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10

MATHEMATICS

Based on New Syllabus

- *Objective Type Questions and Answers*
- *Created Questions with Solution*
- *Practice Questions*

for 1, 2 and 5 Marks

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UNIT 1
RELATIONS AND FUNCTIONS

Objective Type Questions — 1 mark

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is

1) 1 2) 2
3) 3 4) 6 **Ans:** 3)

2. $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$, then $n[(A \cup C) \times B]$ is

1) 8 2) 20
3) 12 4) 16 **Ans:** 3)

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true.

1) $(A \times C) \subset (B \times D)$ 2) $(B \times D) \subset (A \times C)$
3) $(A \times B) \subset (A \times D)$ 4) $(D \times A) \subset (B \times A)$ **Ans:** 1)

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is

1) 3 2) 2
3) 4 4) 8 **Ans:** 2)

5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is

1) $\{2, 3, 5, 7\}$ 2) $\{2, 3, 5, 7, 11\}$
3) $\{4, 9, 25, 49, 121\}$ 4) $\{1, 4, 9, 25, 49, 121\}$ **Ans:** 3)

6. If the ordered pairs $(a+2, 4)$ and $(5, 2a + b)$ are equal then (a,b) is

1) $(2, -2)$ 2) $(5, 1)$
3) $(2, 3)$ 4) $(3, -2)$ **Ans:** 4)

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is

1) m^n 2) n^m
3) $2^{mn} - 1$ 4) 2^{mn} **Ans:** 4)

8. **If {(a, 8), (6, b)} represents an identity function, then the value of a and b are respectively**
1) (8, 6) 2) (8, 8)
3) (6, 8) 4) (6, 6) **Ans:** 1)

9. **Let A ={1, 2, 3, 4}, B = {4, 8, 9, 10}. A function f: A→B given by f ={(1, 4), (2, 8), (3, 9), (4, 10)} is a**
1) Many–one function 2) Identity function
3) One-to-One function 4) Into function **Ans:** 3)

10. **f(x) = $2x^2$ and g(x)= $\frac{1}{3x}$. then (f o g) is**
1) $\frac{3}{2x^2}$ 2) $\frac{2}{3x^2}$
3) $\frac{2}{9x^2}$ 4) $\frac{1}{6x^2}$ **Ans:** 3)

11. **If f: A→B is a bijective function and if n(B) = 7, then n(A) is equal to**
1) 7 2) 49
3) 1 4) 14 **Ans:** 1)

12. **Let f and g be two functions given by f ={(0, 1), (2, 0), (3, -4), (4, 2), (5, 7} g = {(0, 2), (1, 0), (2, 4), (–4, 2), (7, 0} then the range of (f o g) is**
1) {0, 2, 3, 4, 5} 2) {–4, 1, 0, 2, 7}
3) {1, 2, 3, 4, 5} 4) {0, 1, 2} **Ans:** 4)

13. **Let f(x) = $\sqrt{1+x^2}$ then**
1) f(xy) = f(x).f(y) 2) f(xy) ≥f(x).f(y)
3) f(xy) ≤f(x).f(y) 4) None of these **Ans:** 3)

14. **If g ={(1, 1), (2, 3), (3, 5), (4, 7)} is a function given by g(x) = $\alpha x + \beta$ then the values of α and β are**
1) (–1, 2) 2) (2, –1)
3) (–1, –2) 4) (1, 2) **Ans:** 3)

15. **f(x) = $(x + 1)^3 – (x – 1)^3$ represents a function which is**
1) linear 2) cubic
3) reciprocal 4) quadratic **Ans:** 4)

Created Questions | **1 mark**

1. **In n(A) = p, n(B) = q then the total number of relations that exists between A and B is**
   1) 2p 2) 2q
   3) 2p+q 4) 2pq **Ans:** 4)

2. **Which one of the following statement is incorrect?**
   1) Function is nothing but a rule of association between any two variables.
   2) If A and B are any two non-empty sets then any function from A to B represented by the set of ordered pairs is a subset of the Cartesian product B × A.
   3) No two elements in the co-domain have the same pre-image in the domain as far as an injective function is concerned.
   4) The horizontal line test is used to check whether the given function is a one-one function or not. **Ans:** 2)

3. **Read the following statements carefully.**
   i. The graph of a constant function is a straight line parallel to x-axis.
   ii. Composition of functions is always associative.
   iii. The graph of the modulus function exists only in the I and IV quadrant in the Euclidean plane.
   iv. The number of elements in the domain and the range are same in the case of an identity function.

   Now which one of the following is correct?
   1) i, ii, iii only 2) i, ii, iv only
   3) ii, iii, iv only 4) all the four **Ans:** 2)

4. **If A = {4, 9, 16, 36} and B = {1, 2, 3, 4, 5, 6} f: A → B defined by f = {(4, 2) (9, 3), (16, 4), (36, 6)} is**
   1) one-one onto function 2) many-one onto function
   3) one-one into function 4) many-one into function **Ans:** 3)

5. **Identify the wrong statement.**
   i) set of whole numbers is a subset of set of real numbers.
   ii) f : R → R defined by f(x) = $-x$ is one-one and into.
   iii) f : Z → R defined as f(x) = $\frac{1}{x}$ is not a function.
   iv) B – A = B if A and B are disjoint sets.

1) (ii) and (iii)   2) (ii) only
3) (iii) only   4) (ii) and (iv)   **Ans:** 2)

6. f(x) = ax+b and g(x) = 3x+1, if $fog = gof$ then a is

1) 2   2) 3
3) 5   4) 1   **Ans:** 3)

7. If A = {1, 3, 5} B is the set of integers and f : A → B defined by f(x) = $x^2 - 1$ then the range of f is

1) {1, 9, 25}   2) {0, 8, 24}
3) {3, 9, 24}   4) none of three   **Ans:** 2)

8. The pre-image of 2 under the function f = {(0, 1), (2, 2), (3, 2), (5, 8), (4, 8)} are

1) 3 and 5   2) 0 and 2
3) 2 and 3   4) 5 and 4   **Ans:** 3)

9. If f: A → N defined by f(x) = $\frac{x+1}{2}$ is a function then A is equal to

1) N   2) W
3) Z   4) Q   **Ans:** 4)

10. If f: A → A where A = {1, 2, 3} defined by f(x) = x then the function is

1) one-one function   2) constant function
3) one-one onto function   4) identity function   **Ans:** 4)

Two Marks Questions — 2 marks

1. If A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

Solution:

A×B = {(3, 2), (3, 4), (5, 2), (5, 4)} then
A = {3, 5} and B = {2, 4}

2. If B × A={(–2, 3), (–2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.

Solution:

B×A = {(–2, 3), (–2, 4), (0, 3), (0, 4), (3, 3), (3, 4) then
A = {3, 4} B = {–2, 0, 3}

3. Find A × B, A × A and B × A.

i) A = {2, –2, 3} and B = {1, – 4}   (ii) A = {m, n}; B = ϕ

Solution:

i) A = {2, –2, 3}, B = {1, –4}

$A \times B = \{2, -2, 3\} \times \{1, -4\}$

$= \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$

$A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$

$= \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$

$B \times A = \{1, -4\} \times \{2, -2, 3\}$

$= \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$

ii) A = {m, n}, B = φ

$A \times B = \{(m, n) \times \{\} = \{\ \}$

$A \times A = \{(m, n)\} \times \{m, n\}$

$= (m, m), (m, n), (n, m), (n, n)\}$

$B \times A = \{\} \times \{m, n\} = \{\}$

4. Let A = {1, 2, 3} and B = {*x* | *x* is a prime number less than 10}. Find A × B and B × A.

Solution:

$A = \{1, 2, 3\}\ B = \{2, 3, 5, 7\}$

$A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$

$= (1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

$B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$

$= \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

5. Let A={3, 4, 7, 8} and B = {1, 7, 10}. Which of the following sets are relations from A to B?
(i) R_1 = {(3, 7), (4, 7), (7, 10), (8, 1)} (ii) R_2= {(3, 1), (4, 12)}
(iii) R_3 = {(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)}

Solution:

$A \times B = \{(3, 1), (3, 7), (3, 10), (4, 1), (4, 7), (4, 10), (7, 1), (7, 7), (7, 10), (8, 1), (8, 7), (8, 10)\}$

i) We note that, $R_1 \subseteq A\times B$. Thus R_1 is a relation from A and B.

ii) Here $(4, 12) \in R_2$, but $(4, 12) \notin A\times B$. So R_2 is not a relation from A to B.

iii) Here $(7, 8) \in R_3$, but $(7, 8) \notin A\times B$. So R_3 is not a relation from A to B.

6. Let A = {1, 2, 3, 4, ..., 45} and R be the relation defined as " is square of a number" on A. Write R as a subset of A × A. Also, find the domain and the range of R.

Solution:

A = {1, 2, 3, …., 45}, R = {1, 4, 9, 16, 25, 36}

It is clear that R 'is a subset of A'

R × A = {(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)}

$R \subset A$

$\therefore$ Domain of R = {1, 2, 3, 4, 5, 6}

Range of R = {1, 4, 9, 16, 25, 36}

7. A Relation R is given by the set {(x , y) / y = x + 3, $x \in$ {0, 1, 2, 3, 4, 5}. Determine its domain and range.

Solution:

x = {0, 1, 2, 3, 4, 5}

f(x) = y = x +3

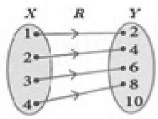
f(0) = 3; f(1) = 4; f(2) = 5; f(3) = 6; f(4) = 7; f(5) = 8

$\therefore$ R = {(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)}

Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {3, 4, 5, 6, 7, 8}

8. Let X= {1, 2, 3, 4} Y = {2, 4, 6, 8, 10} and R = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that R is a function and find its domain, co-domain and range?



Solution:

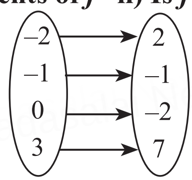
Pictorial representation of R is given diagram,

From the diagram, we see that for each x ∈ X, there exists only one y ∈ Y. Thus all elements in X have only one image in Y. Therefor R is a function.

Domain X = {1, 2, 3, 4}; Co-domain Y = { 2, 4, 6, 8, 10};

Range of f={2, 4, 6, 8}

9. A relation f: X → Y is defined by $f(x) = x^2 - 2$ where, X = {–2, –1, 0, 3} and Y = R. (i) List the elements of f ii) Is f a function?



Solution:

f(x) = $x^2 - 2$ where $x \in$ { –2, –1, 0, 3}

f(–2) = $(-2)^2 - 2 = 2$; f(–1) = $(1)^2 - 2 = -1$

$f(0) = (0)^2 - 2 = -2$; $f(3) = (3)^2 - 2 = 7$

Therefore, $f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

We note that each element in the domain of f has a unique image. Therefore f is a function.

10. Let A = {1, 2, 3, 4} and B=N. Let f:A → B be defined by $f(x) = x^3$ then,

i) Find the range of f (ii) Identify the type of function.

Solution:

$A = \{1, 2, 3, 4\}$, $B = N$

$f : A \to B$, $f(x) = x^3$

$f(1) = (1)^3 = 1$; $f(2) = (2)^3 = 8$;

$f(3) = (3)^3 = 27$; $f(4) = (4)^3 = 64$

i) Range of $f = \{1, 8, 27, 64\}$

ii) It is one-one and into function. [Range ≠ co domain]

11. Let f be a function from R to R defined by $f(x) = 3x - 5$. Find the values of a and b given that (a, 4) and (1, b) belong to f.

Solution:

$f(x) = 3x - 5$ can be written as $f = \{x, (3x - 5) \mid x \in R\}$

(a, 4) means the image of a is 4. That is, $f(a) = 4$

$3a - 5 = 4 \Rightarrow 3a = 9 \Rightarrow a = 3$

(1, b) means the image of 1 is b.

That is, $f(1) = b \Rightarrow 3(1) - 5 = b \Rightarrow b = -2$

12. If $f(x) = x^2 - 1$, $g(x) = x - 2$, find a, if g o f (a) = 1.

Solution:

$f(x) = x^2 - 1$, $g(x) = x - 2$

Given

$$
\begin{aligned}
g \circ f(a) &= 1 \\
g[f(a)] &= 1 \\
g[a^2 - 1] &= 1 \\
a^2 - 1 - 2 &= 1
\end{aligned}
\qquad
\begin{aligned}
a^2 - 3 &= 1 \\
a^2 &= 4 \\
\therefore a &= \pm 2
\end{aligned}
$$

13. Given $f(x) = 2x - x^2$, find (i) f(1) (ii) f(x+1) (iii) f(x) + f(1)

Solution:

i) Replacing x with 1, we get $f(1) = 2(1) - (1)^2 = 2 - 1 = 1$

ii) Replacing x with $x+1$, we get

$f(x+1) = 2(x+1) - (x+1)^2 = 2x+2-(x^2+2x+1) = -x^2+1$

iii) $f(x) + f(1) = (2x-x^2)+1 = -x^2+2x+1$

[Note that $f(x) + f(1) \neq f(x+1)$.

In general, f(a+b) is not equal to f(a) + f(b)]

14. Let X = {3, 4, 6, 8}. Determine whether the relation $R = \{(x, f(x)) \mid x \in X, f(x) = x^2+1\}$ is a function from X to N?

Solution:

Given X = {3, 4, 6, 8}

$f(x) = x^2 + 1$

$f(3) = (3)^2+1 = 9+1 = 10;$ $\quad f(4) = (4)^2+1 = 16+1 = 17$

$f(6) = (6)^2+1 = 36+1 = 37;$ $\quad f(8) = (8)^2+1 = 64+1 = 65$

R = {(3, 10), (4, 17), (6, 37), (8, 65)}.

Yes, R is a function from X to N.

15. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution:

$$\begin{aligned}
f(x) &= 3 - 2x \\
f[x^2] &= [f(x)]^2 \\
3 - 2x^2 &= [3 - 2x]^2 \\
3 - 2x^2 &= 9 + 4x^2 - 12x \\
3 - 2x^2 - 9 - 4x^2 + 12x &= 0 \\
-6x^2 + 12x - 6 &= 0 \div -6 \\
x^2 - 2x + 1 &= 0 \\
(x - 1)(x - 1) &= 0 \\
\boldsymbol{x} &= \mathbf{1, 1}
\end{aligned}$$

16. A plane is flying at a speed of 500 km per hour. Express the distance 'd' travelled by the plane as function of time t in hours.

Solution:

Speed of the plane = 500 km/hr; Time = t hours; Distance = d km

Distance = Time × Speed

d = 500 t

17. Show that the function $f : N \to N$ defined by $f(x) = 2x - 1$ is one-one but not onto.

Solution:

Given $\quad f : N \to N;$ $\quad$ defined by $f(x) = 2x - 1$

$f(1) = 2(1)-1 = 2-1 = 1;$ $f(2) = 2(2)-1 = 4-1 = 3;$
$f(3) = 2(3)-1 = 6-1 = 5\ldots$

i.e Different elements have different images.
More over Range $\neq$ Co-domain.
$\therefore$ $f : N \rightarrow N$ is a one-one function but not onto.

18. Show that the function $f : N \rightarrow N$ defined by $f(m) = m^2 + m + 3$ is one-one function.

Solution:

Given $f : N \rightarrow N$
defined by $f(m) = m^2 + m + 3$

$f(1) = (1)^2+1+3 = 1+1+3 = 5$
$f(2) = (2)^2+2+3 = 4+2+3 = 9$
$f(3) = (3)^2+3+3 = 9+3+3 = 15\ldots\ldots$

i.e Different elements have different images.
Hence $f : N \rightarrow N$ is a one to one function. Hence it is proved.

19. Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Solution:

$f_2(x) = 2x^2-5x+3$ and $f_1(x) = \sqrt{x}$

Then, $f(x) = \sqrt{2x^2 - 5x + 3} = \sqrt{f_2(x)}$
$= f_1|f_2(x)| = f_1 f_2(x) = (f_1 \circ f_2)(x)$

20. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k.

Solution:

$$\begin{aligned} f(x) &= 3x - 2 & g(x) &= 2x + k \\ f \circ g &= f[g(x)] & g \circ f &= g[f(x)] \\ &= f[2x + k)] & g \circ f &= g[3x - 2] \\ &= 3(2x + k) - 2 & &= 2(3x - 2) + k \\ &= 6x + 3k - 2 & &= 6x - 4 + k \end{aligned}$$

$f \circ g = g \circ f \Rightarrow 6x + 3k - 2 = 6x - 4 + k$
$\Rightarrow 3k - k = -4 + 2 \Rightarrow 2k = -2 \Rightarrow k = -1$

21. Find the value of k, such that $f \circ g = g \circ f$
(i) $f(x) = 3x + 2$, $g(x) = 6x - k$ (ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Solution:

i) $f(x) = 3x+2;\ g(x) = 6x-k$

$$\begin{aligned} f\circ g &= g\circ f \\ f[g(x)] &= g[f(x)] \\ f[6x-k] &= g[3x+2] \\ 3(6x-k)+2 &= 6(3x+2)-k \\ 18x-3k+2 &= 18x+12-k \\ -3k+k &= 12-2 \\ -2k &= 10 \\ \Rightarrow k &= -5 \end{aligned}$$

ii) $f(x) = 2x-k;\ g(x) = 4x+5$

$$\begin{aligned} f\circ g &= g\circ f \\ f[g(x)] &= g[f(x)] \\ f[4x+5] &= g[2x-k] \\ 2(4x+5)-k &= 4(2x-k)+5 \\ 8x+10-k &= 8x-4k+5 \\ 4k-k &= 5-10 \\ 3k &= -5 \\ k &= \frac{-5}{3} \end{aligned}$$

22. If $f(x) = x^2 - 1$, find (i) f o f (ii) f o f o f

Solution:

$f(x) = x^2 - 1$

i) **f o f** $= f[f(x)] \Rightarrow f[x^2-1]$

$= [x^2-1]^2-1 = x^4+1-2x^2-1 = x^4-2x^2$

ii) **f o f o f** $= f\circ[x^4-2x^2] = [x^4-2x^2]^2-1$

23. In electrical circuit theory, a circuit C(t) is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = a\,C(t_1) + b\,C(t_2)$, where a, b are constants. Show that the circuit C(t) = 3t is linear.

Solution:

$$\begin{aligned} c(t) &= 3t \\ LHS &= c[at_1 + bt_2] \\ &= 3[at_1 + bt_2] \\ &= 3at_1 + 3bt_2 \\ &= a(3t_1) + b(3t_2) \\ &= a.C(t_1) + b.C(t_2) \\ &= RHS \end{aligned}$$

$\therefore$ c(t) is linear function.

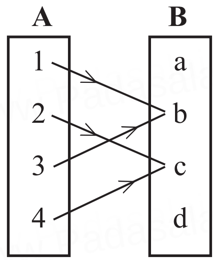
Created Questions with Solution — 2 marks

1. Let A = {1, 2, 3, 4} and B = {a, b, c, d}. Give a function from A → B to each of the following.

i) Neither one-to-one nor onto ii) one-to-one and onto

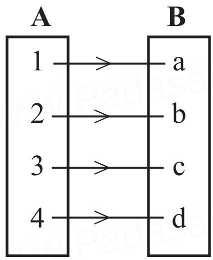
Solution:

i)



Neither one-to-one nor onto

ii)



one-to-one and onto

2. In how many ways can we represent a function? List them.

Solution:

We can represent a function in 4 ways and they are:

i) A set of ordered pairs   ii) An arrow diagram

iii) A table   iv) A graph

3. If A = set of all even prime numbers, B = set of all prime numbers less than 10, C = {x / $x \in$N, x is neither prime nor composite} Find A×(B×C).

Solution:

A = {2}, B = {2, 3, 5, 7}, C = {1}

$$B \times C = \{2, 3, 5, 7\} \times \{1\}$$
$$= \{(2, 1), (3, 1), (5, 1), (7, 1)\}$$
$$A \times (B \times C) = \{2\} \times \{(2, 1), (3, 1), (5, 1), (7, 1)\}$$
$$= \{(2, 2, 1), (2, 3, 1), (2, 5, 1), (2, 7, 1)\}$$

4. i) What is the domain of a logarithmic function?
ii) Say True or False.
"Functions" are also known as Mappings.

Solution:

i) Domain for the logarithmic function = $(0, \infty) = R^+$

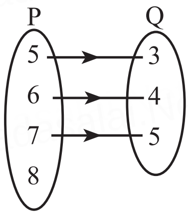
ii) True.

For Practice	2 marks

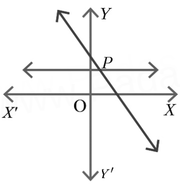
1. If A = {1, 3, 5}, and B = {2, 3} then,
(i) find A × B and B × A (ii) Is A × B = B × A? If not why?
(iii) Show that n(A×B) = n(B×A) = n(A)× n(B)

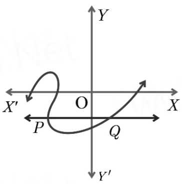
2. Find A × B, A × A and B × A . A = B = {p, q}

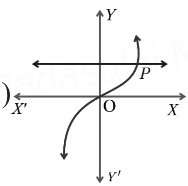
3. The arrow diagram shows a relationship between the sets P and Q.
   Write the relation in
   (i) Set builder form
   (ii) Roster form
   (iii) What is the domain and range of R.



4. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$.
   Show that $A \times A = (B \times B) \cap (C \times C)$.
5. Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$. Which of the following are relation from A to B?
   (i) $R_1 = \{(2, 1), (7, 1)\}$ (ii) $R_2 = \{(-1, 1)\}$
   (iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$
   (iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$
6. Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from Z into Z. Find $f(x)$.
7. Using horizontal line test, determine which of the following functions are one-one.

(i) 

(ii) 

(iii) 

Created Questions — 2 marks

1. Given $f: Z \rightarrow N$ is defined by $f(x) = x+1$, test whether this represents a function or not, Give reason.
2. Given $A = \{1, 2, 4, 8\}$. Write down the set of ordered pairs having the relation "is the divisor of"
3. Define Domain and Range.
4. Given $P = \{-2, -1, 0, 1\}$, $Q = \{1, -2, 6, -3\}$, $R = \{(x, y): y = x^2 - 3, x \in P, y \in Q$
   (i) List the elements of R. (ii) What is the range of R?
   (iii) Is the relation a function if so identify the type of the function.

Five Marks Questions 5 marks

1. Let A = $\{x \in W / x < 2\}$, B = $\{x \in N / 1 < x \leq 4\}$ and C = {3, 5}. Verify that A × (B ∩ C) = (A × B) ∩ (A × C).

Solution:

$A = \{x \in W / x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N / 1 < x \leq 4\} \Rightarrow B = \{2, 3, 4\}$

$C = \{3, 5\}$

To prove $A\times(B\cap C) = (A\times B) \cap (A\times C)$

$B\cap C = \{3\}$

$A\times(B\cap C) = \{(0, 3), (1, 3)\}$ .... (1)

$A\times B = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A\times C = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A\times B) \cap (A\times C) = \{(0, 3), (1, 3)\}$ .... (2)

$\therefore$ (1) = (2) Hence Proved.

2. A = $\{x \in W / x < 2\}$, B = $\{x \in N / 1 < x \leq 4\}$ and C = {3, 5}. Verify that A × (B ∪ C) = (A × B) ∪ (A × C).

Solution:

$A = \{x \in W / x < 2\} \Rightarrow A = \{0, 1\}$

$B = \{x \in N / 1 < x \leq 4\} \Rightarrow B = \{2, 3, 4\}$

$C = \{3, 5\}$

To Prove $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$B\cup C = \{2, 3, 4, 5\}$

$A\times(B\cup C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ .... (1)

$A \times B = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A\times B) \cup (A\times C) = \{(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)\}$ .... (2)

$\therefore$(1) = (2) Hence Proved.

3. Let A = $\{x \in N \mid 1 < x < 4\}$, B = $\{x \in W \mid 0 \leq x < 2\}$ and C = $\{x \in N \mid x<3\}$. Then verify that A×(B∪C) = (A×B) ∪ (A×C).

Solution:

$A = \{2, 3\}$, $B = \{0, 1\}$, $C= \{1, 2\}$

$B \cup C = \{0, 1\} \cup \{1, 2\} = \{0, 1, 2\}$ .... (1)

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$

$= \{(2,0), (2,1), (2,2), (3,0), (3,1), (3,2)\}$ .... (2)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$ .... (3)

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2,1), (2, 2), (3, 1), (3, 2)\}$ .... (4)

$(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cup \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ .... (5)

From (2) = (5). $\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

4. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C=\{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Solution:

$A \cap C = \{3\}$, $B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{3\} \times \{3, 5\} = \{(3, 3), (3, 5)\}$ .... (1)

$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$

$= \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)\}$

$C \times D = \{3, 4\} \times \{1, 3, 5\}$

$= \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ .... (2)

(1), (2) are equal.

$\therefore (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$. Hence it is verified.

5. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C= The set of even prime number. Verify that $A \times (B - C) = (A \times B) - (A \times C)$.

Solution:

$A = \{1, 2, 3, 4, 5, 6, 7\}$ $B = \{2, 3, 5, 7\}$ $C = \{2\}$

$B - C = \{2, 3, 5, 7\} - \{2\} = \{3, 5, 7\}$ .... (1)

$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$

$= \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ .... (2)

$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$

$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7),$

(3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7),
(5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7),
(7, 2) (7, 3), (7, 5), (7, 7)} .... (3)

A × C = {1, 2, 3, 4, 5, 6, 7} × {2}
= {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)} .... (4)

(A × B) – (A × C) = {(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7),
(3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7),
(5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7),
(7, 3), (7,5),(7,7)} .... (5)

(2), (5) are equal. ∴ A × (B – C) = (A × B) – (A × C).
Hence it is verified.

6. **Represent each of the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible.**
i) {(x, y) | x = 2y, x ∈ {2, 3, 4, 5}, y ∈ {1, 2, 3, 4}
ii) {(x, y) | y = x + 3, x, y are natural numbers < 10}.

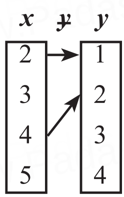
Solution:

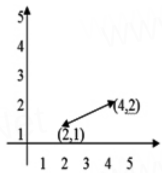
(i) {(x, y) | x = 2y, x ∈ {2, 3, 4, 5}, y ∈ {1, 2, 3, 4}

$x = 2y$

$$f(x) = y = \frac{x}{2};\ f(2) = \frac{2}{2} = 1;\ f(3) = \frac{3}{2};\ f(4) = \frac{4}{2} = 2;\ f(5) = \frac{5}{2}$$

a) An Arrow diagram b) Graph c) Roaster Form





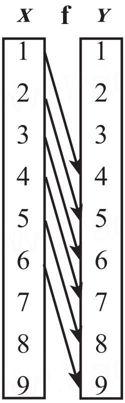
{(2, 1), (4, 2)}

(ii) {(x, y) | $y = x + 3$, x, y are natural numbers < 10

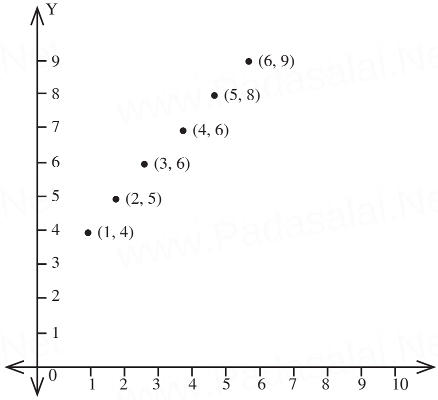
$f(x) = x + 3$

f(1) = 1+3 = 4 f(2) = 2+3 = 5 f(3) = 3+3 = 6
f(4) = 4+3 = 7 f(5) = 5+3 = 8 f(6) = 6+3 = 9

a) An arrow Diagram



b) Graph



c) A set in roster form

{(1, 4), (2, 5), (3, 6) (4, 7), (5, 8), (6, 9)}

7. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A→B be a function given by f(x) = 3x–1. Represent this function. (i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs (iv) in a graphical form.

