q \cup Q^{l}$ where Q^{l} is the set of irrational numbers. $Q^{1} = \left\{\sqrt{2}, \sqrt{5}, \sqrt{-3}\right\}$
6. R × R is the set of all points which we call as the Cartesian plane.
7. (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
8. (i) A × B represent vertices of a square in two dimensions (xy – plane)
   (ii) A × B × C represent vertices of a cube in three dimensions (xyz – plane)
   a) The Cartesian product of two non empty sets provides a shape in two dimensions.
   b) The Cartesian product of two non empty sets provides a shape in three dimensions.

Exercise 1.2

9. a) Relation: Let *A* and *B* be any two non-empty sets. A 'relation' R from *A* to *B* is a subset of A×B satisfying some specified conditions. If x ε A is related to y ε B through R, then we write it as *x* R *y*. *x* R *y* if and only if (x,y) ε R.
   The domain of the relation R= { x ε A / *x* R *y, for some* y ε B }
   The co-domain of the relation R is *B*
   The range of the relation R= { y ε B / *x* R *y, for some* x ε A }
   b) A relation may be represented algebraically either by the roster method or by the set builder method.
10. An arrow diagram is a visual representation of a relation.
11. A relation which contains no element is called a "Null relation".
12. If n(A) = p, n(B) = q then the total number of relations that exist between *A* and *B* is 2^{pq}.

Exercise 1.3

13. Among several relations that exist between two non-empty sets, some special relations are important for further exploration. Such relations are called "Functions".

Definition : A relation f between two non-empty sets X and Y is called aa function from X to Y if, for Each x ϵ X there exists only one y ϵ Y such that (x,y) ϵ f. That is, f = {(x,y)/for all x ϵ X, y ϵ Y.

a) Every function is a relation.

14. Functions are subsets of relations and relations are subsets of Cartesian product..
15. A function is also called as a mapping or transformation.
16. If $f : X \to Y$ is a function then
    (a) The set X is called the domain of the function f and the set Y is called its co-domain.
    (b) If $f(a) = b$, then b is called 'image' of a under f and a is called a 'pre-image' of b.
    (c) The set of all images of the elements of X under f is called the 'range' of f.
    (d) $f : X \to Y$ is a function only if
        (i) every element in the domain of f has an image. (ii) the image is unique.
    (e) The range of a function is a subset of its co-domain.
    (f) Here f is a real valued function
    (g) If A and B are finite sets such that n(A) = p, n(B) = q, then the total number of functions that exist between A and B is q^p.
17. Functions are widely applied in Engineering Sciences.
18. The image of an element should always be unique in a function.

Exercise 1.4

19. A function may be represented by (a) a set of ordered pairs (b) a table form (c) an arrow diagram (d) a graphical form
20. A curve drawn in a graph represents a one-one function, if every vertical line or horizontal line intersects the curve in at most one point.
21. Any equation represented in a graph is usually called a 'curve'.
22. A function $f : A \to B$ is called one – one function if distinct elements of A have distinct images in B.
23. A one-one function is also called an injection.
24. a) A function $f : A \to B$ is called many-one function if two or more elements of A have same image in B.
    b) A function $f : A \to B$ is called many one if f is not one-one.
25. A function is said to be onto function if the range of f is equal to the co-domain of f.
    (If $f : A \to B$ is an onto function then, the range of $f = B$. That is f(A) = B.)
26. An onto function is also called a surjection.
29. a) A function $f : A \to B$ is called an into function if there exists at least one element in B which is not the image of any element of A.
    b) A function $f : A \to B$ is called into if it is not onto.
30. If a function is both one–one and onto, then f is called a bijection from A to B.
31. A one – one and onto function is also called a one – one correspondence.
32. Special cases of a function:
    (i) Constant function (ii) Identity function (iii) Real – valued function
33. A function $f : A \to B$ is called a constant function if the range of f contains only one element.
34. Let A be a non–empty set. Then the function $f : A \to B$ defined by f (x)=x for all x ∈ A is called an identity function on A and is denoted by I_A .
35. A function $f : A \to B$ is called a real valued function if the range of f is a subset of the set of all real numbers R . That is f(A) ⊆ R.

Exercise 1.5

36. Let $f : A \to B$ and $g : B \to C$ be two functions .Then the composition of f and g denoted by $g \circ f$ is defined as the function $g \circ f(x) = g(f(x))$ for all x ∈ A.
37. Generally, $f \circ g \neq g \circ f$ for any two functions f and g. So, composition of functions is not commutative.
38. Composition of three functions is always associative. That is, $f \circ (g \circ h) = (f \circ g) \circ h$
39. Modulus function is not a linear function but it is composed of two linear functions x and $-x$.
    (a) A function $f : R \to R$ defined by $f(x) = mx + c$, m 0 is called a linear equation.
    (b) Linear equation forms a straight line in the graph.
40. Linear functions are always one-one functions (and has applications in Cryptography as well as in several branches of Science and Technology).
41. A function $f : R \to R$ defined by $f(x) = ax^2 + bx + c$, $(a \neq 0)$ is called a quadratic function.
42. A function $f : R \to R$ defined by $f(x) = ax^3 + bx^2 + cx + d$, $(a \neq 0)$ is called a cubic function.
43. A function $f : R - \{0\} \to R$ defined by $f(x) = \frac{1}{x}$ is called a reciprocal function.
44. A function $f : R \to R$ defined by $f(x) = c$, for all x ∈ R is called a constant function.
45. A constant function is a linear function.
46. Quadratic function is a one – one function.
47. Cubic function is a one – one function.
48. The reciprocal function is a bijection.

