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Reg. No. | | | | | | | |

Time : 2.30 hrs. **Half-Yearly Common Examination - 2019** Max. Marks : 100

MATHEMATICS

Instructions : 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately. 2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

Note : This question paper contains four parts.

PART - I

Note : i) Answer all questions. **14 x 1 = 14**

ii) Choose the most suitable answer from the given four alternatives and write the option code with the corresponding answer. iii) Each question carries 1 mark.

1. The range of the relation R = { (x, x^2) / x is a prime number less than 13} is :
a) {2, 3, 5, 7} b) {2, 3, 5, 7, 11} c) {4, 9, 25, 49, 121} d) {1, 4, 9, 25, 49, 121}

2. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then fog is : a) $\frac{3}{2x^2}$ b) $\frac{2}{3x^2}$ c) $\frac{2}{9x^2}$ d) $\frac{1}{6x^2}$

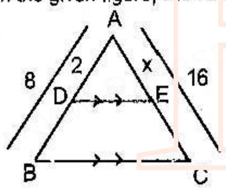
3. If the HCF of 65 and 117 is expressible in the form of 65 m − 117, then the value of m is : a) 4 b) 2 c) 1 d) 3

4. If the sequence $t_1, t_2, t_3, \ldots$ are in A.P, then the sequence $t_6, t_{12}, t_{18}, \ldots$ is :
a) a Geometric Progression b) an Arithmetic Progression c) neither an Arithmetic Progression nor a Geometric Progression d) a Constant Sequence

5. $\frac{x}{x^2-25} - \frac{8}{x^2+6x+5}$ gives : a) $\frac{x^2-7x+40}{(x-5)(x+5)}$ b) $\frac{x^2+7x+40}{(x-5)(x+5)(x+1)}$ c) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$ d) $\frac{x^2+10}{(x^2-25)(x+1)}$

6. Find the matrix X if $2X + \begin{bmatrix}1 & 3\\5 & 7\end{bmatrix} = \begin{bmatrix}5 & 7\\9 & 5\end{bmatrix}$ a) $\begin{bmatrix}-2 & -2\\2 & -1\end{bmatrix}$ b) $\begin{bmatrix}2 & 2\\2 & -1\end{bmatrix}$ c) $\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}$ d) $\begin{bmatrix}2 & 1\\2 & 2\end{bmatrix}$

7. In the given figure, the value of x is :



a) 2 b) 8 c) 4 d) 12

8. The area of triangle formed by the points (−5, 0), (0, −5) and (5, 0) is :
a) 0 sq. units b) 25 sq. units c) 5 sq. units d) none of these

9. (2, 1) is the point of intersection of two lines :
a) x − y −3 = 0 ; 3x − y − 7 = 0 b) x + y = 3 ; 3x + y = 7 c) 3x + y = 3 ; x + y = 7 d) x + 3y − 3 = 0 ; x − y − 7 = 0

10. $\cos 60^\circ \sin 30^\circ + \cos 30^\circ \sin 60^\circ$ = a) 90° b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1

11. The height of a right circular cone whose radius is 3 cm and slant height is 5cm will be :
a) 12 cm b) 4 cm c) 13 cm d) 5 cm

12. The total surface area of a hemisphere is how much times the square of its radius? a) π b) 4π c) 3π d) 2π

13. The standard deviation of a data is 5. If each value is multiplied by 2, then the new variance is :
a) 3 b) 100 c) 10 d) 225

14. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is : a) $\frac{3}{10}$ b) $\frac{7}{10}$ c) $\frac{3}{9}$ d) $\frac{7}{9}$

PART - II

10 x 2 = 20

Note : Answer any ten questions. (Question No.28 is compulsory). Each questions carries 2 marks

15. Let A = {1, 2, 3, 4} and B = N. Let f : A → B be defined by $f(x) = x^3$ then,
i) find the range of f. ii) identify the type of function.

16. a and b are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

17. Show that the sequence described by $a_n = \frac{1}{3}n + \frac{1}{6}$ is an A.P.

18. Find the sum of 1 + 3 + 5 + ... + 55.

19. If α and β are the roots of $x^2 + 6x - 4 = 0$, find the value of $(\alpha - \beta)^2$.

20. If $A = \begin{bmatrix}5 & 2 & 2\\ -\sqrt{17} & 0.7 & \frac{5}{2}\\ 8 & 3 & 1\end{bmatrix}$ then verify $(A^T)^T = A$.

