WAY TO SUCCESS PUBLICATIONS
HALF - YEARLY EXAMINATION
ANSWER KEY
10TH MATHEMATICS

Part – I

Answer all the questions	
1.	c) $\{4, 9, 25, 49,121\}$
2.	c) $\frac{2}{9x^2}$
3.	b) 2
4.	b) An arithmetic progression
5.	c) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$
6.	b) $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$
7.	c) 4
8.	b) 25 sq.units
9.	b) $x + y = 3, 3x + y = 7$
10.	d) 1
11.	b) 4 cm
12.	c) 3 π
13.	b) 100
14.	b) $\frac{7}{10}$

Part – II

15. (i) Range of $f = \{1,8,27,64\}$

(ii) Since distinct elements in A are mapped into distinct images in B, it is a one-one function. $2 \in B$ is not the image of any element of A. So, it is Into function.

16. $800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^5 \times 5^2$

$a^b \times b^a = 2^5 \times 5^2$

$a = 5$ and b = 2.

17. $a_n = \frac{1}{3}n + \frac{1}{6}$

$a_n = \frac{2}{6}n + \frac{3}{6} - \frac{2}{6}$

$a_n = \frac{3}{6} + (n-1)\frac{2}{6}$

It is in the form $a_n = a + (n-1)d$.

Here $a = \frac{3}{6}$, $d = \frac{2}{6}$

Given sequence is an A.P

18. $n = 27 + 1 = 28$

$1 + 3 + 5 + \cdots + 55 = (28)^2 = 784.$

19. $\alpha + \beta = -6, \alpha\beta = -4$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (-6)^2 - 4(-4)$

$= 36 + 16$

$= 52$

20. $A^T = \begin{bmatrix} 5 & -\sqrt{7} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{7} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = A$

21. Slope $m = \frac{y_2-y_1}{x_2-x_1} = \frac{3-a}{9-(-2)} = \frac{3-a}{11}$

$\frac{3-a}{11} = -\frac{1}{2}$ (Given)

$a = \frac{17}{2}$

22. Area of the triangle

Area of the triangle

$= \frac{1}{2}\begin{bmatrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{bmatrix}$

$= \frac{1}{2}[29 + 19] = \frac{1}{2}(48) = 24$ Sq. units

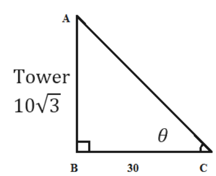
23. $tan\theta = \frac{AB}{BC} = \frac{10\sqrt{3}}{30}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan 30^0 = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^0$

$\therefore$ The angle of elevation $\theta = 30^0$



24. Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Ratio of C.S.A. of balloons $= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{9}{16}$

Ratio of C.S.A. of balloons is 9:16.

25. Volume of the cone= $11088\ cm^3$

$\frac{1}{3}\pi r^2 h = 11088$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

$r^2 = 441$

Therefore, radius of the cone $r = 21\ cm$.

26. $\pi r^2 h = \frac{4}{3}\pi r^3$

$\pi \times 10 \times 10 \times h = \frac{4}{3} \times \pi \times 15 \times 15 \times 15$

$h = 45\ cm$

Height of the cylinder $= 45\ cm$

27. $R = 36.8,\quad S = 13.4,\quad L = ?$

$R + S = L$

$L = 36.8 + 13.4$

$L = 50.2$

28. $S = \{HH, HT, TH, TT\}$

$n(S) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Part – III

29. $A = \{2,3\}, B = \{0,1\}, C = \{1,2\}$

$B \cap C = \{1\}$

$A \times (B \cap C) = \{2,3\} \times \{1\}$

$A \times (B \cap C) = \{(2,1),(3,1)\}$ ……(1)

$A \times C = \{(2,1),(2,2),(3,1),(3,2)\}$

$(A \times B) \cap (A \times C) = \{(2,1),(3,1)\}$ …..(2)

From (1) and (2),

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

30. $f \circ g(x) = f(6x - k) \quad = 18x - 3k + 2$

$g \circ f(x) = g(3x + 2) \quad = 18x + 12 - k$

Given that, $f \circ g = g \circ f$

$18x - 3k + 2 = 18x + 12 - k$

$k = -5$

31. $t_4 = 8 \Rightarrow ar^3 = 8$ …….(1)

$t_8 = \frac{128}{625} \Rightarrow ar^7 = \frac{128}{625}$ …….(2)