2. NUMBERS AND SEQUENCES

Exercise 2.1

1. Euclid is called the "Father of Geometry".
2. (a) Let a and b (a > b) be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r$, $0 \leq r < b$. (Euclid's Division Lamma)
   (b) If r = 0 then a = bq so b divides a.
3. If a and b are any two integers then there exist unique ...rs q and r such that $a = bq + r$, where $0 \leq r < |b|$.
4. If a and b are positive integers such that a = bq + r then every common divisor of a and b is a common divisor of b and r and vice-versa.
5. **Euclid's Division Algorithm**
   To find Highest Common Factor of two positive integers a and b, where $a > b$
   Step1: Using Euclid's division lemma $a = bq + r$; $0 \leq r < b$., where q is the quotient, r is the remainder. If $r = 0$ then b is the Highest Common Factor of a and b.
   Step 2: Otherwise applying Euclid's division lemma divide b by r to get $b = rq_1 + r_1, 0 \leq r_1 < b$
   Step 3: If $r_1 = 0$ then r is the Highest common factor of a and b.
   Step 4: Otherwise using Euclid's division lemma, repeat the process until we get the remainder zero. In that case, the corresponding divisor is the H.C.F of a and b.
   a) HCF of two positive number is denoted by (a, b)
   b) HCF is also called GCD.
6. If a, b are two positive integers with $a > b$ then G.C.D of (a,b) = G.C.D of (a - b, b).
7. **Highest Common Factor of three numbers**:
   We can apply Euclid's Division Algorithm twice to find the Highest Common Factor (H.C.F) of three positive integers using the following procedure.
   Let a, b, c be the given positive integers. (i) Find H.C.F of a,b. Call it as d, $d = (a,b)$
   (ii) Find H.C.F of d and c. This will be the H.C.F of the three given numbers a, b, c .
8. Two positive integers are said to be relatively prime or co prime if their Highest Common Factor is 1.
   a. The difference between any two consecutive terms of an A.P is always constant. That constant value is called the common difference.

Exercise 2.2

9. Every natural number except 1 can be factorized as a product of primes and this factorization is unique except for the order in which the prime factors are written.
10. "Every composite number can be written uniquely as the product of power of primes" is called Fundamental Theorem of Arithmetic.

Exercise 2.3

11. In Mathematics, modular arithmetic is a system of arithmetic for integers where numbers wrap around a certain value.
12. Great German mathematician Carl Friedrich Gauss, is hailed as the "Prince of mathematicians".
13. Two integers a and b are congruence modulo n if they differ by an integer multiple of n. That b - a = kn for some integer k. This can also be written as $a \equiv b \pmod{n}$.
14. Two integers a and b are congruent modulo m, written as $a \equiv b \pmod{m}$, if they leave the same remainder when divided by m.
15. a, b, c and d are integers and m is a positive integer such that if $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then (i) $(a + c) \equiv (b + d) \pmod{m}$ (i) $(a - c) \equiv (b - d) \pmod{m}$ (i) $(a \times c) \equiv (b \times d) \pmod{m}$
16. $a \equiv b \pmod{m}$ then (i) $ac \equiv bc \pmod{m}$ (ii) $a \pm c \equiv b \pm c \pmod{m}$ for any integer c.

Exercise 2.4 Sequences

1. A real valued sequence is a function defined on the set of natural numbers and taking real values.
   a) Each element in the sequence is called a term of the sequence.
2. A sequence can be written as $a_1, a_2, a_3 \ldots\ldots a_n, \ldots.$ here a_1 is the first term, a_2 is the 2nd term, a_n is the nth term.
3. If the number of elements in a sequence is finite then it is called a Finite sequence.
4. If the number of elements in a sequence is infinite then it is called an Infinite sequence.
5. A sequence can be considered as a function defined on the set of natural numbers N.
6. Though all the sequences are functions, not all the functions are sequences.

Exercise 2.5

7. Arithmetic Progression denoted by A.P. is a, a + d, a + 2d, ... a is 1st term, d is common difference.
8. An Arithmetic Progression is a sequence whose successive terms differ by a constant number.
9. An Arithmetic progression having a common difference of zero is called a constant arithmetic progression.
10. nth term of an A.P. is $t_n = a + (n - 1)d$, $d = t_n - t_{n-1}$
11. In a finite A.P. whose first term is a and last term l, then the number of terms in the A.P. is given by $l = a + (n-1)d$ gives $n = \left(\frac{l-a}{d}\right) + 1$.
12. In an Arithmetic Progression
    (i) If every term is added or subtracted by a constant, then the resulting sequence is also an A.P.
    (ii) If every term is multiplied or divided by a non-zero number, then the resulting sequence is also an A.P.
    (iii) If the sum of three consecutive terms of an A.P. is given, then they can be taken as a – d, a and a+d. Here the common difference is d.
    (iv) If the sum of four consecutive terms of an A.P. is given then, they can be taken as a - 3d, a – d, a+d and a + 3d . Here the common difference is $2d$.
13. Three non-zero numbers a, b,, c are in A.P. if and only if 2b = a + c.

Exercise 2.6

14. a) If a series has finite number of terms then it is called a Finite series. If a series has infinite number of terms then it is called an Infinite series.
    b) The sum of the terms of a sequence is called series.
15. A series whose terms are in Arithmetic progression is called Arithmetic series.

17. Sum of upto n terms in an A.P. is $S_n = \frac{n}{2}[2a + (n - 1)d]$.
18. In an A.P. the values of a and d may be either positive or negative. But the value of n is always positive.