21. The line through the points (−2, a) and (9, 3) has slope $-\frac{1}{2}$. Find the value of a. *TEN*

22. Find the area of the triangle formed by the points (1, −1), (−4, 6) and (−3, −5).

23. Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height $10\sqrt{3}$ m.

24. The radius of a spherical balloon increases from 12cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

25. The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

26. An aluminium sphere of radius 15 cm is melted to make a cylinder of radius 10cm. Find the height of the cylinder.

27. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

28. A coin is tossed twice. What is the probability of getting exactly one head?

PART - III

10 x 5 = 50

Note : Answer any 10 questions. (Question number 42 is compulsory). Each questions carries 5 marks

29. Let A = {x ∈ N/1 < x < 4}, B = {x ∈ W/0 ≤ x < 2} and C = {x ∈ N/x < 3}. then verify that
A x (B ∩ C) = (A x B) ∩ (A x C)

30. Find the value of k, such that fog = gof if f(x) = 3x + 2, g(x) = 6x − k

31. In a Geometric Progression the 4^{th} term is 8 and the 8^{th} term is $\frac{128}{625}$. Find the Geometric Progression.

32. Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, ... 24 cm. How much area can be decorated with these colour papers?

33. Solve : x + y + z = 5 ; 2x − y + z = 9 ; x − 2y + 3z = 16.

34. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square find the values of a and b.

35. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

36. Let $A = \begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}, B = \begin{bmatrix}4 & 0\\1 & 5\end{bmatrix}, C = \begin{bmatrix}2 & 0\\1 & 2\end{bmatrix}$, show that $(A - B)^T = A^T - B^T$.

37. State and prove Pythagoras theorem.

38. A(−3, 0), B(10, −2) and C(12, 3) are the vertices of ΔABC. Find the equation of the altitude through A.

39. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. ($\sqrt{3}$ = 1.732)

40. A doll is made by surmounting a cone on a hemisphere of equal radius. The radius of the hemisphere is 7 cm and slant height of the cone is 11 cm. Find the surface area of the doll.

41. Find the standard deviation for the following data.

x	10	15	18	20	25
f	3	2	5	8	2

42. If the roots of $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that 2a = b + c.

PART - IV

2 x 8 = 16

Note : Answer both the questions choosing either of the alternatives. Each question carries 8 marks

43. a) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR. (Scale factor $\frac{3}{5} < 1$) **(OR)** b) Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

44. a) Graph the quadratic equation $x^2 - 9x + 20 = 0$ and state its nature of solutions. **(OR)**
b) Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$

10th Maths 2019

10^{th} std HALF YEARLY EXAM - 2019

ANSWER KEY

PART - I

$14 \times 1m = 14m$

1. c) $\{4, 9, 25, 49, 121\}$
2. c) $\frac{2}{9x^2}$
3. b) 2
4. b) an Arithmetic progression
5. c) $\frac{x^2-7x+40}{(x^2-25)(x+1)}$
6. b) $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$
7. c) 4
8. b) 25 sq.units
9. b) $x+y=3$, $3x+y=7$
10. d) 1
11. b) 4 cm
12. c) 3π
13. b) 100
14. b) $7/10$

PART - II

$10 \times 2m = 20m$

15. (i) range $= \{1, 8, 27, 64\}$

(ii) One to one function.

16. $a = 5$, $b = 2$

17. Creative

$n=0 \Rightarrow t_1 = a_0 = \frac{1}{6}$

$n=1 \Rightarrow t_2 = a_1 = \frac{1}{3}(1)+\frac{1}{6} = \frac{3}{6}$

$n=2 \Rightarrow t_3 = a_2 = \frac{1}{3}(2)+\frac{1}{6} = \frac{5}{6}$

$\vdots$

$n=n \Rightarrow a_n = \frac{1}{3}n+\frac{1}{6}$

$\therefore$ sequence $\frac{1}{6}, \frac{3}{6}, \frac{5}{6}, \frac{7}{6}, \ldots$

here, common difference $d = t_2 - t_1 = t_3 - t_2$

$d = \frac{3}{6}-\frac{1}{6} = \frac{2}{6}$ also $d = \frac{3}{6}-\frac{1}{6} = \frac{2}{6}$

$\boxed{t_2 - t_1 = t_3 - t_2 = 2/6}$

$\therefore a_n = \frac{n}{3}+\frac{1}{6}$ is an A.P.