Solution:

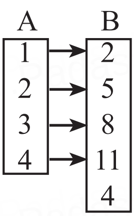
A = {1, 2, 3, 4}, B = {2, 5, 8, 11, 14} f(x) = 3x – 1

f(1) = 3(1)–1 = 3–1 = 2; f(2) = 3(2)–1 = 6–1 = 5

f(3) = 3(3)–1 = 9–1 = 8; f(4) = 3(4)–1 = 12–1 = 11

R = {(1, 2), (2, 5), (3, 8), (4, 11)}

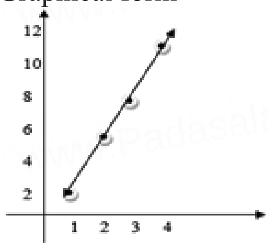
a) Arrow diagram



c) Set of ordered pairs

{(1, 2), (2, 5), (3, 8), (4, 11)}

(iv) Graphical form



(ii) Table

x	1	2	3	4
y	2	5	8	11

8. Let f be a function f : N→ N be defined by f(x) = 3x + 2, $x \in$ N (i) Find the images of 1, 2, 3 (ii) Find the pre-image of 29, 53 (iii) Identify the type of function.

Solution:

The function f : N→ N be defined by f(x) = 3x + 2

(i) If $x = 1$, f(1) = 3(1) + 2 = 5; If $x = 2$, f(2) = 3(2) + 2 = 8; If $x = 3$, f(3) = 3(3) + 2 = 11

The images of 1, 2, 3 are 5, 8, 11 respectively.

(ii) If x is the pre-image of 29, then f(x) = 29.

Hence $3x + 2 = 29$; $3x = 27 \Rightarrow x = 9$.

Similarly, if x is the pre-image of 53 then f(x) = 53.

Hence $3x + 2 = 53$

$3x = 53 - 2 \Rightarrow 3x = 51 \Rightarrow x = 17$.

Thus the pre-image of 29 and 53 are 9 and 17 respectively.

(iii) Since different elements of N have different images in the co-domain, the function f is one-one function. The co-domain of f is N. But the range of f = {5, 8, 11, 14, 17,….} is a proper subset of N . Therefore f is not an onto function. That is, f is an into function. Thus f is one-one and into functions.

9. Forensic scientists can determine the height (in cm) of a person based on length of the thigh bone. They usually do so using the function h(b) = 2.47b + 54.10 where b is the length of the thigh bone.

(i) Verify the function h is one-one or not.

(ii) Also find the height of a person if the length of his thigh bone is 50 cm.

(iii) Find the length of the thigh bone if the height of a person is 147.96 cm.

Solution:

(i) To check if h is one-one, we assume that $h(b_1) = h(b_2)$.

Then we get, $2.47b_1 + 54.10 = 2.47b_2 + 54.10$

$2.47\,b_1 = 2.47b_2 \Rightarrow b_1 = b_2$

Thus, $h(b_1) = h(b_2), \Rightarrow b_1 = b_2$.

So, the function h is one-one.

(ii) If the length of the thigh bone b = 50, then the height is

$$h(50) = (2.47 \times 50) + 54.10 = 177.6 \text{ cms}$$

(iii) If the height of a person is 147.96 cms, then, h(b) = 147.96 and so the length of the thigh bone is given by

$$2.47b + 54.10 = 147.96 \Rightarrow b = \frac{93.86}{2.47} = 38$$

Therefore, the length of the thigh bone is 38 cms.

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (*x*) as y = a*x* + b, where a, b are constants.

Length *x* of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.

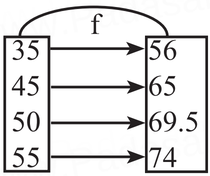
(ii) Find a and b.

(iii) Find the height of a person whose forehand length is 40 cm.

(iv) Find the length of forehand of a person if her height is 53.3 inches.

Solution:

(i)



$y = f(x) = ax + b$

Every element in the domain is uniquely associated with an element in the co-domain. Therefore the relation is a function.

ii) $y = ax + b$

$56 = 35a + b$

$56 - 35a = b \rightarrow (1)$

$y = ax + b$

$65 = 45a + b$

$65 - 45a = b \rightarrow (2)$

From (1) and (2)

$\therefore\ 56 - 35a = 65 - 45a$

$45a - 35a = 65 - 56$

$10a = 9$

$a = 9/10 \Rightarrow a = 0.9$

① $\Rightarrow b = 56 - 35a$

$= 56 - 35 \times 0.9$

$= 56 - 31.5 = 24.5$

iii) $y = ax + b$

$y = 0.9x + 24.5$

If $x = 40$ then $y = 0.9 \times 40 + 24.5$

$= 36 + 24.5 = 60.5$

$\therefore$ Height of the woman = 60.5 inches

iv) $y = 0.9x + 24.5$

If $y = 53.3$

$53.3 = 0.9x + 24.5 \quad \Rightarrow 53.3 - 24.5 = 0.9x \Rightarrow 28.8 = 0.9x$

$$x = \frac{28.8}{0.9} = \frac{288}{9} = 32$$

$\therefore$ Length of the Women's forehand = 32 cm

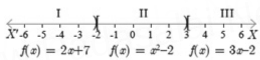
11. If the function f : R→ R is defined by

$$f(x) = \begin{cases} 2x + 7, & x < -2 \\ x^2 - 2, & -2 \le x < 3 \\ 3x - 2, & x \ge 3 \end{cases}$$

(i) f(4) (ii) f(–2) (iii) f(4) + 2f(1) (iv) $\dfrac{f(1) - 3f(4)}{f(-3)}$

Solution:

The function f is defined by three values in I, II, III as shown by the side.



For a given value of x = a, find out the interval at which the point a is located, there after find f(a) using the particular value defined in that interval.

i) First, we see that, $x = 4$ lies in the third interval.

Therefore, $f(x) = 3x - 2$; $f(4) = 3(4) - 2 = 10$

ii) $x = -2$ lies in the second interval.

Therefore, $f(x) = x^2 - 2$; $f(-2) = (-2)^2 - 2 = 2$

iii) From (i), f(4) = 10. To find f(1), first we see that, $x = 1$ lies in the second interval.

Therefore, $f(x) = x^2 - 2 \Rightarrow f(1) = 1^2 - 2 = -1$

So, $f(4) + 2f(1) = 10 + 2(-1) = 8$

iv) We know that f(1) = –1 and f(4) = 10. Find finding f(–3), we see that x = –3 lies in the first interval.
Therefore, f(x) = 2x + 7, thus f(–3) = 2(–3) + 7 = 1

Hence, $\dfrac{f(1)-3f(4)}{f(-3)} = \dfrac{-1-3(10)}{1} = -31$

12. If the function f is defined by $f(x) = \begin{cases} x+2; & x>1 \\ 2; & -1\le x\le 1 \\ x-1; & -3\le x<-1 \end{cases}$

Find the values of (i) f(3) (ii) f(0) (iii) f(–1.5) (iv) f(2) + f(–2)

Solution:

$$f(x) = \begin{cases} x+2; & x>1 \\ 2; & -1\le x\le 1 \\ x-1; & -3\le x<-1 \end{cases}$$

(i) f(3) = x + 2 = 3 + 2 =5

(ii) f(0) = 2

(iii) f(–1.5) = x – 1 = –1.5 – 1 = – 2.5

(iv) f(2) + f(–2) = [x+2] + [x–1]
= [2+2] + [–2–1] = 4+[–3] = 4–3 = 1

13. A function f: [–5, 9]→R is defined as follows.

$$f(x) = \begin{cases} 6x+1; & -5\le x<2 \\ 5x^2-1; & 2\le x<6 \\ 3x-4; & 6\le x\le 9 \end{cases}$$

Find
(i) f(–3) + f(2) (ii) f(7) – f(1)
(iii) 2f(4) + f(8)
(iv) $\dfrac{2f(-2)-f(6)}{f(4)+f(-2)}$

Solution:

(i) f(–3) + f(2) = [6x + 1] + [5x^2 – 1]
= [6(–3) + 1] + [5(2)2 –1]
= [–18 + 1] + [5(4) – 1]
= –17 + [20 – 1] = –17 + 19 = 2

(ii) f(7) – f(1) = [3x – 4] – [6x + 1]
= [3(7) – 4] – [6(1) +1]
= [21 – 4] – [6 + 1] = 17 – 7 = 10

(iii) 2f(4) + f(8) = 2[5x^2 – 1] + [3x – 4]
= 2[5(4)^2 – 1] + [3(8) – 4]

$$= 2[5(16) - 1] + [24 - 4]$$
$$= 2[80 - 1] + [20]$$
$$= 2[79] + 20 = 158 + 20 = 178$$

(iv) $$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2[6x+1] - [3x-4]}{[5x^2 - 1] + [6x+1]}$$
$$= \frac{2[6(-2)+1] - [3(6)-4]}{[5(4)^2 - 1] + [6(-2)+1]}$$
$$= \frac{2[-12+1] - [18-4]}{[5(16)-1] + [-12+1]}$$
$$= \frac{2[-11] - [14]}{[80-1] + [-11]}$$
$$= \frac{-22-14}{79-11} = \frac{-36}{68} = \frac{-9}{17}$$

14. The distance S an object travels under the influence of gravity in time t seconds is given by S(t) = $\frac{1}{2}$gt^2 + at + b where, (g is the acceleration due to gravity), a, b are constants. Verify whether the function S(t) is one-one.

Solution:

$$s(t) = \frac{1}{2}gt^2 + at + b$$

Let t be 1, 2, 3, …… seconds.

$$s(t_1) = s(t_2)$$
$$\frac{1}{2}gt_1^{\,2} + at_1 + b = \frac{1}{2}gt_2^{\,2} + at_2 + b$$
$$\frac{1}{2}g(t_1^{\,2} - t_2^{\,2}) + a(t_1 - t_2) = 0$$
$$\Rightarrow \frac{1}{2}g[(t_1 - t_2)(t_1 + t_2) + a(t_1 - t_2)] = 0$$
$$(t_1 - t_2)[\frac{1}{2}g(t_1 + t_2) + a] = 0$$
$$\therefore t_1 - t_2 = 0$$
$$t_1 = t_2 \quad [\because \frac{1}{2}g[(t_1 + t_2) + a \neq 0]$$

$\therefore$ s(t) is one-one function.

15. **The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C) = F where (F = $\frac{9}{5}$ C+32). Find (i) t(0) (ii) t(28) (iii) t(–10) (iv) the value of C when t(C) = 212 (v) the temperature when the Celsius value is equal to the Fahrenheit value.**

Solution:

$$t(C) = F = \frac{9}{5}C + 32$$

(i) $t(0) = \frac{9}{5}(0) + 32 = 32°F$

(ii) $t(28) = \frac{9}{5}(28) + 32 = 50.4 + 32 = 82.4°F$

(iii) $t(-10) = \frac{9}{5}(-10) + 32 = -18 + 32 = 14°F$

(iv) $t(C) = 212$

$$212 = \frac{9}{5}C + 32 \Rightarrow \frac{9}{5}C + 32 = 212$$

$$\frac{9}{5}C = 212 - 32 \Rightarrow \frac{9}{5}C = 180 \Rightarrow C = 180 \times \frac{5}{9} = 100°C$$

(v) Celsius Value = ∴ Fahrenheit Value

$$C = \frac{9}{5}C + 32 \Rightarrow 5C = 9C + 160 \Rightarrow 9C - 5C = -160$$

$$\Rightarrow 4C = -160; C = \frac{-160}{4} = -40°$$

16. **If f : R → R and g : R → R are defined by f(x)= x^5 and g(x) = x^4 then check if f, g are one-one and fog is one- one?**

Solution:

$f(x) = x^5$ and $g(x) = x^4$

Let A be the domain and B be the co-domain.

Since f is an odd function,every element in the co-domain has only one preimage in the domain.

∴ f is one-one function.

Since g is an even function. There is a possibility that every element in the co-domain has more than one pre-image in the domain.

$\therefore$ g is not one – one function.

$fog = f[g(x)] = f(x^4) = \left[x^4\right]^5 = x^{20}$ = which is an ever function.

$\therefore$ f o g is not one-one.

17. Let A, B, C $\subseteq$ N and a function f : A $\rightarrow$ B be defined by f(x) = 2x + 1 and g : B $\rightarrow$ C be defined by g(x) = x^2. Find the range of f o g and g o f.

Solution:

f : A $\rightarrow$ B g : B $\rightarrow$ C Here A, B, C $\subseteq$ N

$f(x) = 2x + 1$, $g(x) = x^2$

$(f \circ g)(x) = f[g(x)] = f[x^2] = 2x^2 + 1$

Range of f o g $= \{y / y = 2x^2 + 1, x \in N\}$

$\therefore$ $g \circ f(x) = g[f(x)] = g[2x+1] = (2x+1)^2$

Range of g o f $= \{y / y = (2x+1)^2 , x \in N\}$

18. Find x if gff(x) = fgg(x), given f(x) =3x+1 and g(x) = x+3

Solution:

$gff(x) = g[f\{f(x)\}] = g[f(3x+1)] = g[3(3x+1)+1] = g(9x+4)$

$g(9x+4) = [(9x+4)+3] = 9x+7$

$fgg(x) = f[g\{g(x)\}] = f[g(x+3)] = f[(x+3)+3] = f(x+6)$

$f(x+6) = [3(x+6)+1] = 3x+19$

$gff(x) = fgg(x)$

Thus two quantities begin equal we get $9x+7 = 3x+19$

$9x - 3x = 12 \Rightarrow 6x = 12 \Rightarrow x = 2$

19. If f(x) = 2x + 3, g(x) = 1 – 2x and h(x) = 3x. Prove that f o (g o h) = (f o g) o h.

Solution:

$f(x) = 2x + 3$, $g(x) = 1 - 2x$, $h(x) = 3x$

Now $(f \circ g)(x) = f(g(x))$

$= f(1-2x) = 2(1-2x)+3 = 2-4x+3 = 5-4x$

Then, $(f \circ g) \circ h(x) = (f \circ g)(h(x))$

$= (f \circ g)(3x) = 5-4(3x) = 5-12x$ .... (1)

$(g \circ h)(x) = g(h(x)) = g(3x) = 1-2(3x) = 1-6x$

So, f o (g o h) (x) = f$(1-6x)$

$$=2(1-6x)+3 = 2-12x+3 = 5-12x \quad .... (2)$$

From (1) and (2) we get (f o g) o h = f o (g o h). Hence Proved.

Alter:

$$\begin{aligned}
f \circ (g \circ h) &= (f \circ g) \circ h \\
(2x+3) \circ [(1-2x) \circ 3x] &= [(2x+3) \circ (1-2x)] \circ 3x \\
(2x+3) \circ [1-2(3x)] &= [2(1-2x)+3] \circ 3x \\
(2x+3) \circ [1-6x] &= [2-4+3] \circ 3 \\
2(1-6x)+3 &= [5-4x] \circ 3x \\
5-12x &= 5-12 \\
f \circ (g \circ h) &= (f \circ g) \circ h
\end{aligned}$$

Hence it is Proved.

20. Consider the functions f(x), g(x), h(x) as given below. Show that (f o g) o h = f o (g o h) in each case.

(i) f(x) = x–1, g(x) = 3x + 1 and h(x) = x^2

(ii) f(x) = x^2, g(x) = 2x, and h(x) = x+4

(iii) f(x) = x–4, g(x) = x^2 and h(x) = 3x–5

Solution:

(i) f(x) = x–1, g(x) = 3x+1, h(x) = x^2

LHS

f o g = f[g(x)] = f[$3x+1$] = $3x+1-1 = 3x$

(f o g) o h = f o g[h(x)] = fog[x^2] = 3[x^2] = $3x^2$ .... (1)

RHS

g o h = g[h(x)] = g[$3x^2$] = $3x^2+1$

(f o g) o h = f [g o h] = f[$3x^2+1$] = $3x^2+1-1 = 3x^2$ .... (2)

From (1) & (2), we get (f o g) o h = f o (g o h)

Hence it is verified.

Alter:

$$\begin{aligned}
f \circ (g \circ h) &= (f \circ g) \circ h \\
(x-1) \circ [(3x+1) \circ x^2] &= [(x-1) \circ (3x+1)] \circ x^2 \\
(x-1) \circ [3x^2+1] &= [3x+1-1] \circ x^2 \\
3x^2+1-1 &= (3x) \circ 3x^2 \\
3x^2 &= 3x^2
\end{aligned}$$

$$f \circ (g \circ h) = (f \circ g) \circ h$$

Hence it is showed.

(ii) $\mathbf{f(x) = x^2,\ g(x) = 2x,\ h(x) = x+4}$

$fog = f[g(x)] \Rightarrow f[2x]$

$= [2x]^2 = 4x^2$

$$(f \circ g) \circ h = (f \circ g)\, h(x)$$
$$= (f \circ g)\,(x+4)$$
$$= 4(x+4)^2 \Rightarrow 4[x^2+8x+16]$$
$$= 4x^2+32x+64 \quad \text{......... (1)}$$
$$g \circ h = g[h(x)] = g\,(x+4)$$
$$= 2\,(x+4)$$
$$f \circ (g \circ h) = f[2(x+4)]$$
$$= [2(x+4)]^2$$
$$= 4x^2+32x+64 \quad \text{......... (2)}$$

From (1) & (2),

$(f \circ g) \circ h = f \circ (g \circ h)$ Verified.

Alter:

$$f \circ (g \circ h) = (f \circ g) \circ h$$
$$x^2 \circ [(2x) \circ (x+4)] = [x^2 \circ (2x)] \circ (x+4)$$
$$x^2 \circ [2(x+4)] = (2x) \circ (x+4)$$
$$[2(x+4)]^2 = [2(x+4)]^2$$
$$4[x^2+8x+16] = 4[x^2+8x+16]$$
$$f \circ (g \circ h) = (f \circ g) \circ h$$

Hence it is showed.

(iii) $\mathbf{f(x) = x - 4 \quad g(x) = x^2 \quad h(x) = 3x - 5}$

$fog = f[g(x)] = f[x^2] = x^2 - 4$

$(f \circ g) \circ h = f \circ g[h(x)] = f \circ g\,[3x-5] = (3x-5)^2 - 4$ .... (1)

$g \circ h = g[h(x)] = g\,[3x - 5] = (3x - 5)^2$

$f \circ (g \circ h) = f \circ (3x - 5)^2 = (3x - 5)^2 - 4$ .... (2)

From (1) & (2), we get $(f \circ g) \circ h = f \circ (g \circ h)$.

Hence it is verified.

Alter:

$$f \circ (g \circ h) = (f \circ g) \circ h$$

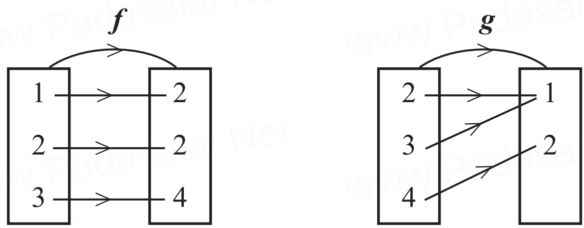
$$
\begin{aligned}
(x-4) \circ [x^2 \circ (3x-5)] &= [(x-4) \circ x^2] \circ (3x-5) \\
(x-4) \circ [(3x-5)^2] &= [x^2-4] \circ (3x-5) \\
(3x-5)^2-4 &= (3x-5)^2-4 \\
\text{f o (g o h)} &= \text{(f o g) o h}
\end{aligned}
$$

Hence it is showed.

Created Questions with Solution — 5 marks

1. **Let f = {(1, 2), (3, 4), (2, 2)} and g = {(2, 1), (3, 1), (4, 2)}. Illustrate "f" and "g" by means of arrow diagrams. Also Find i) f o g as a set of ordered pairs ii) g o f as a set of ordered pairs.**

Solution:



$$
\begin{aligned}
f \circ g\,(2) &= f[g(2)] \\
&= f(1) \\
&= 2 \\
f \circ g\,(3) &= f[g(3)] \\
&= f(1) \\
&= 2 \\
f \circ g\,(4) &= f[g(4)] \\
&= f(2) \\
&= 2
\end{aligned}
\qquad
\begin{aligned}
g \circ f\,(1) &= g\,[f(1)] \\
&= g(2) \\
&= 1 \\
g \circ f\,(2) &= g\,[f(2)] \\
&= g(2) \\
&= 1 \\
g \circ f\,(3) &= g\,[f(3)] \\
&= g(4) \\
&= 2
\end{aligned}
$$

$\therefore$ f o g = {(2, 2), (3, 2), (4, 2)} $\therefore$ g o f = {(1, 1), (2, 1), (3, 2)}

2. **Let $f, g : R \to R$ be defined as $f(x) = 2x-|x|$ and $g(x) = 2x+|x|$. Find f o g and g o f.**

Solution:

$$|x| = \begin{cases} -x & if \quad x<0 \\ x & if \quad x \geq 0 \end{cases}$$

$\therefore f(x) = 2x - |x|$

$$f(x) = \begin{cases} 2x-(-x) & if \quad x<0 \\ 2x-x & if \quad x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} 3x & if \quad x < 0 \\ x & if \quad x \geq 0 \end{cases}$$

$g(x) = 2x+|x|$

$$g(x) = \begin{cases} 2x - x & if \quad x < 0 \\ 2x + x & if \quad x \geq 0 \end{cases}$$

$$g(x) = \begin{cases} x & if \quad x < 0 \\ 3x & if \quad x \geq 0 \end{cases}$$

$$f \circ g[x] = f[g(x)]$$

$$= \begin{cases} f[x] & if \quad x < 0 \\ f[3x] & if \quad x \geq 0 \end{cases}$$

$$= \begin{cases} 3x & if \quad x < 0 \\ 3x & if \quad x \geq 0 \end{cases}$$

$f \circ g[x] = 3x$ for all the values of x

$$g \circ f[x] = g[f(x)]$$

$$= \begin{cases} g(3x) & if \quad x < 0 \\ g(x) & if \quad x \geq 0 \end{cases} \Rightarrow \begin{cases} 3x & if \quad x < 0 \\ 3x & if \quad x \geq 0 \end{cases}$$

$g \circ f[x] = 3x$ for all the values of x

3. If $f : R \to R$ is defined as $f(x) = 2x^2 - 1$. Find the pre-images of 17, 4 and –2.

Solution:

$f(x) = 2x^2-1$ $17 = 2x^2-1$ $17+1 = 2x^2$ $2x^2 = 18$ $x^2 = 9$ $x = \pm 3$	$f(x) = 2x^2-1$ $4 = 2x^2-1$ $4+1 = 2x^2$ $2x^2 = 5$ $x^2 = \frac{5}{2} \Rightarrow x = \pm\sqrt{\frac{5}{2}}$
The pre-images of 17 are 3 and –3	The pre-images of 4 are $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$

$f(x) = 2x^2-1$ $-2 = 2x^2-1$ $1-2 = 2x^2$ $2x^2 = -1$ $x^2 = -\frac{1}{2} \Rightarrow x = \sqrt{-\frac{1}{2}}$	On any account we should not have a negative number inside the square root.

$\therefore$ the element -2 does not have pre-image.

4. Write the domain of the following functions:

i) $f(x) = \dfrac{2x+1}{x-9}$ **ii)** $P(x) = \dfrac{-5}{4x^2+1}$

iii) $g(x) = \sqrt{x-2}$ **iv)** $h(x) = x + 6$

Solution:

i)	Domain $= R - \{9\}$	since the function is defined for all the values of x except when $x = 9$.
ii)	Domain $= R$	Since the function is defined for all the real values of "x".
iii)	Domain $= [2, \infty)$	Since we've a negative number inside the square root when $x<2$ $\therefore x < 2$ won't be possible.
iv)	Domain $= R$	Since the function is defined for all the real values of "x".

For Practice — 5 marks

1. Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \leq x < 2\}$ and $C = \{x \in N \mid x < 3\}$. Then verify that $A\times(B\cap C) = (A\times B) \cap (A\times C)$.
2. Let $f: A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$. Represent f by (i) set of ordered pairs (ii) a table (iii) an arrow diagram (iv) a graph.
3. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$
4. If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$ then which of the following relations are functions from X to Y?

(i) $R_1 = \{(-5, a), (1, a), (3, b)\}$ (ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$
(iii) $R_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

5. Represent the function f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} through (i) an arrow diagram (ii) a table form (iii) a graph.

Created Questions — 5 marks

1. Given $f(x) = \begin{cases} 3x-2, & -5 \le x < 0 \\ x+3, & 0 \le x < 5 \\ 2x-3, & 5 \le x < 10 \end{cases}$

   find (i) $f(-4) + f(4) + f(8)$

   (ii) $\dfrac{2f(-3)+f(2)}{f(7)+2f(-1)}$

2. Given $f(x) = x^2+4$, $g(x) = 3x-2$, $h(x) = x-5$ such that the composition of a functions is associative.

☙★★★❧

UNIT 2

NUMBERS AND SEQUENCES

Objective Type Questions — 1 mark

1. **Euclid's division lemma states that for positive integers *a* and b, then exist unique integers q and r such that *a* = bq + r where r must satisfy.**
   1) $1 < r < b$ 2) $0 < r < b$
   3) $0 \le r < b$ 4) $0 < r \le b$ **Ans:** 3)
2. **Using Euclid's' division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are**
   1) 0, 1, 8 2) 1, 4, 8
   3) 0, 1, 3 4) 1, 3, 5 **Ans:** 1)
3. **If the HCF of 65 and 117 is expressible in the form of 65m – 117 than, the value of m is**
   1) 4 2) 2
   3) 1 4) 3 **Ans:** 2)
4. **The sum of the exponents of prime factors in the prime factorization of 1729 is**
   1) 1 2) 2
   3) 3 4) 4 **Ans:** 3)
5. **The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is**
   1) 2025 2) 5220
   3) 5025 4) 2520 **Ans:** 4)
6. **$7^{4k} \equiv$ ________ (mod 100)**
   1) 1 2) 2
   3) 3 4) 4 **Ans:** 1)
7. **Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is**
   1) 3 2) 5
   3) 8 4) 11 **Ans:** 4)

8. The first term of an arithmetic progression is unity and the common difference is 4. which of the following will be a term of this A.P.

1) 4551 2) 10091
3) 7881 4) 13531 **Ans:** 3)

9. If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P. is

1) 0 2) 6
3) 7 4) 13 **Ans:** 1)

10. An A.P., consists of 31 terms. if its 16^{th} term is m, then the sum of all the terms of this A.P. is

1) 16m 2) 62 m
3) 31 m 4) $\frac{31}{2}$ m **Ans:** 3)

11. In an A.P. The first term is 1 and the common difference is 4. How many terms of the A.P. must be taken for their sum to be equal to 120?

1) 6 2) 7
3) 8 4) 9 **Ans:** 3)

12. If A = 2^{65} and B= $2^{64} + 2^{63} + 2^{62} + \dots + 2^{0}$ which of the following is true?

1) B is 2^{64} more than A 2) A and B are equal
3) B is larger then A by 1 4) A is larger than B by 1
Ans: 4)

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, \dots$ is

1) $\frac{1}{24}$ 2) $\frac{1}{27}$
3) $\frac{2}{3}$ 4) $\frac{1}{81}$ **Ans:** 3)

14. If the sequence $t_1, t_2, t_3 \dots\dots$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, \dots\dots$ is

1) a Geometric Progression
2) an Arithmetic Progression
3) neither an Arithmetic Progression nor a Geometric Progression
4) a constant sequence **Ans:** 2)

15. The value of $(1^3+2^3+3^3+\ldots\ldots+15^3)-(1+2+3+\ldots\ldots+15)$ is

1) 14400 2) 14200
3) 14280 4) 14520 **Ans:** 3)

Created Questions	**1 mark**

1. The sequence –3, –3, –3 ..... is

1) an A.P. only 2) a G.P only
3) neither A.P nor G.P 4) both A.P and G.P **Ans:** 4)

2. Given $f(x) = (-1)^x$ is a function from N to Z. Then the range of f is

1) {1} 2) N
3) {1, –1} 4) Z **Ans:** 3)

3. If $2 + 4 + 6 + \ldots + 2k = 90$, then the value of k is

1) 8 2) 9
3) 10 4) 11 **Ans:** 2)

4. If $\Sigma n = 55$, then $\Sigma n^3 =$ ______.

1) 3025 2) 166375
3) 1540 4) 55 **Ans:** 1)

5. $1 - x + x^2 - x^3 + x^4 =$ ______ $(x \neq 1)$

1) $-x^5$ 2) $\frac{x^5+1}{x+1}$
3) $\frac{x^5-1}{x+1}$ 4) none of these **Ans:** 2)

6. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., then which of the following is in A.P.