Exercise 2.7

19. A Geometric Progression is a sequence in which each term is obtained by multiplying a fixed non-zero number to the preceding term except the first term. The fixed number is called common ratio. The common ratio is usually denoted by r.

    a) G.P is of the form a, ar, ar^2, … … ar^{n-1}. $r = \frac{t_2}{t_1} = \frac{t_3}{t_2}$ … …

20. n th term or general term of Geometric Progression (G.P.) $t_n = ar^{n-1}$
21. When the product of three consecutive terms of a G.P. are given, we can take the three terms as $\frac{a}{r}$, a, ar.
22. When the products of four consecutive terms are given for a G.P. then we can take the four terms as $\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3
23. When each term of a Geometric Progression is multiplied or divided by a non–zero constant then the resulting sequence is also a Geometric Progression.
24. Thus three non-zero numbers a, b, c are in G.P. if and only if $b^2 = ac$, then $\frac{b}{a} = \frac{c}{b}$

Exercise 2.8

25. A series whose terms are in Geometric progression is called Geometric series.
26. The sum of first n terms of the Geometric progression is

    (i) $S_n = \frac{a(1 - r^n)}{1 - r}$ $(r < 1)$ (ii) $S_n = \frac{a(r^n - 1)}{r - 1} = (r > 1)\ (r \neq 1)$

    a) If r = 1, then $S_n = a + a + … … + a = na$.
27. Sum to infinite number of terms of a G.P. is $\frac{a}{1-r}$

Exercise 2.9

28. There are some series whose sum can be expressed by explicit formulae. Such series are called special series.
29. Sum of first 'n' natural numbers (1+2+3+……….+n) $S_n = \frac{n(n+1)}{2}$ are called Triangular Numbers.
30. Sum of first 'n' odd natural numbers (1+3+5+……+(2n-1)) $S_n = n^2$
31. Sum of squares of first 'n' natural numbers (1+2+3+……….+n) $S_n = \frac{n(n+1)(2n+1)}{6}$ are called square Pyramidal Numbers.
32. Sum of cubes of first 'n' natural numbers (1+2+3+……….+n) $S_n = \left[\frac{n(n+1)}{2}\right]^2$
33. Sum of divisions of one number excluding itself is the other. Such pair of number is called Amicable Numbers (or) Friendly Numbers.

3.ALGEBRA

Exercise 3.1

1. Algebra is considered as "Science of determining unknowns".
2. Al-Khwarizmi is hailed as "Father of Algebra".
3. The general form of linear equation in two variables x and y is $ax+by+c = 0$.
4. A linear equation in two variables represent a straight line in xy plane. A linear equation in two variables of the form $ax + by + c = 0$, represents a straight line.
5. A linear equation in three variables of the form $ax + by + cz + d = 0$, represents a plane. The concept is used in GPS systems.
6. If you obtain (the result of a linear equation in three variables) a false equation.
   (a) 0 = 1, then the system has no solution. (b) 0 = 0 then the system has infinitely many solutions

Exercise 3.3

a) Greatest Common Divisar (GCD) of two given polynomials f(x) and g(x).

f(x) = g(x) q(x) + r(x). q (x) – quotient

then deg lg(x) | < deg lg (x) |

8. The product of two polynomials is the product of their LCM and GCD.

Exercise 3.4 – 3.8

9. An expression is called a rational expression if it can be written in the form $\frac{p(x)}{q(x)}$ and $q(x) \neq 0$.

(A rational expression is the ratio of two polynomials)

a) A rational expression $\frac{p(x)}{q(x)}$ is said to be in its lowest form if GCD (p(x), q(x)) = 1.

10. A value that makes a rational expression (in its lowest form) undefined is called an Excluded value.

Exercise 3.9 – 3.12

11. Quadratic Equations $p(x) = ax^2 + bx + c = 0$. (Degree = 2), $a \neq 0$ and a, b, c, are real numbers.

a) p(x) is a polynomial. x = a is called zero of p(x) if p(a) = 0.

12. The values of *x* such that the expression $ax^2 + bx + c$ becomes zero are called roots of the quadratic equation $ax^2 + bx + c = 0$.

13. The formula for finding roots of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

14. Sum of the roots = $\frac{-b}{a} = \frac{-\text{co.eff.of } x}{\text{co.eff.of } x^2}$

15. Product of the roots = $\frac{c}{a} = \frac{\text{constant term}}{\text{co.eff.of } x}$

16. General form of quadratic equation for two given two roots

x^2 - x (sum of the roots) + (product of the roots) = 0.

Exercise 3.13 & 3.14

17.

Values of Discriminant $\Delta = b^2 - 4ac$	Nature of Roots
$\Delta > 0$	Real and Unequal roots
$\Delta = 0$	Real and Equal roots
$\Delta < 0$	No Real root

Exercise 3.15

18. The graph of any second degree polynomial gives a curve called “parabola”.

Solving quadratic equations through intersection of lines

Intersection of parabola with x axis		Solution
(i)	two distinct points	real and unequal roots
(ii)	only one point	real and equal roots
(iii)	doesn’t intersect	no real root

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a – b)^2 + 4ab$	2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)(a - b) = a^2 - b^2$	4. $(x + a)(x + b) = x^2 + (a + b)x + ab$
5. $(x + a)(x - b) = x^2 + (a - b)x – ab$	6. $(a – b)^2 = (a + b)^2 – 4ab$
7. $a^2 + b^2 = (a + b)^2 - 2ab$	8. $a^2 + b^2 = (a - b)^2 + 2ab$
9. $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$	10. $a^2 - \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$
11. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$	
12. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$	
13. $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$	
14. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	15. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
16. $(a - b)^3 = a^3 + 3a^2b + 3ab^2 - b^3$	17. $(a + b)^3 = a^3 - b^3 - 3ab(a - b)$
18. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$	
19. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$	20. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
21. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	22. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
23. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$	24. $(a + b)^2 - (a - b)^2 = 4ab$
25. $a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$	