$(2) \div (1) \Rightarrow r^4 = \left(\frac{2}{5}\right)^4 \Rightarrow r = \frac{2}{5}$

From $(1) \Rightarrow a\left(\frac{2}{5}\right)^2 = 8 \Rightarrow a = 125$

Required G.P, $a, ar, ar^2, ….$

$125, 50, 20, …$

32. $10^2 + 11^2 + 12^2 + \cdots + 24^2$

$= (1^2 + 2^2 + \cdots + 24^2)$

$-(1^2 + 2^2 + 3^2 + \cdots + 9^2)$

$= \frac{24(24+1)(24\times2+1)}{6} - \frac{9(9+1)(2\times9+1)}{6}$

$= 4(25)(49) - 3(5)(19)$

$= 4900 - 285$

$= 4615\ cm^2$

33. $x + y + z = 5$ ………… (1)

$2x - y + z = 9$ ………….. (2)

$x - 2y + 3z = 16$ ………….. (3)

Add (1) + (2)

$(1) \Rightarrow\ x + y + z = 5$

$(2) \Rightarrow 2x - y + z = 9$

$3x + 2z = 14$ ……….. (4)

$(2) \times 2 \Rightarrow 4x - 2y + 2z = 18$

$(3) \Rightarrow x - 2y + 3z = 16$

(−) (+) (−) (−)

$3x - z = 2$ ………….. (5)

$(4) \Rightarrow 3x + 2z = 14$

$(5) \Rightarrow 3x - z = 2$

(−) (+) (−)

$3z = 12$

$3z = 12$

$z = \frac{12}{3} = 4$

$z = 4$ Substituting in (5)

$3x - 4 = 2 \Rightarrow 3x = 6 \Rightarrow x = \frac{6}{3} \Rightarrow x = 2$

$x = 2, z = 4$ Substituting in (1)

$y = 5 - 2 - 4 \Rightarrow y = -1$

$\therefore x = 2, y = -1, z = 4$

34.

	$3x^2 + 2x + 4$
$3x^2$	$9x^4 + 12x^3 + 28x^2 + ax + b$
	$9x^4$
	(−)
$6x^2 + 2x$	$12x^3 + 28x^2$
	$12x^3 + 4x^2$
	(−) (−)
$6x^2 + 4x + 4$	$24x^2 + ax + b$
	$24x^2 + 16x + 16$
	(−) (−) (−)
	0

Given polynomial is a perfect square

$a - 16 = 0, b - 16 = 0$

Therefore $a = 16,\ b = 16$

35. $n(S) = 36$

$A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$B = \{(1,3),(2,2),(3,1)\}$

$A \cap B = \{(2,2)\}$

$n(A) = 6, n(B) = 3, n(A \cap B) = 1$

$P(A) = \frac{6}{36}, P(B) = \frac{3}{36}$

$P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{36}$

$P(A \cup B) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$

36. $A - B = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$

$(A-B)^T = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$............ (1)

$A^T - B^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}$... (2)

From (1) and (2), $(A-B)^T = A^T - B^T$

37.

Pythagoras Theorem

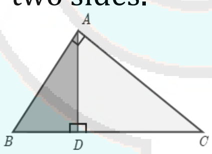
Statement: In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof:

Given : In ΔABC, $\angle A = 90°$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$



No.	Statement	Reason
1.	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \backsim \Delta ABD$ $\frac{AB}{AD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ......(1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \backsim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity

Adding (1) and (2) we get

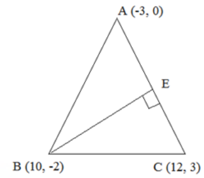
$AB^2 + AC^2 = BC \times BD + BC \times DC$

$= BC\,(BD + DC) = BC \times BC$

$AB^2 + AC^2 = BC^2$

Hence the theorem is proved.

38.



Slope of $BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{12-10} = \frac{5}{2}$

Equation of the altitude passing through the vertex A.

$y - y_1 = m(x - x_1)$

$A(-3,0)$ and $m = \frac{5}{2}$

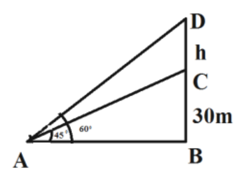
$y - 0 = -\frac{1}{\frac{5}{2}}(x - (-3))$

$y = -\frac{2}{5}(x+3)$

$5y = -2x - 6$

$2x + 5y + 6 = 0$

39. $\tan 45° = \frac{30}{AB}$



$AB = 30m$

$\tan 60^0 = \frac{30+h}{30}$

$30\sqrt{3} = 30 + h$

$h = 30(\sqrt{3} - 1)$

$= 30(1.732 - 1)$

$= 30(0.732)$

$= 21.96$

The height of the tower is 21.96 m.