1) a, b, c 2) a^2, b^2, c^2
3) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ 4) none of these **Ans:** 3)

7. If m^{th} term of an A.P is $\frac{1}{n}$ and n^{th} term of an A.P., is $\frac{1}{m}$ then the sum of the first mn terms is ______.

1) mn+1 2) $\frac{mn+1}{2}$
3) $\frac{mn-1}{2}$ 4) $\frac{mn-1}{3}$ **Ans:** 2)

8. **Sum of n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+....$ is**

1) $\frac{n(n+1)}{2}$ 2) 2n(n+1)

3) $\frac{n(n+1)}{\sqrt{2}}$ 4) 1 **Ans:** 3)

9. **Which of the following is not a prime factor of 3825?**

1) 3 2) 5

3) 11 4) 17 **Ans:** 3)

10. **A radioactive sample decays and the remaining sample at infinite time is given by b = 1 – $\left[\frac{1}{2}+\frac{1}{4}+....\ to\ \infty\right]$**

1) 0 2) 1

3) $\frac{1}{\sqrt{2}}$ 4) $\frac{1}{2}$ **Ans:** 1)

Two Marks Questions — 2 marks

1. **Find the sum of 1 + 3 + 5 + ….. + 55.**

Solution:

$$1+3+5+....+l=\left(\frac{l+1}{2}\right)^2$$

$$1+3+5+....+55=\left(\frac{55+1}{2}\right)^2=\left(\frac{56}{2}\right)^2=28^2=784$$

2. **Find the sum of (i) 1+3+5+...to 40 terms (ii) 2+4+6+.....+80**

Solution:

(i) $1+3+5+...+\text{n terms}=n^2$

$1+3+5+...+40\text{ terms}=(40)^2=1600$

(ii) $2+4+6+...+80=2\,[1+2+3+...+40]$

$$=2\left[\frac{n(n+1)}{2}\right]=40\times 41=1640$$

3. **Find the sum of $1^2+2^2+...+19^2$**

Solution:

(i) $1^2+2^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$

$$1^2+2^2+...+19^2=\frac{19\times 20\times 39}{2\times 3}=19\times 130=2470$$

(ii) $1 + 4 + 9 + \ldots + 225 = 1^2 + 2^2 + 3^2 \ldots + 15^2$

$$= \frac{n(n+1)(2n+1)}{6} = \frac{15 \times 16 \times 31}{2 \times 3}$$

$$= 40 \times 31 = 1240$$

4. Find the sum of 1 + 2 + 3 + …. + 60

Solution:

$$1 + 2 + 3 + \ldots + 60 = \left[\frac{n(n+1)}{2}\right] = \frac{60 \times 61}{2} = 30 \times 61 = 1830$$

5. Find the sum of 3 + 6 + 9 + …. + 96

Solution:

$$3 + 6 + 9 + \ldots + 96 = 3(1 + 2 + 3 + \ldots + 32)$$

$$= 3\left[\frac{32 \times 33}{2}\right] = 3 \times 328 = 1584$$

6. Find the sum of 1 + 3 + 5 + …. + 71

Solution:

$1 + 3 + 5 + \ldots + 71$

$$= \left(\frac{l+1}{2}\right)^2 = \left(\frac{71+1}{2}\right)^2 = \left(\frac{72}{2}\right)^2 = 36^2 = 1296$$

7. Find the sum of $6^2 + 7^2 + 8^2 + \ldots + 21^2$

Solution:

$6^2 + 7^2 + 8^2 + \ldots + 21^2$

$$= (1^2 + 2^2 + 3^2 \ldots + 21^2) - (1^2 + 2^2 + 3^2 \ldots + 5^2)$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{21(21+1)(42+1)}{6} - \frac{5(5+1)(10+1)}{6}$$

$$= \frac{21 \times 22 \times 43}{6} - \frac{5 \times 6 \times 11}{6}$$

$$= 3311 - 55 = 3256$$

8. Find the sum of $1^3 + 2^3 + 3^3 + \ldots + 16^3$

Solution:

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$1^3 + 2^3 + 3^3 + \ldots + 16^3 = \left[\frac{16\times17}{2}\right]^2 = [136]^2 = 18496$$

9. If 1 + 2 + 3 + … + n = 666 then find n.

Solution:

$1 + 2 + 3 + \ldots\ldots + n = 666$

$$\left[\frac{n(n+1)}{2}\right] = 666$$

$$n^2 + n = 1332$$

$$n^2 + n - 1332 = 0$$

$$(n-36)(n+37) = 0$$

$n = -37$ or $n = 36$

But $n \neq -37$ (Since n is a natural number) Hence $n = 36$.

10. If 1 + 2 + 3 + ... + k = 325, then find $1^3 + 2^3 + 3^3 + \ldots + k^3$.

Solution:

$$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} = 325$$

$$1^3 + 2^3 + 3^3 + \ldots + k^3 = \left[\frac{k(k+1)}{2}\right]^2 = (325)^2 = 105625$$

11. If $1^3 + 2^3 + 3^3 + \ldots + k^3 = 44100$ then find 1 + 2 + 3 + ... + k

Solution:

$$1^3 + 2^3 + 3^3 + \ldots + k^3 = 44100 = \left[\frac{k(k+1)}{2}\right]^2$$

$$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} = \sqrt{44100} = 210$$

12. Euclid's Division Lemma.

Solution:

Let a and b ($a > b$) be any two positive integers. Then there exist unique integers q and r such that $a = bq + r,\ 0 \le r < b$.

13. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

Solution:

Using Euclid's Division Algorith

a = bq + r

532 = 21 q + r ⇒ 532 = 21 × 25 + 7

The remainder is 7.

No. of completed rows = 25,

left over flower pots = 7 pots.

$$\begin{array}{r|l} & 25 \\ 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & \;\;7 \end{array}$$

14. Prove that the product of two consecutive positive integers is divisible by 2.

Solution:

Let x and $x + 1$ be two consecutive positive integers. Then their product is $(x + 1)\,x$.

case 1 : when $x = 2k$ Let x be the even number

$\Rightarrow x\,(x + 1) = 2k\,(2k + 1)$ is divisible by 2.

case 2 : when $x = 2k + 1$ Let x be the odd number

$$\begin{aligned} x\,(x + 1) &= (2k + 1)\,(2k + 1 + 1) \\ &= (2k + 1)\,(2k + 2) \\ &= 2(2k + 1)\,(k + 1) \text{ is divisible by } 2 \end{aligned}$$

15. Using Euclid's Division Algorithm to find the Highest Common Factor (H.C.F) of 84, 90 and 120.

Solution:

Euclid's Division Algorithm a = bq + r

$90 = 84q + r\ (b \neq 0)$

$90 = 84 \times 1 + 6 \Rightarrow 84 = 6 \times 14 + 0$

∴ The H.C.F. of 84, 90 is 6. To find the H.C.F of 6 and 120 using Euclid's Division Algorithm.

$120 = 6 \times 20 + 0$. The remainder is zero.

∴ H.C.F. of 84, 90, 120 is 6.

16. If d is the Highest Common Factor of 32 and 60, find x and y satisfying d = 32x + 60y.

Solution:

Applying Euclid's Division Lemma, a = bq+r

$60 = 32 \times 1 + 28 \Rightarrow 32 = 28 \times 1 + 4$

$28 = 4 \times 7 + 0 \Rightarrow$ H.C.F of 32 and 60 is 4

That is d=4. d = $32x + 60y \Rightarrow 4 = 32x + 60y$

$4 = 32(2) + 60(-1) \Rightarrow \therefore x = 2, y = -1$

17. Prove that two consecutive positive integers are always co prime.

Solution:

Two consecutive positive integers be $x+1, x$

H.C.F of (a, b) = H.C.F of (a – b, b)

H.C.F of $(x+1, x)$ = H.C.F of $(x + 1 - x, x)$

H.C.F of $(x+1, x)$ = H.C.F of $(1, x)$

H.C.F of $(x+1, x) = 1 \therefore x + 1, x$ are co-prime numbers.

18. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where, p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4

Solution:

2	113400
2	56700
2	28350
3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

$113400 = 2^3 \times 3^4 \times 5^2 \times 7^1$

$\therefore p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7$

$x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1$

19. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

Solution:

$800 = a^b \times b^a$

$800 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$= 2^5 \times 5^2$

$\therefore a = 2, b = 5$ (or) $a = 5, b = 2$

20. What is the time 100 hours after 7 a.m.?

Solution:

$$100 \equiv x \pmod{24}$$
$$100 - x = 24\,n$$

$100 - x$ is a multiple of 24 ($100 - 4 = 96$)

$\therefore x$ must be 4.

The time 100 hrs after 7 a.m. is $= 7 + 4 = 11$ a.m

$$\begin{array}{r|l} & 4 \\ 24 & 100 \\ & 96 \\ \hline & 4 \end{array}$$

21. What is the time 15 hours before 11 p.m.?

Solution:

11 p.m. = 23 hours

Before 15 hours

$23 - 15 \equiv 8 \pmod{24}$

$\therefore$ The time 15 hours in the past was 8 a.m.

22. Find the least number that is divisible by the first ten natural numbers.

Solution:

The required number is the LCM of (1, 2, 3,……10)

$2 = 2 \times 1$

$3 = 3 \times 1$

$4 = 2 \times 2 = 2^2$

$5 = 5 \times 1$

$6 = 2 \times 3$

$7 = 7 \times 1$

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$10 = 2 \times 5$

$\therefore$ LCM $= 2^3 \times 3^2 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$

23. Solve: $5x \equiv 4 \pmod 6$

Solution:

$5x \equiv 4 \pmod 6$

$5x - 4 = 6k$

$5x = 6k + 4$

$x = \dfrac{6k+4}{5}$ k = 1, 6, 11, ….

If $k = 1 \Rightarrow x = \dfrac{6(1)+4}{5} = \dfrac{6+4}{5} = \dfrac{10}{5} = 2$

If $k = 6 \Rightarrow x = \dfrac{6(6)+4}{5} = \dfrac{36+4}{5} = \dfrac{40}{5} = 8$

If k = 11 $\Rightarrow x = \frac{6(11)+4}{5} = \frac{66+4}{5} = \frac{70}{5} = 14$

$\therefore x = 2, 8, 14, \ldots\ldots$

24. Find the number of integer solutions of $3x \equiv 1 \pmod{15}$

Solution:

$3x \equiv 1 \pmod{15}$ can be written as

$3x - 1 = 15k$ for some integer k

$3x = 15k + 1$

$x = \frac{15k+1}{3} \qquad \Rightarrow x = 5k + \frac{1}{3}$

Since 5k is an integer, $5k + \frac{1}{3}$ cannot be an integer.

So there is no integer solution.

25. Solve $8x \equiv 1 \pmod{11}$

Solution:

$8x - 1 \equiv 11n \Rightarrow 8x = 11n + 1 \Rightarrow x = \frac{11n+1}{8}$

when n = 5 $\Rightarrow x = 7$

when n = 13 $\Rightarrow x = 18\ldots\ldots$

26. Solve $3x - 2 \equiv 0 \pmod{11}$

Solution:

$3x - 2 \equiv 0 \pmod{11}$

$3x - 2 = 11k$

$3x = 11k + 2$

$x = \frac{11k+2}{3}$, k = 2, 5, 8, ...

k = 2 then $x = \frac{11(2)+2}{3} = \frac{22+2}{3} = \frac{24}{3} = 8$

k = 5 then $x = \frac{11(5)+2}{3} = \frac{55+2}{3} = \frac{57}{3} = 19$

k = 8 then $x = \frac{11(8)+2}{3} = \frac{88+2}{3} = \frac{90}{3} = 30$

$\therefore x = 8, 19, 30, \ldots\ldots$

27. Find the remainder when 2^{81} is divided by 17?

Solution:

$2^{81} \equiv (2^9)^9$ .... (1) $[2^9 \equiv 512]$

$2^9 \equiv 2 \pmod{17}$

$(1) \Rightarrow 2^{81} \equiv (2)^9 \pmod{17}$

$= 2 \pmod{17}$

∴ The remainder is 2.

$$\begin{array}{r|l} & 30 \\ 17 & 512 \\ & 510 \\ \hline & 2 \end{array}$$

28. Find the least positive value of x such that $67 + x \equiv 1 \pmod 4$.

Solution:

$67 + x \equiv 1$ (Mod 4)

$67 + x - 1 = 4n$

$66 + x = 4n$

$66 + x$ is a multiple of 4.

68 is the nearest multiple of 4 more than 66.

Therefore the least positive value of x is 2.

29. Find the least positive value of x such that $5x \equiv 4 \pmod 6$

Solution:

$5x \equiv 4 \pmod 6$

$5x - 4 = 6n$

$5x - 4$ is 6 times of n

∴ The least positive value of x is 2 ($\because$ $5(2) - 4 = 6$)

30. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?

Solution:

First Term , a = 20

Common Difference, d = 2

∴ Number of seats in the last row $= t_n = a + (n-1)d$

$$\begin{aligned} t_{30} &= a + 29d \\ &= 20 + 29(2) \\ &= 20 + 58 \\ &= 78 \end{aligned}$$

31. Find the 19^{th} term of an A.P. –11, –15, –19, ...

Solution:

Given a = –11; d = –15+11 = –4; n=19

$t_n = a+(n-1)d$

The 19th term is

$t_{19} = -11 + 18(-4) \Rightarrow -11 - 72$

$t_{19} = -83$

32. Which term of an A.P. 16, 11, 6, 1... is –54?

Solution:

$$n = \frac{l-a}{d} + 1$$

a = 16; d = 11–16= –5; l = –54

$$n = \frac{-54-16}{-5} + 1 \Rightarrow \frac{-70}{-5} + 1$$

n = 14 + 1

n = 15

33. Find the middle term(s) of an A.P. 9, 15, 21, 27, ..., 183 .

Solution:

Given a = 9, d = 6, l = 183

$$n = \frac{l-a}{d} + 1 \Rightarrow \frac{183-9}{6} + 1 \Rightarrow \frac{174}{6} + 1$$

$= 29 + 1 = 30$

$\therefore$ The middle terms are $\frac{30}{2}, \frac{30}{2} + 1$ i.e. 15th and 16th .

$t_n = a + (n-1)\,d$

$\therefore t_{15} = a + 14d = 9 + 14(6) = 9 + 84 = 93$

$t_{16} = a + 15d = 9 + 15(6) = 9 + 90 = 99$

$\therefore$ The 2 middle terms are 93, 99.

34. If 3 + k, 18 – k, 5k + 1 are in A.P, then find k.

Solution:

3 + k, 18 – k, 5k + 1 is an A.P

$$t_2 - t_1 = t_3 - t_2$$

$$(18-k) - (3+k) = (5k+1) - (18-k)$$

$$15 - 2k = 6k - 17$$

$$-2k - 6k = -17 - 15$$
$$-8k = -32$$
$$k = 4$$

35. In a G.P. 729, 243, 81,….. find t_7.

Solution:

$t_n = ar^{n-1}$

$a = 729,\ r = \frac{243}{729} = \frac{1}{3},\ n = 7$

$$t_7 = 729 \times \left(\frac{1}{3}\right)^{7-1} \Rightarrow 729 \times \left(\frac{1}{3}\right)^{6}$$

$$t_7 = 729 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$t_7 = 1$

36. Find the sum of first six terms of G.P. 5, 15, 45….

Solution:

Given $a = 5,\ r = \frac{15}{5} = 3,\ n = 6$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_6 = \frac{5(3^6 - 1)}{2} \Rightarrow \frac{5(729 - 1)}{2} \Rightarrow \frac{5(728)}{2}$$

$$= 5 \times 364$$

$$S_6 = 1820$$

37. Find x so that $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution:

Given $x + 6$, $x + 12$ and $x + 15$ are consecutive terms of a G.P.

$$\frac{t_2}{t_1} = \frac{t_3}{t_2} \Rightarrow \frac{x+12}{x+6} = \frac{x+15}{x+12}$$

$$(x + 12)^2 = (x + 6)(x + 15)$$

$$x^2 + 24x + 144 = x^2 + 21x + 90$$

$$24x - 21x = 90 - 144$$

$$3x = -54$$

$$x = -\frac{54}{3}$$

$$x = -18$$

38. Find the geometric progression whose first term and common ratios are given by a = –7, r = 6.

Solution:

a = –7, r = 6

The General form of Geometric Progression is a, ar, ar^2,…

∴ The required G.P = – 7, – 42, – 252

39. Is 7 × 5 × 3 × 2 + 3, a composite number? Justify your answer.

Solution:

7 × 5 × 3 × 2 + 3 = 213

Sum of the digits in 213 = 2 + 1 + 3 = 6

6 is divisible by 3.

∴ 213 is divisible by "3".

∴ 213 is not a prime number and it is a composite number.

Created Questions with Solution | 2 marks

1. An organization plans to plant saplings in 25 streets in a town in such a way that one sapling for the first street, three for the second, nine for the third and so on. How many saplings are needed to complete the work?

Solution:

Given it is an G.P.

∴ a = 1, r = 3, n = 25.

$$S_n = \frac{a(r^n-1)}{r-1} \Rightarrow \frac{3^{25}-1}{2}$$

2. In a G.P $\frac{1}{4}, -\frac{1}{2}$, 1, –2, ….. Find t_{10}.

Solution:

First Term, $a = \frac{1}{4}$

$$\text{Common ratio}, = \frac{t_2}{t_1} = \frac{-\frac{1}{2}}{\frac{1}{4}} = -2$$

$t_n = ar^{n-1}$

$t_{10} = ar^9$

$= \frac{1}{4}[-2]^9 \Rightarrow \frac{(-1)^9 . 2^9}{4} = -2^7$

3. Which term of an A.P 21, 18, 15,.... is –81? State with reason is there any term 0 in this A.P.

Solution:

First term, a = 21

Common Difference, d = –3

Last term, $l = -81$

Number of terms, $n = \frac{l-a}{d} + 1 \Rightarrow \left[\frac{-81-21}{-3}\right] + \Rightarrow 1 \left[\frac{-102}{-3}\right] + 1$

$= 34 + 1 = 35$

$\therefore$ – 81 is the 35^{th} term

t_n	=	a+(n–1)(d)	3n	=	24
0	=	21+(n–1) (–3)	n	=	24/3
0	=	21 – 3n + 3	n	=	8

$\therefore$ 0 is the 8^{th} term of the given Arithmetic Progression.

4. Find the common difference of an A.P. in which $t_{18} - t_{14} = 32$.

Solution:

$t_{18} - t_{14} = 32$

$t_n = a + (n-1)(d)$

$(a+17d) - (a+13d) = 32$

$a + 17d - a - 13d = 32$

$4d = 32$

$d = 8$

For Practice **2 marks**

1. Find the sum of 51 + 52 + 53 + ... + 92
2. Find the sum of $9^3 + 10^3 + \ldots\ldots 21^3$
3. Find the value of (i) 1+2+3+ ..... +50 (ii) 16+17+18+ ..... +75
4. How many terms of the series $1^3 + 2^3 + 3^3 + \ldots\ldots$ should be taken to get the sum 14400?
5. Find the sum to n term of the series 1 + 11 + 111 + ...
6. Find the indicated terms of the sequences whose n^{th} terms are given by
   (i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13} (ii) $a_n = -(n^2 - 4)$; a_4 and a_{11}
7. Find a_8 and a_{15} whose n^{th} term is $a_n = \begin{cases} \frac{n^2-1}{n+3}; & n \text{ is even}, n \in N \\ \frac{n^2}{2n+1}; & n \text{ is odd}, n \in N \end{cases}$
8. If $a_1 = 1$, $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$, $n \geq 3$, $n \in N$, then find the first six terms of the sequence.
9. First term a and common difference d are given below. Find the corresponding A.P.
   (i) a = 5, d = 6 (ii) a = 7, d = –5 (iii) $a = \frac{3}{4}$, $d = \frac{1}{2}$
10. Find x, y, and z, given that the numbers x, 10, y, 24, z are in A.P.
11. Find the 19^{th} term of an A.P. –11, –15, –19,.....
12. Find the sum of the following. (i) 3, 7, 11,…. upto 40 terms (ii) 102, 97, 92,….upto to 27 terms (iii) 6 + 13 + 20 +….+97
13. Find the 15^{th}, 24^{th} and n^{th} term (general term) of an A.P. given by 3, 15, 27, 39 ......
14. Find the number of terms in the A.P. 3, 6, 9, 12, ... 111.
15. Which term of an A.P. 16, 11, 6, 1….. is –54?
16. In a G.P. 729, 243, 81, …... find t_7.
17. Write the first three terms of the G.P. whose first term and the common ratio are given below.
    (i) a = 6, r = 3 (ii) $a = \sqrt{2}$, $r = \sqrt{2}$ (iii) a = 1000, $r = \frac{2}{5}$

18. Find the number of terms in the following G.P.
(i) 4, 8, 16, ….., 8192 (ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots \frac{1}{2187}$
19. Use Euclid's Division Algorithm to find the Highest Common Factor (H.C.F) of (i) 340 and 412 (ii) 867 and 255
20. If $13824 = 2^a \times 3^b$ then find a and b.
21. Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.
22. A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi?

Created Questions — 2 marks

1. When 2^{256} is divided by 17 what is the remainder?
2. The number $3^{13} - 3^{10}$ is divided by _____.
3. The sum of three non zero prime numbers is 100. One of them exceeds the other 36. Find the largest number.
4. Find the least number when which divided by 12, leaves remainder of 7 when divided by 15 leaves the remainder of 10 and when divided by 16 leaves a remainder of 11.
5. The sum of the p terms of an A.P. is the same as the sum of it's q terms then sum of it's q terms is [where $p \neq q$]

Five Marks Questions — 5 marks

1. Find the sum to n terms of the series 5 + 55 + 555 + ...

Solution:

$$S_n = 5 + 55 + 555 + \ldots + n \text{ terms}$$
$$= 5\,[1 + 11 + 111 + \ldots + n \text{ terms}]$$
$$= \frac{5}{9}\,[9 + 99 + 999 + \ldots + n \text{ terms}]$$
$$= \frac{5}{9}\,[10-1 + 100-1 + 1000-1 + \ldots n \text{ terms}]$$
$$= \frac{5}{9}\,[(10 + 100 + 1000 + \ldots) - (1 + 1 + 1 + \ldots)]$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{9} - n\right] = \frac{50}{81}\left[(10^n-1)\right] - \frac{5}{9}n$$

2. Find the sum to n terms of the series 3 + 33 + 333 + ... to n terms.

Solution:

S_n = 3 + 33 + 333 + ... n

S_n = 3 (1 + 11 + 111 +….. + n terms)

$$= \frac{3}{9}\ (9 + 99 + 999 + \ldots + \text{n term})$$

$$= \frac{3}{9}\ [(10-1) + (100-1) + (1000-1) + \ldots + \text{n term})]$$

$$= \frac{3}{9}[(10 + 100 + 1000 + \ldots + \text{n terms}) - (1 + 1 + 1 + \ldots + \text{n term})]$$

$$= \frac{3}{9}\left[10\left(\frac{10^n-1}{9}\right) - n\right] \qquad = \frac{30}{81}(10^n-1) - \frac{3n}{9}$$

3. Find the sum to n terms of the series 0.4 + 0.44 + 0.444 + ... n

Solution:

0.4 + 0.44 + 0.444 + ... n terms

$$S_n = \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ldots\ n\ terms$$

$$= 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots\ n\ terms\right]$$

$$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ n\ terms\right]$$

$$= \frac{4}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + \left(1-\frac{1}{1000}\right) + \ldots\ n\ terms\right]$$

$$= \frac{4}{9}\left[n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots\ n\ terms\right)\right]$$

$$= \frac{4}{9}\left[n - \frac{1}{10}\left[\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right]\right] = \frac{4}{9}\left[n - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right]$$

4. Find the sum of $15^2 + 16^2 + 17^2 + ... + 28^2$

Solution:

$15^2 + 16^2 + 17^2 + ... + 28^2$

$$= 1^2 + 2^2 + 3^2 ... + 28^2 - 1^2 + 2^2 + 3^2 ..... + 14^2$$

$$= \sum_{n:1}^{28} - \sum_{n:1}^{14}$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{28\times 29\times 57}{2\times 3} - \frac{14\times 15\times 29}{2\times 3}$$

$$= 14 \times 29 \times 19 - 7 \times 5 \times 29$$

$$= 7714 - 1015 = 6699$$

5. Find the sum of the following series $10^3 + 11^3 + 12^3 + ... + 20^3$

Solution:

$10^3 + 11^3 + 12^3 + ... + 20^3$

$$= 1^3 + 2^3 + 3^3 + ... + 20^3 - 1^3 + 2^3 + 3^3 + ... + 9^3$$

$$= \sum_{n:1}^{20} - \sum_{n:1}^{9}$$

$$= \left[\frac{n(n+1)}{2}\right]^2 - \left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{20\times 21}{2}\right]^2 - \left[\frac{9\times 10}{2}\right]^2$$

$$= [210]^2 - (45)^2$$

$$= 44100 - 2025 = 42075$$

6. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm, ….. 24 cm. How much area can be decorated with these colour papers?

Solution:

$$\text{Area} = 10^2 + 11^2 + 12^2 + \ldots\ldots + 24^2$$

$$= \sum_{n:1}^{24} - \sum_{n:1}^{10}$$

$$= (1^2 + 2^2 + 3^2 + \ldots\ldots + 24^2) - (1^2 + 2^2 + \ldots\ldots + 9^2)$$

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6}$$

$$= 4900 - 285 = 4615 \text{ cm}^2$$

Therefore Rekha has 4615 cm^2 colour papers. She can decorate 4615cm^2 area with these colour papers.

7. The sum of the cubes of the first n natural numbers is 2025, then find the value of n.

Solution:

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = 2025$$

$$\left[\frac{n(n+1)}{2}\right]^2 = 2025 \Rightarrow \frac{n(n+1)}{2} = \sqrt{2025} = 45$$

$$n(n+1) = 45 \times 2$$

$$n(n+1) = 90$$

$$n^2+n-90 = 0$$

$$(n+10)(n-9) = 0$$

$$n = 9 \text{ or } -10$$

$$\therefore n = 9$$

8. Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Solution:

$$301 + 308 + 315 + \ldots.. + 595 = ?$$

$$a = 301;\ d = 7;\ l = 595$$

$$n = \frac{l-a}{d} + 1 \qquad n = \frac{595-301}{7} + 1$$

$$n = \frac{294}{7} + 1$$

$$n = 42 + 1$$

$$n = 43$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{43} = \frac{43}{2}(301 + 595)$$

$$= \frac{43}{2}(896)$$

$$= 43 \times 448$$

$$S_{43} = 19264$$

9. The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.

Solution:

$23.30 + 11 \equiv x \pmod{24}$

$34.30 \equiv x \pmod{24}$

$34.30 - x = 24n$

$34.30 - x$ is a multiple of 24. X must be 10.30

Chennai is 4.30 hours ahead to London.

The flight lands at London Airport at 10. 30 – 4.30 Monday 6.00 a.m.

10. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.

Solution:

	1 year	**2 year**
Income	₹15,000	₹16,500
Expenses	₹13,000	₹13,900
Savings	₹2,000	₹2,600

∴ **Annual Savings** ₹ 2,000, ₹ 2,600, ₹ 3,200 ....

here a = 2,000; d = 600; $t_n = 20,000$

$$a + (n-1)d = t_n$$

$$2000 + (n-1)600 = 20,000$$

$$600n - 600 = 18,000$$

$$600n = 18,600$$

$$n = \frac{186}{6} \Rightarrow n = 31 \text{ years}$$

The savings of Priya after 31 years is ₹ 20,000.