Exercise 3.16 – 3.18 Matrices

1. A matrix is a rectangular array of elements. The horizontal arrangements are called rows and vertical arrangements are called columns.
2. The order of the matrix is Number of rows × Number of columns
3. a_{ij} is the element in the i^{th} row and j^{th} column and is referred as $(i, j)^{th}$ element. $A = (a_{ij})\ m \times n$.
   a) The total number of entries in the matrix $A = (a_{ij})\ m \times n$ is mn.
4. (a) A matrix is said to be a row matrix if it has only one row and any number of columns. A row matrix is also called as a row vector.

$$\begin{pmatrix} 1 & 5 & 7 & 4 & 8 \end{pmatrix}$$

(b) A matrix is said to be a column matrix if it has only one column and any number of rows. It is also called as a column vector.

$$\begin{pmatrix} 1 \\ -5 \\ 8 \end{pmatrix}$$

(c) A matrix in which the number of rows is equal to the number of columns is called a square matrix.

$$\begin{pmatrix} 1 & 5 \\ -4 & 7 \end{pmatrix}$$

(d) A square matrix, all of whose elements, except those in the leading diagonal are zero is called a diagonal matrix.

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(e) A diagonal matrix in which all the leading diagonal elements are same is called a scalar matrix.

$$\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

(g) A matrix is said to be a zero matrix or null matrix if all its elements are zero.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(h) The matrix which is obtained by interchanging the elements in rows and columns of the given matrix A is called transpose of A and is denoted by A^T (read as A transpose).

$$\text{If } A = \begin{pmatrix} 1 & 0 & 7 \\ 2 & -4 & 2 \\ 5 & 8 & 6 \end{pmatrix}_{3\times3} \quad \text{then} \quad A^T = \begin{pmatrix} 1 & 2 & 5 \\ 0 & -4 & 8 \\ 7 & 2 & 6 \end{pmatrix}_{3\times3}$$

(i) A square matrix in which all the entries above the leading diagonal are zero is called a lower triangular matrix.

$$\begin{pmatrix} 9 & 0 & 0 \\ 2 & -4 & 0 \\ 3 & 7 & 6 \end{pmatrix}$$

(j) A square matrix in which all the entries below the leading diagonal are zero is called a upper triangular matrix.

$$\begin{pmatrix} 9 & 5 & 7 \\ 0 & -4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$

(k) Equal Matrices : Two matrices A and B are said to be equal if and only if they have the same order and each element of matrix A is equal to the corresponding element of matrix B.

$$A = \begin{pmatrix} 1 & 0 \\ 5 & 7 \end{pmatrix} \quad B = \begin{pmatrix} sin^2\theta + cos^2\theta & 7^2 - 49 \\ 2^2 + 1^2 & \sqrt{25} + 2 \end{pmatrix} \quad \text{A\&B are Equal Matrices}$$

(l) The negative of a matrix $A_{m\times n}$ denoted by $-A_{m\times n}$ is the matrix formed by replacing each element in the Matrix $A_{m\times n}$ with its additive inverse.

$$If\ A = \begin{pmatrix} 5 & -2 & 3 \\ 6 & 7 & -8 \end{pmatrix}_{2\times3} \quad \text{then} \quad -A = \begin{pmatrix} -5 & 2 & -3 \\ -6 & -7 & 8 \end{pmatrix}_{2\times3}$$

5. Addition and subtraction of matrices: Two matrices can be added or subtracted if they have the same order. To add or subtract two matrices, simply add or subtract the corresponding elements.
6. Multiplication of Matrix by a Scalar : We can multiply the elements of the given matrix A by a non-zero number k to obtain a new matrix kA whose elements are multiplied by k. The matrix kA is called scalar multiplication of A.
7. **Properties of Matrix Addition and Scalar Multiplication**

Let A, B, C be m x n matrices and p and q be two non-zero scalars (numbers). Then we have the following properties.

(i) $A + B = B + A$ [Commutative property of matrix addition]

(ii) $A + (B + C) = (A + B) + C$ [Associative property of matrix addition]

(iii) $(pq)A = p(qA)$ [Associative property of scalar multiplication]

(iv) $IA=A$ [Scalar Identity property where I is the unit matrix]

(v) $p(A + B) = p(A) + p(B)$ [Distributive property of scalar and two matrices]

(vi) $(p + q)A = pA + qA$ [Distributive property of two scalars with a matrix]

10. Properties (Multiplication)

a) Matrix multiplication is not Commutative, $AB \neq BA$.

b) Matrix multiplication is distributive over matrix addition.

(i) A (B + C) = AB + AC (Right distributive property) (ii) (A + B)C = AC + BC (Left distributive property)

c) Matrix multiplication is associative (AB) C = A (BC).

d) Multiplication of a matrix is a unit matrix. AI = IA = A.

