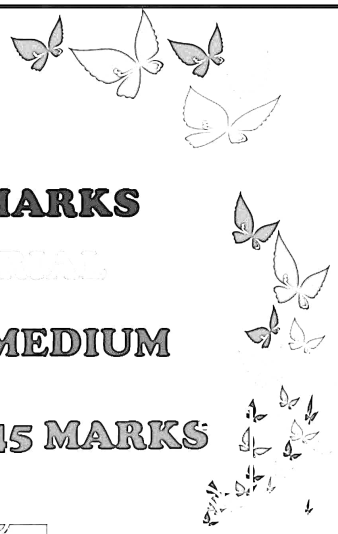
X MATHS

CONFIDENT

FIVE MARKS



ENGLISH MEDIUM

9 QUESTION – 45 MARKS

An equation means nothing to me unless it expresses a thought of God.

WITH YOUR HAPPY.........

1. RELATIONS AND FUNCTION
5 MARKS

If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution : $f(x) = 2x+3$, $g(x) = 1-2x$, $h(x) = 3x$

$(f \circ g) = (2x+3)(1-2x) = 2(1-2x)+3 = 2-4x+3 = 5-4x$

$(f \circ g) \circ h = (5-4x)(3x) = 5-4(3x) = 5-12x$ ....(1)

$(g \circ h) = (1-2x)(3x) = 1-2(3x) = 1-6x$

$f \circ (g \circ h) = (2x+3)(1-6x) = 2(1-6x)+3 = 2-12x+3 = 5-12x$ ....(2)

From (1) and (2) $\Rightarrow f \circ (g \circ h) = (f \circ g) \circ h$.

If $f(x) = \dfrac{x+6}{3}$ **and** $g(x) = 3 - x$, **find** $f \circ g$ **and** $g \circ f$. **Check whether** $f \circ g = g \circ f$.

Solution :

$(f \circ g) = \left(\dfrac{x+6}{3}\right)(3-x) = \dfrac{(3-x)+6}{3} = \dfrac{9-x}{3}$

$(g \circ f) = (3-x)\left(\dfrac{x+6}{3}\right) = 3 - \dfrac{x+6}{3} = \dfrac{9-x-3}{3} = \dfrac{6-x}{3}$

From (1) and (2) $\Rightarrow f \circ g \neq g \circ f$.

If $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$, Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution :

$(f \circ g) = (x-4)(x^2) = x^2 - 4$

$\therefore ((f \circ g) \circ h) = (x^2 - 4)(3x-5) = (3x-5)^2 - 4$ ...(1)

$(g \circ h) = (x^2)(3x-5) = (3x-5)^2$

$\therefore f \circ (g \circ h) = (x-4)(3x-5)^2 = (3x-5)^2 - 4$ ...(2)

From (1) and (2) $\Rightarrow (f \circ g) \circ h = f \circ (g \circ h)$

If $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$, Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution :

$(f \circ g) = (x^2)(2x) = (2x)^2 = 4x^2$

$\therefore ((f \circ g) \circ h) = (4x^2)(x+4) = 4(x+4)^2$ ...(1)

$(g \circ h) = (2x)(x+4) = 2(x+4)$

$\therefore f \circ (g \circ h) = (x^2)(2(x+4)) = (2(x+4))^2 = 4(x+4)^2$ ...(2)

From (1) and (2) $\Rightarrow (f \circ g) \circ h = f \circ (g \circ h)$

If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k.

Solution : $f \circ g = (3x-2)(2x+k) = 3(2x+k) - 2 = 6x + 3k - 2$

$g \circ f = (2x+k)(3x-2) = 2(3x-2) + k = 6x - 4 + k$

$f \circ g = g \circ f \Rightarrow 6x + 3k - 2 = 6x - 4 + k$

$6x - 6x + 3k - k = -4 + 2$

$2k = -2$

$k = -1$

Given the function $f : x \to x^2 - 5x + 6$, **evaluate (i)** $f(-1)$ **(ii)** $f(2a)$ **(iii)** $f(2)$ **(iv)** $f(x-1)$

Solution : $f : x \to x^2 - 5x + 6 \Rightarrow f(x) = x^2 - 5x + 6$

(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

(iv) $f(x-1) = (x-1)^2 - 5(x-1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6 = x^2 - 7x + 12$

If the function $f: R \to R$ defined by $f(x)=\begin{cases}2x+7, x<-2\\ x^2-2, -2\le x<3,\\ 3x-2, x\ge 3\end{cases}$ then find the values of (i) f(4) (ii) f(– 2) (iii) f(4) + 2f(1) (iv) $\frac{f(1)-3f(4)}{f(-3)}$

Solution : (i) $f(4)=3(4)-2 = 12-2=10$

(ii) $f(-2)=(-2)^2-2 = 4-2=2$

(iii) $f(1)=(1)^2-2=1-2=-1$

$\therefore f(4)+2f(1)=10+2(-1)=8$

(iv) $f(-3)=2(-3)+7=1$

$\therefore \frac{f(1)-3f(4)}{f(-3)}=\frac{-1-3(10)}{1}=-31$

A function f : [– 5, 9] → R is defined as follows : $f(x)=\begin{cases}6x+1 & if\ -5\le x<2\\ 5x^2-1 & if\ \ 2\le x<6\\ 3x-4 & if\ \ 6\le x<6\end{cases}$ **Find (i) $f(-3)+f(2)$ (ii) $f(7)-f(1)$ (iii) $2f(4)+f(8)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$**

Solution : $f(-3)=[6(-3)+1]=-18+1=-17$

$f(2)=[5(4)-1]=20-1=19$

$f(4)=[5(16)-1] = 80-1=79$

$f(-2)=[6(-2)+1]=-12+1=-11$

$f(1)=[6(1)+1]=6+1=7$

$f(7)=[3(7)-4]=21-4=17$

$f(8)=[3(8)-4]=24-4=20$

$f(6)=[3(6)-4]=18-4=14$

(i) $f(-3)+f(2) = -17+19=2$

(ii) $f(7)-f(1) = 17-7=10$

(iii) $2f(4)+f(8)=2[79]+20=158+20=178$

(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}=\frac{2(-11)-14}{79+(-11)}=\frac{-22-14}{79-11}=\frac{-9}{17}$

If the function f is defined by $f(x)=\begin{cases}x+2 & if\ x>1\\ 2 & if\ -1\le x\le 1\\ x-1 & if\ -3<x<-1\end{cases}$ **; find the values of (i) $f(3)$ (ii) $f(0)$ (iii) $f(-1,5)$ (iv) $f(2)+f(-2)$**

Solution : (i) $f(3)=3+2=5$

(ii) $f(0)=2$ (iii) $f(-1.5)=-1.5-1=-2.5$ (iv) $f(2)+f(-2)=(2+2)+(-2-1)=4-3=1$

Let f be a function f: N → N be defined by $f(x)=3x+2,\ x\in N$ (i) Find the images of 1, 2, 3 (ii) Find the pre-images of 29, 53 (iii) Identify the type of function

Solution : $f(x)=3x+2, x\in N$

(i) If $x=1$, $f(1)=3(1)+2=5$

If $x=2$, $f(2)=3(2)+2=8$

If $x=3$, $f(3)=3(3)+2=11$

The images of 1, 2, 3 are 5, 8, 11

(ii) $3x+2=29 \Rightarrow 3x=27 \Rightarrow x=9.$

9 is the pre-image of 53

$3x+2=53 \Rightarrow 3x=51 \Rightarrow x=17.$

17 is the pre-image of 53

(iii) Since different elements of N have different images in N f is one-one and into function.

Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that A × (B – C) = (A × B) – (A × C)

Solution : A = {1, 2, 3, 4, 5, 6, 7}, B = {2, 3, 5, 7} and C = {2}

B – C = {3, 5, 7}

∴ A × (B – C) = {(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7),}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7, 7),}

A × C = {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)}

∴ (A × B) – (A × C) = {(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7),}

The functions f and g are defined by $f(x)=6x+8$; $g(x)=\frac{x-2}{3}$ (i) Calculate the value of $gg\left(\frac{1}{2}\right)$ (ii) Write an expression for $gf(x)$ in its simplest form.

Solution :

i) $gg(x)=\left(\frac{x-2}{3}\right)\left(\frac{x-2}{3}\right)=\left(\frac{\frac{x-2}{3}-2}{3}\right)=\left(\frac{\frac{x-2-6}{3}}{3}\right)=\left(\frac{x-8}{9}\right)$

$\therefore gg\left(\frac{1}{2}\right)=\left(\frac{\frac{1}{2}-8}{9}\right)=\frac{-15}{18}=\frac{-5}{6}$

(ii) $gf(x)=\left(\frac{x-2}{3}\right)(6x+8)=\frac{6x+8-2}{3}=\frac{6x+6}{3}=2x+2=2(x+1)$

If $f(x)=2x-1$, $g(x)=\frac{x+1}{2}$, show that $f\circ g=g\circ f=x$.

Solution :

$f\circ g=(2x-1)\left(\frac{x+1}{2}\right)=2\left(\frac{x+1}{2}\right)-1=x+1-1=x$

$g\circ f=\left(\frac{x+1}{2}\right)(2x-1)=\frac{2x-1+1}{2}=x$

$\therefore f\circ g=g\circ f=x$

If $f(x)=2x-k$, $g(x)=4x+5$ such that $f\circ g=g\circ f$. Find the value of k

Solution : $(f\circ g)=(g\circ f) \Rightarrow (2x-k)(4x+5)=(4x+5)(2x-k)$

$2(4x+5)-k = 4(2x-k)+5$

$8x+10-k = 8x-4k+5$

$10-k = -4k+5$

$-k+4k = 5-10$

$3k = -5$

$\Rightarrow k=\frac{-5}{3}$

If $f(x)=3x+2$, $g(x)=6x-k$ such that $f\circ g=g\circ f$. Find the value of k

Solution : $(f\circ g)=(g\circ f) \Rightarrow (3x+2)(6x-k)=(6x-k)(3x+2)$

$3(6x-k)+2=6(3x+2)-k$

$18x-3k+2=18x+12-k$

$-3k+2=12-k$

$-2k=10 \quad \Rightarrow k=\frac{-10}{2}=-5$

Find x if $gff(x)=fgg(x)$, given $f(x)=3x+1$ and $g(x)=x+3$.

Solution :

$gff(x)=(x+3)(3x+1)(3x+1)$
$=(x+3)[3(3x+1)+1]$
$=(x+3)(9x+4)$
$=[(9x+4)+3]$
$=9x+7$

$fgg(x)=(3x+1)(x+3)(x+3)$
$=(3x+1)[(x+3)+3]$
$=(3x+1)(x+6)$
$=[3(x+6)+1]$
$=3x+19$

$gff(x)=fgg(x) \Rightarrow 9x+7=3x+19$
$9x-3x=19-7$
$6x=12$
$x=2$

Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \le 4\}$ and $C = \{3, 5\}$. Verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution : $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$ $B = \{x \in N \mid 1 < x \le 4\} \Rightarrow B = \{2, 3, 4\}$ $C = \{3, 5\}$

$B \cup C = \{2, 3, 4, 5\}$

$\therefore A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(1)

$A \times B = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(2)

$\therefore$ From (1) and (2) $\mathbf{A \times (B \cup C) = (A \times B) \cup (A \times C)}$

Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \le 4\}$ and $C = \{3, 5\}$. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution : $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$, $B = \{x \in N \mid 1 < x \le 4\} \Rightarrow B = \{2, 3, 4\}$ and $C = \{3, 5\}$

$B \cap C = \{3\}$

$\therefore A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ...(1)

$A \times B = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ...(2)

$\therefore$ From (1) and (2), $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \le 4\}$ and $C = \{3, 5\}$. Verify that $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Solution : $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$, $B = \{x \in N \mid 1 < x \le 4\} \Rightarrow B = \{2, 3, 4\}$ and $C = \{3, 5\}$

$A \cup B = \{0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ...(1)

$A \times C = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$B \times C = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$\therefore (A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$\therefore$ From (1) and (2) $\mathbf{(A \cup B) \times C = (A \times C) \cup (B \times C)}$

Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \le x < 2\}$ and $C = \{x \in N \mid x < 3\}$.
verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution : $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$, $B = \{x \in W \mid 0 \le x < 2\} = \{0, 1\}$ and $C = \{x \in N \mid x < 3\} = \{1, 2\}$

$B \cup C = \{0, 1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\} = \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ... (1)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cup \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ... (2)

From (1) and (2), $A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \le x < 2\}$ and $C = \{x \in N \mid x < 3\}$.
verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution : $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$, $B = \{x \in W \mid 0 \le x < 2\} = \{0, 1\}$ and $C = \{x \in N \mid x < 3\} = \{1, 2\}$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$ ... (1)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\} = \{(2, 1), (3, 1)\}$ ... (2)

From (1) and (2), $A \times (B \cap C) = (A \times B) \cap (A \times C)$ is verified.

Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Solution : $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 5, 7\}$ and $C = \{2\}$

$A \cap B = \{2, 3, 5, 7\}$

$\therefore (A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\} = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ... (1)

$A \times C = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$B \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ... (2)

$\therefore$ From (1) and (2), $(A \cap B) \times C = (A \times C) \cap (B \times C)$

If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$. Show that $A \times A = (B \times B) \cap (C \times C)$.

Solution : $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$

$A \times A = \{5, 6\} \times \{5, 6\} = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(1)

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\} = \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\} = \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore (B \times B) \cap (C \times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(2)

$\therefore$ From (1) and (2). $\mathbf{A \times A = (B \times B) \cap (C \times C)}$.

Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true ?

Solution : $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$, $D = \{1, 3, 5\}$

$A \cap C = \{3\}$, $B \cap D = \{3, 5\}$

$\therefore (A \cap C) \times (B \cap D) = \{(3, 3), (3, 5)\}$ ... (1)

$A \times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)\}$

$C \times D = \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$\therefore (A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ...(2)

$\therefore$ From (1) and (2) $\mathbf{(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)}$

If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$ then (i) find $A \times B$ and $B \times A$. (ii) Is $A \times B = B \times A$? If not why ?
(iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

Solution : $A = \{1, 3, 5\}$ and $B = \{2, 3\}$

(i) $A \times B = \{1, 3, 5\} \times \{2, 3\} = \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$

$B \times A = \{2, 3\} \times \{1, 3, 5\} = \{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$

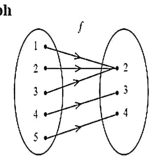
(ii) $(1, 2) \ne (2, 1) \Rightarrow A \times B \ne B \times A$

(iii) $n(A \times B) = n(B \times A) = 6$; $n(B) \times n(A) = 2 \times 3 = 6$

$\therefore n(A \times B) = n(B \times A) = n(A) \times n(B)$.

Represent the function $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$ through (i) an arrow diagram (ii) a table form (iii) a graph

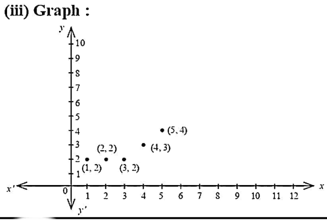
Solution : **(i) Arrow Diagram :**



(ii) Table Form :

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) Graph :



Let $f : A \to B$ be a function define by $f(x) = \frac{x}{2} - 1$ where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}. Represent f by (i) set of ordered pairs ; (ii) a table ; (iii) an arrow diagram ; (iv) a graph

Solution : Given $f(x) = \frac{x}{2} - 1$

$x = 2 \Rightarrow f(2) = 1 - 1 = 0$ $\quad x = 4 \Rightarrow f(4) = 2 - 1 = 1$

$x = 6 \Rightarrow f(6) = 3 - 1 = 2$ $\quad x = 10 \Rightarrow f(10) = 5 - 1 = 4$

$x = 12 \Rightarrow f(12) = 6 - 1 = 5$

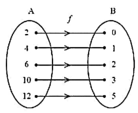
(i) Set of order pairs :

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

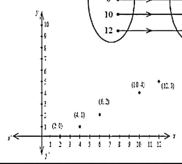
(ii) Table :

x	2	4	6	10	12
f(x)	0	1	2	4	5

(iii) Arrow diagram :



(iv) Graph



Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let $f : A \to B$ be a function given by f(x) = 3x – 1. Represent this function (i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs (iv) in a graphical form

Solution :

A = {1, 2, 3, 4} , B = {2, 5, 8, 11, 14} ;

f (x) = 3x – 1

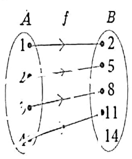
f(1) = 3(1) – 1 = 3 – 1 = 2 ;

f(2) = 3(2) – 1 = 6 – 1 = 5

f(3) = 3(3) – 1 = 9 – 1 = 8 ,

f(4) = 4(3) – 1 = 12 – 1 = 11

(i) Arrow diagram



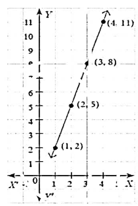
(ii) Table form

x	1	2	3	4
f(x)	2	5	8	11

(iii) Set of ordered pairs

$f = \{(1, 2), (2, 5), (3, 8), (4, 11)\}$

(iv) Graphical form



2 .NUMBERS AND SEQUENCES

5 MARKS

Find the sum of $9^3 + 10^3 + \dots + 21^3$

Solution : $9^3 + 10^3 + \dots + 21^3 = (1^3 + 2^3 + 3^3 + \dots + 21^3) - (1^3 + 2^3 + 3^3 + \dots + 8^3)$

$$= \left[\frac{21 \times (21+1)}{2}\right]^2 = \left[\frac{8 \times (8+1)}{2}\right]^2 = (231)^2 - (36)^2 = 52065 \quad \left[\therefore \sum_{k=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$$

Find the sum of $5^2 + 10^2 + 15^2 + \dots + 105^2$

Solution : $5^2 + 10^2 + 15^2 + \dots + 105^2 = 5^2(1^2 + 2^2 + 3^2 + \dots + 21^2)$

$$= 25 \times \frac{25 \times (21+1)(2 \times 21 + 1)}{6} = \frac{25 \times 21 \times 22 \times 43}{6} = 82775 \quad \left[\therefore \sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

Find the sum of $15^2 + 16^2 + 17^2 + \dots 28^2$

Solution : $15^2 + 16^2 + 17^2 + \dots 28^2 = (1^2 + 2^2 + 3^2 + \dots + 28^2) - (1^2 + 2^2 + 3^2 + \dots + 14^2)$

$$= \frac{28 \times 29 \times 57}{6} - \frac{14 \times 15 \times 29}{6} = 7714 - 1015 = 6699 \left[\therefore \sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,…, 24 cm. How much area can be decorated with these colour papers?

Solution : 10 cm, 11 cm, 12 cm, ... 24 cm

$10^2 + 11^2 + 12^2 + \dots + 24^2 = (1^2 + 2^2 + 3^2 + \dots + 24^2) - (1^2 + 2^2 + \dots + 9^2) \left[\therefore \sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}\right]$

$$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6} = 4900 - 285 = 4615 \text{ cm}^2$$

How many terms of the series $1^3 + 2^3 + 3^3 + \dots$ should be taken to get the sum 14400?