11. A man saved ₹16,500 in ten years. In each year after the first he saved ₹100 more than he did in the preceding year. How much did he save the first year?

Solution:

S_n = ₹16,500; d = ₹100; n = 10

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{10}{2}[2a + 9(100)] = 16500$$

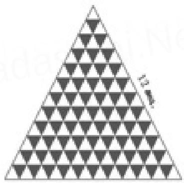
$$2a + 900 = \frac{16500}{5}$$

$$2a = 2400$$

$$\therefore a = 1200$$

$\therefore$ A man saved Rs.1,200 in the first year.

12. A Mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.



Solution:

Since the mosaic is designed in the shape of an equilateral triangle of 12 ft, and the tile is in the shape of an equilateral triangle of 12 inch (1 ft), there will be 12 rows in the mosaic.

From the figure, it is clear that number of white tiles in each row are 1, 2, 3, 4, ..., 12 which clearly forms an A.P.

Similarly the number of blue tiles in each row are

0, 1, 2, 3, ……, 11 which is also an an A.P.

Number of White Tiles $= 1 + 2 + 3 + ... + 12$

$$= \frac{12}{2}(1 + 12) = 78$$

$$\because S_n = \frac{n}{2}[a + l]$$

Number of Blue Tiles $= 0+1+2+3+.....+11$

$= \frac{12}{2}(0+11) = 66$

Total number of tiles in the mosaic = 78 + 66 = 144

13. If $S_1, S_2, S_3, ..., S_m$ are the sums of n terms of m A.P.'s whose first terms are 1, 2, 3, ... m and whose common differences are 1, 3, 5 ...... (2m–1) respectively, then show that $S_1 + S_2 + S_3 + ... + S_m = \frac{1}{2}mn(mn+1)$.

Solution:

$S_1 = \frac{n}{2}[2(1)+(n-1)1]$ $\quad \because S_n = \frac{n}{2}[2a+(n-1)d]$

$S_2 = \frac{n}{2}[2(2)+(n-1)3]$

$S_3 = \frac{n}{2}[2(3)+(n-1)5]$

.
.
.

$S_m = \frac{n}{2}[2(m)+(n-1)(2m-1)]$

$S_1 + S_2 + S_3, ..., + S_m$

$= \frac{n}{2}[2(1+2+3+...+m)+(n-1)(1+3+5+....+(2m-1)]$

$= \frac{n}{2}[2\frac{m(m+1)}{2}+(n-1)m^2]$

$= \frac{n}{2}[m^2+m+m^2n-m^2]$ $\quad \Rightarrow \frac{n}{2}[m^2n+m]$

$= \frac{1}{2}mn[mn+1]$ $\quad \therefore$ Hence Proved.

14. If a, b, c are in A.P. then show that $3^a, 3^b, 3^c$ are in G.P.

Solution:

If a, b, c are in A.P.

$t_2 - t_1 = t_3 - t_2$

$b - a = c - b$

$$2b = a + c \quad \text{.......... (1)}$$

To prove $3^a, 3^b, 3^c$ are in G.P.

If a,b,c are in G.P.

$$\frac{t_2}{t_1} = \frac{t_3}{t_2} \Rightarrow \frac{b}{a} = \frac{c}{b}$$

$$b^2 = ac$$

$$(3^b)^2 = 3^a . 3^c$$

$$\Rightarrow 3^{2b} = 3^{a+c}$$

$\therefore 3^a, 3^b, 3^c$ are in G.P.

15. Sivamani is attending an interview for a job and the company gave two offers to him.

Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years.

Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years. What is his salary in the 4th year with respect to the offers A and B?

Solution:

Offer A:

P = ₹ 20,000; r = 6%;

n = 3 (4th Year)

$$A = P\left(1+\frac{r}{100}\right)^3$$

$$= 20{,}000\left(1+\frac{6}{100}\right)^3$$

$$= 20{,}000\left(\frac{106}{100}\right)^3$$

$$= 20{,}000(1.06)^3$$

$$= ₹\ 23{,}820$$

Offer B:

P = ₹ 22,000

r = 3%

$$A = P\left(1+\frac{r}{100}\right)^3$$

$$= 22{,}000\left(1+\frac{3}{100}\right)^3$$

$$= 22{,}000\left(\frac{103}{100}\right)^3$$

$$= 22{,}000(1.03)^3$$

$$= ₹\ 24{,}040$$

16. If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P., then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

Solution:

a, b, c are three consecutive terms of an A.P.

Let a = a, b = a + d, c = a + 2d

x, y, z are three consecutive terms of an G.P.

$x = a, y = ar, z = ar^2$

LHS $= x^{b-c} \times y^{c-a} \times z^{a-b}$

$= a^{a+d-a-2d} \times (ar)^{a+2d-a} \times (ar^2)^{a-a-d}$

$= a^{-d} \times a^{2d} r^{2d} \times a^{-d} r^{-2d}$

$= a^{-2d} \times a^{2d} \times r^{2d} \times r^{-2d}$

$= a^{-2d+2d} \times r^{2d-2d}$

$= a^0 \times r^0 = 1 \times 1 = 1$

17. If $S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$ n terms then prove that $(x - y) S_n = \left[\dfrac{x^2(x^n-1)}{x-1} - \dfrac{y^2(y^n-1)}{y-1}\right]$

Solution:

$S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$ n terms

$\Rightarrow (x-y) S_n = (x-y)(x+y) + (x-y)(x^2 + xy + y^2) + (x-y)(x^3 + x^2y + xy^2 + y^3) + \ldots\ldots$ n terms

$\Rightarrow (x - y) S_n = (x^2 - y^2) + (x^3 + y^3) + (x^4 - y^4) + \ldots\ldots$ n terms

$= (x^2 + x^3 + x^4 + \ldots\ldots$ n terms$) - (y^2 + y^3 + y^4 + \ldots\ldots$ n terms$)$

Here $a = x^2, r = x$ Here $a = y^2, r = y$

$\Rightarrow (x - y) S_n = \dfrac{x^2(x^n-1)}{x-1} - \dfrac{y^2(y^n-1)}{y-1}$ $\quad \because S_n = \dfrac{a(r^n-1)}{r-1}$

Hence Proved.

18. The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number?

Solution:

Let Senthil's house number be x.

$1+2+3+ \ldots + (x-1) = (x+1) + (x+2) + \ldots + 49$

$1+2+3+ \ldots + (x-1) = [1+2+3+\ldots+49] - [1+2+3+\ldots+x]$

$\dfrac{(x-1)}{2}[1 + (x-1)] = \dfrac{49}{2}[1 + 49] - \dfrac{x}{2}[1+x]$

$\because S_n = \dfrac{n}{2}[a+l]$

$$\frac{x(x-1)}{2} = \frac{49(50)}{2} - \frac{x(x+1)}{2}$$
$$x^2 - x = 2450 - x^2 - x$$
$$2x^2 = 2450$$
$$x^2 = 1225$$
$$x = 35$$

∴ Senthil's house number = 35

19. Find the sum of the Geometric Series 3 + 6 + 12 + ... + 1536

Solution:

3 + 6 + 12 + ... + 1536

a = 3, r = 2

$$t_n = 1536$$
$$ar^{n-1} = 1536$$
$$3(2)^{n-1} = 1536$$
$$3(2)^{n-1} = 3(2)9$$
$$2^{n-1} = 2^9$$

n = 10

$$\therefore S_n = \frac{a(r^n-1)}{r-1} \Rightarrow S_{10} = \frac{3(2^{10}-1)}{2-1}$$
$$= 3\,(1023) = 3069$$

20. A man repays a loan of ₹65,000 by paying ₹400 in the first month and then increasing the payment by ₹300 every month. How long will it take for him to clear the loan?

Solution:

400 + 700 + 1000 + ... n months = ₹65,000

a = 400, d = 300, S_n = 65,000

$$\frac{n}{2}\,[2a + (n-1)d] = 65{,}000$$
$$\frac{n}{2}\,[800 + (n-1)300] = 65{,}000$$
$$n[400 + (n-1)150] = 65{,}000$$
$$n[150n + 250] = 65{,}000$$
$$n[3n + 5] = 1{,}300$$
$$3n^2 + 5n - 1300 = 0$$

$\Rightarrow \quad n = 20, -\frac{65}{3} \qquad \therefore\ n = 20$

21. A man joined a company as Assistant Manager. The company gave him a starting salary of ₹60,000 and agreed to increase his salary 5% annually. What will be his salary after 5 years?

Solution:

Given initial salary = ₹ 60,000

Annual Increment = 5%

Salary increment at the end of 1 year = $60,000 \times \frac{5}{100} = 3000$

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 60,000\left(1+\frac{5}{100}\right)^n \quad = 60,000\left(\frac{105}{100}\right)^5$$

$$= 60,000 \times (1.05)^5 \qquad \approx ₹\ 76,600$$

22. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

Solution:

Let the 3 consecutive terms in an A.P. be a–d, a, a+d

Sum of three terms $a-d + a + a+d = 27$

$3a = 27$

$a = \frac{27}{3}$

$a = 9$

Product of three terms $(a-d)(a)(a+d) = 288$

$a(a^2 - d^2) = 288$

$9(9^2 - d^2) = 288$

$81 - d^2 = \frac{288}{9}$

$81 - d^2 = 32$

$49 = d^2 \quad \therefore d = \pm 7$

$\therefore$ The three terms of A.P are 2, 9, 16 (or) 16, 9, 2

Created Questions with Solution — 5 marks

1. In a G.P. the 9^{th} term is 32805 and term is 1215. Find the 12^{th} term.

Solution:

$t_9 = 32805$, $t_6 = 1215$, $t_{12} = ?$

$t_n = ar^{n-1}$

$t_9 = ar^8 = 32805$ .... (1)

$t_6 = ar^5 = 1215$ .... (2)

$(1) \div (2)\ \dfrac{ar^8}{ar^5} = \dfrac{32805}{1215}$

$r^3 = 27, r = 3$

substitute $r = 3$ in (2)

$ar^5 = 1215$

$a(3)^5 = 1215$

$a \times 243 = 1215, \quad a = \dfrac{1215}{243}$

$a = 5$

$a = 5, r = 3, n = 12$

$t_n = ar^{n-1}$

$t_{12} = 5(3)^{12-1} \Rightarrow t_{12} = 5(3)^{11}$

$\therefore$ The 12^{th} term $= 5 \times 3^{11}$

2. Find the sum of all 3 digit natural numbers which are divisible by 9.

Solution:

The three digit number divisible by 9 are 108, 117, 126, ..., 999

Their sum 108 + 117 + 126 + ... + 999

Here $a = 108$, $d = 9$, $l = 999$

$$n = \frac{l \quad a}{} + 1 \Rightarrow \frac{999-108}{9} + 1 \Rightarrow \frac{891}{9} + 1$$

$$= 99 + 1 \Rightarrow n = 100$$

$$S_n = \frac{n}{2}[a+l] \Rightarrow S_{100} = \frac{100}{2}[108 + 999]$$

$$= 50 \times 1107$$

$$S_{100} = 55350$$

$$S100 = 55350$$

3. **If the sum of the first p terms of an A.P is $ap^2 + bp$. Find its common difference.**

Solution:

$$S_p = ap^2 + bp$$
$$S_{(p-1)} = a(p-1)^2 + b(p-1)$$
$$\begin{aligned} t_p &= S_p - S_{(p-1)} \\ &= ap^2 + bp - [a(p-1)^2 + b(p-1)] \\ &= ap^2 + bp - [a(p^2 + 1 - 2p) + b(p-1)] \\ &= ap^2 + bp - [ap^2 + a - 2ap + bp - b] \\ &= ap^2 + bp - ap^2 - a + 2ap - bp + b \\ t_p &= 2ap + b - a \\ t_1 &= 2a + b - a \quad \Rightarrow \quad a + b \\ t_2 &= 4a + b - a = 3a + b \\ d &= t_2 - t_1 \quad \Rightarrow \quad (3a+b) - (a+b) \\ &= 3a + b - a - b = 2a \end{aligned}$$

4. **In an A.P. if m^{th} term is n and the n^{th} term is m where m n, find the p^{th} term.**

Solution:

We have $t_m = a + (m-1)d = n$ .... (1)

$t_n = a + (n-1)d = m$ .... (2)

Solving (1) and (2), we get

$$\begin{aligned} (m-n)d &= n-m \\ d &= -1 \end{aligned}$$

and

$$\begin{aligned} a &= m+n-1 \\ t_p &= a+(p-1)d \\ &= m+n-1 + (p-1)(-1) \\ &= m+n-1 - p + 1 \\ t_p &= m+n-p \end{aligned}$$

Hence p^{th} term is $m+n-p$

5. **Insert 6 numbers between 3 and 24 such that resulting sequence is an A.P.**

Solution:

Let A_1, A_2, A_3, A_4, A_5 and A_6 be six numbers between 3 and 24 such that $3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ are in A.P.

Here a = 3, t = 24, n = 8

therefore
$$t_n = a+(n-1)d$$
$$24 = 3+7d$$
$$7d = 21$$
$$d = 3$$

$A_1 = a+d = 3+3 = 6$ $\quad$ $A_2 = a+2d = 9$

$A_3 = a+3d = 12$ $\quad$ $A_4 = a+4d = 15$

$A_5 = a+5d = 18$ $\quad$ $A_6 = a+6d = 21$

Hence six numbers between 3 and 24 are 6, 9, 12, 15, 18 and 21.

6. If S_1, S_2, S_3 are the sum of first n natural numbers their squares and their cubes respectively. Show that $9S_2^2 = S_3(1+8S_1)$

Solution:

S_1 = Sum of first n natural number = $\dfrac{n(n+1)}{2}$

S_2 = Sum of first n squares = $\dfrac{n(n+1)(2n+1)}{6}$

S_3 = Sum of first n cubes = $\left[\dfrac{n(n+1)}{2}\right]^2$

We prove that $9S_2^2 = S_3(1+8S_1)$

Now
$$S_2^2 = \frac{n^2(n+1)^2(2n+1)^2}{36}$$

$$\text{L.H.S} = 9S_2^2 = \frac{9n^2(n+1)^2(2n+1)^2}{36}$$
$$= \frac{1}{4}n^2(n+1)^2(2n+1)^2$$

$$1+8S_1 = 1+\frac{8n(n+1)}{2} = 1+4n(n+1)$$

$$\text{R.H.S} = S_3(1+8S_1) = \left[\frac{n(n+1)}{2}\right]^2 [1+4n^2+4n]$$
$$= \frac{1}{4}n^2(n+1)^2(2n+1)^2$$

$$\text{L.H.S} = \text{R.H.S}$$

hence solved

7. If the Sum of n terms of an A.P is nP + $\frac{1}{2}$n(n–1)Q where P and Q are constants, find the common difference.

Solution:

Let $a_1, a_2, a_3, \ldots a_n$ be the given AP then

$$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n = np + \frac{1}{2}n(n-1)Q$$

$$\therefore \quad S_1 = a_1 = P$$

$$S_2 = a_1 + a_2 = 2P + Q$$

$$a_2 = S_2 - S_1 = 2P + Q - P \Rightarrow a_2 = P + Q$$

Common difference

$$d = a_2 - a_1 = P + Q - P = Q$$

$\therefore$ The common difference = Q

8. The sum of n terms of two arithmetic progressions are in the ratio (3n+8) : (7n+15). Find the ratio of their 12th terms.

Solution:

Let a_1, a_2 and d_1, d_2 be the first terms and common difference of the first and second arithmetic progression, respectively, According to the given condition we have

$$\frac{sum\ to\ n\ terms\ of\ first\ A.P}{sum\ to\ n\ terms\ of\ second\ A.P} = \frac{3n+8}{7n+15}$$

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{3n+8}{7n+15}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{3n+8}{7n+15}$$

$$\frac{12^{th}\ term\ of\ first\ A.P}{12^{th}\ term\ of\ second\ A.P} = \frac{a_1 + 11d_1}{a_2 + 11d_2}$$

$$\frac{2a_1 + 22d_1}{2a_2 + 22d_2} = \frac{3(23)+8}{7(23)+15} = \frac{77}{176} \quad \text{[By putting n = 23 in (1)]}$$

$$\frac{a_1 + 11d_1}{a_2 + 11d_2} = \frac{12^{th}\ term\ of\ first\ A.P}{12^{th}\ term\ of\ second\ A.P} = \frac{7}{16}$$

Hence the required ratio is 7:16

9. How many terms of the G.P. 3, $\frac{3}{2}, \frac{3}{4}$ ... are needed to give the sum $\frac{3069}{512}$

Solution:

Let n be the number of terms needed

Given that a = 3, r = $\frac{1}{2}$ and $S_n = \frac{3069}{512}$

Since $S_n = \frac{a(1-r^n)}{1-r} \Rightarrow \frac{3069}{512} = \frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}$

$$\frac{3069}{512} = \frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{\frac{1}{2}} \Rightarrow \frac{3069}{512} = 6\left[1-\left(\frac{1}{2}\right)^n\right]$$

$$\frac{3069}{512\times 6} = 1-\frac{1}{2^n} \Rightarrow \frac{3069}{3072} = 1-\frac{1}{2^n}$$

$$\frac{1}{2^n} = 1 - \frac{3069}{3072} \Rightarrow \frac{1}{2^n} = \frac{3}{3072}$$

$$\frac{1}{2^n} = \frac{1}{1024} \Rightarrow \frac{1}{2^n} = \frac{1}{2^{10}}$$

$$n = 10$$

10. A person has 2 parents, 4 grand parents, 8 great grand parents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Solution:

Here a = 2, r = 2 and n = 10

$$S_n = \frac{a(r^n-1)}{r-1}$$

We have $S_{10} = \frac{2(2^{10}-1)}{2-1} = 2046$

Hence the number of ancestors preceding the person is 2046.

11. How many terms of G.P 3, 3^2, 3^3 . . . are needed to give the sum 120?

Solution:

We have a = 3, r = 3, S_n = 120

$$\text{Therefore} \quad S_n = \frac{a(r^n - 1)}{r-1}$$

$$120 = \frac{3(3^n - 1)}{2}$$

$$3^n - 1 = 80$$

$$3^n = 81$$

$$3^n = 3^4$$

$$n = 4$$

For Practice **5 marks**

1. The sum of first n, 2n, and 3n terms of A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.
2. In a Geometric Progression, the 4^{th} term is $\frac{8}{9}$ and the 7^{th} term is $\frac{64}{243}$. Find the Geometric Progression.
3. The 104^{th} term and 4^{th} term of an A.P. are 125 and 0. Find the sum of first 35 terms.
4. Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.
5. If l^{th}, m^{th} and n^{th} terms of an A.P are x, y, z respectively, then show that (i) $x(m-n) + y(n-l) + z(l-m) = 0$
   (ii) $(x-y)n + (y-z)l + (z-x)m = 0$
6. In an A.P., sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.
7. Find the sum of the series to
   $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + \ldots$ to (i) n terms (ii) 8 terms

Created Questions **5 marks**

1. If (p + 1) the term of an A.P is twice $(q + 1)^{th}$ term, prove that the $(3p + 1)^{th}$ term is twice the $(p + q + 1)^{th}$ term.
2. Solve $1 + 6 + 11 + 16 + \ldots + x = 148$.
3. Find 5 numbers in G.P. such that their product is 32 and the product of the last two numbers is 108.
4. Show that the G.P the product of any two terms equidistant from the beginning and the end is equal to the product of the first and the last term.
5. Find the value of $\sqrt[3]{25\sqrt[3]{25\sqrt[3]{25}}}\ldots\ldots$
6. If $S_1, S_2, S_3 \ldots\ldots S_p$ denote the sums of infinite GPs whose first terms are 1, 2, 3, ... p respectively and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots \frac{1}{(p+1)}$ respectively show that $S_1 + S_2 + S_3 + \ldots\ldots S_p = \frac{p(p+3)}{2}$
7. If the sum of 7 terms of an A.P is 49 and that of 17 terms is 289 find the sum of n terms.
8. Show that one and only one out of n, n+2, n+4 is divisible by 3 when n is any positive integer.

☙★★★❧

UNIT 3 ALGEBRA

Objective Type Questions — 1 mark

1. **A system of three linear equations in three variables is inconsistent if their planes**
   1) intersect only at a point 2) intersect in a line
   3) coincides with each other 4) do not intersect **Ans:** 4)

2. **The solution of the system x+y–3z = –6, –7y+7z=7, 3z=9 is**
   1) x = 1, y = 2, z = 3 2) x = –1, y = 2, z = 3
   3) x = –1, y = –2, z = 3 4) x = 1, y = –2, z = 3 **Ans:** 1)

3. **If (x – 6) is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is**
   1) 3 2) 5
   3) 6 4) 8 **Ans:** 2)

4. $\dfrac{3y-3}{y} \div \dfrac{7y-7}{3y^2}$ **is**
   1) $\dfrac{9y}{7}$ 2) $\dfrac{9y^3}{(21y-21)}$
   3) $\dfrac{21y^2-42y+21}{3y^3}$ 4) $\dfrac{7(y^2-2y+1)}{y^2}$ **Ans:** 1)

5. $y^2 + \dfrac{1}{y^2}$ **is not equal to**
   1) $\dfrac{y^4+1}{y^2}$ 2) $\left[y+\dfrac{1}{y}\right]^2$
   3) $\left[y-\dfrac{1}{y}\right]^2+2$ 4) $\left[y+\dfrac{1}{y}\right]^2-2$ **Ans:** 2)

6. $\dfrac{x}{x^2-25} - \dfrac{8}{x^2+6x+5}$ **gives**
   1) $\dfrac{x - 7x+40}{(x-5)(x+5)}$ 2) $\dfrac{x^2+7x+40}{(x-5)(x+5)(x+1)}$

3) $\dfrac{x^2-7x+40}{(x^2-25)(x+1)}$ 4) $\dfrac{x^2+10}{(x^2-25)(x+1)}$ **Ans:** 3)

7. **The square root of $\dfrac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to**

1) $\dfrac{16}{5}\left|\dfrac{x^2z^4}{y^2}\right|$ 2) $16\left|\dfrac{y^2}{x^2z^4}\right|$

3) $\dfrac{16}{5}\left|\dfrac{y}{xz^2}\right|$ 4) $\dfrac{16}{5}\left|\dfrac{xz^2}{y}\right|$ **Ans:** 4)

8. **Which of the following should be added to make $x^2 + 64$ a perfect square?**

1) $4x^2$ 2) $16x^2$

3) $8x^2$ 4) $-8x^2$ **Ans:** 2)

9. **The solution of $(2x - 1)^2 = 9$ is equal to**

1) –1 2) 2

3) –1, 2 4) None of these **Ans:** 3)

10. **The values of a and b if $4x^4 - 24x^3 + 76x^2 + ax + b$ is a perfect square are**

1) 100, 120 2) 10, 12

3) –120, 100 4) 12, 10 **Ans:** 3)

11. **If the roots equation of the $q^2x^2 + p^2x + r^2 = 0$ are the squares of the roots of the equation $qx^2 + px + r = 0$, then q, p, r are in**

1) A.P 2) G.P

3) both A.P And G.P 4) none of these **Ans:** 2)

12. **Graph of a linear equation is a**

1) straight line 2) circle

3) parabola 4) hyperbola **Ans:** 1)

13. **The number of point of intersection of the quadratic polynomial $x^2 + 4x + 4$ With the X-axis is**

1) 0 2) 1

3) 0 or 1 4) 2 **Ans:** 2)

14. **For the given matrix A = $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{bmatrix}$ the order of the matrix A^T is**

1) 2×3 2) 3×2
3) 3×4 4) 4×3 **Ans:** 3)

15. If A is a 2×3, matrix and B is a 3×4 matrix, how many columns does, AB have

1) 3 2) 4
3) 2 4) 5 **Ans:** 2)

16. If number of columns and rows are not equal in a matrix than it is said to be a

1) diagonal matrix 2) rectangular matrix
3) square matrix 4) identity matrix **Ans:** 2)

17. Transpose of a column matrix is

1) unit matrix 2) diagonal matrix
3) column matrix 4) row matrix **Ans:** 4)

18. Find the matrix X if $2X + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 9 & 5 \end{pmatrix}$

1) $\begin{pmatrix} -2 & -2 \\ 2 & -1 \end{pmatrix}$ 2) $\begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$
3) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ 4) $\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}$ **Ans:** 2)

19. Which of the following can be calculated from the given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$,

(i) A^2 (ii) B^2 (iii) AB (iv) BA
1) (i) and (ii) only 2) (ii) and (iii) only
3) (ii) and (iv) only 4) all of these **Ans:** 3)

20. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$. Which of the following statements are correct?

(i) $AB+C = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$ (ii) $BC = \begin{pmatrix} 0 & 1 \\ 2 & -3 \\ -4 & 10 \end{pmatrix}$

(iii) $BA + C = \begin{pmatrix} 2 & 5 \\ 3 & 0 \end{pmatrix}$ (iv) $(AB)C = \begin{pmatrix} -8 & 20 \\ -8 & 13 \end{pmatrix}$

1) (i) and (ii) only 2) (ii) and (iii) only
3) (iii) and (iv) only 4) all of these **Ans:** 1)

Created Questions — 1 mark

1. **The L.C.M of $x^3 - a^3$ and $(x - a)^2$ is**
   1) $(x^3 - a^3)(x + a)$ 2) $(x^3 - a^3)(x - a)^2$
   3) $(x - a)^2(x^2 + ax + a^2)$ 4) $(x + a)^2(x^2 + ax + a^2)$ **Ans:** 3)

2. **The excluded value of the rational expression $\frac{x^3+8}{x^2-2x-8}$ is**
   1) 8 2) 2
   3) 4 4) 1 **Ans:** 3)

3. **GCD of $6x^2y$, $9x^2yz$, $12x^2y^2z$ is**
   1) $36xy^2z^2$ 2) $36x^2y^2z$
   3) $36x^2y^2z^2$ 4) $3x^2y$ **Ans:** 2)

4. **If a and b are two positive integers where a>0 and b is a factor of a, then HCF of a and b is**
   1) b 2) a
   3) 3ab 4) $\frac{a}{b}$ **Ans:** 1)

5. **If a polynomial is a perfect square then its factors will be repeated ______ number of times.**
   1) odd 2) zero
   3) even 4) none of the above **Ans:** 1)

6. **The solution of $x^2 - 25 = 0$ is**
   1) no real roots 2) real and equal roots
   3) real and unequal roots 4) imaginary roots **Ans:** 3)

7. **For the given matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ the order of the matrix $(A^T)^T$ is**
   1) 2×3 2) 3×2
   3) 3×4 4) 4×3 **Ans:** 1)

8. **Which one of the following is not true?**
   1) If the product of the roots is unity. Then one root of the quadratic equation is the reciprocal of the other.