4.GEOMETRY

Exercise 4.1

1. Two figures are said to be similar if every aspect of one figure is proportional to other figure.
2. Two geometrical figures are congruent, if they have same size and shape.
3. In congruent triangles, the corresponding sides are equal.
4. In similar triangles, the corresponding sides are proportional.
5. If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. (AA similarity criterion is same as the AAA similarity criterion).
6. If one angle of a triangle is equal to one angle of another triangle and if the sides including them are proportional then the two triangles are similar. (SAS Criterion of similarity).
7. If three sides of a triangle are proportional to the three corresponding sides of another triangle, then the two triangles are similar. (SSS Criterion of similarity).
8. A perpendicular line drawn from the vertex of a right angled triangle divides the triangle into two triangles similar to each other and also to original triangle.
9. If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of their corresponding altitudes.
10. If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of the corresponding perimeters.
11. The ratio of the area of two similar triangles are equal to the ratio of the squares of their corresponding sides.
12. If two triangles have common vertex and their bases are on the same straight line, the ratio between their areas is equal to the ratio between the length of their bases.
13. Two triangles are said to be similar if their corresponding sides are proportional.
14. The triangles are equiangular if the corresponding angles are equal.
15. A pair of equiangular triangles are similar.
16. If two triangles are similar, then they are equiangular.

    a) Scale Factor: It measure the ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle.

Exercise 4.2

17. Basic Proportionality Theorem (BPT) or Thales theorem : A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.
18. Converse of Basic Proportionality Theorem : If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.
19. Angle Bisector Theorem :The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.
20. Converse of Angle Bisector Theorem : If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex.
21. Pythagoras Theorem : In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

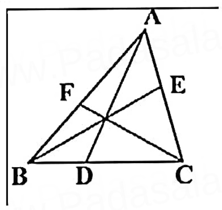
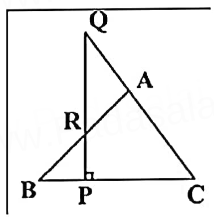
    a) In India Pythagoras Theorem is also referred as Baudhyana Theorem.

    b) (a, b, c) is a Pythagorean Triplet if and only if $c^2 = a^2 + b^2$.

    c) In a right angled triangle, the side opposite to 90° is called the hypotenuse.

Exercise 4.3

23. If a line touches the given circle at only one point, then it is called tangent to the circle.
24. A tangent at any point on a circle and the radius through the point are perpendicular to each other.
25. No tangent can be drawn from an interior point of the circle.
26. Only one tangent can be drawn at any point on a circle.
27. Two tangents can be drawn from any exterior point of a circle.
28. The lengths of the two tangents drawn from an exterior point to a circle are equal,
29. If two circles touch externally the distance between their centers is equal to the sum of their radii.
30. If two circles touch internally, the distance between their centers is equal to the difference of their radii
31. The two direct common tangents drawn to the circles are equal in length.
32. Alternate Segment theorem : If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.
33. A cevian is a line segment that extends from one vertex of a triangle to the opposite side.
34.

Theorem	Statement	Figure
Ceva's Theorem	Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively.Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC}\times\frac{CE}{EA}\times\frac{AF}{FB}=1$ where the lengths are directed.	
Menelaus Theorem	A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC}\times\frac{CQ}{QA}\times\frac{AR}{RB}=1$ where all segments in the formula are directed segments.	

35. In a triangle, the medians are concurrent. Centroid is the point of concurrence of the medians.
36. Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

5. COORDINATE GEOMETRY

Exercise 5.1

1. Apollonius is hailed as "The Great Geometer". Coordinate geometry is also called as Analytical geometry.
2. Coordinate geometry interlinks algebra and geometry'
3. Distance between two points (x_1,y_1), (x_2,y_2) 'd' is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
4. The mid-point M, of the line segment joining (x_1,y_1), (x_2,y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
5. Section Formula:
   (a) Internal Division : The point which divides the line segment joining the points (x_1,y_1) and (x_2,y_2) internally in the ratio m : n = $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
   (b) External Division : The point which divides the line segment joining the points (x_1,y_1) and (x_2,y_2) externally in the ratio m : n = $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

7. Area of a triangle = $[x_1(y_2-y_3)+\ x_2(y_3-y_1)+\ x_3(y_1-y_2)]$ sq.units.

(or) = $[(x_1y_2+x_2y_3+x_3y_1)-(x_2y_1+x_3y_2+x_1y_3)]$ sq.units.

8. If the area of the $\Delta = 0$ then three points $[(x_1,y_1),(x_2,y_2),(x_3,y_3)]$ are collinear.

9. If the points $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are collinear, then

$[x_1(y_2-y_3)+\ x_2(y_3-y_1)+\ x_3(y_1-y_2)]=0$ (or) $x_1y_2+x_2y_3+x_3y_1=x_2y_1+x_3y_2+x_1y_3$

10. Area of the quadrilateral $=\frac{1}{2}\{(x_1-x_3)(y_2-y_4)-(x_2-x_4)(y_1-y_3)\}$ sq.units.

(or) = $\frac{1}{2}\{(x_1y_2+x_2y_3+x_3y_4+x_4y_1)-(x_2y_1+x_3y_2+x_4y_3+x_1y_4)\}$ sq.units.

Exercise 5.2

11. The inclination of X axis and every line parallel to X axis is 0° .

12. The inclination of Y axis and every line parallel to Y axis is 90° .

a) The measure of steepness is called slope or gradient.

13. The slope of the straight line m = tan θ, $0 \le \theta \le 180°$, $\theta \le 90°$ (The slope of a vertical line is undefined.) [Slope = Gradient of the line]

14. The slope of a straight line passing through the given two points (x_1, y_1), (x_2, y_2) is $\frac{y_1-y_2}{x_1-x_2}$

Slope $m = \frac{\text{change in y coordinates}}{\text{change in x coordinates}}$

15. The slope of a straight line passing through the given two points (x_1, y_1), (x_2, y_2) with $x_1 \neq x_2$ is $\frac{y_2-y_1}{x_2-x_1}$.