Solution : $1^3 + 2^3 + 3^3 + \dots + k^3 = 14400$

$$\left(\frac{k(k+1)}{2}\right)^2 = 14400 \Rightarrow \frac{k(k+1)}{2} = 120 \Rightarrow k^2 + k - 240 = 0$$

$$\Rightarrow (k+16)(k-15) = 0 \quad \therefore k = 15$$

The ratio of 6th and 8th term of an A.P. is 7 : 9. Find the ratio of 9th term to 13th term.

Solution :

The ratio of 6th and 8th term of an A.P. is 7 : 9 $\Rightarrow \frac{t_6}{t_8} = \frac{7}{9}$

$$\Rightarrow \frac{a+5d}{a+7d} = \frac{7}{9} \Rightarrow 9a + 45d = 7a + 49d \Rightarrow 2a = 4d \Rightarrow a = 2d \quad .....(1)$$

$$\therefore \frac{t_9}{t_{13}} = \frac{a+8d}{a+12d} = \frac{2d+8d}{2d+12d} = \frac{10d}{14d} = \frac{5}{7} \text{ (from (1))} \qquad \therefore t_9 : t_{13} = 5 : 7$$

The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

Solution : Let the 3 consecutive terms in an A.P. be $a - d, a, a + d$

Sum of 3 terms = 27 $\Rightarrow a - d + a + a + d = 27 \Rightarrow 3a = 27 \Rightarrow a = 9$

Product of 3 terms = 288 $\Rightarrow (a - d) . a . (a + d) = 288 \Rightarrow a^2 (a^2 - d^2) = 288 \Rightarrow 9(81 - d^2) = 288$

$\Rightarrow 81 - d^2 = 32$

$\Rightarrow d^2 = 49$

$\Rightarrow d = \pm 7$

a = 9, d = 7 $\Rightarrow$ the 3 terms are 2, 9, 16

a = 9, d = –7 $\Rightarrow$ the 3 terms are 16, 9, 2

Find the first term of the G.P. whose common ratio 5 and whose sum to first 6 terms is 46872.

Solution : Given $r = 5$, $S_6 = 46872$

$$S_n = a . \frac{r^n - 1}{r - 1} \Rightarrow a \times \frac{5^6 - 1}{4} = 46872$$

$$\Rightarrow a(5^6 - 1) = 46872 \times 4 \Rightarrow a(15624) = 46872 \times 4$$

$$\therefore a = \frac{46872 \times 4}{15624} = 3 \times 4 = 12$$

In an A.P., sum of four consective terms is 28 **and** their sum of their squares is 276. Find the four numbers.

Solution : four consective terms A.P. $(a - 3d), (a - d), (a + d)$ and $(a + 3d)$.

sum of the four terms is 28, $\Rightarrow a - 3d + a - d + a + d + a + 3d = 28 \Rightarrow 4a = 28 \Rightarrow a = 7$

sum of their squares is 276, $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 276$.

$$a^2 - 6ad + 9d^2 + a^2 - 2ad + d^2 + a^2 + 2ad + d^2 + a^2 + 6ad + 9d^2 = 276$$

$$4a^2 + 20d^2 = 276 \Rightarrow 4(7)^2 + 20d^2 = 276.$$

$$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

If a = 7, d = 2 then the four numbers are 1, 5, 9 and 13

If a = 7, d = –2 then the four numbers are 13, 9, 5 and 1

A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹4623. Find the amount received by each child. ***Solution :*** Let the amount form of A.P. $a-d, a, a+d$.

$(a-d)+a+(a+d)=207 \Rightarrow 3a=207 \Rightarrow a=69$

It is given that product of the two least amounts is 4623 $\quad (a-d)\,a=4623$

$(69-d)\,69=4623 \Rightarrow d=2$

Amount given by the mother to her three children are

₹(69 –2), ₹69, ₹(69 + 2). That is, ₹67, ₹69 and ₹71.

The sum of the squares of the first n natural numbers is 285, while the sum of their cubes is 2025. Find the value of n.

Solution :

$$\frac{n(n+1)(2n+1)}{6}=285 \quad \ldots\ldots\ldots\,(1)$$

$$\left(\frac{n(n+1)}{2}\right)^2=2025 \Rightarrow \left(\frac{n(n+1)}{2}\right)=45 \ldots\ldots\,(2)$$

$$(1) \Rightarrow \frac{n(n+1)}{2}\times\frac{2n+1}{3}=285 \Rightarrow 45\times\frac{2n+1}{3}=285 \Rightarrow 2n+1=\frac{285}{15}=19 \Rightarrow 2n=19-1$$

$$\Rightarrow 2n=18 \quad \therefore n=9$$

If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

Solution : Given $9\,(t_9)=15\,(t_{15})$ $\quad$ To Prove : $6\,(t_{24})=0$

$9\,(t_9)=15\,(t_{15})$

$9\,(a+8d)=15\,(a+14d)$

$3\,(a+8d)=5\,(a+14d) \Rightarrow 3a+24d=5a+70d \Rightarrow 2a+46d=0 \Rightarrow 2\,(a+23d)=0$

Multiplying 3 on both sides, $\Rightarrow 6\,(a+23d)=0 \quad \Rightarrow 6\,(t_{24})=0$

The sum of first, n, $2n$ and $3n$ terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3=3(S_2-S_1)$.

Solution : $S_1=t_1=a$, $\quad S_2=t_1+t_2=a+a+d=2a+d$, $\quad S_3=t_1+t_2+t_3=a+a+d+a+2d=3a+3d$

$S_2-S_1=2a+d-a=a+d$

$3(S_2-S_1)=3a+3d=S_3$

Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Solution : $301+308+315+\ldots+595.$ $\quad a=301$; $d=7$; $l=595$.

$$n=\left(\frac{l-a}{d}\right)+1=\left(\frac{595-301}{7}\right)+1=43 \qquad \left[\therefore S_n=\tfrac{n}{2}[a+l]\right]$$

$$S_{57}=\frac{43}{2}[301+595]=19264.$$

How many consecutive odd integers beginning with 5 will sum to 480?

Solution : $5+7+9+\ldots\ldots\, n=480$

$\therefore \quad a=5, d=2, S_n=480 \Rightarrow \tfrac{n}{2}\,[2a+(n-1)d]=480$

$\tfrac{n}{2}\,[10+(n-1)2]=480$

$\tfrac{n}{2}\,[5+(n-1)]=480$

$n\,[n+4]=480$

$n^2+4n-480=0$

$(n+24)(n-20)=0$

$n=-24,\ n=20$

$\therefore n=20$

In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.

Solution : $a, a+d, a+2d, a+3d, a+4d$ are **temperature of Ooty from Monday to Friday to be in A.P.**

Given $a+(a+d)+(a+2d)=0 \Rightarrow 3a+3d=0 \Rightarrow a+d=0 \Rightarrow a=-d$

Given $(a+2d)+(a+3d)+(a+4d)=18 \Rightarrow 3a+9d=18 \Rightarrow -3d+9d=18 \Rightarrow 6d=18$

$\Rightarrow d=3 \quad \therefore a=-3$

The temperature of each of the 5 days -3^0 C, 0^0 C, 3^0 C, 6^0 C, 9^0 C

The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number ?

Solution : Let Senthil's house number be x.

$$1+2+3+\ldots.+(x-1) = (x-1)+(x+2)+\ldots.+49$$

$$1+2+3+\ldots\ldots+(x-1) = [1+2+3+\ldots.+49]-[1+2+3+\ldots.+x]$$

$$\frac{x-1}{2}[1+(x-1)] = \frac{49}{2}[1+49]-\frac{x}{2}[1+x] \qquad \left[\therefore S_n=\tfrac{n}{2}[a+l]\right]$$

$$\frac{x(x-1)}{2}=\frac{49\times 50}{2}-\frac{x(x+1)}{2}$$

$$x^2-x=2450-x^2-x \Rightarrow 2x^2=2450 \qquad x^2=1225 \text{ gives } x=35$$

Senthil's hosue number is 35.

A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.

(i) How many bricks are required for the top most step?

(ii) How many bricks are required to build the stair case?

Solution : $\therefore$ 100, 98, 96, 94, ……… for 30 steps form an A.P. $\quad [t_n=a+(n-1)\,d]$

$a=100, d=-2, n=30$

i) No. of bricks used in the top most step $t_{30}=a+29d=100+29(-2)=100-58=42$

ii) Total no. of bricks used to build the stair case $S_{30}=\frac{30}{2}(100+42)=15\times 142=2130$ $\quad \left[\therefore S_n=\tfrac{n}{2}[a+l]\right]$

If (m + 1)th term of an A.P. is twice the (n + 1)th term, then prove that (3m + 1)th term is twice the (m + n + 1)th term.

Solution : Given $t_{m+1}=2(t_{n+1})$

$a+(m+1-1)\,d=2\,(a+(n+1-1)\,d)$

$a+md=2\,(a+nd) \qquad [t_n=a+(n-1)\,d]$

$a+md=2a+2nd \quad$ —(1)

To Prove : $t_{3m+1}=2\,(t_{m+n+1})$

$t_{3m+1}=a+(3m+1-1)d = a+3md = (a+md)+2md = 2a+2nd+2md$ (from (1))

$=2\,[a+(m+n)d]$

$=2\,[t_{m+n+1}]$

Find the sum of **$10^3+11^3+12^3+\ldots..+20^3$**

$$10^3+11^3+12^3+\ldots..+20^3 = (1^3+2^3+\ldots\ldots+20^3)-(1^3+2^3+\ldots\ldots+9^3)$$

$$=\left(\frac{20\times 21}{2}\right)^2-\left(\frac{9\times 10}{2}\right)^2=(210)^2-(45)^2 \qquad \left(\sum_{k=1}^{n}K^3=\left(\frac{n(n+1)}{2}\right)^2\right)$$

$$=(210+45)(210-45)=(255)\times(165)=42075$$

Find the sum to n terms of the series $5 + 55 + 555 + \ldots$

Solution : $5 + 55 + 555 + \ldots + n$ terms $= 5\,[1 + 11 + 111 + \ldots + n \text{ terms}]$

$$= \frac{5}{9}[9 + 99 + 999 + \ldots + n \text{ terms}]$$

$$= \frac{5}{9}[(10-1) + (100-1) + (1000-1) + \ldots + n \text{ terms}]$$

$$= \frac{5}{9}[(10 + 100 + 1000 + \ldots + n \text{ terms}) - n] \qquad \left[\therefore S_n = a\,.\frac{r^n - 1}{r-1}\right]$$

$$= \frac{5}{9}\left[\frac{10(10^n - 1)}{(10-1)} - n\right] = \frac{50(10^n - 1)}{81} - \frac{5n}{9}$$

In a G.P. the 9[th] term is 32805 and 6[th] term is 1215. Find the 12[th] term.

Solution : Given $t_9 = 32805$, $t_6 = 1215$, $t_{12} = ?$

$a\,.\,r^8 = 32805$ .... (1)

$a\,.\,r^5 = 1215$ .... (2)

(1) ÷ (2) $\Rightarrow \dfrac{\cancel{a}\,.\,r^{\cancel{8}3}}{\cancel{a}\,.\,\cancel{r^5}} = \dfrac{32805}{1215} \Rightarrow r^3 = \dfrac{32805}{1215} \Rightarrow r^3 = 27 \Rightarrow r^3 = 3^3 \Rightarrow r = 3$

Sub. $r = 3$ in (2) $\quad a \times 3^5 = 1215 \Rightarrow a \times 243 = 1215 \Rightarrow a = \dfrac{1215}{243} \Rightarrow a = 5$

$\Rightarrow \therefore t_{12} = a\,.\,r^{11} = 5 \times 3^{11} \qquad [\,t_n = a + (n-1)\,d\,]$

Find the sum to n terms of the series $0.4 + 0.44 + 0.444 + \ldots\ldots$ to n terms

Solution :

$0.4 + 0.44 + 0.444 + \ldots\ldots$ to n terms $= \dfrac{4}{10} + \dfrac{44}{100} + \dfrac{444}{1000} + \ldots\ldots$ to n terms

$$= 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots\ldots n \text{ terms}\right] \qquad \left[\therefore S_n = a\,.\frac{r^n - 1}{r-1}\right]$$

$$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots n \text{ terms}\right]$$

$$= \frac{4}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots\ldots n \text{ terms}\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{10}\left(\frac{1 - \left(\frac{1}{10}\right)^n}{1 - \frac{1}{10}}\right)\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right]$$

Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹ 2 to mail one letter, spent on find the amount postage when 8[th] set of letters is mailed.

Solution : $\therefore$ The total cost $= (4 \times 2) + (16 \times 2) + (64 \times 2) + \ldots\ldots\ldots$ 8[th] set

$= 8 + 32 + 28 + \ldots\ldots\ldots$ 8[th] set

$S_8 = 8\,.\dfrac{4^8 - 1}{3} = 8 \times \dfrac{65535}{3} \qquad \therefore a = 8, r = 4, n = 8 \qquad \left[\therefore S_n = a\,.\dfrac{r^n - 1}{r-1}\right]$

$= 8 \times 21845 = ₹\,174760$

If $S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots$ n terms then prove that

$(x - y)\,S_n = \left|\dfrac{x^2\,(x^n - 1)}{x - 1} - \dfrac{y^2\,(y^n - 1)}{y - 1}\right|$

Solution : $(x - y)\,S_n = (x^2 - y^2) + (x^3 + y^3) + (x^4 - y^4) + \ldots\ldots n$ terms

$= (x^2 + x^3 + x^4 + \ldots\ldots n \text{ terms}) - (y^2 + y^3 + y^4 + \ldots\ldots n \text{ terms})$

$= \dfrac{x^2(x^n - 1)}{x - 1} - \dfrac{y^2(y^n - 1)}{y - 1} \qquad \left(a = x^2, r = x \ \&\ a = y^2, r = y \ \because S_n = a\,.\dfrac{r^n - 1}{r - 1}\right)$

Find the sum to n terms of the series $3 + 33 + 333 + \ldots\ldots$ to n terms

Solution $\quad 3\,(1 + 11 + 111 + \ldots\ldots + n \text{ terms}) = \dfrac{3}{9}(9 + 99 + 999 + \ldots\ldots + n \text{ terms}) \qquad \left[S_n = a\,.\dfrac{r^n - 1}{r - 1}\right]$

$$= \frac{3}{9}[(10-1) + (100-1) + (1000-1) + \ldots\ldots n \text{ terms}]$$

$$= \frac{3}{9}[(10 + 100 + 1000 + \ldots\ldots n \text{ terms}] - [(1 + 1 + 1 + \ldots\ldots n \text{ terms}]$$

$$= \frac{3}{9}\left[10\,.\left(\frac{10^n - 1}{n}\right) - n\right] = \frac{30}{81}(10^n - 1) - \frac{3n}{9} = \frac{10}{27}(10^n - 1) - \frac{n}{3}$$

Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.

Solution :

Case (i) $\quad a = 603, d = 1, l = 901 \Rightarrow n = \dfrac{901 - 603}{1} + 1 = 298 + 1 = 299$

$S_{299} = \dfrac{299}{2} \times 1504 = 299 \times 752 = 224848 \qquad \left[\therefore n = \dfrac{l - a}{d} + 1 \ \therefore S_n = \frac{n}{2}[a + l]\right]$

Case (ii) $\quad a = 604, \quad d = 4, \quad l = 900 \Rightarrow n = \dfrac{900 - 604}{4} + 1 = \dfrac{296}{4} + 1 = 74 + 1 = 75$

$S_{75} = \dfrac{75}{2} \times 1504 = 75 \times 752 = 56400$

$\therefore$ **Sum of all natural numbers between 602 and 902 which are not divisible by 4**

$= 224848 - 56400 = 168448$

Find the greatest number consisting of 6 digits which is exactly divisible by 24, 15, 36?

Solution : L.C.M of 24, 15, 36

3	24, 15, 36
2	8, 5, 12
2	4, 5, 6
	2, 5, 3

L.C.M $= 5 \times 3^2 \times 2^3 = 5 \times 9 \times 8 = 360$

The greatest 6 digit no. is 999999

360) 999999 (277
720
2799
2520
2799
2520
279

$\therefore$ Required greatest number $= 999999 - 279 = 999720$

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3 . ALGEBRA

5 MARKS

Find the square root of $\left[\sqrt{15}x^2 + (\sqrt{3} + \sqrt{10})\,x + \sqrt{2}\right]\left[\sqrt{5}x^2 + (2\sqrt{5} + 1)\,x + 2\right]\left[\sqrt{3}x^2 + (\sqrt{2} + 2\sqrt{3})x + 2\sqrt{2}\right]$

Solution : $\sqrt{15}x^2 + (\sqrt{3} + \sqrt{10})\,x + \sqrt{2} = (\sqrt{5}x + 1) \times (\sqrt{3}x + \sqrt{2})$

$\sqrt{5}x^2 + (2\sqrt{5} + 1)\,x + 2 = (\sqrt{5}x + 1)\,(x + 2)$

$\sqrt{3}x^2 + (\sqrt{2} + 2\sqrt{3})\,x + 2\sqrt{2} = (x + 2)\,(\sqrt{3}x + \sqrt{2})$

$\sqrt{(\sqrt{5}x + 1)(\sqrt{3}x + \sqrt{2})(\sqrt{5}x + 1)(x + 2)(\sqrt{3}x + \sqrt{2})(x + 2)} = \left|(\sqrt{5}x + 1)(\sqrt{3}x + \sqrt{2})(x + 2)\right|$

Find the GCD of the following by division algorithm $2x^4+13x^3+27x^2+23x+7$, x^3+3x^2+3x+1, x^2+2x+1

Solution :

Quotient: 1 1
Divisor 1 2 1 | 1 3 3 1
1 2 1
(–) (–) (–)
1 2 1
1 2 1
(–) (–) (–)
0

Quotient: 2 9 7
Divisor 1 2 1 | 2 13 27 23 7
(–)2 (–)4 (–)2
9 25 23
(–)9 (–)18 (–)9
7 14 7
7 14 7
(–) (–) (–)
0

∴ G.C.D. = x^2+2x+1

Find the GCD of the following by division algorithm $3x^4+6x^3-12x^2-24x$, $4x^4+14x^3+8x^2-8x$

Solution : Let $f(x) = 4x^4+14x^3+8x^2-8x = 2x(2x^3+7x^2+4x-4)$

$g(x) = 3x^4+6x^3-12x^2-24x = 3x(1x^3+2x^2-4x-8)$ GCD of 2x , 3x = x

Quotient: 2
Divisor 1 2 –4 –8 | 2 7 4 –4
2 4 –8 –16
(–) (–) (+) (+)
3 12 12

∴ $3(x^2+4x+4)\neq 0$

3 is not a divisor of f(x)

Quotient: 1 –2
Divisor 1 4 4 | 1 2 –4 –8
1 4 4
(–) (–) (–)
–2 –8 –8
–2 –8 –8
(+) (+) (+)
0

∴ G.C.D = $x(x^2+4x+4)$

Find the GCD of x^4+3x^3-x-3, x^3+x^2-5x+3

Solution : $f(x) = x^4+3x^3-x-3$ $g(x) = x^3+x^2-5x+3$

Quotient: 1 2
Divisor 1 1 –5 3 | 1 3 0 –1 –3
1 1 –5 3
(–) (–) (+) (–)
2 5 –4 –3
(–)2 (–)2 (+)–10 (–)6
3 6 –9