18. w.k.T $n = \frac{(l-a)}{d}+1$

$n = 28$

$\Sigma(1+3+5+\cdots+55) = n^2 = (28)^2 = 784$

19. w.k.T $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

Given data we get $\alpha+\beta = -6$, $\alpha\beta = -4$

$\therefore (\alpha-\beta)^2 = (-6)^2 - 4(-4) = 36+16 = 52$.

20. $A^T = \begin{bmatrix} 5 & 2 & \cancel{4} \\ -\sqrt{17} & 0.7 & 5/2 \\ 8 & 3 & 1 \end{bmatrix}$

$A = (A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & 5/2 \\ 8 & 3 & 1 \end{bmatrix}$

Ideamaiyam YouTube channel

11^{th} and 12^{th} std Online Tution.

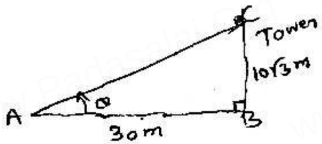
21. slope $= \frac{y_2-y_1}{x_2-x_1} = -\frac{1}{2}$

$a = \frac{17}{2}$

22. $A = \frac{1}{2}\left\{\begin{matrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{matrix}\right\}$ sq.units.

$A = 24$ sq.units

23. $\tan\theta = \frac{BC}{AB} = \frac{10\sqrt{3}}{30}$



$= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

24. let r_1 and r_2 be the radii of the balloons

$\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

ratio of C.S.A of balloons $= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$\therefore$ The required ratio of CSA is $9:16$.

25. $\frac{1}{3}\pi r^2 h = 11088$

$r^2 = 441 \Rightarrow r = 21$ cm

26. Creative

Given $r_1 = 15cm$, $r_2 = 10cm$

Volume of sphere $= \frac{4}{3}\pi r_1^3$ cu.units

Volume of cylinder $= \pi r_2^2 h$ cu.units

$\frac{4}{3}\cancel{\pi} r_1^3 = \cancel{\pi} r_2^2 h$

$4r_1^3 = 3r_2^2 h$

$\therefore h = \frac{4r_1^3}{3r_2^2} = \frac{4 \times 15 \times 15 \times 15}{3 \times 10 \times 10} = 45$ cm.

27. Largest value = Range + Smallest value

$L = 36.8 + 13.4 = 50.2$

28. Sample space $S = \{HH, HT, TH, TT\}$ $n(S) = 2^2 = 4$

$P(\text{getting exactly one head}) = \frac{2}{4} = \frac{1}{2}$.

PART - III

$\boxed{10 \times 5m = 50m}$

29. $A = \{2, 3\}$, $B = \{0, 1\}$, $C = \{1, 2\}$.

$B \cap C = \{1\}$

$LHS = A \times (B \cap C) = \{(2,1), (3,1)\}$

$A \times B = \{(2,0), (2,1), (3,0), (3,1)\}$

$A \times C = \{(2,1), (2,2), (3,1), (3,2)\}$

$RHS = (A \times B) \cap (A \times C) = \{(2,1), (3,1)\}$

30. $f \circ g = 3(6x - k) + 2$

$g \circ f = 6(3x + 2) - k$

$3(6x - k) + 2 = 6(3x+2) - k$

$-10 = 2k$

$\boxed{k = -5}$

31. $t_4 = ar^3 = 8 \rightarrow ①$

$t_7 = ar^6 = \frac{128}{625} \rightarrow ②$

$\frac{②}{①} \Rightarrow \frac{\cancel{a}r^{\cancel{6}3}}{\cancel{a}\cancel{r^3}} = \frac{\cancel{128}16}{625} \times \frac{1}{\cancel{8}} \Rightarrow r^3 = \frac{8}{125} = \frac{(2)^3}{(5)^3}$

$\Rightarrow \boxed{r = \frac{2}{5}}$

Put $r = 2/5$ in ① we get

$a\left(\frac{8}{125}\right) = 8 \Rightarrow \boxed{a = 125}$

$\therefore$ The required G.P is, 125, 50, 20, ...