16. Two non-vertical lines are parallel if and only if their slopes are equal. $(m_1 = m_2)$.

17. Two non-vertical lines with slopes $m1$ and $m2$ are perpendicular if and only if $m_1 \times m_2 = -1$

18. The General form of the equation of a straight line is ax + by + c = 0. (First degree)

19. Values of slopes:

S.No	Condition	Slope
1.	$\theta = 0°$	The line is parallel to the positive direction of X axis.
2.	$0° < \theta < 90°$	The line has positive slope (A line with positive slope rises from left to right).
3.	$90° < \theta < 180°$	The line has negative slope (A line with negative slope falls from left to right).
4.	$\theta = 180°$	The line is parallel to the negative direction of X axis.
5.	$\theta = 90°$	The slope is undefined.

a) In any triangles exterior angle is equal to sum of the opposite interior angles.

b) If the slopes of both the pairs of opposite sides are equal then the quadrilateral is a parallelogram.

Exercise 5.3 & 5.4

20. Equation of X axis is y = 0

21. Equation of Y axis is x = 0

22. Equation of a straight line parallel to X axis is y = b

23. Equation of a straight line parallel to Y axis x = c

a. If $c > 0$, then the line $x=c$ lies right to the side of the Y axis

b. If $c < 0$, then the line $x=c$ lies left to the side of the Y axis

c. If $c = 0$, then the line $x=c$ is the Y axis itself.

24. Slope-Intercept Form:

A line with slope m and y intercept c can be expressed through the equation $y=mx+c$.

25. $y = mx$ is the equation of a straight line passing through the origin.

a) Convert: Celsius to Fahrenheit. $F = \frac{9}{5}C + 32$.

26. Point-Slope form : $y - y_1 = m(x - x_1)$

28. Intercept form of a line: $\frac{x}{a} + \frac{y}{b} = 1$.

29. The equation of all lines parallel to the line $ax+by+c=0$ can be put in the form $ax+by+k=0$ for different values of k.

30. The equation of all lines perpendicular to the line $ax+by+c=0$ can be put in the form $bx - ay + k = 0$ for different values of k.

31. Two straight lines $a_1x + b_1y + c_1 = 0$ and; $a_2x + b_2y + c_2 = 0$ where the coefficients are non-zero, are

   (i) parallel if and only if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$; That is $a_1b_2 - a_2b_1 = 0$ gives $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$.

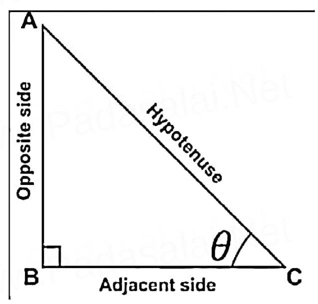
   (ii) Perpendicular if and only if $a_1a_2 + b_1b_2 = 0$.

32. The straight line $ax + by + c = 0$ (a) Slope $(m) = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -a/b$

   (b) y intercept (c) $= \frac{-\text{constant term}}{\text{coefficient of } y} = -c/b$

6. TRIGONOMETRY

1. Canon Mathematicus describes about trigonometry.
2. Viete introduced the term "coefficient" in mathematics.
3. Hipparchus is considered as "The Father of Trigonometry".
   a) A table of chord lengths for a circle of circumference = 360 × 60 = 21600 units.
   b) The word "Trigonometry" was invented by German mathematician Bartholomaeus Pitiscus. (17^{th} Century AD)
4.



$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{opposite side}}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$ $\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}$ $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

5. Complementary angle

$\sin(90° - \theta) = \cos\theta$	$\text{cosec}(90° - \theta) = \sec\theta$	$\tan(90° - \theta) = \cot\theta$
$\cos(90° - \theta) = \sin\theta$	$\sec(90° - \theta) = \text{cosec}\,\theta$	$\cot(90° - \theta) = \tan\theta$

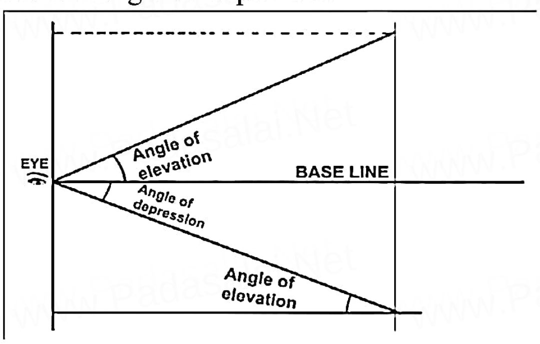
$sin^2\theta + cos^2\theta = 1$	$1 + tan^2\theta = sec^2\theta$	$1 + cot^2\theta = cosec^2\theta$
$sin^2\theta = 1 - cos^2\theta$	$1 = sec^2\theta - tan^2\theta$	$1 = cosec^2\theta - cot^2\theta$
$cos^2\theta = 1 - sin^2\theta$	$tan^2\theta = sec^2\theta - 1$	$cot^2\theta = cosec^2\theta - 1$

a) An equation involving triganometrix ratio's of an angle is called a trigonometric identity if it is true for all values of the angle.

7. Table of Trigonometric Ratios for 0°, 30°, 45°, 60°, 90°

	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
θ	**0°**	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\propto$
cosec	$\propto$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$\propto$
cot	$\propto$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

8. Angle of Elevation & Angle of Depression



(a) Angle of Elevation : The angle of elevation is an angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.

(b) Angle of Depression : V The angle of depression is an angle formed by the line of sight with the horizontal when the point is below the horizontal level.

(c) The angle of elevation and depression are usually measured by a device called clinometer.
Angle of Depression and Angle of Elevation are equal because they are alternate angles.