$3(x^2+2x-3)\neq 0$

3 is not a divisor of f(x) , g(x)

Quotient: 1 –1
Divisor 1 2 –3 | 1 1 –5 3
1 2 –3
(–) (–) (+)
–1 –2 3
–1 –2 3
(+) (+) (–)
0

∴ GCD = (x^2+2x-3)

Find the GCD of the following by division algorithm $3x^3+3x^2+3x+3$, $6x^3+12x^2+6x+12$

Solution : Let $f(x) = 6x^3+12x^2+6x+12 = 6(x^3+2x^2+x+2)$

$g(x) = 3x^3+3x^2+3x+3 = 3(x^3+x^2+x+1)$ GCD of 6, 3 = 3

Quotient: 1
Divisor 1 1 1 1 | 1 2 1 2
1 1 1 1
(–) (–) (–) (–)
1 0 1

∴ $x^2+1\neq 0$

Quotient: 1 1
Divisor 1 0 1 | 1 1 1 1
1 0 1 0
(–) (–)
1 0 1
1 0 1
(–) (–) (–)
0

∴ G.C.D = $3(x^2+1)$

Find the square root of the expression
$\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}$

Solution :

Quotient: 1 –5 1
1 | 1 –10 27 –10 1
1
(–)
2 –5 | –10 27
–10 25
(+) (–)
2 –10 1 | 2 –10 1
2 –10 1
(–) (+) (–)
0

∴ $\sqrt{\frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{y}{x}+\frac{y^2}{x^2}} = \left|\frac{x}{y}-5+\frac{y}{x}\right|$

If $4x^4-12x^3+37x^2+bx+a$ is perfect square, Find the values of a and b

Solution :

Quotient: 2 –3 7
2 | 4 –12 37 b a
4
(–)
4 –3 | –12 37
–12 9
(+) (–)
4 –6 7 | 28 b a
28 –42 49
0

∴ a = 49 , b = –42

If $ax^4+bx^3+361x^2+220x+100$ is perfect square, Find the values of a and b

Solution :

Quotient: 10 11 12
10 | 100 220 361 b
100
(–)
20 11 | 220 361
220 121
(–) (–)
20 22 12 | 240 b a
240 264 144
0

∴ a = 144 , b = 264

If $\frac{1}{x^4}-\frac{6}{x^3}+\frac{13}{x^2}+\frac{m}{x}+n$ is perfect square, Find the values of m and n

Solution :

Quotient: 1 –3 2
1 | 1 –6 13 m n
1
(–)
2 –3 | –6 13
–6 9
(+) (–)
2 –6 2 | 4 m n
4 –12 4
0

∴ m = –12 n = 4

If $x^4-8x^3+mx^2+nx+16$ is perfect square, Find the values of m and n

Solution :

Quotient: 1 –4 4
1 | 1 –8 m n 16
1
(–)
2 –4 | –8 m
–8 16
(+) (–)
2 –8 4 | (m – 16) n 16
8 –32 16
0

∴ m = 24 , n = –32

Find $\sqrt{64x^4-16x^3+17x^2-2x+1}$

Solution :

Quotient: 8 –1 1
8 | 64 –16 17 –2 1
64
(–)
16 –1 | –16 17
–16 1
(+) (–)
16 –2 1 | 16 –2 1
16 –2 1
(–) (+) (–)
0

$\sqrt{64x^4-16x^3+17x^2-2x+1} = |8x^2-x+1|$

If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution :

	3	2	4		
3	9	12	28	a	b
	9 (–)				
6 2		12	28		
		12 (–)	4 (–)		
6 4 4			24	a	b
			24	16	16
			0		

$\therefore$ a = 16, b = 16.

Find the square root by division method $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Solution :

	11	–9	–12		
11	121	198	183	216	144
	121 (–)				
22 –9		–198	–183		
		–198 (+)	81 (–)		
22 –18 –12			–264	216	144
			–264 (+)	216 (–)	144
			0		

$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = |11x^2 - 9x - 12|$

Find the square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$ by division method

Solution :

	1	–6	3		
1	1	–12	42	–36	9
	1 (–)				
2 –6		–12	42		
		–12 (+)	36 (–)		
2 –12 3			6	–36	9
			6 (–)	–36 (+)	9 (–)
			0		

$\therefore \sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = |x^2 - 6x + 3|$

Find the square root of $37x^2 - 28x^3 + 4x^4 + 42x + 9$ by division method

Solution :

	2	–7	–3		
2	4	–28	37	42	9
	4 (–)				
4 –7		–28	37		
		–28 (+)	49 (–)	(+)	
4 –14 –3			–12	42	9
			–12 (+)	42 (–)	9 (–)
			0		

$\therefore \sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = |2x^2 - 7x - 3|$

Find the square root of the expression $\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$

Solution :

	2	5	–3		
2	4	20	13	–30	9
	4 (–)				
4 5		20	13		
		20 (–)	25 (–)		
4 10 –3			–12	–30	9
			–12 (+)	–30 (+)	9 (–)
			0		

$\sqrt{\frac{4x^2}{y} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}} = \left|\frac{2x}{y} + 5 - \frac{3y}{x}\right|$

Find the square root of $289x^4 - 612x^3 + 970x^2 - 684x + 361$

Solution :

	17	–18	19		
17	289	–612	970	–684	361
	289 (–)				
34 –18		–612	970		
		–612 (+)	324 (–)		
34 –36 19			646	–684	361
			646 (–)	–684 (+)	361 (–)
			0		

$\sqrt{289x^4 - 612x^3 + 970x^2 - 684x + 361} = |17x^2 - 18x + 19|$

Find the GCD of $6x^3 - 30x^2 + 60x - 48$ and $3x^3 - 12x^2 + 21x - 18$

Solution : Let $f(x) = 6x^3 - 30x^2 + 60x - 48 = 6(x^3 - 5x^2 + 10x - 8)$

$g(x) = 3x^3 - 12x^2 + 21x - 18 = 3(x^3 - 4x^2 + 7x - 6)$ GCD of 3 and 6 is 3.

	1			
1 –5 10 –8	1	–4	7	–6
	1 (–)	–5 (+)	10 (–)	–8 (+)
		1	–3	2

	1	–2		
1 –3 2	1	–5	10	–8
	1 (–)	–3 (+)	2 (–)	
		–2	8	–8
		–2 (+)	6 (–)	–4 (+)
			2	–4

$2(x - 2) \neq 0$

2 is not a divisor of g(x)

	1	–1	
1 –2	1	–3	2
	1 (–)	–2 (+)	
		–1	2
		–1 (+)	2 (–)
		0	

GCD $= 3(x - 2)$

Find the GCD of the polynomials $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

Solution : Let $f(x) = 2x^3 - 5x^2 + 5x - 3$ and $g(x) = x^3 + x^2 - x + 2$

	2			
1 1 –1 2	2	–5	5	–3
	2 (–)	2 (–)	–2 (+)	4 (–)
		–7	7	–7

$-7(x^2 - x + 1) \neq 0$

–7 is not a divisor of g(x)

	1	2		
1 –1 1	1	1	–1	2
	1 (–)	–1 (+)	1 (–)	
		2	–2	2
		2 (–)	–2 (+)	2 (–)
		0		

GCD $= x^2 - x + 1$

Find the GCD of the following by division algorithm $x^3 - 11x^2 + x - 11$, $x^4 - 1$

Solution : Let $f(x) = x^4 - 1$ $g(x) = x^3 - 11x^2 + x - 11$

	1	11			
1 –11 1 –11	1	0	0	0	–1
	1 (–)	–11 (+)	1 (–)	–11 (+)	
		11	–1	11	–1
		11 (–)	–121 (+)	11 (–)	–121 (+)
			120	0	120

$120(x^2 + 0x + 1) \neq 0$

120 is not a divisor of f(x), g(x)

	1	0	–11
1 0 1	1 –11	1 –11	
	1 (–)	0 (–)	1 (–)
	–11	0	–11
	–11 (+)	0 (–)	–11 (+)
		0	

GCD $= x^2 + 1$

LCM and GCD of the two polynomials $(x^2 + y^2)(x^4 + x^2y^2 + y^4)$ and $(x^2 - y^2)$ q(x) is $(x^4 - y^4)(x^2 + y^2 - xy)$ find the p(x)

Solution : LCM $= (x^2 + y^2)(x^4 + x^2y^2 + y^4)$ $q(x) = (x^4 - y^4)(x^2 + y^2 - xy)$

GCD $= (x^2 - y^2)$ $p(x) = ?$

$$p(x) = \frac{\text{LCM x GCD}}{q(x)} = \frac{(x^2 + y^2)(x^4 + x^2y^2 + y^4)(x^2 - y^2)}{(x^2 + y^2)(x^2 - y^2)(x^2 + y^2 - xy)}$$

$$= \frac{(x^4 + x^2y^2 + y^4)}{(x^2 + y^2 - xy)}$$

$$= \frac{(x^2 + y^2 - xy)(x^2 + y^2 + xy)}{(x^2 + y^2 - xy)} = (x^2 + y^2 + xy)$$

Find the LCM of $(2x^2 - 3xy)^2$, $(4x - 6y)^2$, $8x^3 - 27y^3$

Solution : $(2x^2 - 3xy)^2 = (x(2x - 3y))^2 = x^2 (2x - 3y)^2$

$(4x - 6y)^3 = (2(2x - 3y))^3 = (2)^3 (2x - 3y)^3 = 8(2x - 3y)^3$

$8x^3 - 27y^3 = (2x)^3 - (3y)^3 = (2x-3y)(4x^2 + 6xy + 9y^2)$

$\therefore$ LCM $= 8x^2 (2x - 3y)^3 (4x^2 + 6xy + 9y^2)$

Find the square root of $(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$

Solution : $\sqrt{(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)} = \sqrt{(3x-1)(2x+1)(3x-1)(x+1)(2x+1)(x+1)}$

$= |(3x-1)(2x+1)(x+1)|$

Find the square root of $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

Solution : $\sqrt{(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)} = \sqrt{(4x-1)(x-2)(7x+1)(x-2).(7x+1)(4x-1)}$

$= |(7x+1)(4x-1)(x-2)|$

Find the square root of the following $\left(2x^2 + \frac{17}{6}x + 1\right)\left(\frac{3}{2}x^2 + 4x + 2\right)\left(\frac{4}{3}x^2 + \frac{11}{3}x + 2\right)$

Solution :

$$\sqrt{(2x^2 + {}^{17}\!/_6 x + 1)({}^3\!/_2 x^2 + 4x + 2)({}^4\!/_3 x^2 + {}^{11}\!/_3 x + 2)} = \sqrt{\frac{(12x^2 + 17x + 6)}{6} \cdot \frac{(3x^2 + 8x + 4)}{2} \cdot \frac{(4x^2 + 11x + 6)}{3}}$$

$$= \sqrt{\frac{(4x+3)(3x+2).(3x+2)(x+2).(4x+3)(x+2)}{36}}$$

$$= {}^1\!/_6 \sqrt{(4x+3)^2.(3x+2)^2.(x+2)^2}$$

$$= {}^1\!/_6 \,|(4x+3)(3x+2)(x+2)|$$

Simplify $\frac{1}{x^2 - 5x + 6} + \frac{1}{x^2 - 3x + 2} - \frac{1}{x^2 - 8x + 15}$

Solution :

$$\frac{1}{x^2 - 5x + 6} + \frac{1}{x^2 - 3x + 2} - \frac{1}{x^2 - 8x + 15} = \frac{1}{(x-2)(x-3)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5) + (x-3)(x-5) - (x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x^2 - 6x + 5) + (x^2 - 8x + 15) - (x^2 - 3x + 2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2 - 11x + 8}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)} = \frac{x-9}{(x-1)(x-3)(x-5)}$$

The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5. Find the number of rows in the hall at the beginning.

Solution : Let the number of rows be x.

$\therefore$ Number of seats in each row = x $\quad\therefore$ Total number of seats in the hall $= x^2$

$\therefore$ By the data given, $2x \times (x - 5) = x^2 + 375$

$2x^2 - 10x = x^2 + 375$

$x^2 - 10x - 375 = 0$

$(x - 25)(x + 15) = 0$

$\therefore \quad x = 25, -15 \quad \therefore$ No. of rows at the beginning = 25.

A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/ hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

Solution : Time taken $= \frac{90}{x+15}$ $\quad\therefore$ By Data given $\frac{90}{x+15} - \frac{90}{x} = \frac{1}{2}$ $\left(\because 30 \text{ min} = \frac{1}{2}\text{hr}\right)$

$\Rightarrow 90\left(\frac{1}{x+15} - \frac{1}{x}\right) = \frac{1}{2} \Rightarrow \frac{x - x - 15}{x(x+15)} = \frac{1}{180} \Rightarrow 180 \times (-15) = x^2 + 15x \Rightarrow x^2 + 15x - 2700 = 0$

$\Rightarrow (x+60)(x-45) = 0$

$\therefore x = -60, 45 \quad \therefore$ Original speed = 45 Km/hr

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A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.

Solution : Let the present ages of the girl and her sister be x, y

By data given, i) $x = 2y$ $\quad$ ii) $(x + 5)(y + 5) = 375$

$(2y + 5)(y + 5) = 375$

$2y^2 + 15y - 350 = 0 \Rightarrow y = {}^{-35}\!/_2, 10$

y can't be -ve

15	-700
$\frac{35}{2}$	$\frac{-20}{2}$
$\frac{35}{2}$	-10

$\therefore y = 10 \quad \therefore x = 2y \Rightarrow x = 20 \quad \therefore$ Their present ages are 20, 10 years old.

A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains

Solution : Let the average speed of passenger train be x km/hr.

Then the average speed of express train will be (x + 20) km/hr

Time taken by passenger train **and** express train $\Rightarrow T_1 = \frac{240}{x}$ hr **and** $T_2 = \frac{240}{x+20}$ hr

$T_1 - T_2 = 1 \Rightarrow \frac{240}{x} - \frac{240}{x+20} = 1 \Rightarrow \frac{240}{x} = \frac{240}{x+20} + 1 \Rightarrow 240\left|\frac{1}{x} - \frac{1}{x+20}\right| = 1$

$\Rightarrow x^2 - 2x - 4800 = 0 \Rightarrow (x + 80)(x - 60) = 0$

$x = -80$ or 60.

Average speed of the passenger train is 60 km/hr **and** Average speed of the express train is 80 km/hr.

From a group of $2x^2$ black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total ?

Solution : Given number of black bees $= 2x^2$

By the data given, $2x^2 - x - {}^8\!/_9 (2x^2) = 2 \Rightarrow 2x^2 - 9x = 18 \Rightarrow 2x^2 - 9x - 18 = 0$

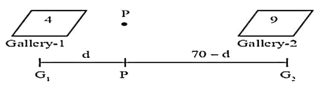
$x = 6, {}^{-3}\!/_2 \quad \therefore x = 6$

$\therefore$ Total number of bees $= 2x^2 = 2(36) = 72$

Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70 m. Where should a person stand for hearing the same intensity of the singers voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).

Solution :

$\frac{4}{9} = \frac{d^2}{(70-d)^2} \Rightarrow \frac{2^2}{3^2} = \frac{d^2}{(70-d)^2} \Rightarrow \frac{2}{3} = \frac{d}{(70-d)}$



$(70 - d)2 = 3d \Rightarrow \therefore d = 28$m

The person should stand 28m from gallery 1 (or) 42m from gallery-2 to hear the same intensity of the singers voice.

If α, β are the roots of the equation $2x^2 - x - 1 = 0$, then form the equation whose roots are $2\alpha + \beta, 2\beta + \alpha$

Solution : $2x^2 - x - 1 = 0$ here, $a = 2, b = -1, c = -1$

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-1)}{2} = \frac{1}{2} \; ; \; \alpha\beta = \frac{c}{a} = -\frac{1}{2}$$

Sum of the roots $= 2\alpha + \beta + 2\beta + \alpha = 3(\alpha + \beta) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$

Product of the roots $= (2\alpha + B)(2\beta + \alpha) = 4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta = 5\alpha\beta + 2(\alpha^2 + \beta^2)$

$= 5\alpha\beta + 2[(\alpha+\beta)^2 - 2\alpha\beta]$

$= 5\left(-\frac{1}{2}\right) + 2\left[\frac{1}{4} - 2 \times -\frac{1}{2}\right] = 0$

The required equation is $x^2 -$ (Sum of the roots)x + (Product of the roots) = 0

$$x^2 - \frac{3}{2}x + 0 = 0 \Rightarrow 2x^2 - 3x = 0$$

If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are real and equal prove that either $a = 0$ (or) $a^3 + b^3 + c^3 = 3abc$.

Solution : The **roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are real and equal**

Here $A = c^2 - ab$, $B = -2(a^2 - bc)$ $C = b^2 - ac$

$\Delta = B^2 - 4AC = 0 \Rightarrow 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow (a^2 - bc)^2 - (c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow (a^4 + b^2c^2 - 2a^2bc) - (b^2c^2 - ab^3 - ac^3 + a^2bc) = 0$

$\Rightarrow a^4 - 3a^2bc + ab^3 + ac^3 = 0$

$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0 \Rightarrow a = 0$ (or) $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Hence proved.

If the roots of $(a - b)x^2 + (b - c)x + (c - a) = 0$ are real and equal, then prove that b, a, c are in arithmetic progression.

Solution : The **roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are real and equal**

Here $A = a - b$, $B = b - c$, $C = c - a$

$\therefore \Delta = B^2 - 4AC = 0 \Rightarrow (b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow (b^2 + c^2 - 2bc) - 4(ac - bc - a^2 + ab) = 0$

$\Rightarrow b^2 + c^2 - 2bc - 4ac + 4bc + 4a^2 - 4ab = 0$

$\Rightarrow 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$

$\Rightarrow (2a - b - c)^2 = 0$

$\Rightarrow 2a - b - c = 0$

$\Rightarrow b + c = 2a$

$\therefore$ b, a, c are in A.P.

A flock of swans contained x^2 members. As the clouds gathered, 10x went to a lake and one eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

Solution : A flock of swans contained x^2 members.

given data $x^2 - 10x - \frac{1}{8}x^2 = 6 \Rightarrow 7x^2 - 80x - 48 = 0$ here, $a = 7, b = -80, c = -48$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{80 \pm \sqrt{6400 - 4(7)(-48)}}{14} = \frac{80 \pm 88}{14} \Rightarrow x = 12, -\frac{4}{7}$$

Hence, x = 12. Total number of swans is $x^2 = 144$.

A boat takes 1.6 hours longer to go 36 kms up a river than down the river. If the speed of the water current is 4 km per hr, what is the speed of the boat in still water?

Solution : Let the speed of the boat in still water be x km/hr. Distance = 36 kms, Time difference = 1.6 hrs.

$\therefore$ By data given,

$$\frac{36}{x-4} - \frac{36}{x+4} = \frac{8}{5} \Rightarrow 36\left(\frac{1}{x-4} - \frac{1}{x+4}\right) = \frac{8}{5} \Rightarrow \frac{x+4-x+4}{x^2-16} = \frac{8}{36 \times 5} \quad \left(\because 1.6 \text{ hrs} = \frac{8}{5}\text{hrs}\right)$$

$$\Rightarrow \frac{8}{x^2-16} = \frac{8}{180} \Rightarrow x^2 - 16 = 180 \Rightarrow x^2 = 196$$

$\therefore x = 14$ $\therefore$ Speed of boat in still water = 14km / hr.