2) If $A = [a_{ij}]_{m\times n}$ and if m = 1, then the matrix "A" is called a row matrix.
3) Multiplication of a square matrix "A" with the unit matrix of the same order won't make any change in "A".
4) Matrix multiplication is ever commutative. **Ans:** 4)

9. The pair of linear equations $x+2y = 5$ and $3x+12y = 10$ has
1) unique solution 2) no solution
3) more than two solutions 4) infinitely many solution
Ans: 1)

10. The pair of linear equations $3^{x+y} = 81$ and $81^{x-y} = 3$ has
1) no solution
2) infinitely many solution
3) the solution $x = 2\frac{1}{8}$ $y = 1\frac{7}{8}$
4) none of these **Ans:** 3)

Two Marks Questions — 2 marks

1. Find the L.C.M of the following $x^3 - 27, (x-3)^2, (x^2-9)$.

Solution:

$x^3 - 27 = x^3 - 3^3 = (x-3)(x^2+3x+3^2) = (x-3)(x^2+3x+9)$

$(x-3)^2 = (x-3)^2$

$(x^2-9) = x^2 - 3^2 = (x+3)(x-3)$

L.C.M. $= (x-3)^2 (x+3)(x^2+3x+9)$

2. Find the L.C.M of the given expressions $p^2 - 3p + 2, p^2 - 4$

Solution:

$p^2 - 3p + 2 = (p-1)(p-2)$

$p^2 - 4 = p^2 - 2^2 = (p+2)(p-2)$

L.C.M. $= (p-2)(p+2)(p-1) = (p^2-4)(p-1)$

3. Find the L.C.M. of the given expressions $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

Solution:

$(2x^2-3xy)^2 = [x(2x-3y)]^2 = x^2(2x-3y)^2$

$8x^3-27y^3 = (2x)^3 - (3y)^3 = (2x-3y)[(2x)^2 + (3y)^2 + 2x(3y)]$

$= (2x-3y)(4x^2+9y^2+6xy)$

$(4x-6y)^3 = (2)^3(2x-3y)^3 = 8(2x-3y)^3$

L.C.M $= 8x^2(2x-3y)^3 \; [4x^2+9y^2+6xy]$

4. **Find the square root of 256 $(x-a)^8 (x-b)^4 (x-c)^{16} (x-d)^{20}$.**

Solution:

square root = $\sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$

$= 16\,|(x-a)^4 (x-b)^2 (x-c)^8 (x-d)^{10}|$

5. **Find the square root of $\dfrac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$.**

Solution:

$$\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \frac{12}{9}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$$

6. **Find the square root of the following expression**

$$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}.$$

Solution:

$$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}} = \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4(b-c)^4}{(b-c)^2(a-b)^6(b-c)^2}\right|$$

$$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4}{(a-b)^6}\right|$$

7. **Find the square root of $4x^2 + 20x + 25$.**

Solution:

$\sqrt{4x^2+20x+25} = \sqrt{(2x+5)^2} = |2x+5|$

8. **Find the square root of $9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$**

Solution:

$9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$

$= (3x)^2 + (-4y)^2 + (5z)^2 + 2(3x)(-4y) + 2(-4y)(5z) + 2(5z)(3x)$

$= [3x - 4y + 5z]^2 \quad \therefore \quad \sqrt{(3x-4y+5z)^2} = |3x - 4y + 5z|$

9. **Find the square root of $\sqrt{16x^4 + 8x^2 + 1}$**

Solution:

$\sqrt{16x^4+8x^2+1} = \sqrt{[4x^2]^2 + 8x^2 + 1^2}$

$= \sqrt{[4x^2+1]^2}$

$= |4x^2+1|$

10. Find the square root of the following rational expression $16x^2 + 9y^2 - 24xy + 24x - 18y + 9$

Solution:

$16x^2 + 9y^2 - 24xy + 24x - 18y + 9$

$= (4x)^2 + (-3y)^2 + 3^2 + 2(4x)(-3y) + 2(-3y)(3) + 2(3)(4x)$

$= (4x - 3y + 3)^2$

$\therefore \sqrt{(4x-3y+3)^2} = |4x - 3y + 3|$

11. Determine the quadratic equations, whose sum and product of roots are $-9, 20$.

Solution:

Required Equation is

$x^2 - [\alpha + \beta]x + \alpha\beta = 0$

$x^2 - [-9]x + 20 = 0 \Rightarrow x^2 + 9x + 20 = 0$

12. Determine the quadratic equations, whose sum and product of roots are $\frac{5}{3}, 4$

Solution:

Required Equation is

$x^2 -$ (sum of the roots x + products of the roots) = 0

$x^2 - \frac{5}{3}x + 4 = 0$

Multiply both sides by 3

$3x^2 - 5x + 12 = 0$

13. Determine the quadratic equations, whose sum and product of roots are $\frac{-3}{2}, -1$

Solution:

Required Equation is

$x^2 - [\alpha + \beta]x + \alpha\beta = 0$

$x^2 - \left[-\frac{3}{2}\right]x - 1 = 0 \quad \Rightarrow x^2 + \frac{3}{2}x - 1 = 0$

Multiply both sides by 2

$2x^2 + 3x - 2 = 0$

14. Determine the quadratic equations, whose sum and product of roots are $-(2 - a)^2, (a + 5)^2$

Solution:

Required Equation is

$x^2 - [\alpha+\beta]x + \alpha\beta = 0 \quad \Rightarrow x^2 - [-(2-a)^2]x + (a+5)^2 = 0$

$x^2 + (2-a)^2 x + (a+5)^2 = 0$

15. Find the sum and product of the roots of the following quadratic equation $x^2 + 3x - 28 = 0$.

Solution:

$x^2 + 3x - 28 = 0$

$a = 1, b = 3, c = -28$

Sum of the roots $= \alpha + \beta = -\dfrac{b}{a} = -\dfrac{3}{1} = -3$

Product of the roots $= \alpha\beta = \dfrac{c}{a} = -\dfrac{28}{1} = -28$

16. Find the sum and product of the roots of the following quadratic equation $x^2 + 3x = 0$

Solution:

$x^2 + 3x = 0$

$a = 1, b = 3, c = -0$

Sum of the roots $= \alpha + \beta = -\dfrac{b}{a} = -\dfrac{3}{1} = -3$

Product of the roots $= \alpha\beta = \dfrac{c}{a} = \dfrac{0}{1} = 0$

17. Find the sum and product of the roots of the following quadratic equation $3 + \dfrac{1}{a} = \dfrac{10}{a^2}$

Solution:

$3 + \dfrac{1}{a} = \dfrac{10}{a^2} \Rightarrow \dfrac{3a+1}{a} = \dfrac{10}{a^2}$

$\Rightarrow 3a + 1 = \dfrac{10}{a}$

$3a^2 + a = 10 \Rightarrow 3a^2 + a - 10 = 0$

$a = 3, b = 1, c = -10$

Sum of the roots $= \alpha+\beta = -\dfrac{b}{a} = -\dfrac{1}{3}$

Product of the roots $= \alpha\beta = \dfrac{c}{a} = -\dfrac{10}{3}$

18. Find the sum and product of the roots of the following quadratic equation $3y^2 - y - 4 = 0$

Solution:

$3y^2 - y - 4 = 0$

$a = 3, b = -1, c = -4$

Sum of the roots $= \alpha+\beta = -\frac{b}{a} = \frac{1}{3}$

Product of the roots $= \alpha\beta = \frac{c}{a} = -\frac{4}{3}$

19. Determine the nature of the roots for the following quadratic equation $15x^2 + 11x + 2 = 0$

Solution:

$15x^2 + 11x + 2 = 0$ comparing with $ax^2 + bx + c = 0$

$a = 15,\ b = 11, c = 2$

$\Delta = b^2 - 4ac$

$= 11^2 - 4 \times 15 \times 2 = 121 - 120 = 1 = (+)$ ve

$\therefore$ The roots are real and unequal.

20. Determine the nature of the roots for the following quadratic equation $x^2 - x - 1 = 0$

Solution:

$x^2 - x - 1 = 0$

$a = 1,\ b = -1, c = -1$

$\Delta = b^2 - 4ac = (-1)^2 - 4(1)(-1) = 1 + 4 = 5$

$\therefore$ The roots are real and unequal.

21. Find the value(s) of 'k' for which the roots of the following equation are real and equal $(5k - 6)x^2 + 2kx + 1 = 0$

Solution:

$(5k - 6)\, x^2 + 2kx + 1 = 0$

$a = 5k - 6, b = 2k, c = 1$

The roots are real and equal. So

$b^2 - 4ac = 0$

$(2k)^2 - 4(5k-6)1 = 0 \Rightarrow 4k^2 - 20k + 24 = 0 \quad \div 4$

$k^2 - 5k + 6 = 0 \Rightarrow (k - 2)(k - 3) = 0$

$k = 2$ or $k = 3$

22. Solve $2x^2 - 2\sqrt{6}\,x + 3 = 0$

Solution:

$2x^2 - 2\sqrt{6}\ x + 3 = 0 = 2x^2 - \sqrt{6}\ x - \sqrt{6}\ x + 3$

(by splitting the middle term)

$= \sqrt{2}\ x(\sqrt{2}\ x - \sqrt{3}) - \sqrt{3}(\sqrt{2}\ x - \sqrt{3})$

$= (\sqrt{2}\ x - \sqrt{3})(\sqrt{2}\,x - \sqrt{3})$

Now equating the factors to zero to we get

$$(\sqrt{2}\,x - \sqrt{3})(\sqrt{\ }\ x - \sqrt{3}) = 0$$

$$(\sqrt{2}\,x - \sqrt{3})^2 = 0$$

$$\sqrt{2}\,x - \sqrt{3} = 0$$

Therefore the solution is $x = \dfrac{\sqrt{3}}{\sqrt{2}}$ twice

23. Solve: $x^4 - 13x^2 + 42 = 0$

Solution:

$x^4 - 13x^2 + 42 = 0$

Let $x^2 = y$

$y^2 - 13y + 42 = 0$

$y = 6$ (or) $y = 7$

$x^2 = 6$ (or) $x^2 = 7$

$\therefore\ x = \pm\sqrt{6}$ (or) $x = \pm\sqrt{7}$

24. If the difference between a number and its reciprocal is $\dfrac{24}{5}$ find the number.

Solution:

Let x be the required number

$\dfrac{1}{x}$ be its reciprocal

Given $x - \dfrac{1}{x} = \dfrac{24}{5} \quad \Rightarrow \dfrac{x^2 - 1}{x} = \dfrac{24}{5}$

$5x^2 - 5 = 24x$

$5x^2 - 24x - 5 = 0$

$5x^2 - 25x + x - 5 = 0$

$x = 5,\ -\dfrac{1}{5}$

25. Determine the quadratic equation, whose sum and product of roots are $-\frac{3}{2}$, -1

Solution:

Sum of the roots $= \alpha + \beta = -\frac{3}{2}$

Product of the roots, $\alpha\beta = -1$

$\therefore$ The required quadratic equation is

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - \left(\frac{-3}{2}\right)x - 1 = 0$

$\Rightarrow \quad x^2 + \frac{3x}{2} - 1 = 0$

Multiply both sides by 2

$2x^2 + 3x - 2 = 0$

26. The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

Solution:

Let the present age of Kumaran be x years.

Two years ago, his age is $(x - 2)$ years.

Four years from now, his age $= (x + 4)$ years

Given $(x - 2)(x + 4) = 1 + 2x$

$x^2 + 2x - 8 = 1 + 2x$ gives $(x - 3)(x + 3) = 0$

Then, $x = \pm 3$ (Rejecting -3 as age cannot be negative)

Kumaran's present age is 3 years.

27. Find the excluded values of $\frac{y}{y^2 - 25}$

Solution:

Consider $\quad y^2 - 5^2 = 0$

$y^2 - 5^2 = 0$

$(y + 5)(y - 5) = 0$

$y + 5 = 0$, $y - 5 = 0$

$y = -5$, $y = 5$

The expression is undefined if $y = -5$, $y = 5$

$\therefore$ The excluded values are $5, -5$

28. Find the excluded values of $\dfrac{t}{t^2-5t+6}$

Solution:

Consider $t^2 - 5t + 6 = 0$

$$t^2 - 5t + 6 = 0$$
$$(t-2)(t-3) = 0$$
$$t-2=0,\ t-3=0$$
$$t=2,\ t=3$$
$$(t-2)(t-3)=0 \Rightarrow t=2,\ t=3$$

The expression is undefined if $t = 2$, $t = 3$

$\therefore$ The excluded values are 2, 3

29. Find the excluded values of $\dfrac{x}{x^2+1}$

Solution:

For all the values of x, $x^2 \geq 0$

Hence, $x^2 + 1 \geq 0 + 1 = 1$

For each value of x, $x^2 + 1 \neq 0$

Therefore, there can be no real excluded values for the given rational expression.

30. Reduce the rational expression to its lowest form $\dfrac{x-3}{x^2-9}$

Solution:

$$\frac{x-3}{x^2-9} = \frac{x-3}{(x+3)(x-3)} = \frac{1}{(x+3)}$$

31. Reduce the rational expression to its lowest form $\dfrac{x^2-16}{x^2+8x+16}$

Solution:

$$\frac{x^2-16}{x^2+8x+16} = \frac{x^2-4^2}{(x+4)(x+4)} = \frac{(x+4)(x-4)}{(x+4)(x+4)} = \frac{x-4}{(x+4)}$$

32. Reduce the rational expression to its lowest form $\dfrac{x^2-1}{x^2+x}$

Solution:

$$\frac{x^2-1}{x^2+x} = \frac{x^2-1^2}{x(x+1)} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{(x-1)}{x}$$

33. Multiply $\dfrac{x^3}{9y^2}$ by $\dfrac{27y}{x^5}$

Solution:

$$\frac{x^3}{9y^2}\times\frac{27y}{x^5}=\frac{27}{9yx^2}=\frac{3}{x^2y}$$

34. Multiply $\dfrac{x^4b^2}{x-1}$ by $\dfrac{x^2-1}{a^4b^3}$

Solution:

$$\frac{x^4b^2}{x-1}\times\frac{x^2-1}{a^4b^3}=\frac{x^4}{x-1}\times\frac{(x+1)(x-1)}{a^4b}=\frac{x^4(x+1)}{a^4b}$$

35. Find $\dfrac{x^2-16}{x+4}\div\dfrac{x-4}{x+4}$

Solution:

$$\frac{x\quad -16}{x+4}\times\frac{x+4}{x-4}=\frac{(x+4)(x-4)}{x+4}\times\frac{x+4}{x-4}=\quad +$$

36. Find $\dfrac{p^2-10p+21}{p-7}\times\dfrac{p^2+p-12}{(p-3)^2}$

Solution:

$$\frac{p^2-10p+21}{p-7}\times\frac{p^2+p-12}{(p-3)^2}=\frac{(p-7)(p-3)}{(p-7)}\times\frac{(p+4)(p-3)}{(p-3)^2}$$

$$=(p+4)$$

37. Multiply $\dfrac{5t^3}{4t-8}\times\dfrac{6t-12}{10t}$

Solution:

$$\frac{5t^3}{4t-8}\times\frac{6t-12}{10t}=\frac{5t^3}{4(t-2)}\times\frac{6(t-2)}{10t}=\frac{3t^2}{4}$$

38. The number of volleyball games that must be scheduled in a league with n teams is given by G(n) = $\dfrac{n^2-n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

Solution:

$$G(n) = \frac{n^2 - n}{2} = 15$$

$$n^2 - n = 30$$

$$n^2 - n - 30 = 0$$

$$(n-6)(n+5) = 0$$

$$n = 6, -5$$

Number of Teams in the league = 6

39. A ball rolls down a slope and travels a distance $d = t^2 - 0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet.

Solution:

$d = t^2 - 0.75t$ here $d = 11.25$

$$t^2 - 0.75t = 11.25$$

$$t^2 - 0.75t - 11.25 = 0$$

$$(t - 3.75)(t + 3) = 0$$

$$t - 3.75 = 0; \qquad t + 3 = 0$$

$$t = 3.75; \qquad t = -3,\ t \neq -3$$

So $t = 3.75$ seconds.

40. In a matrix $A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$

write (i) The number of elements (ii) The order of the matrix (iii) Write the elements $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$

Solution:

$$A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$$

i) Number of elements $= 4 \times 4 = 16$

ii) The order of the matrix $= 4 \times 4$

iii) $a_{22} = \sqrt{7}$; $a_{23} = \dfrac{\sqrt{3}}{2}$; $a_{24} = 5$; $a_{34} = 0$; $a_{43} = -11$; $a_{44} = 1$

41. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution:

Matrix having 18 elements.

1×18 (OR) 2×9 (OR) 3×6 (OR) 6×3 (OR) 9×2 (OR) 18×1

Matrix having 6 elements.

1×6 (OR) 2×3 (OR) 3×2 (OR) 6×1

42. Construct a 3 × 3 matrix whose elements are given by

(i) $a_{ij} = |i-2j|$ (ii) $a_{ij} = \dfrac{(i+j)^3}{3}$

Solution:

(i) $a_{ij} = |i - 2j|$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} |1-2| & |1-4| & |1-6| \\ |2-2| & |2-4| & |2-6| \\ |3-2| & |3-4| & |3-6| \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$$

(ii) $a_{ij} = \dfrac{(i+j)^3}{3}$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{bmatrix}$$

43. If $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

Solution:

$$A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

44. If $A = \begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of –A.

Solution:

$$-A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix} \Rightarrow (-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

45. If $A = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $[A^T]^T = A$

Solution:

$$A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix} \Rightarrow (A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$$

$$\therefore (A^T)^T = A$$

46. $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ find the value of 3A – 9B.

Solution:

$$3A - 9B = 3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} - 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} - \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} + \begin{pmatrix} -63 & -27 & -72 \\ -9 & -36 & -81 \end{pmatrix}$$

$$= \begin{pmatrix} 0-63 & 12-27 & 27-72 \\ 24-9 & 9-36 & 21-81 \end{pmatrix}$$

$$= \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

47. Find the values of x, y and z from the following equation.

$$\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} 12 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix} \quad \Rightarrow y = 12 \; ; \quad z = 3 \; ; \quad x = 3$$

48. Find the values of x, y and z from the following equation.

$$\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$$

$5 + z = 5$

$z = 5 - 5$

$z = 0$

$x + y = 6$; $xy = 8$

$y = 6 - x$; $x(6 - x) = 8$

$6x - x^2 - 8 = 0 \Rightarrow x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x - 2 = 0$ OR $x - 4 = 0$

$x = 2$ OR $x = 4$

If $x = 2$, $y = \dfrac{8}{x} = \dfrac{8}{2} = 4$; If $x = 4$, $y = \dfrac{8}{4} = 2$

49. Find the values of x, y and z from the following equation.

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

x+y+z = 9 .... (1) x+z = 5 .... (2) y+z=7 .... (3)

Substitute (3) in (1)

$$x + 7 = 9 \Rightarrow x = 9 - 7 = 2$$

Put $x = 2$ in (2)

$$2 + z = 5 \Rightarrow z = 5 - 2 = 3$$

Substitute z = 3 in (3)

$$y + 3 = 7 \Rightarrow y = 7 - 3 \Rightarrow y = 4$$

50. If $A = \begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ **then Find 2A + B.**

Solution:

$$2A + B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+4 & 16+11 & 12-3 \\ 2-1 & 6+2 & 18+4 \\ -8+7 & 6+5 & -2+0 \end{pmatrix} \Rightarrow \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

51. If $A = \begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$**, find 4A - 3B.**

Solution:

$$4A - 3B = 4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix} - 3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} + \begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}$$

$$= \begin{pmatrix} 20+21 & 16-12 & -8+9 \\ 2-\frac{3}{4} & 3-\frac{21}{2} & 4\sqrt{2}-9 \\ 4-15 & 36+18 & 16-27 \end{pmatrix}$$

$$= \begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

52. If A is of order p × q and B is order q × r what is the order of AB and BA?

Solution:

Given A is of order p × q

B is or order q × r

∴ Order of AB = p × r

Order of BA is not defined.

(∵ No. of columns in B & No. of rows in A are not equal)

53. If a polynomial p(x) = x^2 – 5x – 14 is divided by another polynomial q(x) we get $\frac{x-7}{x+2}$ find q(x).

Solution:

p(x) = x^2 – 5x – 14

Given $\frac{p(x)}{q(x)} = \frac{x-7}{x+2}$

$$\frac{x^2-5x-14}{q(x)} = \frac{x-7}{x+2}$$

$$\frac{(x-7)(x+2)}{q(x)} = \frac{x-7}{x+2}$$

∴ q(x) = (x + 2)^2

54. If one root of the equation $3x^2 + kx + 81 = 0$ (having real roots) is the square of the other then find k.

Solution:

Let α,β be the roots of $3x^2 + kx + 81 = 0$

$\alpha + \beta = -\dfrac{k}{3}$ .... (1)

$\alpha\beta = 27$ .... (2)

Given $\alpha = \beta^2$

From (2) $\Rightarrow \beta^3 = 27 \quad \Rightarrow \beta = 3$

$\therefore \alpha = 9$

(1) $\Rightarrow 9 + 3 = -\dfrac{k}{3} \quad \Rightarrow 12 = -\dfrac{k}{3}$

$k = -36$

55. Which rational expression should be subtracted from $\dfrac{x^2+6x+8}{x^3+8}$ to get $\dfrac{3}{x^2-2x+4}$

Solution:

$$\text{Required expression} = \frac{x^2+6x+8}{x^3+8} - \frac{3}{x^2-2x+4}$$

$$= \frac{(x+2)(x+4)}{(x+2)(x^2-2x+4)} - \frac{3}{x^2-2x+4}$$

$$= \frac{(x+4)}{(x^2-2x+4)} - \frac{3}{x^2-2x+4}$$

$$= \frac{x+4-3}{x^2-2x+4} \Rightarrow \frac{x+1}{x^2-2x+4}$$

56. If $x = \dfrac{a^2+3a-4}{3a^2-3}$ and $y = \dfrac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^{-2}

Solution:

$$x = \frac{a^2+3a-4}{3a^2-3} = \frac{(a+4)(a-1)}{3(a+1)(a-1)} = \frac{(a+4)}{3(a+1)}$$

$$x^2 = \frac{(a+4)^2}{9(a+1)^2}$$

$$y = \frac{a^2+2a-8}{2a^2-2a-4} = \frac{(a+4)(a-2)}{2(a+1)(a-2)} = \frac{(a+4)}{2(a+1)}$$

$$y^2 = \frac{(a+4)^2}{4(a+1)^2} \qquad \Rightarrow y^{-2} = \frac{4(a+1)^2}{(a+4)^2}$$

$$x^2 y^{-2} = \frac{(a+4)^2}{9(a+1)^2} \cdot \frac{4(a+1)^2}{(a+4)^2} \quad \Rightarrow \frac{4}{9}$$

Created Questions with Solution — 2 marks

1. If P(x+1) (x+2) + Q (x–1) = x^2+6x–1. Find the value of P and Q.

Solution:

Put $x = 1$

$$P(2)(3) = (1)^2+6(1)-1$$
$$6P = 6$$
$$P = 1$$

Put $x = -1$

$$Q(-2) = (-1)^2+6(-1)-1$$
$$-2Q = 1-6-1$$
$$Q = 3$$

2. Find the G.C.D of $(x-1)^3$, x^4-x^3+2x-2.

Solution:

$(x-1)^3 = (x-1)(x-1)(x-1)$

$$x^4-x^3+2x-2 = x^3(x-1)+2(x-1)$$
$$= (x^3+2)(x-1)$$
$$\text{G.C.D} = (x-1)$$

3. Find the L.C.M of $2(x^3+x^2-x-1)$ and $3(x^3+3x^2-x-3)$.

Solution:

$$2(x^3+x^2-x-1) = 2[x^2(x+1) - 1(x+1)]$$
$$= 2[(x^2-1)(x+1)]$$
$$3(x^3+3x^2-x-3) = 3[x^2(x+3) - 1(x+3)]$$
$$= 3[(x^2-1)(x+3)]$$
$$\text{L.C.M} = 6(x^2-1)(x+1)(x+3)$$

4. Simplify: $\frac{a^3}{a-b} + \frac{b^3}{b-a}$

Solution:

$$\frac{a^3}{a-b} + \frac{b^3}{b-a} = \frac{a^3}{a-b} - \frac{b^3}{a-b} \Rightarrow \frac{a^3-b^3}{a-b}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{a-b}$$

$$= a^2+ab+b^2$$

5. If A $(x–1)$ $(x–2)$ $(x–3)$ + B$(x–1)$ $(x–2)$ + C$(x–1)$ + D ≡ x^3 find the value of A, B, C, D.

Solution:

Put $x = 1$

$D = (1)^3 \Rightarrow D = 1$

Put $x = 2$

$$\begin{aligned} C+D &= (2)^3=8 \\ C+1 &= 8 \\ C &= 7 \end{aligned}$$

Put $x = 3$

$$\begin{aligned} 2B+2C+D &= 27 \\ 2B+14+1 &= 27 \\ 2B+15 &= 27 \\ 2B &= 12 \\ B &= 6 \end{aligned}$$

Equating the co-efficient of x^3

A = 1, B = 6, C = 7, D = 1

6. Find two consecutive positive odd numbers, the sum of whose squares is 802.

Solution:

Let the consecutive positive odd numbers be a and a+2.

Given

$$\begin{aligned} a^2+(a+2)^2 &= 802 \\ a^2+a^2+4a+4 &= 802 \\ 2a^2+4a-798 &= 0 \\ a^2+2a-399 &= 0 \\ (a+21)(a-19) &= 0 \end{aligned}$$

$a = 19$ $\quad a = -21$ (not possible)

∴ The numbers are 19, 21

7. If $\cos\theta \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \sin\theta \begin{pmatrix} x & -\cos\theta \\ \cos\theta & x \end{pmatrix} = I_2$

Solution:

$$\cos\theta \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \sin\theta \begin{pmatrix} x & -\cos\theta \\ \cos\theta & x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix} + \begin{pmatrix} x\sin\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & x\sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + x\sin\theta & 0 \\ 0 & \cos^2\theta + x\sin\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cos^2\theta + x\sin\theta = 1$$

$$x\sin\theta = 1-\cos^2\theta$$

$$x\sin\theta = \sin^2\theta$$

$$\therefore x = \sin\theta$$

8. The value of $\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ .....

Solution:

Let $x = \sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ .....

Squaring on both sides

$x^2 = 6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ .....

$x^2 = 6+x$

$x^2-x-6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ (or) $x = -2$ (not possible)

$\therefore \sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}$ ..... $= 3$

For Practice **2 marks**

1. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. Prove that $AA^T = I$
2. Verify that $A^2 = I$ when $A = \begin{pmatrix} 5 & -4 \\ 6 & -5 \end{pmatrix}$
3. Find the square root of $\dfrac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$
4. Find the LCM of $x^4 - 27a^3x$, $(x-3a)^2$ whose GCD is $(x-3a)$.
5. Determine the nature of the roots for $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$, $a \neq 0, b \neq 0$
6. Find the L.C.M of the following (x^4-1), x^2-2x+1
7. Find the G.C.D of the given polynomials x^4-1, x^3-11x^2+x-11
8. Find the sum and product of the roots for $kx^2 - k^2x - 2k^3 = 0$.
9. Find the values of k for which the roots of the following equation is real and equal $kx^2 + (6k+2)x + 16 = 0$
10. If a matrix has 16 elements, what are the possible orders it can have?
11. Construct a 3×3 matrix whose elements are $a_{ij} = i^2j^2$
12. Find the value of a, b, c, d from the equation $\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$
13. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find A + B.
14. If $A = \begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$, find A + B.
15. If $A = \begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$ find AB, BA and verify AB = BA?

16. Show that the matrices A = $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, B = $\begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ satisfy commutative property AB = BA.

17. If A = $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}$, B = $\begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ find AB.

18. Find the zeros of the quadratic expression $x^2+8x+12$.

Created Questions — 2 marks

1. If A = $\begin{pmatrix} 9 & 1 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$ find the matrix X such that 3A + 5B + 2X = 0.
2. The sides of a right angled triangle are $2x - 1$, $2x$ and $2x + 1$, find x.
3. If α and β be the two zeros of the quadratic polynomial | $P(x) = 2x^2 - 3x + 7$. Evaluate $\alpha^3 + \beta^3$
4. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
5. Find the H.C.F. of $x^3 + x^2 + x + 1$ and $x^4 - 1$
6. Find the L.C.M of $x^3 - y^3$ and $x^2 - y^2$
7. Simplify: $\dfrac{x^4 - 8x}{2x^2 + 5x - 3} \times \dfrac{2x-1}{x^2 + 2x + 4} \times \dfrac{x+3}{x^2 - 2x}$

Five Marks Questions — 5 marks

1. **Find the square root of the following expression $(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$**

Solution:

$(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$

$6x^2 + x - 1 = (3x - 1)(2x + 1)$

$3x^2 + 2x - 1 = (3x - 1)(x + 1)$

$2x^2 + 3x + 1 = (2x + 1)(x + 1)$

$\therefore \sqrt{(6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)}$

$= \sqrt{(3x-1)(2x+1)(3x-1)(x+1)(2x+1)(x+1)}$

$= |(3x-1)(2x+1)(x+1)|$

2. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$.