40. Hemisphere : $r = 7$

Cone: $r = 7, l = 11$

Surface area of doll$= 2\pi r^2 + \pi r l$

$= \left(2 \times \frac{22}{7} \times 7 \times 7\right) + \left(\frac{22}{7} \times 7 \times 11\right)$

$= \frac{22}{7} \times 7(14 + 11)$

$= 550 cm^2$

41.

x	f	$d = x - A$	fd	fd^2
10	3	-8	-24	192
15	2	-3	-6	18
18	5	0	0	0
20	8	2	16	32
25	2	7	14	98
	$N = 20$		0	340

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2} = \sqrt{\frac{340}{20}} = \sqrt{17}$$

$\cong 4.1$

42. $\Delta = b^2 - 4ac$

$(b-c)^2 - 4(a-b)(c-a) = 0$

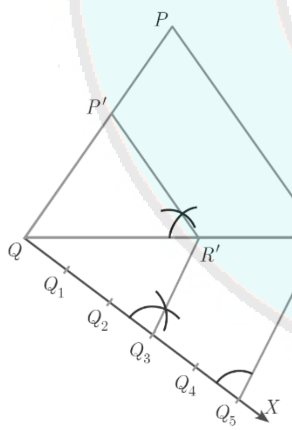
$b^2 + c^2 + 4a^2 + 2bc - 4ac - 4ab = 0$

$(2a - b - c)^2 = 0$

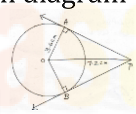
$2a = b + c$

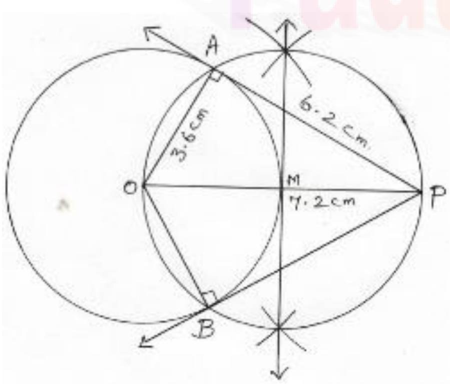
Part – IV

43. a)



b) Rough diagram





Verification: $PT = \sqrt{OP^2 - OT^2}$

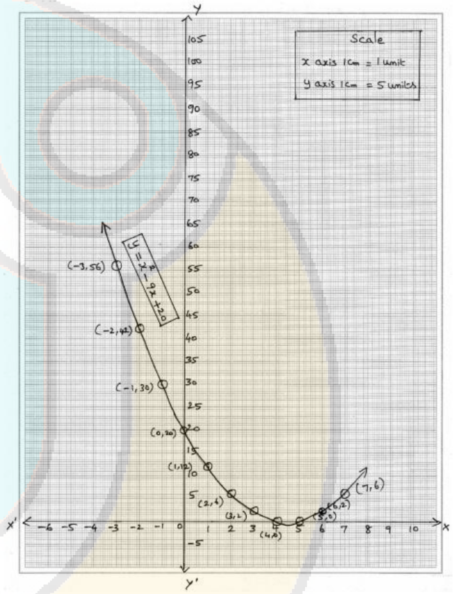
$= \sqrt{(7.2)^2 - (3.6)^2} = \sqrt{38.7} \simeq 6.2\ cm.$

44. a) $x^2 - 9x + 20 = 0$

$y = x^2 - 9x + 20$

x	-3	-2	-1	0	1	2	3	4	5	6
x^2	9	4	1	0	1	4	9	16	25	36
$-9x$	27	18	9	0	-9	-18	-27	-36	-45	54
20	20	20	20	20	20	20	20	20	20	20
y	56	42	30	20	12	6	2	0	0	2

Points: $(-3, 56), (-2,42), (-1,30), (0,20), (1,12), (2,6), (3,2), (4,0), (5,0), (6,2)$



Solution $x = \{4, 5\}$

Real and unequal roots.

44.b) $y = x^2 + x - 2$

x	-3	-2	-1	0	1	2
y	4	0	-2	-2	0	4

Solve

$y = x^2 + x - 2$

$0 = x^2 + x - 2 \quad (-)$

$y = 0$