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4 . GEOMETRY

5 MARKS

Show that in a triangle, the medians are concurrent.

Solution : The medians are the cevians where D, E, F are midpoints of BC, CA and AB

D is a mid point of $\frac{BD}{DC} = 1 \ldots (1)$

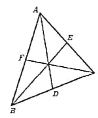
E is a midpoint of $\frac{CE}{EA} = 1 \ldots (2)$

F is a midpoint of $\frac{AF}{FB} = 1 \ldots (3)$

(1), (2) and (3) we get, $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \times 1 \times 1 = 1$

Ceva's theorem is satisfied.

Hence the Medians are concurrent.



P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that $4(AQ^2 + BP^2) = 54 AB^2$.

Solution : ΔAQC is a right triangle at C, $\Rightarrow AQ^2 = AC^2 + QC^2 \ldots\ldots (1)$

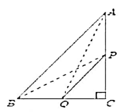
ΔBPC is a right triangle at C, $\Rightarrow BP^2 = BC^2 + CP^2 \ldots (2)$

From (1) and (2), $AQ^2 + BP^2 = AC^2 + QC^2 + BC^2 + CP^2$

$4(AQ^2 + BP^2) = 4AC^2 + 4QC^2 + 4BC^2 + 4CP^2 = 4AC^2 + (2QC)^2 + 4BC^2 + (2CP)^2$

$= 4AC^2 + BC^2 + 4BC^2 + AC^2$ (P and Q are mid points)

$= 5(AC^2 + BC^2) = 5AB^2$ (By Pythagoras Theorem)



Show that the angle bisectors of a triangle are concurrent.

Solution: In ΔABC angular bisectors of A and B meet at 'O'.

From O, draw perpendicular OD, OE, OF to BC, CA, AB

Consider $\Delta BOD \equiv BOF$

$\angle ODB = \angle OFB = 90^0$

$\angle OBD = \angle OBF$

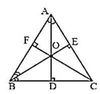
$\therefore OD = OF$

Consider $\Delta OCD, \Delta OCE$

$\angle ODC = \angle OEC = 90^0$

$\angle OCD = \angle OCE$

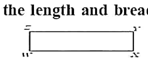
$\therefore OD = OE$



CO is angle bisector of $\angle C$. $\therefore$ Angle bisectors of a triangle are concurrent.

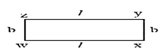
In the rectangle WXYZ, XY + YZ = 17 cm, and XZ + YW = 26 cm. Calculate the length and breadth of the rectangle?

Solution: **XZ + YW = 26 cm $\Rightarrow$ XZ = YW = $\sqrt{l^2 + b^2}$**



$\sqrt{l^2 + b^2} + \sqrt{l^2 + b^2} = 26$

$2\sqrt{l^2 + b^2} = 26$



$\sqrt{l^2 + b^2} = \frac{26}{2} = 13 \Rightarrow l^2 + b^2 = 169 \Rightarrow 12^2 + 5^2 = 169$

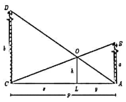
$\therefore$ Length = 12 cm, Breadth = 5m

Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

Solution : $\frac{CA}{CL} = \frac{AB}{LO} \Rightarrow x = \frac{ph}{a}$ ...... (1)

$\frac{AL}{CA} = \frac{LO}{CD} \Rightarrow y = \frac{ph}{b}$ .... (2)

$(1) + (2) \Rightarrow x + y = \frac{ph}{a} + \frac{ph}{b} \Rightarrow h = \frac{ab}{a+b}$



(Given $AB = AD$)

In ΔABC if DE || BC, **AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1, find the value of x.**

Solution : Given AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1

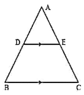
DE || BC, By Thales theorem $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$

$$(8x-7)(3x-1) = (4x-3)(5x-3)$$

$$24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

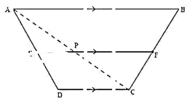
$$4x^2 - 2x - 2 = 0 \Rightarrow 2x^2 - x - 1 = 0 \Rightarrow x = 1, -\tfrac{1}{2} \qquad \therefore x = 1$$



In trapezium ABCD, AB || DC, E and F are points on non-parallel sides AD and BC respectively, such that EF || AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

Solution : In trapezium ABCD, AB || DC || EF

Join AC to meet EF at P



In ΔADC, EP || DC By Thales theorem $\Rightarrow \frac{AE}{ED} = \frac{AP}{PC}$ ....(1)

In ΔABC, PR || AB By Thales theorem $\Rightarrow \frac{BF}{FC} = \frac{AP}{PC}$ ..... (2)

From (1) & (2) $\frac{AE}{ED} = \frac{BF}{FC}$ Hence proved

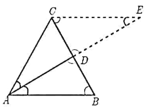
Statement and prove **Angle Bisector Theorem**

Statement The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Given : In ΔABC, AD is the internal bisector

To Prove : $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw CE parallel to AB. Extend AD to E



Proof : In $\Delta ABD \sim \Delta ECD$, $AB \parallel CE$

$\angle AEC = \angle BAE$ Alternate angles equal.

In ΔACE is isosceles

$\angle CAE = \angle CEA$, AC = CE ....(1)

By AA Similarity $\Delta ABD \sim \Delta ECD$

$\frac{AB}{CE} = \frac{BD}{CD} \Rightarrow \frac{AB}{AC} = \frac{BD}{CD}$ From (1) AC = CE. Hence proved.

Statement and prove Basic Proportionality Theorem or Thales theorem

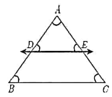
Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Given: In ΔABC, D is a point on AB and E is a point on AC.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw a line DE || BC



Proof : In ΔADE and ΔABC, DE || BC

$\angle A$ common angle

$\angle ADE = \angle ABC$ Corresponding angles are equal

By AA similarity $\Delta ADE \sim \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC}$$

$$\frac{AD}{AD} + \frac{AD}{DB} = \frac{AE}{AE} + \frac{AE}{EC}$$

$$1 + \frac{AD}{DB} = 1 + \frac{AE}{EC} \Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved

Statement and prove **Pythagoras Theorem**

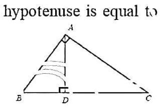
Statement

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Given : In ΔABC, $A = 90^0$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$



Proof : In ΔABC and ΔABD $\angle B$ is common

$\angle BAC = \angle BDA = 90°$

By AA similarity, $\Delta ABC \sim \Delta ABD$

$\frac{AB}{BD} = \frac{BC}{AB} \Rightarrow AB^2 = BC \times BD$ .... (1)

In ΔABC and ΔADC $\angle C$ is common

$\angle BAC = \angle ADC = 90°$

By AA similarity, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC} \Rightarrow AC^2 = BC \times DC$ .... (2)

Adding, (1) and (2)

$$AB^2 + AC^2 = BC \times BD + BC \times DC = BC(BD + DC) = BC \times BC = BC^2$$

Hence proved.

5 . COORDINATE GEOMETRY

5 MARKS

Show that the points P(–1.5, 3) , Q(6, –2) , R(–3, 4) are collinear.

Solution :

The area of the triangle $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} = \frac{1}{2}\begin{Bmatrix} -1.5 & 6 & -3 & -3 \\ 3 & -2 & 4 & 5 \end{Bmatrix}$

$$= \frac{1}{2}\{(3 + 24 - 9) - (18 + 6 - 6)\} = \frac{1}{2}\{18 - 18\} = 0$$

$\therefore$ The given points are collinear.

The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹1300 per square feet. What will be the total cost for making the parking lot?

Solution :

Area of parking lot $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{Bmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{Bmatrix}$

$= \frac{1}{2}\{(10+45+28+2)-(10+20+9+14)\}$

$= \frac{1}{2}\{85-53\} = \frac{1}{2}(32) = 16$ sq. units.

Total cost for constructing the parking lot $= 16 \times 1300 = ₹20800$

Find the area of the triangle formed by the points (–10, – 4), (–8, –1) and (–3, –5)

Solution:

$\therefore$ Area of triangle $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{Bmatrix} -10 & -3 & -8 & -10 \\ -4 & -5 & -1 & -4 \end{Bmatrix}$

$= \frac{1}{2}[(50+3+32)-(12+40+10)] = \frac{1}{2}[85-62] = \frac{23}{2} = 11.5$ sq.units

Find the area of the triangle whose vertices are (-3,5), (5,6) and (5,-2).

Solution :

The area of the triangle $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} = \frac{1}{2}\begin{Bmatrix} -3 & 5 & 5 & -3 \\ 5 & 6 & -2 & 5 \end{Bmatrix}$

$= \frac{1}{2}\{(-18-10+25)-(25+30+6)\}$

$= \frac{1}{2}\{-3-61\} = \frac{1}{2}(-64) = 32$ sq. units

A triangular shaped glass with vertices at A(-5,-4), B(1,6) and C(7,-4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution : $\therefore$ Area of triangle $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{bmatrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{bmatrix}$

$= \frac{1}{2}[(-30-4-28)-(-4+42+20)]$

$= \frac{1}{2}[-62-(58)] = \frac{1}{2}[-120] = 60$ sq. units

$\therefore$ No. of paint cans needed $= \frac{60}{6} = 10$

Find the area of a triangle formed by the lines 3x +y –2 = 0 , 5x +2y –3 = 0 and 2x – y –3 = 0.

Solution :

$3x+y-2=0$ .... (1)

$5x+2y-3=0$ .... (2)

$2x-y-3=0$ .... (3)

$(1)\times 2 \Rightarrow 6x+2y=4$

$(2) \Rightarrow 5x+2y=3$

$x = 1$

Sub. in (1) $3+y-2=0 \Rightarrow y=-1$

$\therefore$ A (1, –1)

$3x+y=2$

$2x-y=3$

$5x = 5$

$x=1 \quad \therefore y=-1$

$\therefore$ B is (1, –1)

$(2) \Rightarrow 5x+2y=3$

$(3)\times 2 \Rightarrow 4x-2y=6$

$9x = 9$

$x=1$

$\therefore (3) \Rightarrow 2-y-3=0$

$\Rightarrow -y=1 \quad \therefore y=-1$

$\therefore$ C is (1, –1)

$\therefore$ A (1, – 1), B (1, – 1), C (1, – 1) $\therefore$ All point line on the same line $\therefore$ Area of $\Delta = 0$ sq. units

Without using distance formula, show that the points (-2,-1) , (4, 0) , (3, 3) and (-3,2) are vertices of a parallelogram.

Solution : Slope of $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{0+1}{4+2} = \frac{1}{6}$ Slope of $CD = \frac{3-2}{3+3} = \frac{1}{6}$ $\therefore$ AB & CD are parallel

Slope of $AD = \frac{2+1}{-3+2} = \frac{3}{-1} = -3$ Slope of $BC = \frac{0-3}{4-3} = \frac{-3}{1} = -3$ $\therefore$ AD & BC are parallel

$\therefore$ ABCD is a parallelogram

Let P(11,7) , Q(13.5, 4) and R(9.5, 4) be the mid- points of the sides AB, BC and AC respectively of ΔABC . Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR .

Solution : **ΔABC the points P,Q,R be the mid- points of the sides AB, BC and AC respectively**

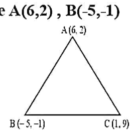
Area of ΔPQR is equal to $\frac{1}{4}$ Area of ΔABC

Area of $\Delta ABC = \frac{1}{2}\begin{bmatrix} 7 & 15 & 12 & 7 \\ 7 & 7 & 1 & 7 \end{bmatrix} = \frac{1}{2}[(49+15+84)-(105+84+7)]$

$= \frac{1}{2}[148-196] = \frac{1}{2}(-48) = 24$ sq. units

Area of $\Delta PQR = \frac{1}{2}(24) = 6$ sq. units $\therefore$ Area of $\Delta ABC = 4$ (Area of ΔPQR).

Find the equation of the median and altitude of ΔABC through A where the vertices are A(6,2) , B(-5,-1) and C(1,9)



Solution : i) Equation of the median through A. mid point of BC $= \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$

$= D(-2, 4)$

Equation of AD is $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6} \Rightarrow \frac{y-2}{2} = \frac{x-6}{-8} = \frac{y-2}{1} = \frac{x-6}{-4} \Rightarrow x-6 = -4y+8 \Rightarrow x+4y-14=0$

ii) Equation of altitude through 'A'

Slope of BC $= \frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$ $\therefore$ AD $\perp$ BC, slope of AD $= \frac{-3}{5}$ and A is (6, 2).

$\therefore$ Equation of altitude AD is $y - y_1 = m(x-x_1)$

$\Rightarrow y-2 = \frac{-3}{5}(x-6) \Rightarrow 5y-10 = -3x+18 \Rightarrow 3x+5y-28=0$

Find the area of the quadrilateral whose vertices are (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution :

Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 & x_1 \\ y_1 & y_2 & y_3 & y_1 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$

$= \frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$

$= \frac{1}{2}[33-(-35)] = \frac{1}{2}[68] = 34$ sq. units

Find the area of the quadrilateral whose vertices are (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution : Area of quadrilateral

$= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 & x_1 \\ y_1 & y_2 & y_3 & y_1 & y_1 \end{Bmatrix} = \frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$

$= \frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$

$= \frac{1}{2}[58-(-12)] = \frac{1}{2}[70] = 35$ sq. units

Find the area of the quadrilateral whose vertices are (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution :

Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{Bmatrix} = \frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$

$= \frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$

$= \frac{1}{2}[33-(-35)] = \frac{1}{2}[68] = 34$ sq. units

Find the area of the triangle formed by the points (1, –1), (–4, 6) and (–3, –5)

Solution:

$\therefore$ Area of triangle $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{Bmatrix} 1 & -4 & -3 & 3 \\ -1 & 6 & -5 & -1 \end{Bmatrix}$

$= \frac{1}{2}\{(6+20+3)-(4-18-5)\}$

$= \frac{1}{2}[29+19] = \frac{1}{2}(48) = 24$ sq. units

Find the area of the quadrilateral whose vertices are (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution :

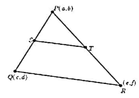
Area of quadrilateral $= \frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 & x_1 \\ y_1 & y_2 & y_3 & y_1 & y_1 \end{Bmatrix}$

$= \frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$

$= \frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$

$= \frac{1}{2}[58-(-12)] = \frac{1}{2}[70] = 35$ sq. units

Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length.



Solution : Let P (*a, b*) Q (*c, d*) and R (e, f) be the vertices of a triangle.

Let S be the mid-point of PQ and T be the mid-point of PR

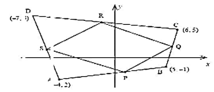
$S = \left(\frac{a+c}{2}, \frac{b+d}{2}\right)$ and $T = \left(\frac{a+e}{2}, \frac{b+f}{2}\right)$

slope of ST $= \frac{f-d}{e-c}$ and slope of QR $= \frac{f-d}{e-c}$ $\therefore$ ST is parallel to QR.

$ST = \sqrt{\left(\frac{a+e}{2}-\frac{a+c}{2}\right)^2+\left(\frac{b+f}{2}-\frac{b+d}{2}\right)^2}$

$= \frac{1}{2}\sqrt{(e-c)^2+(f-d)^2}$ $\qquad ST = \frac{1}{2}QR$

A quadrilateral has vertices at A (-4,-2) B(5, -1) , C(6, 5) and D(–7, 6). Show that the mid-points of its sides form a parallelogram.



Solution : Mid point of AB $= \left(\frac{-4+5}{2}, \frac{-2-1}{2}\right) = P\left(\frac{1}{2}, \frac{-3}{2}\right)$

Mid point of BC $= \left(\frac{5+6}{2}, \frac{-1+5}{2}\right) = Q\left(\frac{11}{2}, 2\right)$

Midpoint of CD $= \left(\frac{-7+6}{2}, \frac{6+5}{2}\right) = R\left(\frac{-1}{2}, \frac{11}{2}\right)$ $\qquad$ Midpoint of AD $= \left(\frac{-7-4}{2}, \frac{6-2}{2}\right) = S\left(\frac{-11}{2}, 2\right)$

Slope of PQ $= \dfrac{2+\frac{3}{2}}{\frac{11}{2}-\frac{1}{2}} = \dfrac{7/2}{10/2} = \frac{7}{10}$

Slope of SR $= \dfrac{2-\frac{11}{2}}{\frac{-11}{2}+\frac{1}{2}} = \dfrac{-7/2}{-10/2} = \frac{7}{10}$ $\therefore$ Slope of PQ = Slope of SR $\therefore$ PQ and SR are parallel.

Slope of PS $= \dfrac{2+\frac{3}{2}}{\frac{-11}{2}-\frac{1}{2}} = \dfrac{7/2}{-12/2} = \frac{-7}{12}$

Slope of QR $= \dfrac{11/2-2}{-1/2-11/2} = \dfrac{7/2}{-12/2} = \frac{-7}{12}$ $\therefore$ Slope of PS = Slope of QR $\therefore$ PS and QR are parallel.

$\therefore$ PQRS is a parallelogram.

Find the equation of a line passing through the point A(1, 4) and perpendicular to the line joining points (2,5) and (4,7) .

Solution : The slope of line joining points (2,5) and (4,7)

$$\frac{y_2-y_1}{x_2-x_1}=\frac{(7)-(5)}{(4)-(2)}=\frac{2}{2}=1$$

The required line slope is -1

The equation of the required straight line is $y-y_1=m(x-x_1)$

$$y-4=-1(x-1)\Rightarrow$$
$$y-4=-x+1$$
$$x+y-5=0.$$

Find the equation of a line passing through (6,–2) and perpendicular to the line joining the points (6,7) and (2,–3).

Solution : $\therefore$ Slope of the line joining (6, 7) and (2, −3) $=\frac{y_2-y_1}{x_2-x_1}=\frac{(-3)-(7)}{(2)-(6)}=\frac{-3-7}{2-6}=\frac{-10}{-4}=\frac{5}{2}$

$\therefore$ Slope of the line perpendicular to $\frac{5}{2}$ is $\frac{-2}{5}$

$\therefore$ Equation of the required line is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{-2}{5}$, $(x_1,y_1)=$ **(6,–2)**

$$\Rightarrow y+2=\frac{-2}{5}(x-6)\Rightarrow 5y+10=-2x+12\Rightarrow 2x+5y-2=0$$

Find the equation of a straight line passing through the point P(-5,2) and parallel to the line joining the points Q(3,-2) and R(-5, 4) .