Solution:

	8	–1	1			
8	64	–16	17	–2	1	
	(–) 64					
16 –1		–16	17			$-\frac{16}{16} = -1$
		(+) –16	(–) 1			
16 –2 1			+16	–2	1	$\frac{16}{16} = 1$
			(–) +16	(+) –2	(–) 1	
			0			

Therefore, $\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = \left|8x^2 - x + 1\right|$

3. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution:

	3	2	4			
3	9	12	28	a	b	
	(–) 9					
6 2		12	28			$\frac{12}{6} = 2$
		(–) 12	(–) 4			
6 4 4			24	a	b	$\frac{24}{6} = 4$
			(–) 24	(–) 16	(–)16	
			a = 16, b = 16			

4. Find the square root of the following polynomial by division method $x^4 - 12x^3 + 42x^2 - 36x + 9$.

Solution:

	1	–6	3			
1	1	–12	42	–36	9	
	(–) 1					
2 –6		–12	42			$\frac{-12}{2} = -6$
		(+) –12	(–) 36			
2 –12 3			6	–36	9	$\frac{6}{2} = 3$
			(–) 6	(+) –36	(–) +9	
			0			

Solution is $|x^2 - 6x + 3|$

5. Find the square root of the following polynomial by division method $37x^2 - 28x^3 + 4x^4 + 42x + 9$

Solution:

	2	−7	−3			
2	4	−28	37	42	9	
	(−) 4					
4 −7		−28	37			
		(+) −28	(−) 49			$\frac{-28}{4} = -7$
4 −14 −3			−12	42	9	
			(+) −12	(−) 42	(−)9	$\frac{-12}{4} = -3$
			0			

Solution is $|2x^2 - 7x - 3|$

6. Find the square root of the following polynomial by division method $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Solution:

	11	−9	−12			
11	121	−198	−183	216	144	
	(−) 121					
22 −9		−198	−183			
		(+) −198	(−) + 81			$-\frac{198}{22} = -9$
22 −18 −12			−264	216	144	
			(+) −264	(−) 216	(−)144	$-\frac{264}{22} = -12$
			0			

Solution is = $|11x^2 - 9x - 12|$

7. Find the values of a and b if the following polynomial is a perfect squares $4x^4 - 12x^3 + 37x^2 + bx + a$.

Solution:

	2	−3	7			
2	4	−12	37	b	a	
	(−) 4					
4 −3		−12	37			
		(+) −12	(−) 9			$-\frac{12}{4} = -3$
4 −6 7			+ 28	b	a	
			(−) 28	(+) −42	(−)49	$\frac{28}{4} = 7$
			a = 49, b = −42			

8. **Find the values of a and b if the following is a perfect square $ax^4 + bx^3 + 361x^2 + 220x + 100$.**

Solution:

	10	11	12			
10	100	220	361	b	a	
	(–) 100					
20 11		220	361			$\frac{220}{20} = 11$
		(–) 220	(–)121			
20 22 12			240	b	a	$\frac{240}{20} = 12$
			(–) 240	(–) 264	(–)144	
		a = 144,	b = 264			

9. **Find the values of m and n if the following polynomial are perfect square $x^4 - 8x^3 + mx^2 + nx + 16$.**

Solution:

	1	–4	4			
1	1	–8	m	n	16	
	(–) 1					
2 –4		–8	m			$-\frac{8}{2} = -4$
		(+) –8	(–) 16			
2 –8 4			m–16	n	16	
			(–) 8	(+) –32	16	
			0			

m – 16 = 8 n = –32

m = 8+16

m = 24

10. **If A = (1 –1 2), B = $\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$ and C = $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ show that (AB)C = A(BC)**

Solution:

$$AB = (1\ \ -1\ \ 2)\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix} = (1-2+2 \quad -1-1+6) = (1 \quad 4)$$

$$(AB)C = (1 \quad 4)\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} = (1+8 \quad 2-4) = (9 \quad -2)$$

$$BC = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1-2 & 2+1 \\ 2+2 & 4-1 \\ 1+6 & 2-3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$$

$$A(BC) = (1 \ -1 \ 2) \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix} (-1-4+14 \quad 3-3-2) \quad = (9 \ \ -2)$$

$\therefore$ LHS = RHS

11. If $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ **verify that A(B+C) = AB + AC.**

Solution:

$$B + C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-7 & 2+6 \\ -4+3 & 2+2 \end{pmatrix} = \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

LHS = A (B + C)

$$= \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix} = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

RHS = AB + AC

$$= \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -3-4 & 4+8 \\ -13+16 & 4+0 \end{pmatrix} \Rightarrow \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

$\therefore$ LHS = RHS

12. If $A=\begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}$ show that $(AB)^T = B^TA^T$.

Solution:

$$AB = \begin{pmatrix}1 & 2 & 1\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}2 & -1\\ -1 & 4\\ 0 & 2\end{pmatrix}$$

$$= \begin{pmatrix}2-2+0 & -1+8+2\\ 4+1+0 & -2-4+2\end{pmatrix} = \begin{pmatrix}0 & 9\\ 5 & -4\end{pmatrix} \quad \Rightarrow AB^T = \begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix}$$

$$B^T = \begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix}; \qquad A^T = \begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}$$

$$B^TA^T = \begin{pmatrix}2 & -1 & 0\\ -1 & 4 & 2\end{pmatrix}\begin{pmatrix}1 & 2\\ 2 & -1\\ 1 & 1\end{pmatrix}$$

$$= \begin{pmatrix}2-2+0 & 4+1+0\\ -1+8+2 & -2-4+2\end{pmatrix} = \begin{pmatrix}0 & 5\\ 9 & -4\end{pmatrix}$$

$\therefore$ LHS = RHS

13. If $A=\begin{bmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{bmatrix}$, $B=\begin{bmatrix}2 & 3 & 4\\ 1 & 9 & 2\\ -7 & 1 & -1\end{bmatrix}$, and $C=\begin{bmatrix}8 & 3 & 4\\ 1 & -2 & 3\\ 2 & 4 & -1\end{bmatrix}$ then verify that $A+(B+C) = (A+B)+C$.

Solution:

$$A+(B+C) = \begin{bmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{bmatrix} + \left(\begin{bmatrix}2 & 3 & 4\\ 1 & 9 & 2\\ -7 & 1 & -1\end{bmatrix} + \begin{bmatrix}8 & 3 & 4\\ 1 & -2 & 3\\ 2 & 4 & -1\end{bmatrix}\right)$$

$$= \begin{bmatrix}4 & 3 & 1\\ 2 & 3 & -8\\ 1 & 0 & -4\end{bmatrix} + \begin{bmatrix}10 & 6 & 8\\ 2 & 7 & 5\\ -5 & 5 & -2\end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{bmatrix} \qquad \text{.......... (1)}$$

$$(A+B)+C = \left(\begin{bmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{bmatrix}\right) + \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{bmatrix} \qquad \text{.......... (2)}$$

From (1) and (2) LHS = RHS

14. Find the value of a, b, c, d from the following matrix equation. $\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix} + \begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$

Solution:

$$\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix} + \begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{pmatrix} = \begin{pmatrix} 2+0 & 2a+1 \\ b-5 & 4c+0 \end{pmatrix}$$

$d + 3 = 2$	$8 + a = 2a + 1$	$3b - 2 = b - 5$	$a - 4 = 4c$
$d = 2 - 3$	$8 - 1 = 2a - a$	$3b - b = -5 + 2$	$7 - 4 = 4c$
$d = -1$	$7 = a$	$b = -\dfrac{3}{2}$	$c = \dfrac{3}{4}$

$$a = 7,\ b = -\frac{3}{2},\quad c = \frac{3}{4},\ d = -1$$

15. Find the non-zero values of x satisfying the matrix equation

$$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$

Solution:

$$x\begin{pmatrix}2x & 2\\ 3 & x\end{pmatrix}+2\begin{pmatrix}8 & 5x\\ 4 & 4x\end{pmatrix} = 2\begin{pmatrix}x^2+8 & 24\\ 10 & 6x\end{pmatrix}$$

$$\begin{pmatrix}2x^2 & 2x\\ 3x & x^2\end{pmatrix}+\begin{pmatrix}16 & 10x\\ 8 & 8x\end{pmatrix} = \begin{pmatrix}2x^2+16 & 48\\ 20 & 12x\end{pmatrix}$$

$$\begin{pmatrix}2x^2+16 & 12x\\ 3x+8 & x^2+8x\end{pmatrix} = \begin{pmatrix}2x^2+16 & 48\\ 20 & 12x\end{pmatrix}$$

$\therefore\ 12x = 48 \Rightarrow\ x = 4$ $\qquad$ $x^2 + 8x = 12x$

$3x+8 = 20 \Rightarrow 3x = 12$ $\qquad$ $\Rightarrow x^2 - 4x = 0$

$\Rightarrow\ x = 4$ $\qquad$ $\Rightarrow x(x-4) = 0,\quad x = 0,\ x = 4$

$\therefore\ x = 4$

16. If $A = \begin{bmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{bmatrix}$, $B = \begin{bmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{bmatrix}$ then show that $A^2 + B^2 = I$.

Solution:

$A = \begin{bmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{bmatrix}$, $B = \begin{bmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{bmatrix}$

To show: $A^2 + B^2 = I$

$$A^2 = A\,.\,A$$

$$= \begin{bmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta+0 & 0+0\\ 0+0 & 0+\cos^2\theta\end{bmatrix} = \begin{bmatrix}\cos^2\theta & 0\\ 0 & \cos^2\theta\end{bmatrix}$$

$$B^2 = \begin{bmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{bmatrix}\begin{bmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{bmatrix}$$

$$= \begin{bmatrix}\sin^2\theta+0 & 0+0\\ 0+0 & 0+\sin^2\theta\end{bmatrix} = \begin{bmatrix}\sin^2\theta & 0\\ 0 & \sin^2\theta\end{bmatrix}$$

$$\therefore A^2+B^2 = \begin{bmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$$

$\therefore A^2 + B^2 = I$ Hence Proved.

17. Solve the following quadratic equation by completing the square method $\frac{5x+7}{x-1} = 3x+2$.

Solution:

$$\frac{5x+7}{x-1} = 3x+2$$
$$5x + 7 = (x - 1)(3x + 2)$$
$$5x + 7 = 3x^2 - x - 2$$
$$3x^2 - x - 5x = 9$$
$$3x^2 - 6x = 9 \quad \text{(Dividing both sides by 3)}$$
$$x^2 - 2x = 3$$
$$x^2 - 2x + 1^2 = 3 + 1^2 \quad \text{(Add (1)2 on both sides)}$$
$$(x - 1)2 = 4$$
$$x - 1 = \pm 2$$
$$x = \pm 2 + 1$$
$$x = 2 + 1 \text{ (or) } -2 + 1$$
$$x = 3 \text{ (or) } - 1$$

Solution $x = \{3, -1\}$

18. Find the LCM of the following polynomial $a^2 + 4a - 12$, $a^2 - 5a+6$ whose GCD is $a - 2$.

Solution:

$f(x) = a^2 + 4a - 12 \Rightarrow (a + 6)(a - 2)$

$g(x) = a^2 - 5a + 6 \Rightarrow (a - 3)(a - 2)$

GCD $= a - 2$

$$\text{LCM} = \frac{f(x)\times g(x)}{GCD} \Rightarrow \frac{(a+6)(a-2)\times(a-3)(a-2)}{a-2}$$
$$= (a + 6)(a - 3)(a - 2)$$

19. Find the LCM and GCD for the following and verify that $f(x) \times g(x) = \text{LCM} \times \text{GCD}$. $(x^3 - 1)(x + 1)$, $(x^3 + 1)$.

Solution:

$f(x) = (x^3 - 1)(x + 1)$

$= (x-1)(x^2+1^2+x\,.1)(x+1)$

$= (x-1)(x^2+x+1)(x+1)$

$g(x) = x^3+1$

$= x^3+1^3 = (x+1)(x^2+1^2-x\,.1)$

$= (x+1)(x^2-x+1)$

G.C.D $= (x+1)$

L.C.M $= (x+1)(x-1)(x^2+x+1)(x^2-x+1)$

$f(x) \times g(x) = \text{L.C.M} \times \text{G.C.D}$

$[(x-1)(x^2+x+1)(x+1)] \times [(x+1)(x^2-x+1)]$

$= [(x+1\,(x-1)(x^2+x+1)(x^2-x+1)] \times (x+1)$

20. The roots of the equation $x^2+6x-4=0$ are α, β. Find the quadratic equation whose roots are

i) α^2 and β^2 ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$ iii) $\alpha^2\beta$ and $\beta^2\alpha$

Solution:

i) α^2 and β^2

$x^2+6x-4=0$

a = 1, b = 6, c = –4

$\alpha+\beta = -\frac{6}{1} = -6,\ \alpha\beta = \frac{-4}{1} = -4$

Sum : $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$= (-6)^2-2(-4)$

$= 36+8 = 44$

Product : $\alpha^2\beta^2 = (\alpha\beta)^2 = (-4)^2 = 16$

$x^2 - (\text{Sum})\,x + \text{Product} = 0$

$\therefore x^2-44x+16=0$

ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

Sum $= \frac{2}{\alpha}+\frac{2}{\beta} = \frac{2\alpha+2\beta}{\alpha\beta} = \frac{2(-6)}{-4} = \frac{-12}{-4} = 3$

Sum = 3

Product $= \frac{2}{\alpha}\times\frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{-4} = -1$

$x^2 - (\text{Sum})\,x + \text{Product} = 0$

$\therefore x^2-3x-1=0$

iii) **$\alpha^2\beta$ and $\beta^2\alpha$**

Sum = $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = (-4)(-6) = 24$

Product = $(\alpha^2\beta)(\beta^2\alpha) = \alpha^3\beta^3 = (\alpha\beta)^3 = (-4)^3 = -64$

$x^2 - (\text{Sum})\,x + \text{Product} = 0$

$\therefore x^2 - 24x - 64 = 0$

21. Solve : $\dfrac{1}{2x}+\dfrac{1}{4y}-\dfrac{1}{3z}=\dfrac{1}{4};\ \dfrac{1}{x}=\dfrac{1}{3y};\ \dfrac{1}{x}-\dfrac{1}{5y}+\dfrac{4}{z}=2\dfrac{2}{15}$

Solution:

Let $\dfrac{1}{x}=p,\ \dfrac{1}{y}=q,\ \dfrac{1}{z}=r$

The given equations are written as

$$\frac{p}{2}+\frac{q}{4}-\frac{r}{3} = \frac{1}{4} \Rightarrow \frac{6p+3q-4r}{12} = \frac{1}{4}$$

$6p + 3q - 4r = 3$ .... (1)

$p = \dfrac{q}{3} \Rightarrow 3p = q$ .... (2)

$$p - \frac{q}{5} + 4r = 2\frac{2}{15} = \frac{32}{15}$$

$15p - 3q + 60r = 32$ .... (3)

substituting (2) in (1) and (3) we get,

$15p - 4r = 3$ .... (4)

$6p + 60r = 32$ reduces to $3p + 30r = 16$ .... (5)

Solving (4) and (5)

$$\begin{array}{rcrcr} & 15p & - & 4r & = & 3 \\ (-) & 15p & - & (+)\ 150r & = & 80\ (-) \\ \hline & & & -154r & = & -77 \\ \hline \end{array}$$

We get, $r = \dfrac{1}{2}$

Substituting $r = \dfrac{1}{2}$ in (4) we get, $15p - 2 = 3$ gives $p = \dfrac{1}{3}$

From (2), $q = 3p$, we get $q = 1$

Therefore, $x = \dfrac{1}{p} = 3,\ y = \dfrac{1}{q} = 1,\ z = \dfrac{1}{r} = 2$

That is $x = 3, y = 1, z = 2$.

22. Solve the following system of linear equations in three variables: $x + y + z = 5$; $2x - y + z = 9$; $x - 2y + 3z = 16$.

Solution:

$x + y + z = 5$ .... (1)

$2x - y + z = 9$ .... (2)

$x - 2y + 3z = 16$ .... (3)

$(1) - (3) \Rightarrow 3y - 2z = -11$ .... (4)

$$\begin{array}{rrrrrrr} (2) \Rightarrow & 2x & - & y & + & z = & 9 \\ (1) \times 2 \Rightarrow & 2x & + & 2y & + & 2z = & 10 \\ (-) & (-) & & (-) & & & (-) \\ \hline \text{sub} & & - & 3y & - & z = & -1 \end{array} \quad \text{—— (5)}$$

From (4) & (5)

$$\begin{array}{rrrrr} 3y & - & 2z & = & -11 \\ -3y & - & z & = & -1 \ (+) \\ \hline \text{Add} & & -3z & = & -12 \end{array}$$

$$z = 4$$

Sub $z = 4$ in (5)

$-3y - 4 = -1$

$\Rightarrow -3y = 3$

$\Rightarrow y = -1$

Sub $y = -1, z = 4$ (1) in (1)

$\Rightarrow x - 1 + 4 = 5$

$\Rightarrow x = 2$

Solution set : $x = 2, y = -1, z = 4$

23. If $A = \dfrac{x}{x+1}$ and $B = \dfrac{1}{x+1}$,

prove that $\dfrac{(A+B)^2 + (A-B)^2}{A \div B} = \dfrac{2(x^2+1)}{x(x+1)^2}$

Solution:

LHS

$$= \frac{(A+B)^2 + (A-B)^2}{\frac{A}{B}} = \frac{A^2 + B^2 + 2AB + A^2 + B^2 - 2AB}{\frac{A}{B}}$$

$$= [2A^2+2B^2]\frac{B}{A} \Rightarrow 2(A^2+B^2)\frac{B}{A}$$

$$= 2\left[\left(\frac{x}{x+1}\right)^2+\left(\frac{1}{x+1}\right)^2\right]\frac{1/x+1}{x/x+1}$$

$$= 2\left[\left(\frac{x}{x+1}\right)^2+\left(\frac{1}{x+1}\right)^2\right]\frac{1}{x+1}.\frac{x+1}{x}$$

$$= 2\left[\frac{x^2+1}{(x+1)^2}\right].\frac{1}{x} = \frac{2(x^2+1)}{x(x+1)^2}$$

24. Simplify: $\dfrac{1}{x^2-5x+6}+\dfrac{1}{x^2-3x+2}-\dfrac{1}{x^2-8x+15}$

Solution:

$$= \frac{1}{(x-2)(x-3)}+\frac{1}{(x-1)(x-2)}-\frac{1}{(x-3)(x-5)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-2)(x-1)}{(x-2)(x-1)(x-3)(x-5)}$$

$$= \frac{x^2-5x-x+5+x^2-5x-3x+15-[x^2-x-2x+2]}{(x-2)(x-1)(x-3)(x-5)}$$

$$= \frac{x^2-6x+5+x^2-8x+15-x^2+3x-2}{(x-2)(x-1)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-2)(x-1)(x-3)(x-5)}$$

$$= \frac{(x-2)(x-9)}{(x-2)(x-1)(x-3)(x-5)} = \frac{(x-9)}{(x-1)(x-3)(x-5)}$$

25. If $A=\dfrac{2x+1}{2x-1}$, $B=\dfrac{2x-1}{2x+1}$ **find** $\dfrac{1}{A-B}-\dfrac{2B}{A^2-B^2}$

Solution:

$$\frac{1}{A-B}-\frac{2B}{A^2-B^2} = \frac{1}{A-B}-\frac{2B}{(A+B)(A-B)}$$

$$= \frac{A+B-2B}{(A+B)(A-B)} \Rightarrow \frac{(A-B)}{(A+B)(A-B)}$$

$$= \frac{1}{(A+B)} \Rightarrow \frac{1}{\frac{2x+1}{2x-1}+\frac{2x-1}{2x+1}}$$

$$= \frac{1}{\frac{(2x+1)^2+(2x-1)^2}{(2x+1)(2x-1)}}$$

$$= \frac{(2x+1)(2x-1)}{(2x+1)^2+(2x-1)^2}$$

$$= \frac{[2x]^2-1^2}{4x^2+1+4x+4x^2+1-4x}$$

$$= \frac{4x^2-1}{8x^2+2} \Rightarrow \frac{4x^2-1}{2(4x^2+1)}$$

26. Vani, her father and her grand father have an average age of 53. One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now?

Solution:

Let the present age of Vani, her father and grand father be x, y, z respectively.

By data given,

$$\frac{x+y+z}{3} = 53 \Rightarrow x+y+z = 159 \qquad \text{.......... (1)}$$

$$\frac{1}{2}z+\frac{1}{3}y+\frac{1}{4}x = 65 \Rightarrow \frac{6z+4y+3x}{12} = 65$$

$$3x+4y+6z = 780 \qquad \text{.......... (2)}$$

$$(z-4) = 4(x-4) \Rightarrow 4x-z = 12 \qquad \text{.......... (3)}$$

From (1) & (2)

$$\begin{array}{lccccccccc} (1)\times(4) \Rightarrow & 4x & + & 4y & + & 4z & = & 636 \\ (2) \Rightarrow & 3x & + & 4y & + & 6z & = & 780 \\ \hline \text{(subtracting)} & x & & & - & 2z & = & -144 \end{array} \qquad \text{.......... (4)}$$

From (3) & (4)

$$\begin{array}{llrcrcr} (3)\times(2) \Rightarrow & 8x & - & 2z & = & 24 \\ (4) \Rightarrow & x & - & 2z & = & -144 \\ \hline \text{(subtracting)} & 7x & & & = & 168 \\ \hline \end{array} \quad \text{..........} \ (5)$$

$$x = \frac{168}{7} = 24$$

Substitute $x = 24$ in (3)

$96 - z = 12$

$z = 84$

(1) $\Rightarrow 24 + y + 84 = 159$

$\Rightarrow y = 51$

$\therefore$ Vani's Present Age = 24 years

Father's Present Age = 51years

Grand father's Age = 84 years

27. A girl is twice as old as her sister. Five years hence, the product of their ages(in years) will be 375. Find their present ages.

Solution:

Let the present age of the girl and her sister be x, y.

i) $x = 2y$

ii) $(x+5)(y+5) = 375$

$\Rightarrow (2y+5)(y+5) = 375$

$\Rightarrow 2y^2 + 15y - 350 = 0$

$\Rightarrow y = -\frac{35}{2}, 10$

y can`t be Negative,

$\therefore y = 10$

$\therefore x = 2y \quad \Rightarrow x = 20$

$\therefore$ The present ages are 20, 10 years old.

28. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are real and equal prove that either $a = 0$ (or) $a^3 + b^3 + c^3 = 3abc$

Solution:

$(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

$A = c^2 - ab,\ B = -2(a^2 - bc),\ C = b^2 - ac$

$\Delta = B^2 - 4AC$

Since the roots are real and equal $\Delta = 0$

$[-2(a^2 - bc)]^2 - 4[c^2 - ab]\,[b^2 - ac] = 0$

$4(a^2 - bc)^2 - 4\,(c^2 - ab)\,(b^2 - ac) = 0$

$4[a^4 + b^2c^2 - 2a^2bc] - 4\,[\,b^2c^2 - ac^3 - ab^3 + a^2bc] = 0$

$4a^4 + 4b^2c^2 - 8a^2bc - 4\,b^2c^2 + 4ac^3 + 4ab^3 - 4a^2bc = 0$

$4a^4 + 4ab^3 + 4ac^3 - 12a^2bc = 0$

$4a\,[a^3 + b^3 + c^3 - 3abc] = 0$

$a = 0 \quad (\text{OR}) \quad a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

29. If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = -\frac{13}{7}$. Find the values of a.

Solution:

Given α, β are the roots of $7x^2 + ax + 2 = 0$

$\alpha + \beta = -\frac{a}{7}; \quad \alpha\beta = \frac{2}{7}$

$$\beta - \alpha = -\frac{13}{7}$$

$$\alpha - \beta = \frac{13}{7}$$

$$(\alpha - \beta)^2 = \frac{169}{49}$$

$$(\alpha + \beta)^2 - 4\alpha\beta = \frac{169}{49}$$

$$\left(\frac{-a}{7}\right)^2 - 4\left(\frac{2}{7}\right) = \frac{169}{49}$$

$$\frac{a^2}{49} - 4\left(\frac{2}{7}\right) = \frac{169}{49}$$

$$\frac{\quad}{49} - \frac{8}{7} = \frac{169}{49}$$

$$\frac{a^2 - 56}{49} = \frac{169}{49}$$

$$a^2 - 56 = 169$$

$$a^2 = 225$$

$$\therefore\ a = 15, -15$$

Created Questions with Solution	5 marks

1. A motor boat whose speed is 18 km hr in still water takes 1 hour more to go to 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution:

Speed of the water = x km/h

Speed of the boat = 18 km / hour

Speed of the boat in the direction of the water = $18 + x$

Speed of the boat in the opposite direction of the water = $18 - x$

Time taken by the boat to cross 24 km in the along the direction of the

water $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{24}{18+x}$

Time taken by the boat to cross 24 km in the opposite the direction of the water $= \dfrac{24}{18-x}$

$$\text{By Given data} \quad \frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24\left(\frac{1}{18-x} - \frac{1}{18+x}\right) = 1$$

$$24\left(\frac{18+x-18+x}{(18-x)(18+x)}\right) = 1$$

$$24\left(\frac{2x}{324-x^2}\right) = 1$$

$$48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

$$(x+54)(x-6) = 0$$

$x + 54 = 0$ OR $x - 6 = 0$

$x = -54$ which is not possible OR $x = 6$

$\therefore$ Speed of the water = 6 km/hour

2. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 \\ 1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 5 \\ 1 & 3 \end{pmatrix}$,

Prove that A(BC) = (AB) C.

Solution:

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 3 \\ -1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 5 \\ 1 & 3 \end{pmatrix}$

$$BC = \begin{pmatrix} 0+3 & 0+9 \\ 1+5 & -5+15 \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ 6 & 10 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 9 \\ 6 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3+12 & 9+20 \\ 9+24 & 27+40 \end{pmatrix} = \begin{pmatrix} 15 & 29 \\ 33 & 67 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ -1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 0-2 & 3+10 \\ 0-4 & 9+20 \end{pmatrix} = \begin{pmatrix} -2 & 13 \\ -4 & 29 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} -2 & 13 \\ -4 & 29 \end{pmatrix}\begin{pmatrix} -1 & 5 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2+13 & -10+39 \\ 4+29 & -20+87 \end{pmatrix} = \begin{pmatrix} 15 & 29 \\ 33 & 67 \end{pmatrix}$$

$\therefore$ LHS = RHS.

3. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then, Prove that $A^2 - 4A + 5I_2 = 0$.