(d) $Speed = \frac{distance}{\text{time}}$

7.MENSURATION

1. Mensuration can be thought as applied geometry.
2. Surface Area Surface area is the measurement of all exposed area of a solid object.
3. A right circular cylinder is a solid generated by the revolution of a rectangle about one of its sides as axis.
4. A solid cylinder is an object bounded by two circular plane surfaces and a curved surface.
5. The area between the two circular bases is called its 'Lateral Surface Area' (L.S.A.) or 'Curved Surface Area' (C.S.A.).
6. Total surface area refers to the sum of areas of the curved surface area and the two circular regions at the top and bottom.
7. The term 'surface area' refers to 'total surface area'.
8. An object bounded by two co-axial cylinders of the same height and different radii is called a 'hollow cylinder'.
9. A right circular cone is a solid generated by the revolution of a right angled triangle about one of the sides containing the right angle as axis.
10. A sphere is a solid generated by the revolution of a semicircle about its diameter as axis.
11. Every plane section of a sphere is a circle.
12. The line of section of a sphere by a plane passing through the centre of the sphere is called a great Circle, all other plane sections are called small circles.
13. A section of the sphere cut by a plane through any of its great circle is a hemisphere.
14. Hemisphere is exactly half of the sphere.

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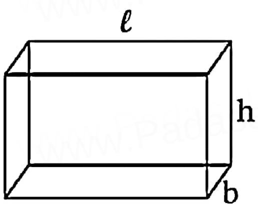
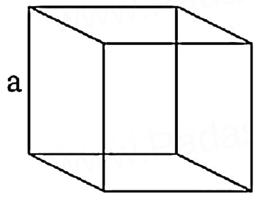
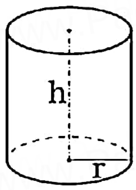
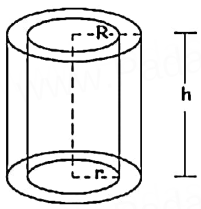
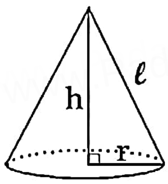
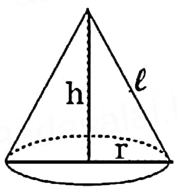
S.NO	SHAPE	CURVED SURFACE AREA (C.S.A.) / LATERAL SURFACE AREA (L.S.A) (in sq.units)	TOTAL SURFACE AREA (T.S.A.) (in sq.units)	VOLUME (in cub.units)
1	CUBOID 	$2h(\ell + b)$	$2(\ell b + bh + h\ell)$	ℓbh
2	CUBE 	$4a^2$	$6a^2$	a^3
3	RIGHT CIRCULAR CYLINDER 	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
4	HOLLOW CYLINDER 	$2\pi(R+r)h$	$2\pi(R+r)(R-r+h)$	$\pi(R^2-r^2)h$
5	RIGHT CIRCULAR CONE 	$\pi r\ell$	$\pi r(\ell + r)$	$\frac{1}{3}\pi r^2 h$



ℓ = slant height

h = height

r = radius

$\ell = \sqrt{h^2+r^2}$

$h = \sqrt{\ell^2 - r^2}$

$r = \sqrt{\ell^2 - h^2}$

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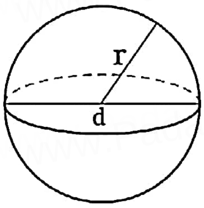
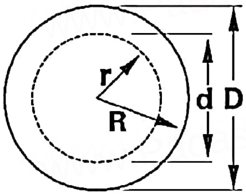
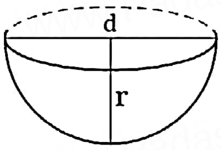
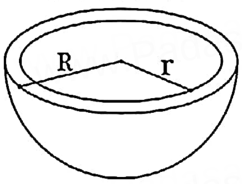
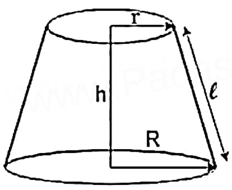
S.NO	SHAPE	CURVED SURFACE AREA (C.S.A.) / LATERAL SURFACE AREA (L.S.A) (in sq.units)	TOTAL SURFACE AREA (T.S.A.) (in sq.units)	VOLUME (in cub.units)
6	SPHERE 	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
7	HOLLOW SPHERE 	$4\pi r^2$	$4\pi(R^2+r^2)$	$\frac{4}{3}\pi(R^3-r^3)$
8	HEMISPHERE 	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
9	HOLLOW HEMISPHERE 	$2\pi(R^2+r^2)$	$\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
10	FRUSTUM OF RIGHT CIRCULAR CONE 	$\pi(R+r)\ell$ where $\ell=\sqrt{h^2+(R-r)^2}$	$\pi(R+r)\ell+\pi R^2+\pi r^2$ where $\ell=\sqrt{h^2+(R-r)^2}$	$\frac{\pi h}{3}(R^2+Rr+r^2)$

8.STATISTICS AND PROBABILITY

Exercise 8.1 & 8.2

1. Prasanta Chandra Mahalanobis is hailed as "Father of Indian Statistics".
2. 29th June is celebrated as "National Statistics Day".
3. The Measures of Central Tendency usually will be near to the middle value of the data.
4. The Arithmetic Mean or Mean of the given values is sum of all the observations divided by the total number of observations.
5. Arithmetic Mean $\bar{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$
6. Dispersion is a measure which gives an idea about the scatteredness of the values.
7. Measures of Variation (or) Dispersion of a data provide an idea of how observations spread out (or) scattered throughout the data.
8. Different Measures of Dispersion are 1. Range 2. Mean deviation 3. Quartile deviation 4. Standard deviation 5. Variance 6. Coefficient of Variation
9. The difference between the largest value and the smallest value is called Range. Range = L - S.
10. Coefficient of range = $\frac{L-S}{L+S}$
    a) If the frequency of initial class is zero, then the next class will be considered for the calculation of Range.
11. The mean of the squares of the deviations from the mean is called Variance. It is denoted by σ^2
12. The positive square root of Variance is called Standard deviation.
13. Comparing two (or) more data for corresponding changes the relative measure of S.D called coefficient of variation.