Solution : Slope of QR $=\frac{y_2-y_1}{x_2-x_1}=\frac{(4)-(-2)}{(-5)-(3)}=\frac{4+2}{-5-3}=\frac{6}{-8}=\frac{-3}{4}$

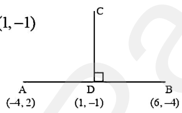
$\therefore$ Equation of the required line is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{-3}{4}$, $(x_1,y_1)=$ (-5,2)

$$y-2=\frac{-3}{4}(x+5)\Rightarrow 4y-8=-3x-15\Rightarrow 3x+4y+7=0$$

Find the equation of the perpendicular bisector of the line joining the points A(-4,2) and B(6,-4)

Solution :

D is the midpoint of AB $\therefore D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-4+6}{2},\frac{2-4}{2}\right)=(1,-1)$



Slope of AB $=\frac{y_2-y_1}{x_2-x_1}=\frac{(-4)-(2)}{(6)-(-4)}=\frac{-4-2}{6+4}=\frac{-6}{10}=\frac{-3}{5}$

$\therefore$ Slope of CD $=\frac{5}{3}$ ($\because CD\perp AB$)

$\therefore$ Equation of perpendicular bisector CD is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{5}{3}$, $(x_1,y_1)=(1,-1)$

$$y+1=\frac{5}{3}(x-1)\Rightarrow 3y+3=5x-5\Rightarrow 5x-3y-8=0$$

PROBLEMS FOR PRACTICE

Find the equation of a straight line passing through (1,-4) and has intercepts which are in the ratio 2:5

Find the equation of a line which passes through (5,7) and makes intercepts on the axes equal in magnitude but opposite in sign.

A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through (–3, 8). Find its equation.

Find the equation of a straight line through the intersection of lines 7x + 3y = 10 , 5x −4y = 1 and parallel to the line 13x + 5y +12 = 0.

Solution : $7x+3y=10$ .... (1)

$5x-4y=1$ .... (2)

$(1)\times 4 \Rightarrow 28x+12y=40$

$(2)\times 3 \Rightarrow 15x-12y=3$

$43x=43$

$x=1$

Sub x = 1 in (1) $\Rightarrow 7(1)+3y=10\Rightarrow 3y=10-7\Rightarrow 3y=3\Rightarrow y=1$

$\therefore$ The required line parallel to the line $13x+5y+12$ is $13x+5y+k=0$

it passes through (1, 1) $\Rightarrow 13+5+k=0\Rightarrow k=-18$ $\therefore 13x+5y-18=0$

Find the equation of a straight line through the intersection of lines 5x −6y = 2 , 3x + 2y = 10 and perpendicular to the line 4x −7y +13 = 0.

Solution : $5x-6y=2$ .... (1)

$3x+2y=10$ .... (2)

(1) $\Rightarrow 5x-6y=2$

$(2)\times 3\Rightarrow 9x+6y=30$

$14x=32\Rightarrow x=\frac{16}{7}$

Sub in (2) $\frac{48}{7}+2y=10\Rightarrow 2y=10-\frac{48}{7}$

$$\Rightarrow 2y=\frac{22}{7}\Rightarrow y=\frac{11}{7}$$

The required line is perpendicular to $4x-7y+13=0$ is $7x+4y+k=0$

$\therefore$ Since it passes through $\left(\frac{16}{7},\frac{11}{7}\right)\Rightarrow 7\left(\frac{16}{7}\right)+4\left(\frac{11}{7}\right)+k=0\Rightarrow 16+\frac{44}{7}+k=0\Rightarrow k=-16-\frac{44}{7}$

$$\Rightarrow k=\frac{-156}{7}$$

$\therefore$ The required line is $7x+4y-\frac{156}{7}=0\Rightarrow 49x+28y-156=0$

Find the equation of a straight line joining the point of intersection of 3x +y +2 = 0 and x −2y −4 = 0 to the point of intersection of 7x −3y = −12 and 2y = x + 3.

Solution :

$3x+y=-2$ ...... (1)

$x-2y=4$ ...... (2)

$(1)\times 2\Rightarrow 6x+2y=-4$

$(2)\Rightarrow x-2y=4$

$7x=0\Rightarrow x=0$

$x=0$ Sub in (1) $\Rightarrow 2y=-4\Rightarrow y=-2$

$\therefore$ The point of int. is $(0,-2)$

$7x-3y=-12$ .... (3)

$x-2y+3=0$ ...... (4)

$(3)\Rightarrow 7x-3y=-12$

$(4)\times 7\Rightarrow 7x-14y=-21$

$11y=9\Rightarrow y=\frac{9}{11}$

Sub in (4) $x-\frac{18}{11}+3=0\Rightarrow x=\frac{18}{11}-3=\frac{-15}{11}$

The point of int. is $\left(\frac{-15}{11},\frac{9}{11}\right)$

The required equation of the line joining $(0,-2)$, $\left(\frac{-15}{11},\frac{9}{11}\right)$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}\Rightarrow\frac{y+2}{\frac{9}{11}+2}=\frac{x-0}{\frac{-15}{11}}\Rightarrow\frac{y+2}{31}=\frac{x}{-15}\Rightarrow 31x=-15y-30$$
$$\Rightarrow 31x+15y+30=0$$

Find the equation of a straight line through the point of intersection of the lines 8x + 3y = 18, 4x + 5y = 9 and bisecting the line segment joining the points (5,–4) and (–7,6).

Solution : $8x+3y=18$ .... (1)

$4x+5y=9$ .... (2)

(1) $\Rightarrow 8x+3y=18$

$(2)\times 2\Rightarrow 8x+10y=18$

$-7y=0\Rightarrow y=0$

$(2) \Rightarrow \quad 4x = 9 \quad \therefore x = \frac{9}{4}$

$\therefore$ The point of int. $\left(\frac{9}{4}, 0\right)$

Mid point of the line joining $(5, -4), (-7, 6) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5-7}{2}, \frac{-4+6}{2}\right) = (-1, 1)$

Equation of the required line joining $\left(\frac{9}{5}, 0\right), (-1, 1) \Rightarrow \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$$\frac{y-0}{1} = \frac{x-\frac{9}{4}}{-1-\frac{9}{4}}$$

$$4x+13y-9=0$$

Find the equation of a line passing through the point of intersection of the lines $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.

Solution : $4x+7y=3$ .... (1)

$2x-3y=-1$ .... (2)

$(1) \Rightarrow 4x+7y = +3$

$(2)\times 2 \Rightarrow 4x-6y = -2$

$13y = 5$

$y = 5/13$

Sub in (1) $4x+35/13 = 3 \Rightarrow 4x = 3-35/13 \Rightarrow 4x = 4/13$

$\Rightarrow x = 1/13$

$\therefore$ The point is $(1/13, 5/13)$

Equation of plane in intercept form is $\frac{x}{a}+\frac{y}{b}=1$, where $a=b$

$\Rightarrow \frac{x}{a}+\frac{y}{a}=1 \Rightarrow x+y=a$ .... (1)

Since (1) passes through $(1/13, 5/13)$

$a = 1/13 + 5/13 = 6/13$

$\therefore x+y = 6/13 \Rightarrow 13x+13y-6=0$

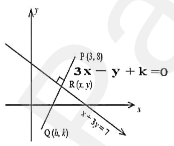
Find the image of point (3,8) with respect to the line $x+3y=7$ assuming the line to be a plane mirror.

Solution :

The line is perpendicular to $x+3y=7$ is $3x-y+k=0$

$\therefore$ Since it passes through (3,8) $\Rightarrow 3(3)-(8)+k=0 \Rightarrow 9-8+k=0 \Rightarrow k=-1$

$\therefore \quad 3x-y-1=0$



$x+3y=7$ ..... (1)

$3x-y=1$ ..... (2)

$(1) \Rightarrow x+3y=7$

$(2)\times 3 \Rightarrow 9x-3y=3$

$10x = 10$

$x = 1$

$x=1$ Sub in (1) $\Rightarrow 1+3y=7 \Rightarrow 3y=7-1 \Rightarrow y=6$

$\therefore$ (1,6) is the midpoint of $3x-y=1$ and $x+3y=7$

Let Q (h, k) be the image of P (3, 8)

$\therefore (1,6) = \left(\frac{h+3}{2}, \frac{k+8}{2}\right)$

$1 = \frac{h+3}{2}, \quad 6 = \frac{k+8}{2} \quad \Rightarrow \quad \therefore h=-1, \quad k=-4$

$\therefore (-1,-4)$ is the image of P (3, 8)

Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines $4x+5y=13$ and $x-8y+9=0$.

Solution : Given lines $4x+5y-13=0$ ...(1)

$x-8y+9=0$ ...(2)

x	y	1	
5	−13	4	5
−8	9	1	−8

$\frac{x}{45-104} = \frac{y}{-13-36} = \frac{1}{-32-5}$

$\frac{x}{-59} = \frac{y}{-49} = \frac{1}{-37} \qquad x = \frac{59}{37}, y = \frac{49}{37}$

The equation of line parallel to Y axis is x = c.

The equation of the line is $x = \frac{59}{37} \Rightarrow 37x-59=0$.

6 . TRIGONOMETRY
5 MARKS

If $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$, then prove that $\frac{a^2-1}{a^2+1} = \sin\theta$

Solution :

$a = \frac{1+\sin\theta}{\cos\theta}$ .... (1)

$\frac{1}{a} = \frac{\cos\theta}{1+\sin\theta}$ .... (2)

$a+\frac{1}{a} = \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$\frac{a^2+1}{a} = \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$ .... (3)

$a = \frac{1+\sin\theta}{\cos\theta}$ .... (1)

$\frac{1}{a} = \frac{\cos\theta}{1+\sin\theta}$ .... (2)

$a-\frac{1}{a} = \frac{1+\sin\theta}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta}$

$\frac{a^2-1}{a} = \frac{1+\sin\theta}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta}$ .... (4)

$\frac{(4)}{(3)}$ we get, $\frac{a^2-1}{a^2+1} = \sin\theta$

If $\cos\theta+\sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Solution : $\cos\theta+\sin\theta = \sqrt{2}\cos\theta$ Squaring both sides, $(\cos\theta+\sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta = 2\cos^2\theta$

$2\sin\theta\cos\theta = 2\cos^2\theta-\cos^2\theta-\sin^2\theta$

$2\sin\theta\cos\theta = \cos^2\theta-\sin^2\theta$

$2\sin\theta\cos\theta = (\cos\theta+\sin\theta)(\cos\theta-\sin\theta)$

$\cos\theta-\sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta+\sin\theta} = \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta} = \sqrt{2}\sin\theta$

If $a\cos\theta - b\sin\theta = c$, then prove that $(a\sin\theta+b\cos\theta) = \pm\sqrt{a^2+b^2-c^2}$

Solution : $a\cos\theta - b\sin\theta = c$ Squaring on both sides $(a\cos\theta - b\sin\theta)^2 = c^2$

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta = a^2+b^2-c^2$

$(a\sin\theta + b\cos\theta)^2 = a^2+b^2-c^2$

$\therefore a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

Hence proved.

Prove $\dfrac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\dfrac{\sin^3 A-\cos^3 A}{\sin A-\cos A}=2$

Solution : $\dfrac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\dfrac{\sin^3 A-\cos^3 A}{\sin A-\cos A}=\dfrac{\cancel{(\sin A+\cos A)}.(\sin^2 A-\sin A\cos A+\cos^2 A)}{\cancel{\sin A+\cos A}}+\dfrac{\cancel{(\sin A-\cos A)}.(\sin^2 A+\sin A\cos A+\cos^2 A)}{\cancel{\sin A-\cos A}}$

$=(1-\cancel{\sin A\cos A})+(1+\cancel{\sin A\cos A}) \quad =2$

If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$

Solution : $\sin\theta+\cos\theta=\sqrt{3}$ Squaring both sides, $(\sin\theta+\cos\theta)^2=(\sqrt{3})^2$

$(\sin\theta+\cos\theta)^2=3$

$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=3$

$1+2\sin\theta\cos\theta=3$

$\sin\theta\cos\theta=1$ .... (1)

$\tan\theta+\cot\theta=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}=\dfrac{1}{1}=1$ (from (1)) Hence proved.

Prove $\dfrac{\sin A-\sin B}{\cos A+\cos B}+\dfrac{\cos A-\cos B}{\sin A+\sin B}=0$

Solution :

$\dfrac{\sin A-\sin B}{\cos A+\cos B}+\dfrac{\cos A-\cos B}{\sin A+\sin B}=\dfrac{(\sin^2 A-\sin^2 B)+(\cos^2 A-\cos^2 B)}{(\cos A+\cos B).(\sin A+\sin B)}=\dfrac{(\sin^2 A+\cos^2 A)-(\sin^2 B+\cos^2 B)}{(\cos A+\cos B).(\sin A+\sin B)}$

$=\dfrac{1-1}{(\cos A+\cos B).(\sin A+\sin B)}=0$

If $\operatorname{cosec}\theta+\cot\theta=P$, then prove that $\cos\theta=\dfrac{P^2-1}{P^2+1}$

Solution : $\operatorname{cosec}\theta+\cot\theta=P$ .... (1)

$\operatorname{cosec}\theta-\cot\theta=\dfrac{1}{P}$ ...... (2)

$2\operatorname{cosec}\theta=P+\dfrac{1}{P}$

$2\operatorname{cosec}\theta=\dfrac{P^2+1}{P}$ ...... (3)

$\operatorname{cosec}\theta+\cot\theta=P$ .... (1)

$\operatorname{cosec}\theta-\cot\theta=\dfrac{1}{P}$ .... (2)

$2\cot\theta=P-\dfrac{1}{P}$

$2\cot\theta=\dfrac{P^2-1}{P}$ .... (4)

$\dfrac{(4)}{(3)}\quad \dfrac{2\cot\theta}{2\operatorname{cosec}\theta}=\dfrac{P^2-1}{P}\times\dfrac{P}{P^2+1}\Rightarrow\cos\theta=\dfrac{P^2-1}{P^2+1}$

If $\sqrt{3}\sin\theta-\cos\theta=0$, then show that $\tan 3\theta=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

Solution : $\sqrt{3}\sin\theta-\cos\theta=0\Rightarrow\theta=30^0$

$\tan 3(30^0)=\dfrac{3\left(\frac{1}{\sqrt{3}}\right)-\left(\frac{1}{\sqrt{3}}\right)^3}{1-3\left(\frac{1}{\sqrt{3}}\right)^2}$

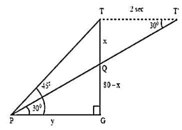
$\tan 90^0=\dfrac{\sqrt{3}-\frac{1}{3\sqrt{3}}}{1-1}$

undefined = undefined

Hence proved.

A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Determine the speed at which the bird flies. ($\sqrt{3}$ = 1.732)

Solution :

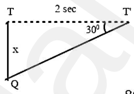


$\tan 45^0=\dfrac{TG}{PG}\Rightarrow 1=\dfrac{80}{y}\Rightarrow y=80$ ..... (1)

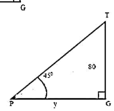
$\tan 30^0=\dfrac{80\text{-}x}{y}\Rightarrow\dfrac{1}{\sqrt{3}}=\dfrac{80\text{-}x}{80}$ (from (1)) $\Rightarrow 80=80\sqrt{3}-\sqrt{3}x$

$\Rightarrow\sqrt{3}x=80(\sqrt{3}-1)$

$\tan 30^0=\dfrac{TQ}{TT'}$



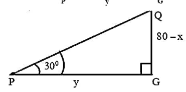
$\therefore\ x=\dfrac{80(\sqrt{3}-1)}{\sqrt{3}}$ ...(2)



$\Rightarrow\dfrac{1}{\sqrt{3}}=\dfrac{x}{TT'}\Rightarrow TT'=\sqrt{3}x=\sqrt{3}\times\dfrac{80(\sqrt{3}-1)}{\sqrt{3}}=80(\sqrt{3}-1)$ (from (2))

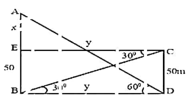
$\therefore$ Speed of the bird $=\dfrac{\text{Distance}}{\text{Time}}=\dfrac{80(\sqrt{3}-1)}{2}=40(\sqrt{3}-1)=40\times 1.732$

$=29.28$ m/sec



The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.

Solution :



$\tan 30^0=\dfrac{CD}{BD}\Rightarrow\dfrac{1}{\sqrt{3}}=\dfrac{50}{y}$

$\Rightarrow y=50\sqrt{3}\ m$ .... (1)

$\tan 60^0=\dfrac{AB}{BD}\Rightarrow\sqrt{3}=\dfrac{x+50}{y}\Rightarrow y=\dfrac{x+50}{\sqrt{3}}$

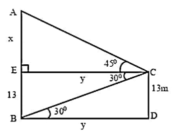
$50\sqrt{3}=\dfrac{x+50}{\sqrt{3}}$ (from (1)) $\Rightarrow 150=x+50\quad\therefore x=100$m

$\therefore$ Height of cell phone tower $=100+50=150\text{m}>120$ m $\therefore$ The tower does not meet the radiation norms.

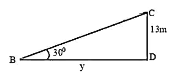
From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3}$ = 1.732)

Solution:

$\tan 30^0=\dfrac{CD}{BD}\Rightarrow\dfrac{1}{\sqrt{3}}=\dfrac{13}{y}\Rightarrow y=13\sqrt{3}$ m



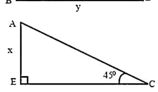
$\tan 45^0=\dfrac{AE}{EC}\Rightarrow 1=\dfrac{x}{y}\Rightarrow y=x\Rightarrow x=13\sqrt{3}$ m



$\therefore$ Height of the second tree $=x+13=13\sqrt{3}+13$

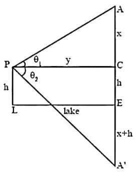
$=13(\sqrt{3}+1)$

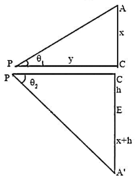
$=13\times 2.732=35.52$ m



If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\dfrac{h(\tan\theta_1+\tan\theta_2)}{\tan\theta_2+\tan\theta_1}$

Solution :





$$\tan\theta_1=\frac{x}{y} \Rightarrow y=\frac{x}{\tan\theta_1} \;.....(1)$$

$$\tan\theta_2=\frac{x+2h}{y} \Rightarrow y=\frac{(x+2h)}{\tan\theta_2} \;....(2)$$

$\therefore$ From (1) & (2)

$$\frac{x}{\tan\theta_1}=\frac{x+2h}{\tan\theta_2} \Rightarrow x\tan\theta_2=x\tan\theta_1+2h\tan\theta_1 \Rightarrow x(\tan\theta_2-\tan\theta_1)=2h\tan\theta_1$$

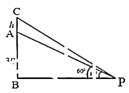
$$\Rightarrow x=\frac{2h\tan\theta_1}{\tan\theta_2-\tan\theta_1}$$

The height that the cloud is located from the ground is

$$\therefore AE=h+x=h+\frac{2h\tan\theta_1}{\tan\theta_2-\tan\theta_1}=h\left[1+\frac{2\tan\theta_1}{\tan\theta_2-\tan\theta_1}\right]=h\left[\frac{\tan\theta_2+\tan\theta_1}{\tan\theta_2-\tan\theta_1}\right]$$

From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. **($\sqrt{3}=1.732$)**

Solution:



$$\tan 60^0=\frac{30+h}{BP} \Rightarrow \sqrt{3}=\frac{30+h}{BP} \;.......(1)$$

$$\tan 45^0=\frac{30}{BP} \Rightarrow BP=30 \;.......(2)$$

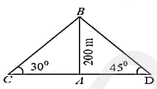
Substituting (2) in (1), we get

$$\sqrt{3}=\frac{30+h}{30} \Rightarrow h=30(\sqrt{3}-1)=30(1.732-1)=30(0.732)=21.96$$

The height of the tower is 21.96 m.

Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3}=1.732$)

Solution :



$$\tan 30^0=\frac{AB}{AC} \Rightarrow \frac{1}{\sqrt{3}}=\frac{200}{AC} \Rightarrow AC=200\sqrt{3} \;........(1)$$

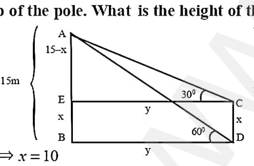
$$\tan 45^0=\frac{AB}{AD} \Rightarrow 1=\frac{200}{AD} \Rightarrow AD=200 \;........(2)$$

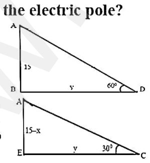
CD $=$ AC $+$ AD $=200\sqrt{3}+200=200(\sqrt{3}+1)=200\times 2.732=546.4$

Distance between two ships is 546.4 m.

The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

Solution :





$$\tan60^0=\frac{AB}{BD} \Rightarrow \sqrt{3}=\frac{15}{y} \Rightarrow y=\frac{15}{\sqrt{3}}=5\sqrt{3} \;.....(1)$$

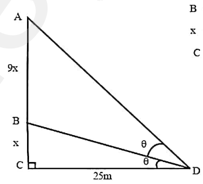
$$\tan30^0=\frac{AE}{EC} \Rightarrow \frac{1}{\sqrt{3}}=\frac{15-x}{y}$$

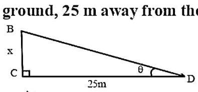
$\Rightarrow 5\sqrt{3}=(15-x)\sqrt{3} \Rightarrow 5=15-x$ (From (1)) $\Rightarrow x=10$

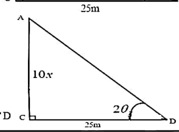
$\therefore$ Height of the pole = 10m

A vertical pole fixed to the ground is divided in the ratio 1 : 9 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a place on the ground, 25 m away from the base of the pole, what is the height of the pole?

Solution :







$\tan\theta=\dfrac{x}{25}$ | $\tan 2\theta=\dfrac{10x}{25}$

$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta} \Rightarrow \frac{10x}{25}=\frac{2\left(x/25\right)}{1-x^2/25}$$

$$\Rightarrow 625-x^2=125 \Rightarrow x^2=500 \Rightarrow x=10\sqrt{5}$$

$\therefore$ Height of the pole $=10x=10(10\sqrt{5})=100\sqrt{5}$ m

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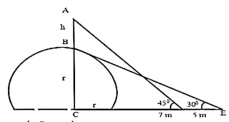
A flag pole 'h' metres is on the top of the hemispherical dome of radius 'r' metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle 45° and moving 5 m away from the dome and seeing the bottom of the pole at an angle 30°. Find (i) the height of the pole (ii) radius of the dome. ($\sqrt{3}=1.732$)

Solution :



$$\tan45^0=\frac{AC}{CD}=\frac{h+r}{r+7} \Rightarrow 1=\frac{h+r}{r+7} \Rightarrow r+7=h+r \Rightarrow h=7$$

$\therefore$ Height of the pole = 7m

$$\tan30^0=\frac{BC}{CE} \Rightarrow \frac{1}{\sqrt{3}}=\frac{r}{r+7+5} \Rightarrow \sqrt{3}r=r+12 \Rightarrow \sqrt{3}r-r=12$$

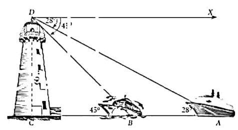
$$\Rightarrow r(\sqrt{3}-1)=12$$

$$r=\frac{12}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{12(\sqrt{3}+1)}{2}=6(2.732)=16.392 \text{ m}$$

$\therefore$ Radius of dome = 16.39 m

As observed from the top of a 60 m high light house from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. ($\tan 28°=0.5317$)

Solution :



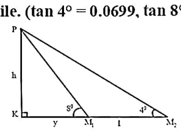
$$\tan45^0=\frac{DC}{BC} \Rightarrow 1=\frac{60}{BC} \Rightarrow BC=60 \text{ m}$$

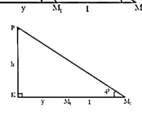
$$\tan28^0=\frac{DC}{AC} \Rightarrow 0.5317=\frac{60}{AC} \Rightarrow AC=112.85$$

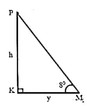
Distance between the two ships AB = AC − BC = 52.85 m

A traveler approaches a mountain on highway. He measures the angle of elevation to the peak at each milestone. At two consecutive milestones the angles measured are 4° and 8°. What is the height of the peak if the distance between consecutive milestones is 1 mile. ($\tan 4°=0.0699$, $\tan 8°=0.1405$)

Solution :







$$\tan 8^0=\frac{h}{y} \Rightarrow y=\frac{h}{\tan 8^0} \;........(1)$$

$$\tan 4^0=\frac{h}{y+1} \Rightarrow y+1=\frac{h}{\tan 4^0} \Rightarrow y=\frac{h}{\tan 4^0}-1 \;....(2)$$

$\therefore$ From (1) & (2)

$$\frac{h}{\tan 8^0}=\frac{h}{\tan 4^0}-1 \Rightarrow \frac{h}{\tan 4^0}-\frac{h}{\tan 8^0}=1 \Rightarrow$$

$$h\left[\frac{\tan 8^0-\tan 4^0}{\tan 8^0\tan 4^0}\right]=1$$

$$h=\frac{0.14\times 0.07}{0.14-0.07}=0.14 \text{ miles}$$

A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in km / hr), assuming that it is sailing in still water? ($\sqrt{3}$ = 1.732)

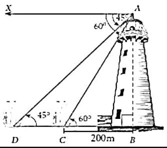
Solution : $\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{200} \Rightarrow AB = 200\sqrt{3}$ ...... (1)

$\tan 45° = \frac{AB}{BD} \Rightarrow 1 = \frac{200\sqrt{3}}{BD}$ [by (1)] $\Rightarrow BD = 200\sqrt{3}$

Now, $CD = BD - BC \Rightarrow CD = 200\sqrt{3} - 200 = 200(\sqrt{3}-1) = 146.4$

speed of the boat $= \frac{\text{distance}}{\text{time}} = \frac{146.4}{10} = 14.64$ m/s

$= 14.64 \times \frac{3600}{1000}$ km/hr $= 52.704$ km/hr



An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. ($\sqrt{3}$ = 1.732)

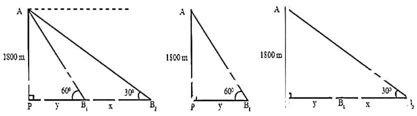
Solution: $\tan 60^0 = \frac{1800}{y} \Rightarrow \sqrt{3} = \frac{1800}{y}$

$\Rightarrow y = \frac{1800}{\sqrt{3}} = 600\sqrt{3}$ ... (1)

$\tan 30^0 = \frac{1800}{y+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{1800}{y+x}$

$y + x = 1800\sqrt{3} \Rightarrow 600\sqrt{3} + x = 1800\sqrt{3}$ (From (1))

the distance between the two boats $= 1200\sqrt{3} = 1200\,(1.732) = 2078.4$ m



From the top of a light house, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Solution:

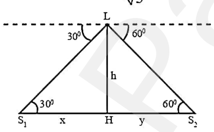
$\tan 30^0 = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$ .... (1)

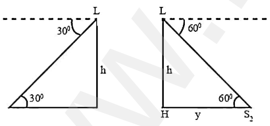
$\tan 60^0 = \frac{h}{y} \Rightarrow \sqrt{3} = \frac{h}{y}$

$\Rightarrow y = \frac{h}{\sqrt{3}}$ .... (2)



∴ Adding (1) & (2)

$x + y = \sqrt{3}h + \frac{h}{\sqrt{3}}$



∴ Adding (1) & (2) $\quad x + y = \sqrt{3}h + \frac{h}{\sqrt{3}} = \frac{3h+h}{\sqrt{3}} = \frac{4h}{\sqrt{3}}$

Distance between 2 ships $= \frac{4h}{\sqrt{3}}$ m

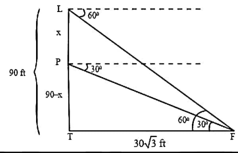
A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending.

Solution:

$\tan 30^0 = \frac{90-x}{30\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{90-x}{30\sqrt{3}}$

$\Rightarrow 30 = 90 - x \Rightarrow x = 60$ ft

∴ Speed of the lift $= \frac{\text{Dist.}}{\text{Time}} = \frac{60}{2} = 30$ ft/min.

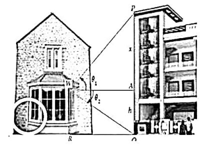


From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ_1 and θ_2 respectively. Show that the height of the opposite house is $h\left(1+\frac{\cot\theta_2}{\cot\theta_1}\right)$

Solution : $\tan\theta_1 = \frac{x}{AW} \quad AW = x\cot\theta$ ...... (1)

$\tan\theta_2 = \frac{h}{AW} \quad AW = h\cot\theta_2$ ...... (2)

From (1) and (2) we get, $x\cot\theta_1 = h\cot\theta_2 \Rightarrow x = h\frac{\cot\theta_2}{\cot\theta_1}$



height of the opposite house $= x + h = h\frac{\cot\theta_2}{\cot\theta_1} + h = h\left(1+\frac{\cot\theta_2}{\cot\theta_1}\right)$

7.MENSURATION
5 MARKS

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.

Solution: Right Circular Cone ⇒ r = 7 cm and h = 8 cm

Hollow Sphere ⇒ R = 5 cm and r = ?

Volume of Hollow Sphere = Vol. of Right Circular Cone $\Rightarrow \frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi r^2 h$

$4\,(125 - r^3) = 49 \times 8$

$r^3 = 27 \Rightarrow \therefore r = 3$

∴ Internal diameter of hollow sphere = 6 cm

A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.

Solution: Solid Sphere ⇒ r = 6 cm

Hollow Cylinder ⇒ R = 5 cm H = 32 cm t = ?

Volume of Hollow Cylinder = Volume of Solid Sphere $\Rightarrow \pi(R^2 - r^2)H = \frac{4}{3}\pi r^3$

$(25 - r^2)32 = \frac{4}{\cancel{3}} \times \cancel{6}^{2} \times 6 \times 6 \Rightarrow 25 - r^2 = \frac{\cancel{4} \times \cancel{2} \times \cancel{6}^{3} \times \cancel{6}^{3}}{\cancel{32}\,\cancel{4}}$

$25 - r^2 = 9$

$r^2 = 16 \Rightarrow r = 4$

∴ Thickness = R – r = 5 – 4 = 1 cm

A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

Solution : **Cylinder** $\Rightarrow$ $h_1 = 15$ cm, $r_1 = 6$ cm

cones (**Cone** + hemispherical cap) $\Rightarrow$ $r_2 = 3$ cm, $h_2 = 9$ cm , radius hemispherical cap $r_2 = 3$ cm

$$\text{Number of ice cream cones needed} = \frac{\text{Volume of the cylinder}}{\text{Volume of the cone} + \text{Volume of the hemispherical cap}} = \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3}$$

$$= \frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{1}{\cancel{3}}\times\frac{22}{7}\times\cancel{3}\times 3\times 9+\frac{2}{\cancel{3}}\times\frac{22}{7}\times\cancel{3}\times 3\times 3} = \frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{22}{7}\times 9(3+2)} = 12$$

Number of ice cream cones needed $= 12$

A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Solution : **Hemispherical hollow bowl has material of volume** $\Rightarrow$ $\frac{436\pi}{3}$

Diameter is 14 cm $\Rightarrow$ $R = 7$ cm

$$\Rightarrow \frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3} \Rightarrow 7^3 - r^3 = 218 \Rightarrow 343 - r^3 = 218$$

$\therefore$ $r^3 = 125$ $\Rightarrow$ $r = 5$ cm

$\therefore$ Thickness $= R - r = 7 - 5 = 2$ cm

A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.

Solution : **Solid cylinder** $\Rightarrow$ $h = 12$ cm $d = ?$

Hollow Cylinder $\Rightarrow$ $R = 4.3$ cm $r = 1.1$ cm $H = 4$ cm

Volume of hollow cylinder = Volume of solid cylinder

$$\cancel{\pi} H(R^2 - r^2) = \cancel{\pi} r^2 h$$

$$4[(4.3)^2 - (1.1)^2] = r^2\times 12 \Rightarrow r^2 = \frac{4(17.28)}{12} = 5.76 \Rightarrow \therefore\ r = 2.4$$

$\therefore$ Diameter of solid cylinder $= 2r = 4.8$ cm

A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Solution: **Hemisphere** $\Rightarrow$ Radius = r and **Cylinder** $\Rightarrow$ Radius = r $= h + \frac{1}{2}h = \frac{3}{2}h$

$$\therefore \text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\times\left(\frac{3}{2}h\right)^3 = \frac{2}{3}\pi\times\frac{27}{8}h^3 = \frac{9}{4}\pi h^3$$

$$\therefore \text{Volume of Cylinder} = \pi r^2 h = \pi\times\left(\frac{3}{2}h\right)^2 h = \pi\times\frac{9}{4}h^2 h = \frac{9}{4}\pi h^3$$

$\therefore$ Vol. of Hemisphere = Vol. of Cylinder

$\therefore$ % of juice that can be trasnferred to the cylindrical vessel = 100 %

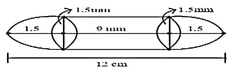
A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?

Solution:

Cylinder $\Rightarrow$ $H = 9$ mm , $r = 1.5$ mm $= \frac{3}{2}$

Hemisphere $\Rightarrow$ $r = 1.5$ mm $= \frac{3}{2}$

$\therefore$ Volume of the Capsule = Vol. of Cylinder + 2 (Vol. of hemisphere)



$$= \pi r^2 H + 2\left(\frac{2}{3}\pi r^3\right) = \frac{22}{7}\left[\frac{9}{4}\times 9+\frac{4}{3}\times\frac{27}{8}\right]$$

$$= \frac{22}{7}\left[\frac{81}{4}+\frac{9}{2}\right]$$

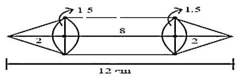
$$= \frac{22}{7}\left[\frac{81+18}{4}\right] = \frac{22\times 99}{28} = \frac{11\times 99}{14} = 77.78\ \text{mm}^3$$

Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.

Solution:

Cone $\Rightarrow$ $h = 2$cm , $r = 1.5\ \text{cm} = \frac{3}{2}$

Cylinder $\Rightarrow$ $H = 8$cm , $r = 1.5\ \text{cm} = \frac{3}{2}$



$\therefore$ Volume of the model = 2 (Vol. of Cone) + Vol of Cylinder

$$= \frac{2}{3}\pi r^2 h + \pi r^2 H = \pi r^2\left[\frac{2h}{3}+H\right]$$

$$= \frac{22}{7}\times\frac{9}{4}\left[\frac{4}{3}+8\right] = \frac{11\times 9}{7\times 2}\left[\frac{28}{3}\right] = \frac{11\times 3\times \cancel{14}}{\cancel{7}} = 66\ \text{cm}^3$$

The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Solution : Let $r = 100$ %

old surface area $= 4\pi(100)^2$

If the radius increases by 25% $\Rightarrow$ New radius $= 125$ %

New surface area $= 4\pi(125)^2$

$$\therefore \text{Increment in SA} = 4\pi(125)^2 - 4\pi(100)^2$$

$$= 4\pi\left((125)^2 - (100)^2\right)$$

$$= 4\pi\left((125)^2 - (100)^2\right)$$

$$= 4\pi\, 225\times 25$$

$$\therefore \text{Percentage inc. in SA} = \frac{4\pi\, 225\times 25}{4\pi(100)^2}\times 100 = \frac{225}{4} = 56.25\%$$

If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

Solution : h = 45 cm, R = 28 cm, r = 7 cm

$$\text{Volume of the frustum} = \frac{1}{3}\pi[R^2 + Rr + r^2]h$$

$$= \frac{1}{3} \times \frac{22}{7} \times [28^2 + (28 \times 7) + 7^2] \times 45$$

$$= 48510 \text{ cm}^3$$

A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm.

Solution : **cone** ⇒ r = 5 cm, h = 12 cm

$$\therefore\ l = \sqrt{h^2 + r^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$\therefore \text{ CSA of cone} = \pi r l = \frac{22}{7} \times 5 \times 13 = \frac{110 \times 13}{7} \text{ cm}^2$$

Area of sheet of paper = 5720 cm²

$$\therefore \text{ Number of caps} = \frac{5720 \times 7}{110 \times 13} = 28 \text{ caps}$$

Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.

Solution: **Cylindrical Pipe** ⇒ Speed of water in the pipe = 15 Km/hr ⇒ H = 15000 m

$$\text{Radius of pipe } r = 7 \text{ cm} = \frac{7}{100} \text{ m}$$

Rectangular Tank ⇒ $l = 50$ m $b = 44\ m$ $h = 21 \text{cm} = \frac{21}{100} m$

$$\therefore \text{ Required time} = \frac{\text{Volume of tank}}{\text{Volume of pipe}} = \frac{lbh}{\pi r^2 H} = \frac{50 \times 44 \times {}^{21}\!/_{100}}{\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000} = 2 \text{ hrs}$$

A shuttle cock used for playing badminton has the shape of a frustum of a cone is mounted on a hemisphere. The diam-eters of the frustum are 5 cm and 2 cm. The height of the entire shuttle cock is 7 cm. Find its external surface area.

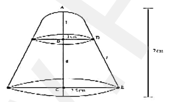
Solution: Radius of hemisphere = 1 cm

Radius of frustum r = 1 cm

Total length of cock = 7cm

∴ Height of frustum = 7 − 1 = 6 cm

Radius of frustum R = 2.5 cm



$$\therefore l = \sqrt{h^2 + (R - r)^2} = \sqrt{26 + (1.5)^2} = 6.18$$

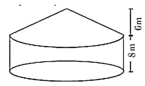
$$\therefore \text{ External Surface Area} = \text{CSA of Frustum} + \text{CSA of hemisphere} = \pi(R + r)l + 2\pi r^2$$

$$= \pi[(2.5 + 1)\,6.18 + 2 \times 1]$$

$$= \frac{513.7}{7} = 73.39 \text{ cm}^2$$

Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq. m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?.

Solution : Let h_1 and h_2 be the height of cylinder and cone



Area for one person = 4 sq. m and Total number of persons = 150

Total base area = 150 × 4 ⇒ $\pi r^2 = 600$

$$r^2 = 600 \times \frac{7}{22} = \frac{2100}{11} \quad \text{........ (1)}$$

Volume of air required for 1 person = 40 m³

Total Volume of air required for 150 persons = 150 × 40 = 6000 m³

$$\pi r^2 h_1 + \frac{1}{3}\pi r_2 h_2 = 6000 \Rightarrow \pi r^2\left(h_1 + \frac{1}{3}h_2\right) = 6000$$

$$\frac{22}{7} \times \frac{2100}{11}\left(8 + \frac{1}{3}h_2\right) = 6000 \quad \text{[using (1)]}$$

$$8 + \frac{1}{3}h_2 = \frac{6000 \times 7 \times 11}{22 \times 2100} \Rightarrow \frac{1}{3}h_2 = 10 - 8 = 2$$

Height of the conical tent h_2 is 6 m

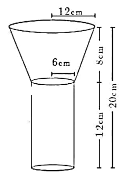
A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.