Solution:

$$A^2 = A \times A$$

$$= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

$$\text{LHS} = A^2 - 4A + 5I_2$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} - 4\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ -8 & -12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4+5 & -4+4+0 \\ 8-8+0 & 7-12+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{RHS}$$

4. Solve: $2x + y + 4z = 15$, $x - 2y + 3z = 13$, $3x + y - z = 2$.

Solution:

Given:
$$2x + y + 4z = 15 \quad \text{.......... (1)}$$
$$x - 2y + 3z = 13 \quad \text{.......... (2)}$$
$$3x + y - z = 2 \quad \text{.......... (3)}$$

$$\begin{array}{rrcl} (1) \times 2 \Rightarrow & 4x + 2y + 8z & = & 30 \\ (2) \times 1 \Rightarrow & x - 2y + 3z & = & 13 \\ \hline \text{Add} & 5x \quad + 11z & = & 43 \\ \hline \end{array} \quad \text{.......... (4)}$$

$$\begin{array}{rrcl} (2) \times 1 \Rightarrow & x - 2y + 3z & = & 13 \\ (3) \times 2 \Rightarrow & 6x + 2y - 2z & = & 4 \\ \hline \text{Add} & 7x \quad + z & = & 17 \end{array} \quad \text{.......... (5)}$$

From (4) and (5)

$$\begin{array}{rrcl} (4) \times 1 \Rightarrow & 5x + 11z & = & 43 \\ (5) \times 11 \Rightarrow & 77x + 11z & = & 187 \\ \text{Sub} & (-) \quad (-) & & (-) \\ \hline & -72x & = & -144 \end{array}$$

$$x = \frac{144}{72} = 2$$

Substitute the value of x in eqn (5)

$$\begin{aligned} 7x + z &= 17 \\ 7(2) + z &= 17 \\ 14 + z &= 17 \\ z &= 17 - 14 \\ z &= 3 \end{aligned}$$

Substitute the values of x and z in eqn (1)

$$\begin{aligned} 2x + y + 4z &= 15 \\ 2(2) + y + 4(3) &= 15 \\ 4 + y + 12 &= 15 \\ y + 16 &= 15 \\ y &= 15 - 16 \\ y &= -1 \end{aligned}$$

5. Find the values of a and b if $16x^4 - 24x^3 + (a-1)x^2 + (b+1)x + 49$ is a perfect square.

Solution:

	4	−3	−7		
4	16	−24	(a−1)	b+1	49
	(−) 16				
8 −3		−24	(a−1)		
		(+) −24	(−) 9		
8 −6 −7			(a−10)	b+1	49
			(+) −56	(−) 42	(−) 49
			0		

$\frac{-24}{8} = -3$

$$\therefore a - 10 = -56 \qquad b + 1 = 42$$
$$a = -56 + 10 \qquad b = 42 - 1$$
$$a = -46 \qquad b = 41$$

6. Solve the equation $1/(x+1) + 2/(x+2) = 4/(x+4)$, where $x+1 \neq 0$, $x+2 \neq 0$ and $x+4 \neq 0$ using quadratic formula.

Solution:

Given
$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$$
$$\frac{(x+2)+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$\frac{x+2+2x+2}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$\frac{3x+4}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$(3x+4)(x+4) = 4(x+1)(x+2)$$
$$3x^2 + 16x + 16 = 4[x^2 + 3x + 2]$$
$$3x^2 + 16x + 16 = 4x^2 + 12x + 8$$
$$4x^2 + 12x + 8 - 3x^2 - 16x - 16 = 0$$
$$x^2 - 4x - 8 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)} = \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2}$$

$$\Rightarrow \frac{4 \pm \sqrt{16 \times 3}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$$

$\therefore$ The Solution is $(2 + 2\sqrt{3})$, $(2 - 2\sqrt{3})$

7. Simplify: $\dfrac{a^2 - 16}{a^3 - 8} \times \dfrac{2a^2 - 3a - 2}{2a^2 + 9a + 4} \div \dfrac{3a^2 - 11a - 4}{a^2 - 2a + 4}$.

Solution:

Given

$$\frac{a^2 - 16}{a^3 - 8} \times \frac{2a^2 - 3a - 2}{2a^2 + 9a + 4} \div \frac{3a^2 - 11a - 4}{a^2 - 2a + 4}$$

$$= \frac{a^2 - 16}{a^3 - 8} \times \frac{2a^2 - 3a - 2}{2a^2 + 9a + 4} \times \frac{a^2 - 2a + 4}{3a^2 - 11a - 4}$$

$$= \frac{a^2 - 4^2}{a^3 - 2^3} \times \frac{(a-2)(2a+1)}{(a+4)(2a+1)} \times \frac{a^2 - 2a + 4}{(3a+1)(a-4)}$$

$$= \frac{(a+4)(a-4)}{(a-2)(a^2 + 2a + 4)} \times \frac{(a-2)(2a+1)}{(2a+1)(a+4)} \times \frac{a^2 - 2a + 4}{(3a+1)(a-4)}$$

$\because a^2 - b^2 = (a - b)(a + b)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$= \frac{(a^2 - 2a + 4)}{(3a+1)(a^2 + 2a + 4)}$$

8. A train covered a certain distance at a uniform speed. If the train would have been 10km/hr faster it would have taken 2 hour less than the scheduled time and if the train were slower by 10km/hr, it would have taken 3 hour more than the scheduled time. Find the distance covered by the train.

Solution:

Let Speed $= x$ km /hr

Distance $=$ y km

Time $=$ z hrs

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$\therefore$ y $= xz$ .... (1)

Given $\quad z - 2 = \dfrac{y}{x + 10}$

$$(x + 10)(z - 2) = y$$

$$xz - 2x + 10z - 20 = y$$

$$y - 2x + 10z - 20 = y$$

$$-2x + 10z - 20 = 0$$
$$x - 5z + 10 = 0$$
$$x - 5z = -10 \quad \text{.......... (2)}$$
$$z + 3 = \frac{y}{x-10}$$
$$(z + 3)(x - 10) = y$$
$$xz + 3x - 10z - 30 = y$$
$$y + 3x - 10z - 30 = y$$
$$3x - 10z - 30 = 0 \quad \text{.......... (3)}$$

From (2) and (3)

$$(2) \times 2 \Rightarrow \quad 2x - 10z = -20$$
$$(3) \times 1 \Rightarrow \quad 3x - 10z = 30$$
$$\text{(Subtract)} \quad -x = -50$$
$$x = 50$$

Substitute $x = 50$ in equation (2)

$$x - 5z = -10$$
$$50 - 5z = -10$$
$$-5z = -60$$
$$z = 12$$
$$(1) \Rightarrow \quad y = xz$$
$$y = 50 \times 12$$
$$y = 600 \text{ km}$$

9. A car left 30 minutes later than the scheduled time. In order to reach its destination 150 km away in time, it has to increase its speed by 25 km/hr from its usual speed. Find its usual speed.

Solution:

Let the usual speed of the car be x km / hr

Thus the increased speed of the car is $(x + 25)$ km / hr

Total Distance = 150 km; Time Taken = $\dfrac{\text{Distance}}{\text{Speed}}$

Let T_1 and T_2 be the time taken in hours by the car to cover the given distance in scheduled time and decreased time (as the speed is increased) respectively.

$$T_1 - T_2 = \frac{1}{2} \text{ hr} \qquad \frac{150}{x} - \frac{150}{x+25} = \frac{1}{2}$$

$$150\left[\frac{1}{x}-\frac{1}{x+25}\right] = \frac{1}{2}$$

$$150\left[\frac{x+25-x}{x(x+25)}\right] = \frac{1}{2}$$

$$150\left[\frac{25}{x^2+25x}\right] = \frac{1}{2}$$

$$x^2+25x-7500 = 0$$

$$(x+100)(x-75) = 0$$

$x = 75$ (or) $x = -100$ (is not an admissible value)

$\therefore$ The usual speed of the car is 75 km/hr

10. Solve: $x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

Solution:

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1 = 0$$

$$x^2+\frac{ax}{a+b}+\left(\frac{a+b}{a}\right)x+\frac{a}{a+b}\times\frac{a+b}{a} = 0$$

$$x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right) = 0$$

$$\left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right) = 0$$

$$x+\frac{a}{a+b} = 0 \text{ (or) } x+\frac{a+b}{a} = 0$$

$$x = -\frac{a}{a+b} \text{ (or) } x = -\left(\frac{a+b}{a}\right)$$

$$\text{hence } x = \left\{\frac{-a}{a+b}, -\left(\frac{a+b}{a}\right)\right\}$$

11. Solve: $5^{(x+1)}+5^{(2-x)} = 5^3+1$

Solution:

$$5^{x+1}+5^{2-x} = 5^3+1$$

$$5^x.5+5^2 5^{-x} = 125+1$$

$$5^x.5+\frac{25}{5^x} = 126$$

Let $5^x = y$

$$5y+\frac{25}{y} = 126$$

$$5y^2+25 = 126y$$

$$5y^2-126y+25 = 0$$

$$(5y-1)(y-25) = 0$$

$$5y-1 = 0 \text{ (or) } y-25 = 0$$

$$y = \frac{1}{5} \text{ (or) } y = 25$$

Now $5^x = y$

$5^x = \frac{1}{5}$ (or) $5^x = 25$

$5^x = 5^{-1}$ (or) $5^x = 5^2$

$x = -1$ (or) $x = 2$

Hence $x = \{2, -1\}$

12. The altitude of a right triangle is 7 cm less than it's base. If the hypotenuse is 13cm. Find the other two sides.

Solution:

Let base = x cm

It's height = $(x - 7)$ cm

$$\text{Hypotenuse} = \sqrt{(Base)^2 + (Height)^2}$$

$$13 = \sqrt{(x)^2 + (x-7)^2}$$

$$169 = x^2 + (x-7)^2$$

$$169 = x^2 + x^2 - 14x + 49$$

$$169 = 2x^2 - 14x + 49$$

$$2x^2 - 14x + 49 - 169 = 0$$

$$2x^2 - 14x - 120 = 0$$

$$x^2 - 7x - 60 = 0$$

$$(x-12)(x+5) = 0$$

$x = 12$ (or) $x = -5$ (not admissible)

i.e $x = 12$

base = 12 cm; height = $x-7 = 12-7 = 5$ cm

13. Two water taps together can fill a tank in $9\frac{3}{8}$ hrs. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately find the time in which each tap can separately fill the tank.

Solution:

Let the smaller tap fill the tank in x hours.

The larger tap fill the tank in $(x-10)$ hours.

By given data

$$\frac{1}{x} + \frac{1}{x-10} = \frac{1}{9\frac{3}{8}} \Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\frac{2x-10}{x^2-10x} = \frac{8}{75} \Rightarrow \frac{x-5}{x^2-10x} = \frac{4}{75}$$

$$75(x-5) = 4(x^2-10x)$$

$$75x-375 = 4x^2-40x$$
$$4x^2-75x+375-40x = 0$$
$$4x^2-115x+375 = 0$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{115 \pm \sqrt{13225-4(4)(375)}}{2(4)}$$
$$x = \frac{115 \pm \sqrt{13225-6000}}{8} \Rightarrow \frac{115 \pm \sqrt{7225}}{8}$$
$$x = \frac{115 \pm 85}{8} \Rightarrow \frac{115+85}{8} \text{ (or) } \frac{115-85}{8}$$
$$x = \frac{200}{8} \text{ (or) } \frac{30}{8} \Rightarrow x = 25 \text{ (or) } \frac{15}{4}$$

Thus, time to fill the tank by the smaller tap alone = 25 hours and by larger tape alone $x-10$ = 15 hours.

14. If the roots of the equation $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$

Solution:

$(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$

Given that the roots are real and equal

$$\therefore b^2-4ac = 0$$
$$[-2(ac+bd)]^2 - 4[a^2+b^2][c^2+d^2] = 0$$
$$4(a^2c^2+b^2d^2+2abcd) - 4[a^2c^2+a^2d^2+b^2c^2+b^2d^2] = 0$$

$\div$ by 4 $\quad a^2c^2+b^2d^2+2abcd-a^2c^2-a^2d^2-b^2c^2-b^2d^2 = 0$

$$a^2d^2+b^2c^2-2abcd = 0$$
$$(ad-bc)^2 = 0$$
$$ad-bc = 0$$
$$ad = bc$$
$$\frac{a}{b} = \frac{c}{d} \text{ hence proved.}$$

15. Simplify: $\frac{2x^2+3x+1}{3x^2+4x+1} \times \frac{4x^2+5x+1}{5x^2+6x+1} \times \frac{15x^2+8x+1}{8x^2+6x+1}$

Solution:

$$\frac{(2x^2+2x+x+1)(4x^2+4x+x+1)(15x^2+5x+3x+1)}{(3x^2+3x+x+1)(5x^2+5x+x+1)(8x^2+4x+2x+1)}$$

$$= \frac{[2x(x+1)+1(x+1)][4x(x+1)+1(x+1)]}{[3x(x+1)+1(x+1)][5x(x+1)+1(x+1)]} \frac{[5x(3x+1)+1(3x+1)]}{[4x(2x+1)+1(2x+1)]}$$

$$= \frac{\cancel{(x+1)}\,\cancel{(2x+1)}\,\cancel{(x+1)}\,\cancel{(4x+1)}\,\cancel{(3x+1)}\,\cancel{(5x+1)}}{\cancel{(x+1)}\,\cancel{(3x+1)}\,\cancel{(x+1)}\,\cancel{(5x+1)}\,\cancel{(2x+1)}\,\cancel{(4x+1)}}$$

$= 1$

16. Find the value of p, when $px^2 + \left(\sqrt{3}-\sqrt{2}\right)x - 1 = 0$ and $x = \frac{1}{\sqrt{3}}$ is one root of the equation.

Solution:

Sum of the roots $= -\frac{b}{a}$

$$\alpha + \frac{1}{\sqrt{3}} = \frac{-\left(\sqrt{3}-\sqrt{2}\right)}{p} \qquad \ldots\ldots\ldots\ (1)$$

Product of the roots $= \frac{c}{a}$

$$\alpha \cdot \frac{1}{\sqrt{3}} = -1/p \qquad \Rightarrow \qquad \alpha = -\frac{\sqrt{3}}{p}$$

Sub $\alpha = -\frac{\sqrt{3}}{p}$ in eqn (1)

$$-\frac{\sqrt{3}}{p} + \frac{1}{\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{p} \qquad \Rightarrow \qquad = \frac{\sqrt{2}}{p} - \frac{\sqrt{3}}{p}$$

$$p = \sqrt{3}\ \sqrt{2} = \sqrt{6}$$

For Practice	**5 marks**

1. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
   Show that $A^2 - (a+d)A = (bc - ad)\, I_2$
2. $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$

3. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$.

4. If $A = \begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$

find the value of (i) B – 5A (ii) 3A – 9B

5. Find the values of x, y, z if

(i) $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

(ii) $(x \quad y-z \quad z+3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)$

6. Find x and y if $x\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

7. Find the non-zero values of x satisfying the matrix equation

$$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$

8. Solve for x, y: $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

9. If $A = \begin{pmatrix} 1 & 3 \\ 5 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 5 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 1 & 3 \end{pmatrix}$

Verify that A(B+C) = AB + AC

10. If $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ find AB and BA.

Verify AB = BA.

11. Write down the quadratic equation in general form for which the sum and product of the roots are given below.

(i) 9, 14 (ii) $-\frac{7}{2}, \frac{5}{2}$ (iii) $-\frac{3}{5}, -\frac{1}{2}$

12. Find the sum and product of the roots for each of the following quadratic equations: (i) $x^2+8x-65 = 0$ (ii) $2x^2+5x+7 = 0$

13. Find the square root of $(4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$

14. If α and β are the roots of $x^2 + 7x + 10 = 0$ find the values of

(i) $(\alpha - \beta)$ (ii) $\alpha^2 + \beta^2$

(iii) $\alpha^3 - \beta^3$ (iv) $\alpha^4 + \beta^4$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

15. If α, β are the roots of equation $3x^2 + 7x - 2 = 0$ find the values of (i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

16. Find the G.C.D of the polynomials $x^3 + x^2 - x + 2$ and $2x^3 - 5x^2 + 5x - 3$.

17. If α, β are the roots of the equation $2x^2 - x - 1 = 0$, then form the equation whose roots are

(i) $\frac{1}{\alpha}, \frac{1}{\beta}$ (ii) $\alpha^2\beta, \beta^2\alpha$ (iii) $2\alpha+\beta, 2\beta+\alpha$

18. The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. Without solving for the roots, find $\frac{1}{\alpha} + \frac{1}{\beta}, \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

19. The roots of the equation $x^2 + 6x - 4 = 0$ are α, β. Find the quadratic equation whose roots are (i) α^2 and β^2 (ii) $\alpha^2\beta$ and $\beta^2\alpha$

20. If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \frac{-13}{7}$.

Find the values of a.

21. Find the G.C.D of $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$.

22. Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹1800 worth of apples and ₹600 worth bananas, then how many kgs of each fruit did she buy?

23. Solve $3p^2 + 2\sqrt{5}\,p - 5 = 0$ by formula method.

24. A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains.

Created Questions — 5 marks

1. Simplify: $\dfrac{1}{a^2+3a+2}+\dfrac{1}{a^2+5a+6}-\dfrac{1}{a^2+4a+3}$

2. If $A=\begin{pmatrix}3 & 2 & 0\\ 1 & 4 & 0\\ 0 & 0 & 5\end{pmatrix}$ show that $A^2-7A+10I_3=0$

3. The area of rectangle gets reduced by 9 square units. If it's length is reduced by 5 units and breadth is increased by 3 units, if we increase the length by 3 units and breadth by 2 units the area increases by 67 square units. Find the dimensions of the rectangle.

4. The students of a class are to stand in rows. If 3 students are extra in a row, their would be 1 row less. If 3 students are less in a row, their would be 2 rows more find the number of students in the class.

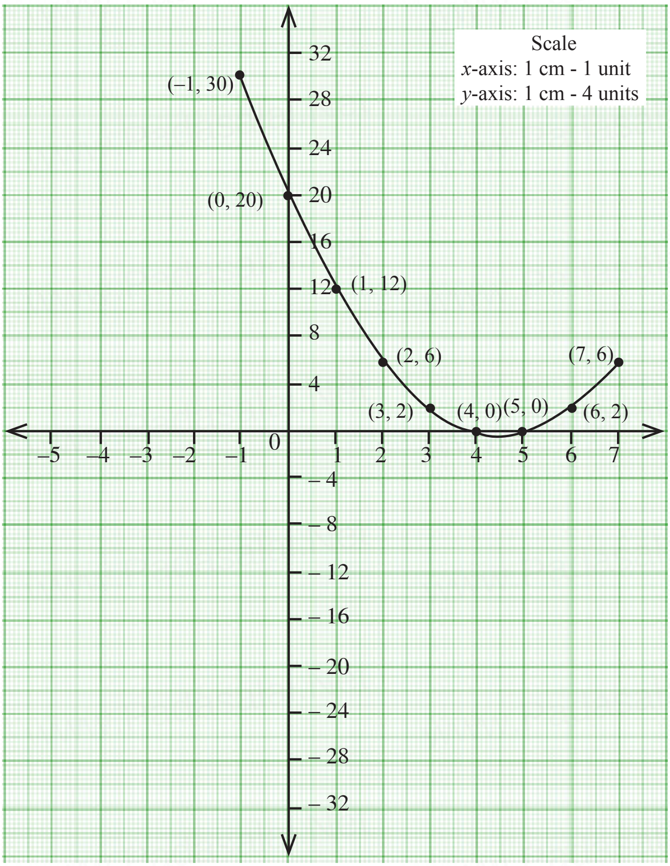
Quadratic Graphs — 5 marks

1. State the nature of solution of the following quadratic equation $x^2 - 9x + 20 = 0$.

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-9x$	36	27	18	9	0	–9	–18	–27	–36
+20	20	20	20	20	20	20	20	20	20
y	72	56	42	30	20	12	6	2	0

Points: (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1,12), (2, 6), (3, 2), (4, 0)



Solution:

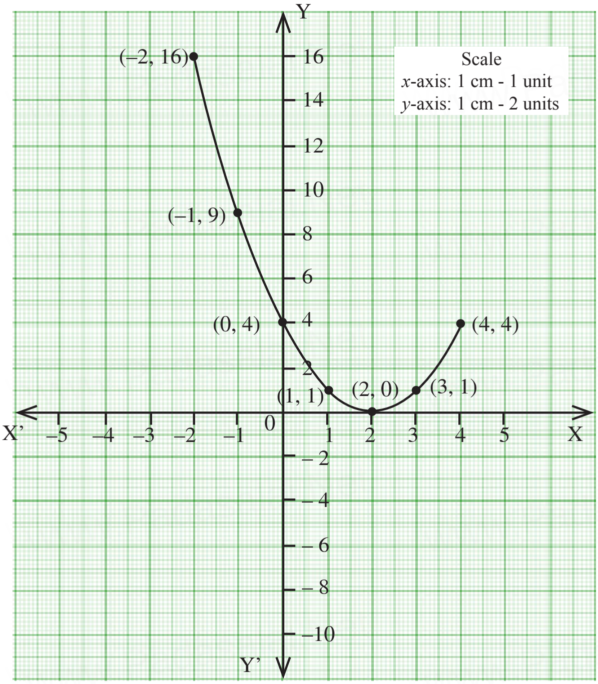
Since there are two points of intersection with the X-axis, the roots are real and unequal.

2. State the nature of solution of the following quadratic equation: $x^2 - 4x + 4 = 0$.

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	–4	–8	–12	–16
+4	4	4	4	4	4	4	4	4	4
y	36	25	16	9	4	1	0	1	4

Points: (–4, 36), (–3, 25), (–2, 16), (–1, 9), (0, 4), (1,1), (2, 0), (3, 1), (4, 4)



Solution:

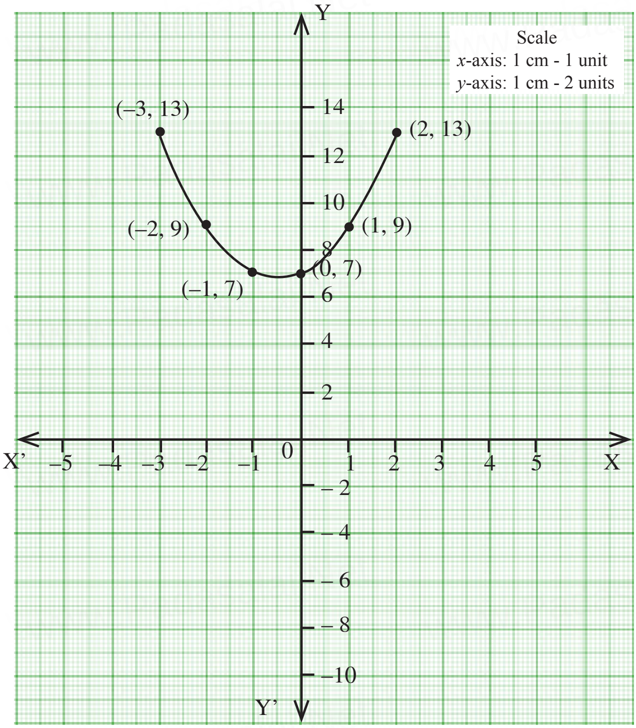
Since there is only one point of intersection with the X-axis, the roots are real and equal.

3. State the nature of solution of the following quadratic equation : $x^2 + x + 7 = 0$.

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$+x$	–4	–3	–2	–1	0	1	2	3	4
+7	7	7	7	7	7	7	7	7	7
y	19	13	9	7	7	9	13	19	27

Points: (–4, 19), (–3, 13), (–2, 9), (–1, 7), (0, 7), (1,9), (2, 13), (3, 19), (4, 27)



Solution:

Here the parabola does not intersect or touch the x-axis.
So there is no real root.

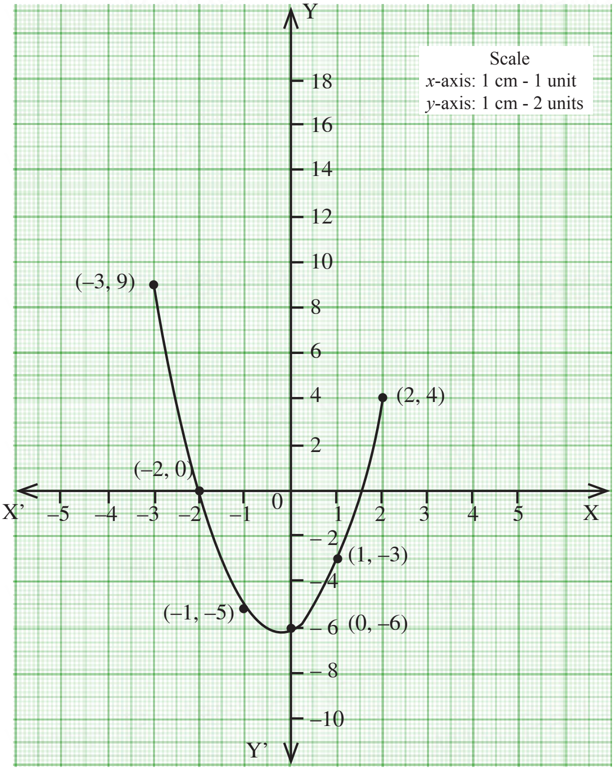
4. State the nature of solution of the following quadratic equation: $(2x - 3)(x + 2) = 0$.

$y = (2x - 3)(x + 2) = 2x^2 + 4x - 3x - 6 = 2x^2 + x - 6$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$+x$	–4	–3	–2	–1	0	1	2	3	4
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	22	9	0	–5	–6	–3	4	15	30

Points: (–4, 22), (–3, 9), (–2, 0), (–1, –5), (0, –6), (1, –3), (2, 4), (3, 15), (4, 30)



Solution:

Since the parabola intersects the x-axis at two different points the roots are real and distinct.