S.No.	Method	Standard Deviation	
		Ungrouped Data	Grouped Data
1.	Direct method	$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$	---------
2.	Mean method	$\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$	$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$
3.	Assumed Mean method	$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma f_i d_i}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$
4.	Step Deviation method	$\sigma = c\sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$	$\sigma = c\sqrt{\frac{\Sigma f_i d_i}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$
5.	Range	L – S L= Largest Value S= Smallest Value	Coefficient of range = $\frac{L-S}{L+S}$
6.	Variance	$\sigma^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$	
7.	Standard Deviation	$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$	
8..	Coefficient of variation	C.V.= $\frac{\sigma}{\bar{x}}$ x 100%	

Exercise 8.3 Probability

1. A random experiment is an experiment in which (i) Th e set of all possible outcomes are known (ii)Exact outcome is not known.
2. The set of all possible outcomes in a random experiment is called a sample space. It is generally denoted by *S*.
3. Each element of a sample space is called a sample point.
4. Tree diagram allow us to see visually all the possible outcomes of an experiment. Each branch in a tree diagram represent a possible outcome.
5. In a random experiment each possible outcome is called an event.
   a) Performing an experiment once is called a trial.

Events	Explanation	Example
1.Equally likely events	Two or more events are said to be equally likely if each one of them has an equal chance of occurring.	Head and tail are equally likely events in tossing a coin.
2.Certain events	In an experiment, the event which surely occur is called certain event.	When we roll a die, the event of getting any natural number from one to six is a certain event.
3.Impossible events	In an experiment if an event has no scope to occur then it is called an impossible event.	When we toss two coins, the event of getting three heads is an impossible event.
4.Mutually exclusive events	Two or more events are said to be mutually exclusive if they don't have common sample points. i.e., events *A, B* are said to be mutually exclusive if $A \cap B = \phi$.	When we roll a die the events of getting odd numbers and even numbers are mutually exclusive events.
5.Exhaustive events	The collection of events whose union is the whole sample space are called exhaustive events.	When we toss a coin twice, events of getting two heads, exactly one head, no head are exhaustive events.
6.Complementary events	The complement of an event *A* is the event representing collection of sample points not in *A*. It is denoted $A^{/}$ or A^{o} or $\bar{A}$. The event *A* and its complement $A^{/}$ are mutually exclusive and exhaustive.	When we roll a die, the event 'rolling a 5 or 6' and the event of rolling a 1, 2, 3 or 4 are complementary events.

6. If an event *E* consists of only one outcome then it is called an <u>elementary event</u>.
7. In a random experiment, let S be the sample space and $E \subseteq S$. Then it *E* is an event. The probability of occurrence of *E* is defined as

$$P(E) = \frac{\text{Numbers of outcomes favourable to occurence of } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}.$$

8. The probability of sure event is 1. The probability of impossible event is 0.
9. *E* is a subset of *S*. The probability value always lies from 0 to 1.
10. The complement event of E is $\bar{E}$. $P(E) + P(\bar{E}) = 1$.

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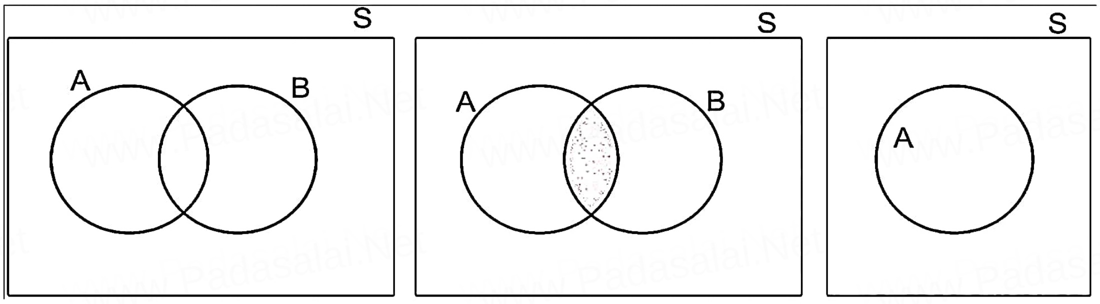
Exercise 8.4

11. (i) $(A \cap B)$ is an event that occurs only when both A and B occurs. (ii) $(A \cup B)$ is an event that occurs only when at least one of A or B occurs. (iii) $\bar{A}$ is an event that occurs only when A doesn't occur.



$A \cap B$ $\quad$ $A \cup B$ $\quad$ $\bar{A}$

12. $A \cap \bar{A} = \phi$.
13. $A \cup \bar{A} = S$.
14. If A, B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \phi$
15. $P(Union\ of\ events) = \sum(Probability\ of\ events)$
16. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, for any two events A,B.
    $(i)\ P(A \cap \bar{B}) = P(only\ A) = P(A) - P(A \cap B)$
    $(ii) P(\bar{A} \cap B) = P(only\ B) = P(B) - P(A \cap B)$
    $(iii) P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
17. $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$.

**** Do OR Die ****