Solution : Let R, r be the top and bottom radii of the frustum.

R = 12 cm, r = 6 cm, h_2 = 12 cm

Let h_1, h_2 be the heights of the frustum and cylinder.

h_1 = 20 − 12 = 8 cm



Slant height of the frustum.

$$l = \sqrt{(R - r)^2 + h_1^2} = \sqrt{36 + 64} = 10 \text{ cm}$$

Outer surface area of the funnel = CSA of Frustum + CSA of Cylinder

= $2\pi r h_2 + \pi(R + r)\,l$ sq. units

= $\pi[2rh_2 + (R + r)\,l]$

= $\pi[(2 \times 6 \times 12) + (18 \times 10)]$

= $\pi[144 + 180]$

= $\frac{22}{7} \times 324 = 1018.28 \text{ cm}^2$

An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion be 8cm and the diameter of the top of the funnel be 18 cm, then find the area of the tin sheet required to make the funnel.

Solution : R = 9 cm r = 4 cm, H = 10 cm

$$l = \sqrt{(R - r)^2 + h^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Area of tin sheet required to make the funnel = CSA of Frustum + CSA of Cylinder

= π (R + r) l + 2πrH

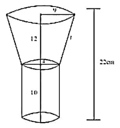
= π[(R + r)l + 2 r H]

= π [13 × 13 + 2 × 4 × 10]

$$= \frac{22}{7}[169+80]$$

$$= \frac{22}{7} \times 249$$

$$= \frac{5478}{7} = 782.57 \text{ cm}^3$$



The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone.

Solution : volume of a cone = $1005\ \frac{5}{7}$ cm³ & base area = $201\ \frac{1}{7}$ cm²

$$\therefore \frac{1}{3}\pi r^2 h = \frac{7040}{7} \quad \& \quad \pi r^2 = \frac{1408}{7}$$

$$\Rightarrow \frac{1}{3} \times \frac{1408}{\cancel{7}} \times h = \frac{7040}{\cancel{7}} \Rightarrow h = \frac{7040}{1408} \times 3 \Rightarrow h = 5 \times 3 \Rightarrow h = 15$$

$$\Rightarrow \pi r^2 = \frac{1408}{7} \Rightarrow \frac{22}{7} \times r^2 = \frac{1408}{7} \Rightarrow r^2 = \frac{1408}{7} = 64 \Rightarrow \therefore r = 8$$

$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{15^2 + 8^2} = \sqrt{225+64} = \sqrt{289} = 17 \text{ cm}$$

$\therefore$ Slant height = 17cm

Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution : (smaller cylinder) diameter of coin = 1.5 cm

$$\therefore r = \frac{1.5}{2} = 0.75 \text{ cm}, \quad h = 2 \text{ mm} = 0.2 \text{ cm}$$

diameter of bigger cylinder = 4.5 cm $\Rightarrow$ R = 2.25 cm , H = 10 cm

$$\therefore \text{Number of Coins} = \frac{\text{Volume of largest cylinder}}{\text{Volume of smallest cylinder}} = \frac{\pi R^2 H}{\pi r^2 h} = \frac{\frac{9}{4} \times \frac{9}{4} \times 10}{\frac{3}{4} \times \frac{3}{4} \times \frac{2}{10}} = 450 \text{ coins}$$

The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. Howmany words can be written using a bottle of ink containing one fifth of a litre?

Solution : Given height of the pen = 7 cm = 70 mm , radius $= \frac{5}{2}$ mm

$$\therefore \text{Volume of the pen} = \pi r^2 h = \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{25}{\cancel{4}} \times \overset{5}{\cancel{70}} = 1375 \text{ mm}^3 = 1.375 \text{ cm}^3$$

By data given 1.375 cm³ $\rightarrow$ 330 words

$$\Rightarrow \frac{1}{5} \text{ of a litre} = \frac{1}{5}(1000 \text{ cm}^3)$$

200 cm³ $\rightarrow$ x words

$$\therefore x = \frac{200 \times 330}{1.375} = 48000 \text{ words}$$

A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

Solution : Given radius of sector = 21 cm, θ = 216° ie R = 21 = l (slant height of cone)

Length of arc of the sector = Perimeter of base of cone $\Rightarrow \frac{\theta}{360} \times 2\pi R = 2\pi r$

$$\Rightarrow \frac{216}{360} \times \cancel{2\pi}\, 21 = \cancel{2\pi} r \Rightarrow r = \frac{216}{360} \times 21 \Rightarrow r = \frac{63}{5} = 12.6 \text{ cm}$$

$$\therefore h = \sqrt{l^2 - r^2} = \sqrt{21^2 - (12.6)^2} = 16.8$$

$$\therefore \text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{4.2}{\cancel{12.6}} \times \overset{1.8}{\cancel{12.6}} \times 16.8 = 2794.18 \text{ cm}^3$$

8. STATISTICS AND PROBABILITY
5 MARKS

Find the mean and variance of the first n natural numbers.

Solution :

$$\text{Mean } \bar{x} = \frac{\sum x_i}{n} = \frac{\cancel{n}(n+1)}{2 \times \cancel{n}} = \frac{n+1}{2}$$

$$\text{Variance } \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{\cancel{n}(n+1)(2n+1)}{6 \times \cancel{n}} - \left[\frac{\cancel{n}(n+1)}{2 \times \cancel{n}}\right]^2 = \frac{(n+1)(2n+1)}{6} - \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2-1}{12}$$

48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Solution :

x	f	$d = x - 9$	d^2	$f.d$	$f.d^2$
6	3	−3	9	−9	27
7	6	−2	4	−12	24
8	9	−1	1	−9	9
9	13	0	0	0	0
10	8	1	1	8	8
11	5	2	4	10	20
12	4	3	9	12	36
	48			0	124

$$\sigma = \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

$$= \sqrt{\frac{124}{48}}$$

$$= 1.6$$

The marks scored by the students in a slip test are given below.

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

Solution :

x	f	$d = x - 8$	d^2	$f.d$	$f.d^2$
4	7	−4	16	−28	112
6	3	−2	4	−6	12
8	5	0	0	0	0
10	9	2	4	18	36
12	5	4	16	20	80
	7			4	240

$$\sigma = \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = 2.87$$

Find the variance and standard deviation of the wages of 9 workers given below:
₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.

Solution :

x	$d = x - 300$	d^2
280	−20	400
280	−20	400
290	−10	100
290	−10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
	$\Sigma d = 0$	$\Sigma d^2 = 2000$

variance $\sigma^2 = \frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2$

$= \frac{2000}{9} - \left(\frac{0}{9}\right)^2$

$= \frac{2000}{9} = 222.2$

$S.D = \sqrt{222.2} = 14.91$

Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Solution : Given data is 24, 26, 33, 37, 29, 31. $\bar{x} = \frac{24+26+33+37+29+31}{6} = \frac{180}{6} = 30$

x	$d = x - 30$	d^2
24	−6	36
26	−4	16
29	−1	1
31	1	1
33	3	9
37	7	49
	0	112

$\therefore \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{112}{6} - \left(\frac{0}{6}\right)^2} = 4.31$

$\therefore C.V = \frac{4.31}{30} \times 10 = 14.36$

The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

Solution : Given data is 38, 40, 47, 44, 46, 43, 49, 53.

$\bar{x} = \frac{38+40+47+44+46+43+49+53}{8} = \frac{360}{8} = 45$

x	$d = x - 45$	d^2
38	−7	49
40	−5	25
43	−2	4
44	−1	1
46	1	1
47	2	4
49	4	16
53	8	64
8	0	172

$\therefore \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{164}{8} - \left(\frac{0}{8}\right)^2} = 4.53$

$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.53}{45} \times 100 = 10.07$

The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.

Solution : $n = 7, \bar{x} = 8, \sigma^2 = 16$

5 of the observerations are 2, 4, 10, 12, 14

Let the remaining 2 observations be *a, b*.

$\therefore \bar{x} = 8 \Rightarrow \frac{\Sigma x}{n} = 8$

$\Rightarrow \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \sigma^2 \quad \Rightarrow \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = 16$

$\Rightarrow \frac{\Sigma x^2}{7} - 8^2 = 16 \quad \Rightarrow \frac{\Sigma x^2}{7} - 64 = 16 \quad \Rightarrow \frac{\Sigma x^2}{7} = 80 \quad \Rightarrow \Sigma x^2 = 560$

$\Rightarrow 2^2 + 4^2 + 10^2 + 12^2 + 10^2 + a^2 + b^2 = 560$

$\Rightarrow 460 + a^2 + b^2 = 560$

$\therefore a^2 + b^2 = 100$

$\Rightarrow 8^2 + 6^2 = 100$

$\therefore a = 8, \quad b = 6$

In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50years. What is the probability that a chosen individual from the town is either a female or over 50 years?

Solution : Let A - Female , B - Over 50 years

$n(S) = 8000, n(A) = 3000, n(B) = 1300 \quad n(A \cap B) = \frac{30}{100} \times 3000 = 900$

$\therefore P(A) = \frac{3000}{8000}, P(B) = \frac{1300}{8000}, P(A \cap B) = \frac{900}{8000}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3000 + 1300 - 900}{8000} = \frac{3400}{8000} = \frac{34}{80} = \frac{17}{40}$

Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S) = 36

Let A even number on the 1[st] die.

A = {(2, 1),(2, 2),(2, 3),(2, 4),(2, 5),(2, 6),(4, 1),(4, 2),(4, 3),(4, 4),(4, 5),(4, 6),(6, 1),(6, 2),(6, 3),(6, 4),(6, 5),(6, 6)} $n(A) = 18 \Rightarrow P(A) = \frac{18}{36}$

Let B - Total of face sum as 8.

B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} $n(B) = 5 \Rightarrow P(B) = \frac{5}{36}$

$A \cap B$ = {(2, 6), (4, 4), (6, 2)} $n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{3}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.

Solution : $n(S) = 52 - 3 = 49$

i) Let A - a diamond card $n(A) = 13 \quad \therefore P(A) = \frac{13}{49}$

ii) Let B - a queen card $n(B) = 3$ (except spade queen out of 4) $\therefore P(B) = \frac{3}{49}$

iii) Let C - a spade card $n(C) = 10 \ (13 - 3 = 10) \quad \therefore P(C) = \frac{10}{49}$

iv) Let D - 5 of heart $n(D) = 1 \quad \therefore P(D) = \frac{1}{49}$

If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$, $P(A\cup B\cup C)=\frac{9}{10}$, $P(A\cap B\cap C)=\frac{1}{15}$, then find P(A),P(B) and P(C) ?

Solution : $P(B)=2.P(A)$, $P(C)=3.P(A)$, $P(A\cap B)=\frac{1}{6}$, $P(B\cap C)=\frac{1}{4}$, $P(A\cap C)=\frac{1}{8}$,

$P(A\cup B\cup C)=\frac{9}{10}$, $P(A\cap B\cap C)=\frac{1}{15}$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$$\frac{9}{10}=P(A)+2.P(A)+3.P(A)-\frac{1}{6}-\frac{1}{4}-\frac{1}{8}+\frac{1}{15}$$

$\therefore P(A)=\frac{11}{48}$ $\therefore P(B)=2.P(A)=2\times\frac{11}{48}=\frac{11}{24}$ $\therefore P(C)=3.P(A)=3\times\frac{11}{48}=\frac{11}{16}$

The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Solution : By the data given, $n(S)=52-2-2-2=46$

i) Let A clubber card. $n(A)=13 \Rightarrow P(A)=\frac{13}{46}$

ii) Let B - queen of red card. $n(B)=0 \Rightarrow P(B)=0$ (queen diamond and heart are included in S)

iii) Let C - King of black cards $n(C)=1$ (excluding spade king) $\Rightarrow P(C)=\frac{1}{46}$

Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution : $S=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5)(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$ $n(S)=36$

Let A a doublet
$A=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$ $n(A)=6 \Rightarrow P(A)=\frac{6}{36}$

Let B face sum 4.
$B=\{(1,3),(2,2),(3,1)\}$ $n(B)=3 \Rightarrow P(B)=\frac{3}{36}$

$A\cap B=\{(2,2)\} \Rightarrow n(A\cap B)=1 \Rightarrow P(A\cap B)=\frac{1}{36}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{6}{36}+\frac{3}{36}-\frac{1}{36}=\frac{8}{36}=\frac{2}{9}$

Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution : S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)} $n(S)=8$

Let A - at most 2 tails
A = {(HHT), (HTH), (THH), (HTT), (THT), (TTH), (HHH)} $n(A)=7 \Rightarrow P(A)=\frac{7}{8}$

Let B - atleast 2 heads
B = {(HHH), (HHT), (HTH), (THH)} $n(B)=4 \Rightarrow P(B)=\frac{4}{8}$

$\therefore$ A∩B = {(HHH), (HHT), (HTH), (THH)} $n(A\cap B)=4 \Rightarrow P(A\cap B)=\frac{4}{8}$

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{7}{8}+\frac{4}{8}-\frac{4}{8}=\frac{7}{8}$

A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution : $n(S)=52$

Let A king card, Let B heart card, Let C red card.

$P(A)=\frac{4}{52}$, $P(B)=\frac{13}{52}$, $P(C)=\frac{26}{52}$, $P(A\cap C)=\frac{2}{52}$, $P(A\cap B)=\frac{1}{52}$, $P(B\cap C)=\frac{13}{52}$

$P(A\cap B\cap C)=\frac{1}{52}$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$$=\frac{4}{52}+\frac{13}{52}+\frac{26}{52}-\frac{1}{52}-\frac{13}{52}-\frac{2}{52}+\frac{1}{52}=\frac{28}{52}=\frac{7}{13}$$

A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black

Solution : S = {5R, 6W, 7G, 8B}

i) Let A - White ball $n(A)=6 \Rightarrow P(A)=\frac{6}{26}=\frac{3}{13}$

ii) Let B - Black (or) red $n(B)=5+8=13 \Rightarrow P(B)=\frac{13}{26}=\frac{1}{2}$

iii) Let C - not white $n(C)=20 \Rightarrow P(C)=\frac{20}{26}=\frac{10}{13}$

iv) Let D - Neither white nor black $n(D)=12 \Rightarrow P(D)=\frac{12}{26}=\frac{6}{13}$

What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution : Total number of cards = 52

Let A king card $n(A)=4 \Rightarrow P(A)=\frac{4}{52}$

Let B queen card $n(B)=4 \Rightarrow P(B)=\frac{4}{52}$ $P(A\cap B)=\frac{0}{52}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{4}{52}+\frac{4}{52}-\frac{0}{52}=\frac{2}{13}$

A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i) then find x.

Solution : Total number of balls in the bag $n(S)=x+12$. ($x\rightarrow$ red $12\rightarrow$ black)

i) Let A red balls $n(A)=x$, $P(A)=\frac{x}{x+12}$

ii) If 8 more red balls are added in the bag. $n(S)=x+20$

By the problem, $\frac{x+8}{x+20}=2\left(\frac{x}{x+12}\right)$

$(x+8)(x+12)=2x^2+40x$

$x^2+20x-96=0$

$(x+24)(x-4)=0 \Rightarrow$

$\therefore x=-24, 4$ $\therefore x=4$ $\therefore P(A)=\frac{4}{16}=\frac{1}{4}$

Two unbiased dice are rolled once. Find the probability of getting (i) a doublet (equal numbers on both dice) (ii) the product as a prime number (iii) the sum as a prime number (iv) the sum as 1

Solution : $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(S) = 36$

i) Let A a doublet
$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$ $n(A) = 6$ $\therefore P(A) = \frac{6}{36} = \frac{1}{6}$

ii) Let B the product as a prime number.
$B = \{(1, 2), (1, 3), (1, 5), (2, 1), (3, 1), (5, 1)\}$ $n(B) = 6$ $\therefore P(B) = \frac{6}{36} = \frac{1}{6}$

iii) Let C be the sum of numbers on the dice is prime.
$C = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)\}$ $n(C) = 14$ $\therefore P(C) = \frac{7}{36}$

iv) Let D be the sum of numbers is 1. $n(D) = 0$ $\therefore P(D) = 0$

If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Solution : $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$
$\therefore n(S) = 36$

Let A - Product of face value is 6.
$A = \{(1, 6), (2, 3), (3, 2), (6, 1)\} \Rightarrow n(A) = 4 \Rightarrow P(A) = \frac{4}{36}$

Let B - Difference of face value is 5. $B = \{(6, 1)\} \Rightarrow n(B) = 1 \Rightarrow P(B) = \frac{1}{36}$

$A \cap B = \{(6, 1)\} \Rightarrow n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{4}{36} + \frac{1}{36} - \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$

In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that
(i) The student opted for NCC but not NSS. (ii) The student opted for NSS but not NCC.
(iii) The student opted for exactly one of them.

Solution: Let A and B be NCC and NSS
Total number of students n(S)= 50.