32. $10^2 + 11^2 + 12^2 + \cdots + 24^2 = (1^2 + 2^2 + \cdots + 24^2) - (1^2 + 2^2 + \cdots + 9^2)$

$= \frac{24(25)(49)}{6} - \frac{9(10)(19)}{6} = \frac{29400 - 1710}{6}$

$= \frac{27690}{6} = 4615 \text{ cm}^2.$

33. $x = 2$, $y = -1$, $z = 4$.

34. $a - 16 = 0 \Rightarrow a = 16$. $\sqrt{9x^4 + 12x^3 + 28x^2 + ax + b} = |3x^2 + 2x + 4|$.

$b - 16 = 0 \Rightarrow b = 16$.

35. $n(S) = 36$ Sample space எழுதுதல்

$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

$P(A) = P(\text{getting a doublet}) = \frac{6}{36}$

$P(B) = P(\text{getting sum of faces as } 4) = \frac{3}{36}$

$P(A \cap B) = \frac{1}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6 + 3 - 1}{36} = \frac{8}{36} = \frac{2}{9}$

36. $A - B = \begin{bmatrix} -3 & 2 \\ 0 & -2 \end{bmatrix}$ $(A - B)^T = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}$

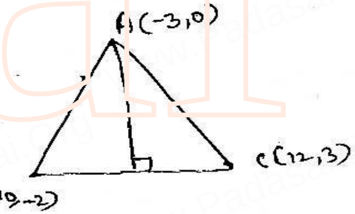
$A^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ $B^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$

$(A - B)^T = A^T - B^T = \begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}$

37. Diagram $\rightarrow$ 1 mark

Statement $\rightarrow$ 1 mark

Proof $\rightarrow$ 3 marks



Creative
38. Diagram $\rightarrow$ 1 mark
(Use Any Alternate method)

eqn. of BC is $5x - 2y - 54 = 0$ $\rightarrow$ $2x + 5y + k = 0 \rightarrow ①$

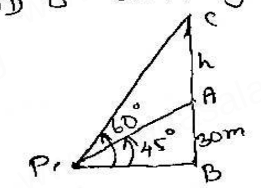
$AD \perp BC \Rightarrow$ eqn of AD is

It lie on $A(-3,0)$

$\therefore -6 + 0 + k = 0 \Rightarrow \boxed{k = 6}$

$\therefore$ required equation of AD is $2x + 5y + 6 = 0$

39. Diagram $\rightarrow$ 1 mark



$\tan 60° = \frac{30 + h}{BP} = \sqrt{3} \rightarrow ①$

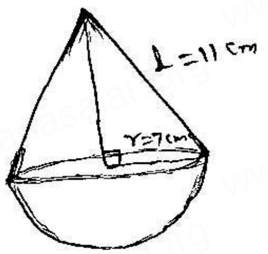
$\tan 45° = \frac{30}{BP} \Rightarrow \boxed{BP = 30} \rightarrow ②$

Put ② in ① we get

$h = 30(\sqrt{3} - 1) = 21.96\text{m}.$

(40) diagram - 1 mark

Given $r = 7$ cm
$l = 11$ cm
$h = 8.5$ cm



$l^2 = h^2 + r^2$
$h^2 = l^2 - r^2$
$h = \sqrt{121-49}$
$= \sqrt{72}$
$h = 8.5$ cm

T.S.A of a doll = C.S.A of the Cone + C.S.A of the hemisphere

$= \pi r l + 2\pi r^2$ sq.units

$= \frac{22}{7} \times 7 [11 + 2(7)] = 22[11+14]$

$= 22[25]$

$= 550$ sq.cm.

(41) $\bar{x} = 18$

$N = \Sigma f = 20$, $\Sigma fx = 360$, $\Sigma fd = -2$, $\Sigma fd^2 = 340$

$\Sigma fd = 30$.

$S.D = \sigma = \sqrt{\frac{\Sigma fd^2}{N}} = \sqrt{\frac{340}{20}} = \sqrt{17} \simeq 4.12.$

(42) W.K.T If the roots are equal then

$B^2 - 4AC = 0$

$(b-c)^2 - 4(a-b)(c-a) = 0$

$b^2 + c^2 - 2bc - 4[ac - a^2 - bc + ab] = 0$

$(-2a + b + c)^2 = 0$

$\boxed{b + c = 2a}$

PART - IV

$2 \times 8m = 16m.$

(43) We know that

(44)