$n(A) = 28$, $n(B) = 30$, $n(A \cap B) = 18$ $\Rightarrow P(A) = \frac{28}{50}$, $P(B) = \frac{30}{50}$ and $P(A \cap B) = \frac{18}{50}$

(i) Probability of the students opted for NCC but not NSS $P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{28}{50} - \frac{18}{50} = \frac{1}{5}$

(ii) Probability of the students opted for NSS but not NCC. $P(\overline{A} \cap B) = P(B) - P(A \cap B) = \frac{30}{50} - \frac{18}{50} = \frac{6}{25}$

(iii) Probability of the students opted for exactly one of them $P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$

From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Solution : $n(S) = 52$

Let A - Red King $n(A) = 2$ $\Rightarrow P(A) = \frac{2}{52}$ Let B - Black Queen $n(B) = 2$ $\Rightarrow P(B) = \frac{2}{52}$

$n(A \cap B) = 0 \Rightarrow P(A \cap B) = \frac{0}{52}$ $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{52} + \frac{2}{52} - \frac{0}{52} = \frac{4}{52} = \frac{1}{13}$

Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

Solution : $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$ $n(S) = 36$

(i) Let A the sum of outcome values equal to 4.
$A = \{(1,3),(2,2),(3,1)\}$; $n(A) = 3$. $P(A) = \frac{3}{36} = \frac{1}{12}$

(ii) Let B the sum of outcome values greater than 10.
$B = \{(5,6),(6,5),(6,6)\}$; $n(B) = 3$ $P(B) = \frac{3}{36} = \frac{1}{12}$

(iii) Let C the sum of outcomes less than 13.
$n(C) = n(S) = 36$ $P(C) = \frac{36}{36} = 1$

From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Solution : $n(S) = 52$

(i) Let A red card. $n(A) = 26$ $\Rightarrow P(A) = \frac{26}{52} = \frac{1}{2}$

(ii) Let B heart card. $n(B) = 13$ $\Rightarrow P(B) = \frac{13}{52} = \frac{1}{4}$

(iii) Let C red king card. $n(C) = 2$ $\Rightarrow P(C) = \frac{2}{52} = \frac{1}{26}$

(iv) Let D face card.
The face cards are Jack (J), Queen (Q), and King (K). $n(D) = 4 \times 3 = 12$ $\Rightarrow P(D) = \frac{12}{52} = \frac{3}{13}$

(v) Let E a number card.
The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10. $n(E) = 4 \times 9 = 36$ $\Rightarrow P(E) = \frac{36}{52} = \frac{9}{13}$

Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails

Solution : When 3 fair coins are tossed,
$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$ $n(S) = 8$

i) Let A all heads. $A = \{(HHH)\}$ $n(A) = 1$ $\therefore P(A) = \frac{1}{8}$

ii) Let B atleast one tail.
$B = \{(HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$ $n(B) = 7$ $\Rightarrow P(B) = \frac{7}{8}$

iii) Let C at most one head.
$C = \{(HTT), (THT), (TTH), (TTT)\}$ $n(C) = 4$ $\Rightarrow P(C) = \frac{4}{8} = \frac{1}{2}$

iv) Let D - atmost 2 tails
$D = \{(HHH), (HHT), (HTT), (HTH), (THH), (THT), (TTH)\}$ $n(D) = 7$ $\Rightarrow P(D) = \frac{7}{8}$

A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution : S = {3, 5, 7, 9, ... 35, 37} $\Rightarrow$ n(S) = 18

Let A - multiple of 7. A = {7, 14, 21, 28, 35} n(A) = 5 $\Rightarrow$ P(A) = $\frac{5}{18}$

Let B - a prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37} n(B) = 11 $\Rightarrow$ P(B) = $\frac{11}{18}$

A∩B = {7} n(A∩B) = 1 $\Rightarrow$ P(A∩B) = $\frac{1}{18}$

$\therefore$ P(A∪B) = P(A) + P(B) – P(A∪B) = $\frac{5}{18}+\frac{11}{18}-\frac{1}{18} = \frac{15}{18} = \frac{5}{6}$

A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution : S = {(HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)} n(S) = 8

Let A - exactly 2 heads , A = {(HHT), (HTH), (THH)} n(A) = 3 $\Rightarrow$ P(A) = $\frac{3}{8}$

Let B - atleast one tail B = {(HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)} n(B) = 7 $\Rightarrow$ P(B) = $\frac{7}{8}$

Let C - Consecutively 2 heads , C = {(HHH), (HHT), (THH)} n(C) = 3 $\Rightarrow$ P(C) = $\frac{3}{8}$

A∩B = {(HHT), (HTH), (THH)} n(A∩B) = 3 $\Rightarrow$ P(A∩B) = $\frac{3}{8}$

B∩C = {(HHT), (THH)} n(B∩C) = 2 $\Rightarrow$ P(B∩C) = $\frac{2}{8}$

C∩A = {(HHT), (THH)} , n(C∩A) = 2 $\Rightarrow$ P(C∩A) = $\frac{2}{8}$

P(A∪B∪C) = P(A) + P(B) – P(C) – P(A∩B) – P(B∩C) – P(C∩A) + P(A∩B∩C)

$= \frac{3}{8}+\frac{7}{8}+\frac{3}{8}-\frac{3}{8}-\frac{2}{8}-\frac{2}{8}+\frac{2}{8} = \frac{8}{8} = 1$

The frequency distribution is given below.

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a variance of 160. Determine the value of k.

Solution : Given variance = 160

x	f	$d=\frac{x-A}{k}$	d^2	$f.d$	$f.d^2$
k	2	−3	9	−6	18
$2k$	1	−2	4	−2	4
$3k$	1	−1	1	−1	1
$4k$	1	0	0	0	0
$5k$	1	1	1	1	1
$6k$	1	2	4	2	4
	7			−6	28

$$\therefore k^2\left(\frac{\sum fd^2}{\sum f}-\left(\frac{\sum fd}{\sum f}\right)^2\right)=160$$

$$\Rightarrow k^2\left[\frac{28}{7}-\left(\frac{-6}{7}\right)^2\right]=160 \Rightarrow k^2\left[4-\frac{36}{49}\right]=160$$

$$\Rightarrow k^2\left[\frac{160}{49}\right]=160 \Rightarrow k^2=\frac{16\times 40}{16} \Rightarrow k^2=49$$

$\therefore k=7$ ($\because k$ is positive)

The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?

Solution : Let A - electrification contract , $\overline{B}$ - not plumbing contract , B - plumbing contract

$P(A)=\frac{3}{5},\ P(\overline{B})=\frac{5}{8} \Rightarrow P(B)=1-\frac{5}{8}=\frac{3}{8}$, $P(A\cup B)=\frac{5}{7}$

$\therefore P(A\cap B)=P(A)+P(B)-P(A\cup B) = \frac{3}{5}+\frac{3}{8}-\frac{5}{7} = \frac{73}{280}$

The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Solution :

x	d = x − 35	d^2
25	−10	100
29	6	36
30	−5	25
33	−2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
n = 10	9	453

$$\therefore \sigma=\sqrt{\frac{\sum d^2}{n}-\left(\frac{\sum d}{n}\right)^2}$$

$$=\sqrt{\frac{453}{10}-\left(\frac{9}{10}\right)^2}$$

$= 6.67$

The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.

Solution : Mean $=\frac{11.4+12.5+12.8+16.3+17.8+19.2}{6}=\frac{90}{6}=15$

x	d = x − 15	d^2
11.4	−3.6	12.96
12.5	−2.5	6.25
12.8	−2.2	4.84
16.3	1.3	1.69
17.8	2.8	7.84
19.2	4.2	17.64
6	0	51.22

$$\sigma=\sqrt{\frac{\sum d^2}{n}-\left(\frac{\sum d}{n}\right)^2}$$

$$=\sqrt{\frac{51.22}{6}-\left(\frac{0}{6}\right)^2}$$

$= 2.9$

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3. MATRICES 5 MARKS

Find X and Y if X + Y = $\begin{pmatrix}7 & 0\\3 & 5\end{pmatrix}$ and X − Y = $\begin{pmatrix}3 & 0\\0 & 4\end{pmatrix}$

Solution :

$X+Y=\begin{pmatrix}7 & 0\\3 & 5\end{pmatrix}$ .... (1) $X-Y=\begin{pmatrix}3 & 0\\0 & 4\end{pmatrix}$ .... (2)

(1) + (2) $\Rightarrow 2X=\begin{pmatrix}10 & 0\\3 & 9\end{pmatrix} \Rightarrow X=\begin{pmatrix}5 & 0\\3/2 & 9/2\end{pmatrix}$ (1) − (2) $\Rightarrow 2Y=\begin{pmatrix}4 & 0\\3 & 1\end{pmatrix} \Rightarrow Y=\begin{pmatrix}2 & 0\\3/2 & 1/2\end{pmatrix}$

If $A=\begin{pmatrix}2 & 1\\1 & 3\end{pmatrix}, B=\begin{pmatrix}2 & 0\\1 & 3\end{pmatrix}$ find AB and BA. Check if AB = BA.

Solution : $AB=\begin{pmatrix}2 & 1\\1 & 3\end{pmatrix}\times\begin{pmatrix}2 & 0\\1 & 3\end{pmatrix}=\begin{pmatrix}(2\ 1)\begin{pmatrix}2\\1\end{pmatrix} & (2\ 1)\begin{pmatrix}0\\3\end{pmatrix}\\(1\ 3)\begin{pmatrix}2\\1\end{pmatrix} & (1\ 3)\begin{pmatrix}0\\3\end{pmatrix}\end{pmatrix}=\begin{pmatrix}4+1 & 0+3\\2+3 & 0+9\end{pmatrix}=\begin{pmatrix}5 & 3\\5 & 9\end{pmatrix}$

$BA=\begin{pmatrix}2 & 0\\1 & 3\end{pmatrix}\times\begin{pmatrix}2 & 1\\1 & 3\end{pmatrix}=\begin{pmatrix}(2\ 0)\begin{pmatrix}2\\1\end{pmatrix} & (2\ 0)\begin{pmatrix}1\\3\end{pmatrix}\\(1\ 3)\begin{pmatrix}2\\1\end{pmatrix} & (1\ 3)\begin{pmatrix}1\\3\end{pmatrix}\end{pmatrix}=\begin{pmatrix}4+0 & 2+0\\2+3 & 1+9\end{pmatrix}=\begin{pmatrix}4 & 2\\5 & 10\end{pmatrix}$

AB ≠ BA

If $A=\begin{pmatrix}1 & -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}$ and $C=\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$ show that (AB)C = A(BC).

Solution :

$$(AB)=\begin{pmatrix}1 & -1 & 2\end{pmatrix}\times\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}=\begin{pmatrix}1-2+2 & -1-1+6\end{pmatrix}=\begin{pmatrix}1 & 4\end{pmatrix}$$

$$(AB)\,C=\begin{pmatrix}1 & 4\end{pmatrix}\times\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}=\begin{pmatrix}1+8 & 2-4\end{pmatrix}=\begin{pmatrix}9 & -2\end{pmatrix}\ ......(1)$$

$$BC=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}\times\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}=\begin{pmatrix}1-2 & 2+1\\ 2+2 & 4-1\\ 1+6 & 2-3\end{pmatrix}=\begin{pmatrix}-1 & 3\\ 4 & 3\\ 7 & -1\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1 & -1 & 2\end{pmatrix}\times\begin{pmatrix}-1 & 3\\ 4 & 3\\ 7 & -1\end{pmatrix}=\begin{pmatrix}-1-4+14 & 3-3-2\end{pmatrix}=\begin{pmatrix}9 & -2\end{pmatrix}\ ....(2)$$

From (1) and (2), (AB)C = A(BC).

If $A=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}$, $B=\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}$, $C=\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}$ verify that A (B + C) = AB + AC.

Solution :

$$B+C=\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}+\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}=\begin{pmatrix}-6 & 8\\ -1 & 4\end{pmatrix}$$

$$A(B+C)=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}-6 & 8\\ -1 & 4\end{pmatrix}=\begin{pmatrix}-6-1 & 8+4\\ 6-3 & -8+12\end{pmatrix}=\begin{pmatrix}-7 & 12\\ 3 & 4\end{pmatrix}\ ...(1)$$

$$AB=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}=\begin{pmatrix}1-4 & 2+2\\ -1-12 & -2+6\end{pmatrix}=\begin{pmatrix}-3 & 4\\ -13 & 4\end{pmatrix}$$

$$AC=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}=\begin{pmatrix}-7+3 & 6+2\\ 7+9 & -6+6\end{pmatrix}=\begin{pmatrix}-4 & 8\\ 16 & 0\end{pmatrix}$$

$$AB+AC=\begin{pmatrix}-3 & 4\\ -13 & 4\end{pmatrix}+\begin{pmatrix}-4 & 8\\ 16 & 0\end{pmatrix}=\begin{pmatrix}-7 & 12\\ 3 & 4\end{pmatrix}\ .....(2)$$

From (1) and (2), A (B + C) = AB + AC. Hence proved.

If $A=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}$ show that $(AB)^T = B^T A^T$.

Solution :

$$AB=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}\times\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}=\begin{pmatrix}2-2+0 & -1+8+2\\ 4+1+0 & -2-4+2\end{pmatrix}=\begin{pmatrix}0 & 9\\ 5 & -4\end{pmatrix}$$

$$(AB)^T=\begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix}\ ....(1)$$

$$B^T=\begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix},\quad A^T=\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix}\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}2-2+0 & 4+1+0\\ -1+8+2 & -2-4+2\end{pmatrix}=\begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix}\ .....(2)$$

From (1) and (2), $(AB)^T = B^T A^T$. Hence proved.

Let $A=\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$, $C=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ **show that** $(A-B)^T = A^T - B^T$

Solution :

$$(A-B)^T=\begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}^T=\begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix}\ ....(1)$$

$$A^T-B^T=\begin{pmatrix}1 & 1\\ 2 & 3\end{pmatrix}-\begin{pmatrix}4 & 1\\ 0 & 5\end{pmatrix}=\begin{pmatrix}-3 & 0\\ 2 & -2\end{pmatrix}\ ......(2)$$

$\therefore$ From (1) & (2) $(A-B)^T = A^T - B^T$

If $A=\begin{pmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{pmatrix}$, $B=\begin{pmatrix}2 & 3 & 4\\ 1 & 9 & 2\\ -7 & 1 & -1\end{pmatrix}$ and $C=\begin{pmatrix}8 & 3 & 4\\ 1 & -2 & 3\\ 2 & 4 & -1\end{pmatrix}$ **then verify that A + (B + C) = (A + B) + C**

Solution :

$$B+C=\begin{pmatrix}10 & 6 & 8\\ 2 & 7 & 5\\ -5 & 5 & -2\end{pmatrix}$$

$$A+(B+C)=\begin{pmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{pmatrix}+\begin{pmatrix}10 & 6 & 8\\ 2 & 7 & 5\\ -5 & 5 & -2\end{pmatrix}=\begin{pmatrix}14 & 9 & 9\\ 4 & 10 & -3\\ -4 & 5 & -6\end{pmatrix}\ .....(1)$$

$$A+B=\begin{pmatrix}6 & 6 & 5\\ 3 & 12 & -6\\ -6 & 1 & -5\end{pmatrix}$$

$$\therefore (A+B)+C=\begin{pmatrix}6 & 6 & 5\\ 3 & 12 & -6\\ -6 & 1 & -5\end{pmatrix}+\begin{pmatrix}8 & 3 & 4\\ 1 & -2 & 3\\ 2 & 4 & -1\end{pmatrix}=\begin{pmatrix}14 & 9 & 9\\ 4 & 10 & -3\\ -4 & 5 & -6\end{pmatrix}\ ....(2)$$

$\therefore$ From (1) & (2) **A + (B + C) = (A + B) + C**

If $A=\begin{pmatrix}2&5\\4&3\end{pmatrix}, B=\begin{pmatrix}1&-3\\2&5\end{pmatrix}$ **find AB, BA and check if AB = BA?**

Solution :

$$AB=\begin{pmatrix}2&5\\4&3\end{pmatrix}\begin{pmatrix}1&-3\\2&5\end{pmatrix}=\begin{pmatrix}2+10&-6+25\\4+6&-12+15\end{pmatrix}=\begin{pmatrix}12&19\\10&3\end{pmatrix}$$

$$BA=\begin{pmatrix}1&-3\\2&5\end{pmatrix}\begin{pmatrix}2&5\\4&3\end{pmatrix}=\begin{pmatrix}2-12&5-9\\4+20&10+15\end{pmatrix}=\begin{pmatrix}-10&-4\\24&25\end{pmatrix}\ \ldots\ldots(2)$$

$\therefore$ From (1) & (2) AB $\neq$ BA

Given that $A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}, B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}, C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$ **verify that A(B + C) = AB + AC**

Solution :

$$A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}, B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}, C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$$

$$(B+C)=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}+\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}=\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}$$

$$A(B+C)=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}=\begin{pmatrix}2-3&2+18&4+15\\10+1&10-6&20-5\end{pmatrix}=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix}\ \ldots(1)$$

$$AB=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}$$

$$AB=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}$$

$$AC=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}=\begin{pmatrix}1-12&3+3&2+9\\5+4&15-1&10-3\end{pmatrix}$$

$$AB+AC=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}+\begin{pmatrix}-11&6&11\\9&14&7\end{pmatrix}=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix}\ \ldots\ldots(2)$$

$\therefore$ From (1) & (2) **A(B + C) = AB + AC**

If $A=\begin{pmatrix}3&1\\-1&2\end{pmatrix}$ **show that** $A^2-5A+7I_2=0$

Solution : **If** $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ and $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $A^2-(a+d)A=(bc-ad)I_2$.

$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}3&1\\-1&2\end{pmatrix}$ $\therefore A^2-(3+2)A=((1)(-1)-(3)(2))I_2$ $\therefore A^2-5A+7I_2$

Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}, C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$ **Show that A(BC) = (AB)C**

Solution : $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}, C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$

$$BC=\begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}8+0&0+0\\2+5&0+10\end{pmatrix}=\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}8&0\\7&10\end{pmatrix}=\begin{pmatrix}8+14&0+20\\8+21&0+30\end{pmatrix}=\begin{pmatrix}22&20\\29&30\end{pmatrix}\ \ldots\ldots(1)$$

$$AB=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}4&0\\1&5\end{pmatrix}=\begin{pmatrix}4+2&0+10\\4+3&0+15\end{pmatrix}=\begin{pmatrix}6&10\\7&15\end{pmatrix}$$

$$(AB)C=\begin{pmatrix}6&10\\7&15\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}12+10&0+20\\14+15&0+30\end{pmatrix}=\begin{pmatrix}22&20\\29&30\end{pmatrix}\ \ldots\ldots(2)$$

$\therefore$ From (1) & (2) **A(BC) = (AB)C**

Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}, C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$ **Show that (A − B) C = AC − BC**

Solution : $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}, C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$

$$(A-B)C=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}-6+2&0+4\\0-2&0-4\end{pmatrix}=\begin{pmatrix}-4&4\\-2&-4\end{pmatrix}\ \ldots\ldots(1)$$

$$AC=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}2+2&0+4\\2+3&0+6\end{pmatrix}=\begin{pmatrix}4&4\\5&6\end{pmatrix}$$

$$BC=\begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}8+0&0+0\\2+5&0+10\end{pmatrix}=\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$\therefore AC-BC=\begin{pmatrix}4&4\\5&6\end{pmatrix}-\begin{pmatrix}8&0\\7&10\end{pmatrix}=\begin{pmatrix}-4&4\\-2&-4\end{pmatrix}\ \ldots\ldots(2)$$

$\therefore$ From (1) & (2), (A − B) C = AC − BC

Solve for x, y $\begin{pmatrix}x^2\\y^2\end{pmatrix}+2\begin{pmatrix}-2x\\y\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$

Solution :

$$\begin{pmatrix}x^2\\y^2\end{pmatrix}+2\begin{pmatrix}-2x\\y\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$$

$\Rightarrow x^2-4x=5$	$y^2-2y=8$
$\Rightarrow x^2-4x-5=0$	$\Rightarrow y^2-2y-8=0$
$\Rightarrow (x-5)(x+1)=0$	$\Rightarrow (y-4)(y+2)=0$
$\Rightarrow x=5,-1$	$\Rightarrow \therefore y=4, y=-2$

If $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **show that** $\mathbf{A^2-(a+d)A=(bc-ad)I_2}$.

Solution :

$$A^2 = A.A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$$(a+d)A = \begin{pmatrix} a^2+ad & ab+bd \\ ca+cd & ad+d^2 \end{pmatrix}$$

$$A^2-(a+d)A = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} - \begin{pmatrix} a^2+ad & ab+bd \\ ca+cd & ad+d^2 \end{pmatrix} = \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix} = (bc-ad)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \mathbf{(bc-ad)\,I_2}.$$

If $A=\begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}, B=\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ **verify that** $\mathbf{(AB)^T=B^TA^T}$

Solution :

$$AB = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix} = \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$\therefore (AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \quad \ldots\ldots\ldots (1)$$

$$B^TA^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix} = \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix} = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix} \ldots (2)$$

$\therefore$ From (1) & (2), $(AB)^T = B^TA^T$ Hence proved.