5. Draw the graph of $y = x^2+x-2$ and hence solve $x^2+x-2 = 0$.

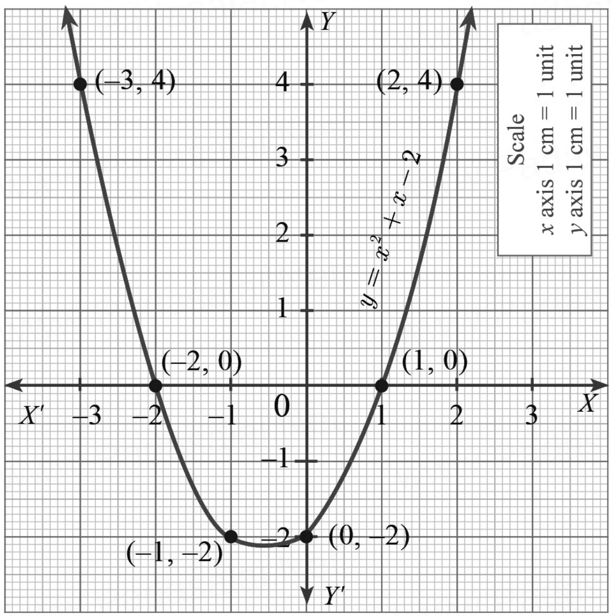
Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$+x$	–4	–3	–2	–1	0	1	2	3	4
–2	–2	–2	–2	–2	–2	–2	–2	–2	–2
y	10	4	0	–2	–2	0	4	10	18

Points: (–4, 10), (–3,4), (–2, 0), (–1, –2), (0, –2), (1, 0), (2, 4), (3, 10), (4, 18)

Subtraction:

$$\begin{array}{rcl} y & = & x^2 + x - 2 \\ 0 & = & x^2 + x - 2 \\ (-) & & (-)\ (-)\ (+) \\ \hline y & = & 0 \\ \hline \end{array}$$



Solution:

–2 and 1

6. Draw the graph of y =x^2–5x–6 and hence solve x^2–5x–14=0.

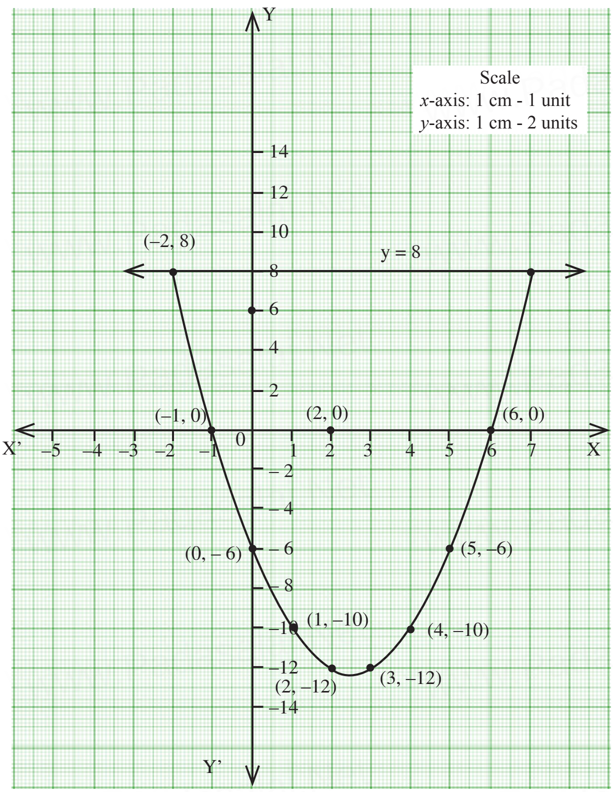
Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–5x	20	15	10	5	0	–5	–10	–15	–20
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	30	18	8	0	–6	–10	–12	–12	–10

Points: (–4, 30), (–3, 18), (–2, 8), (–1,0), (0,–6), (1,–10), (2, –12), (3, –12), (4, –10)

Subtraction:

$$\begin{array}{ccccc} y & = & x^2 & -\ 5x & -\ 6 \\ 0 & = & x^2 & -\ 5x & -\ 14 \\ (-) & & (-) & (+) & (+) \\ \hline \mathbf{y} & = & & \mathbf{8} & \\ \hline \end{array}$$



Solution: –2 and 7

7. Draw the graph of y = $2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$.

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$-3x$	12	9	6	3	0	–3	–6	–9	–12
–5	–5	–5	–5	–5	–5	–5	–5	–5	–5
y	39	22	9	0	–5	–6	–3	4	15

Points: (–4, 39), (–3, 22), (–2, 9), (–1,0), (0, –5), (1, –6), (2, –3), (3, 4), (4, 15)

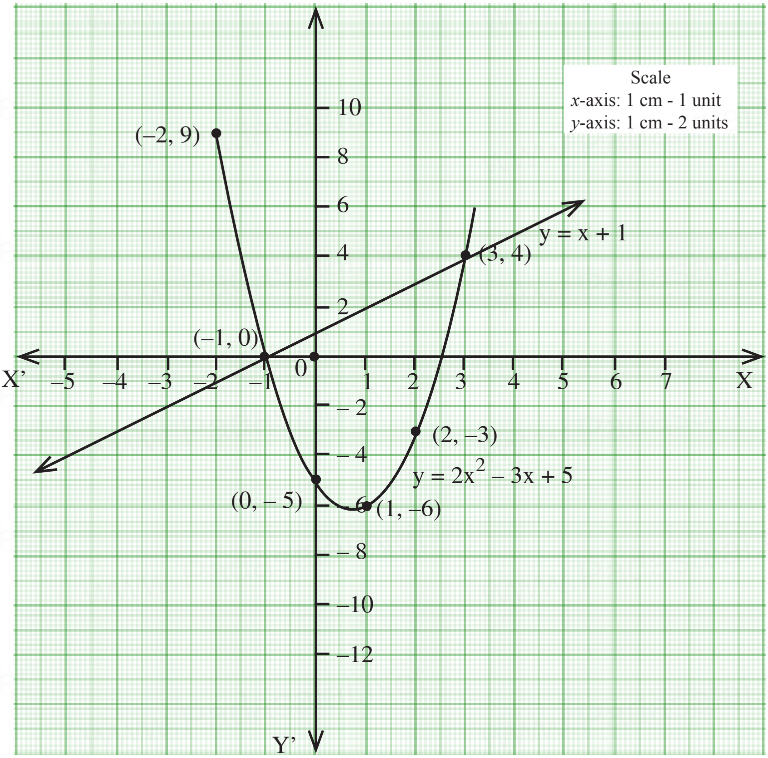
Subtraction:

$$\begin{array}{rcl} y & = & 2x^2 - 3x - 5 \\ 0 & = & 2x^2 - 4x - 6 \\ (-) & & (-)\quad (+)\quad (+) \\ \hline \mathbf{y} & = & \boldsymbol{x+1} \\ \hline \end{array}$$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x	–4	–3	–2	–1	0	1	2	3	4
+1	1	1	1	1	1	1	1	1	1
y	–3	–2	–1	0	1	2	3	4	5

The solution is {–1, 3}



8. Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
y	32	18	8	2	0	2	8	18	32

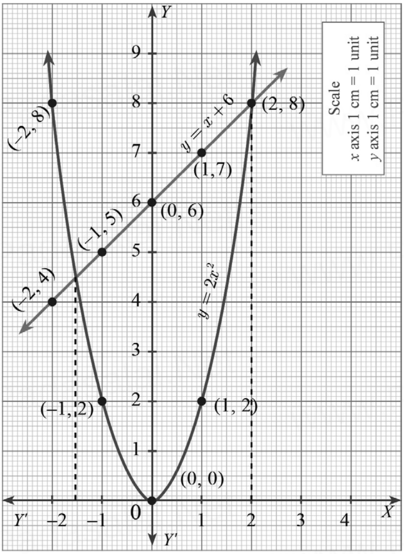
Points: (–4, 32), (–3, 18), (–2, 8), (–1, 2), (0, 0), (1, 2), (2, 8), (3, 18), (4, 32)

Subtraction:

$$\begin{array}{rcl} y & = & 2x^2 \\ 0 & = & 2x^2 \; - x \; - 6 \\ (-) & & (-) \quad (+) \quad (+) \\ \hline \mathbf{y} & = & \mathbf{x + 6} \end{array}$$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x	–4	–3	–2	–1	0	1	2	3	4
+6	6	6	6	6	6	6	6	6	6
y	2	3	4	5	6	7	8	9	10



Solution: –1.5 and 2

9. Draw the graph of y = $x^2 - 4x + 3$ and hence use it to solve $x^2 - 6x + 9 = 0$.

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	–4	–8	–12	–16
+3	3	3	3	3	3	3	3	3	3
y	35	24	15	8	3	0	-1	0	3

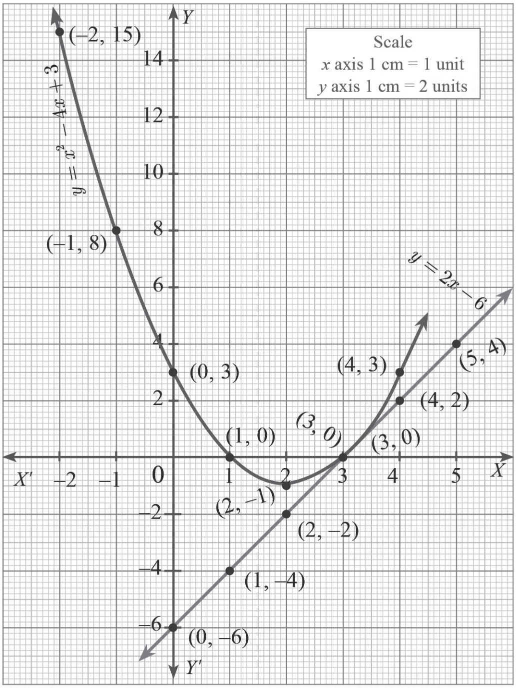
Points: (–4, 35), (–3, 24), (–2, 15), (–1, 8), (0, 3), (1, 0), (2, –1), (3, 0), (4, 3)

Subtraction:

$$\begin{array}{cccccc} y & = & x^2 & - & 4x & +3 \\ 0 & = & x^2 & - & 6x & +9 \\ (-) & & (-) & & (+) & (-) \\ \hline \mathbf{y} & = & & \mathbf{2x - 6} & & \end{array}$$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
$2x$	–8	–6	–4	–2	0	2	4	6	8
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	–14	–12	–10	–8	–6	–4	–2	0	2



Solution: 3 twice

10. Draw the graph of y = $(x - 1)(x + 3)$ and hence solve $x^2 - x - 6 = 0$.

y = $(x - 1)(x + 3)$ $= x^2 + 3x - x - 3$ $= x^2 + 2x - 3$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$+2x$	–8	–6	–4	–2	0	2	4	6	8
–3	–3	–3	–3	–3	–3	–3	–3	–3	–3
y	5	0	–3	–4	–3	0	5	12	21

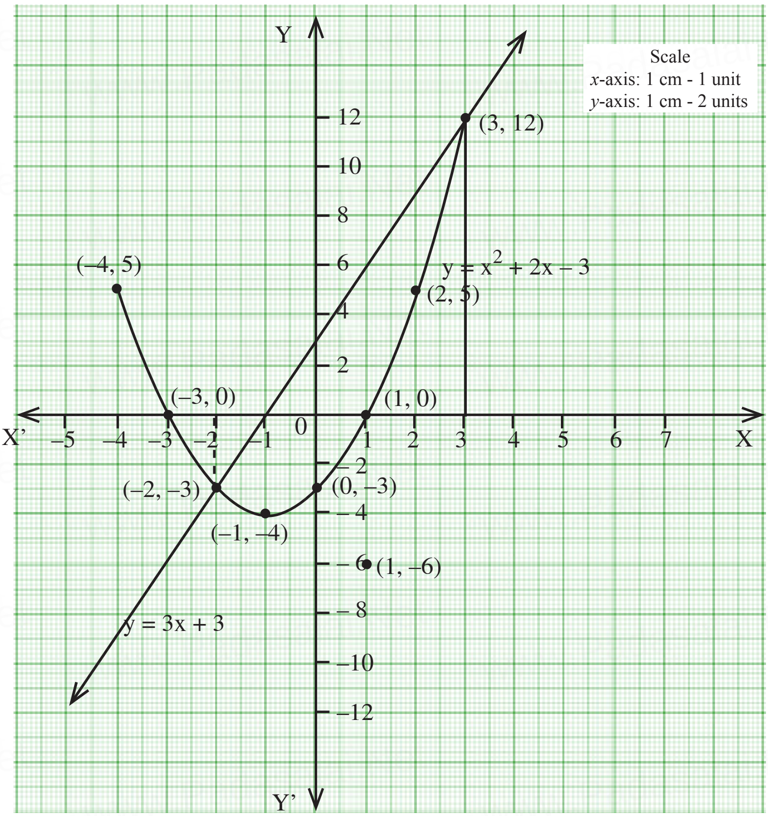
Points: (–4, 5), (–3, 0), (–2, –3), (–1, –4), (0, –3), (1, 0), (2, 5), (3, 12), (4, 21)

Subtraction:

$$\begin{array}{rcl} y &=& x^2 + 2x - 3 \\ 0 &=& x^2 - x - 6 \\ (-) & & (-)\ (+)\ (+) \\ \hline \mathbf{y} &=& \mathbf{3x + 3} \\ \hline \end{array}$$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
$3x$	–12	–9	–6	–3	0	3	6	9	12
3	3	3	3	3	3	3	3	3	3
y	–9	–6	–3	0	3	6	9	12	15



Solution:

–2 and 3

11. **Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and circumference of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.**

1. **Table:**

Diameter (x) cm	1	2	3	4	5
Circumference (y) cm	3.1	6.2	9.3	12.4	15.5

Solution:

2. **Variation:**
From the table, we found that as x increases, y also increases. Thus, the variation is a direct variation.

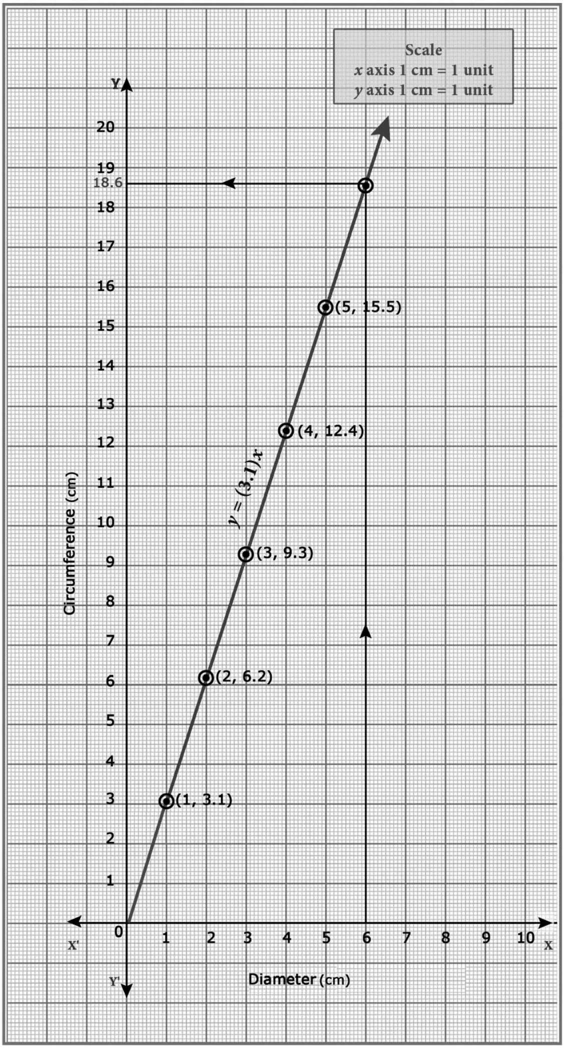
3. **Equation:**
Let $y = kx$, where k is the constant of proportionality.
From the given values, we have,

$$k = \frac{y}{x} = \frac{3.1}{1} = \frac{6.2}{2} = \frac{9.3}{3} = \frac{12.4}{4}$$

$\therefore$ i.e $y = 3.1\,x$

4. **Points to be plotted:**
When you plot the points (1, 3.1) (2, 6.2) (3, 9.3), (4, 12.4), (5, 15.5), you find the relation $y = (3.1)x$ forms a straight-line graph.

6. **Solution:**
Clearly, from the graph, when diameter is 6 cm, its circumference is 18.6 cm.



12. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find

(i) the constant of variation

(ii) how far will it travel in $1\frac{1}{2}$ hr

(iii) the time required to cover a distance of 300 km from the graph.

Solution:

1. **Table:**

Let x be the time taken in minutes and y be the distance travelled in km.

Time taken x (in minutes)	60	120	180	240
Distance y (in km)	50	100	150	200

2. **Variation:**

(i) Observe that as time increases, the distance travelled also increases. Therefore, the variation is a direct variation.

3. **Equation:**

$y = kx$

Constant of variation

$$k = \frac{y}{x} = \frac{50}{60} = \frac{100}{120} = \frac{150}{180} = \frac{200}{240} = \frac{5}{6}$$

4. **Points to be plotted:**

(60, 50), (120, 100), (180, 150), (240, 200)

$$y = kx \Rightarrow y = \frac{5}{6}x$$

(ii) From the graph, $y = \frac{5}{6}x$ if $x = 90$,

then $y = \frac{5}{6} \times 90 = 75$ km

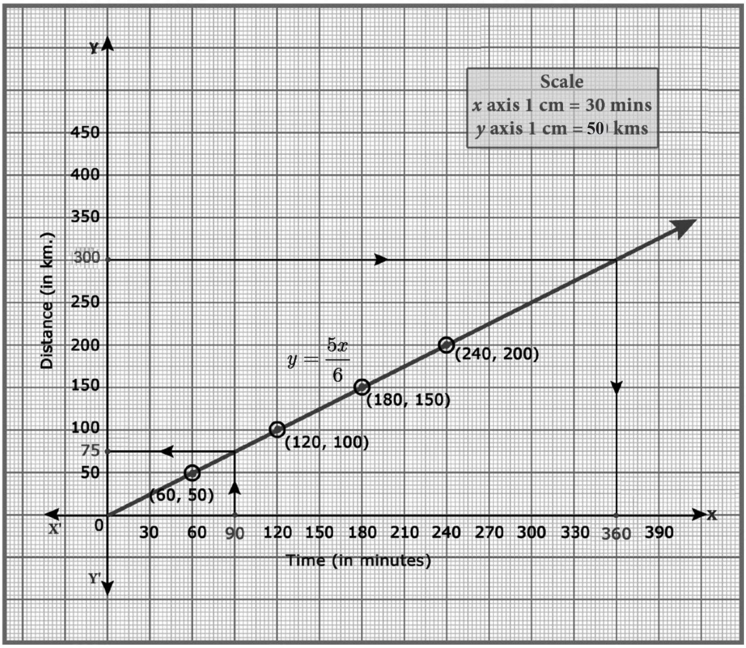
The distance travelled for $1\frac{1}{2}$ hours.

(i.e.,) 90 minutes is 75 km.

(iii) From the graph, $y = \frac{5}{6}x$, if y = 300

then $x = \frac{6y}{5} = \frac{6}{5} \times 300 = 360$ minutes (or) 6 hours.

The time taken to cover 300 km is 360 minutes, that is 6 hours.



13. A company initially started with 40 workers to complete the work by 150 days. Later, it decided to fasten up the work increasing the number of workers as shown below.

Number of workers (x)	40	50	60	75
Number of days (y)	150	120	100	80

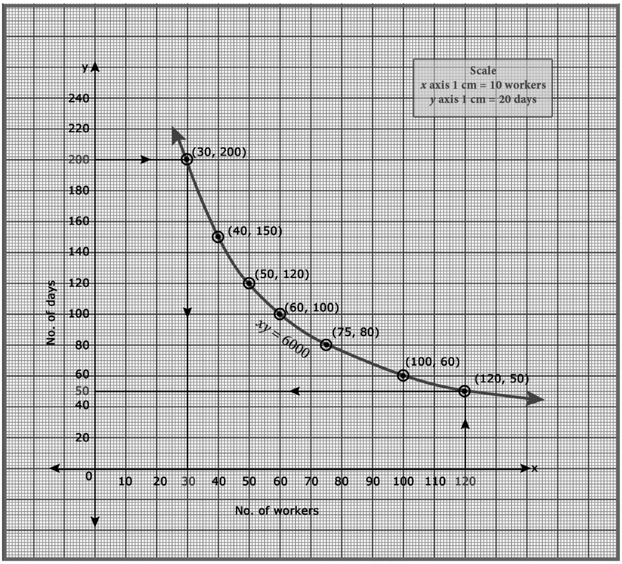
(i) Graph the above data and identify the type of variation.

(ii) From the graph, find the number of days required to complete the work if the company decides to opt for 120 workers?

(iii) If the work has to be completed by 30 days, how many workers are required?

Solution:

i)



From the given table, we observe that as x increases, y decreases. Thus, the variation is an inverse variation.

Let $y = \frac{k}{x}$

$\Rightarrow xy = k$, $k > 0$ is called the constant of variation.

From the table, $k = 40\times150 = 50\times120 = ... = 75\times80 = 6000$

Therefore, $xy = 6000$

Plot the points (40, 150), (50, 120), (60, 100) of (75, 80) and join to get a free hand smooth curve (Rectangular Hyperbola).

(ii) From the graph, the required number of days to complete the work when the company decides to work with 120 workers is 50 days.

Also, from $xy = 6000$ if $x = 120$, then $y = \frac{6000}{120} = 50$

(iii) From the graph, if the work has to be completed by 30 days, the number of workers required is 200.

Also, from $xy = 6000$ if $y = 30$, then $x = \frac{6000}{30} = 200$

14. **Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km/hr and reached the destination in 1 hour. He was followed by Aradhana, Ponmozhi, Jeyanth, Sathya and Swetha with their respective speed of 6 km/hr, 4 km/hr, 3 km/hr and 2 km/hr. And, they covered the distance in 2 hrs, 3 hrs, 4 hrs and 6 hours respectively.**

Draw the speed-time graph and use it to find the time taken to Kaushik with his speed of 2.4 km/hr.

Solution:

1. **Table:**

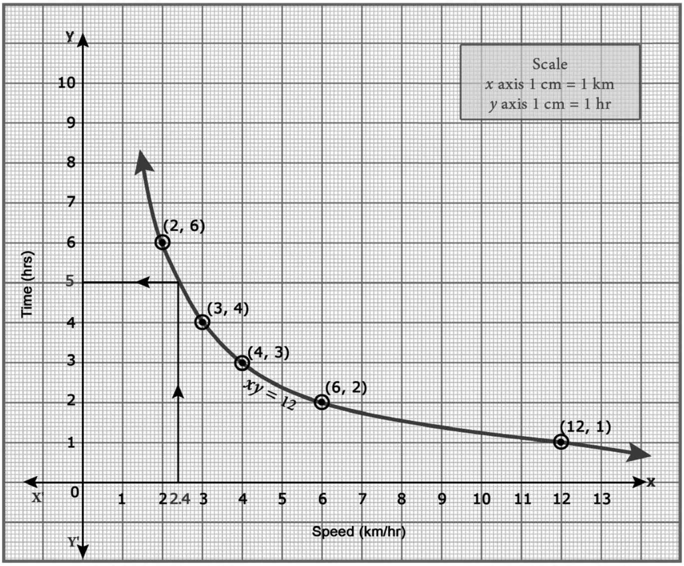
Speed x (km/hr)	12	6	4	3	2
Time y (hours)	1	2	3	4	6

2. **Variation:**
From the table, we observe that as x decreases, y increases. Hence, the type is on inverse variation.

3. **Equation:**
Let $y = \frac{k}{x}$
$\Rightarrow xy = k$, $k > 0$ is called the constant of variation.
From the table $k = 12\times1 = 6\times2 = \ldots.. = 2\times6 = 12$
Therefore, $xy = 12$.

4. **Points to be plotted:**
(12, 1), (6, 2), (4, 3), (3, 4), (2, 6)

5. **Solution:**
From the graph, we observe that Kaushik takes 5 hrs with a speed of 2.4 km/hr.



15. **A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find**

(i) the marked price when a customer gets a discount of ₹ 3250 (from graph).

(ii) the discount when the marked price is ₹ 2500.

Solution:

x = Marked price, y = Discount

$$\text{Discount} = 50\% = \frac{50}{100} = \frac{1}{2}$$

It is clear that the given problem is related with Direct variation

$\therefore$ y = kx

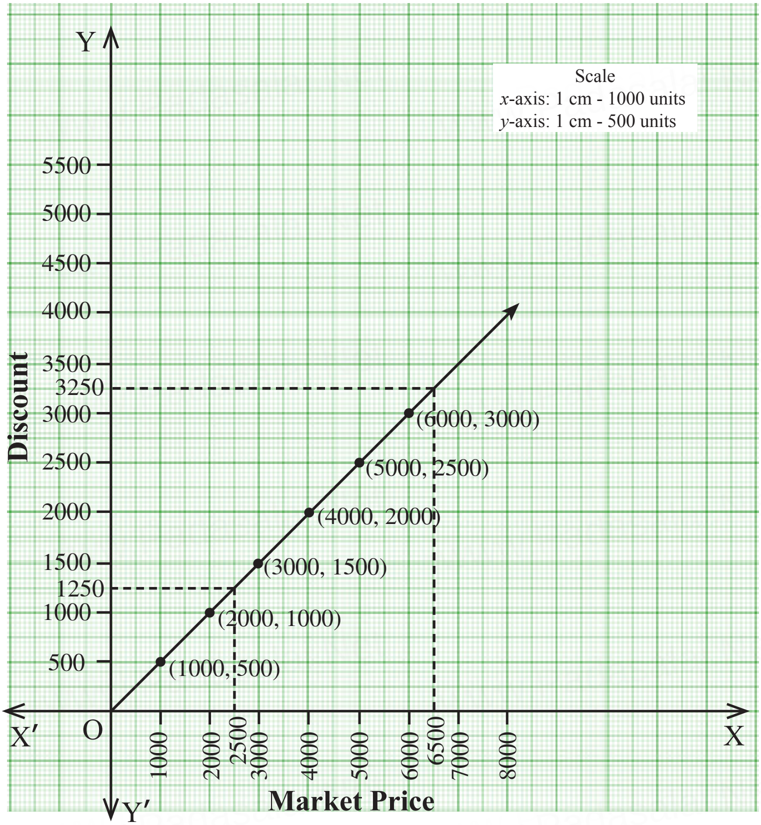
i.e $y = \frac{1}{2}x$

Marked Price (x)	1000	2000	3000	4000	5000
Discount (y)	500	1000	1500	2000	2500

Points to be plotted = { (1000, 500), (2000, 1000), (4000, 2000), (5000, 2500), (6000, 3000)}

(i) From the graph we see that
when y = ₹3250 we've x = ₹6500
The marked price = ₹6500

(ii) Similarly if x = ₹2500, we've y = ₹1250
∴ The Discount = ₹1250



16. Draw the graph of $xy = 24$, $x, y > 0$. Using the graph find, (i) y when $x = 3$ and (ii) x when y = 6.

Solution:

x	1	2	3	4	6	8	12	24
y	24	12	8	6	4	3	2	1

From the table we see that as x increases y decreases so it is indirect variation

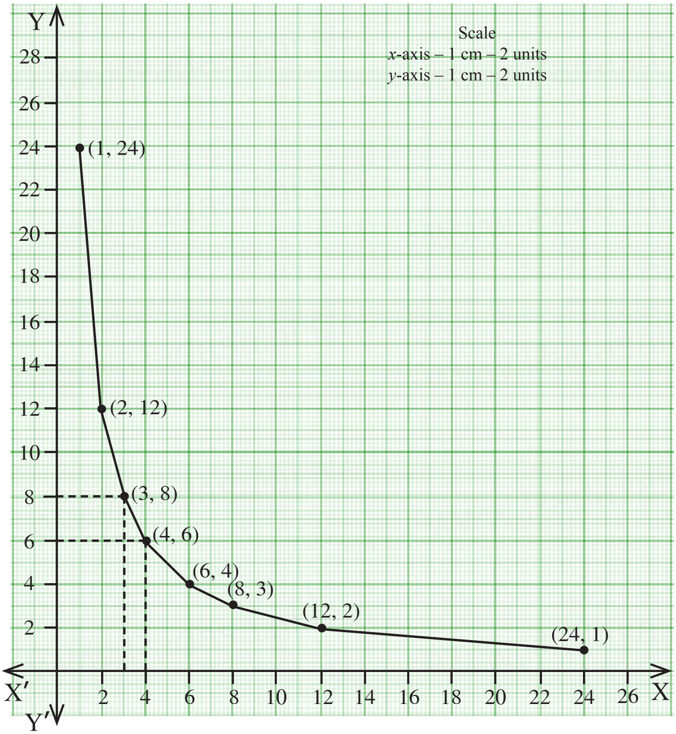
We have $xy = k$

i.e $xy = 24$

Points to be plotted = {(1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1)}

For: (i) When $x = 3$ we've $y = 8$

(ii) When $y = 6$ we've $x = 4$



17. Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also (i) find y when $x = 9$ (ii) find x when $y = 7.5$

Solution:

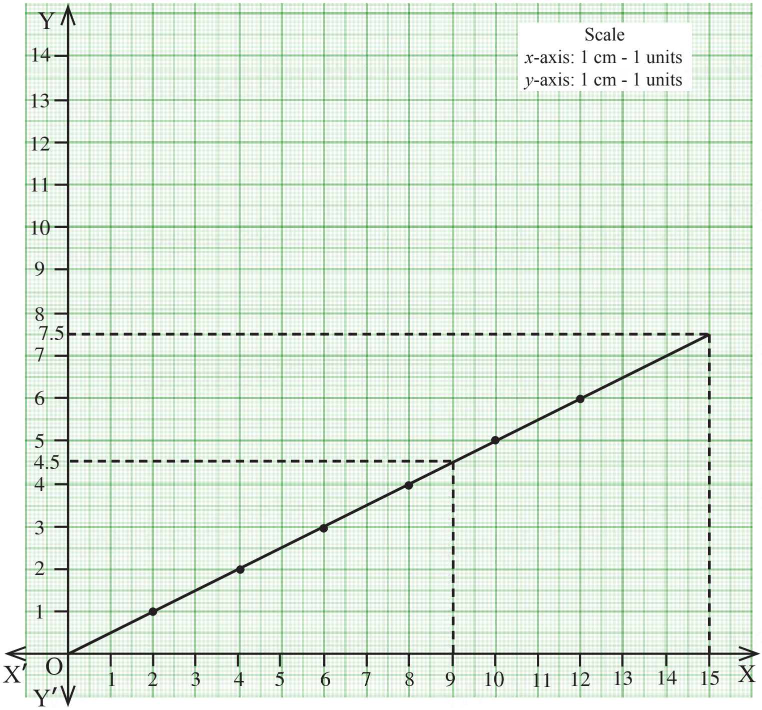
$$y = \frac{1}{2}x$$

x	2	4	6	8	10	12
y	1	2	3	4	5	6

From the table we see that as x increases, y decreases. So it is inverse variation.

$$k = \frac{1}{2}$$

Points to be plotted: {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6)}



(i) When $x = 9$ we've y = 4.5

(ii) When y = 7.5 we've $x = 15$

18. The following table shows the data about the number of pipes and the time taken to fill the same tank.

No. of pipes (x)	2	3	6	9
Time Taken (in min) (y)	45	30	15	10

Draw the graph for the above data and hence

(i) Find the time taken to fill the tank when five pipes are used.

(ii) Find the number of pipes when the time is 9 minutes.

Solution:

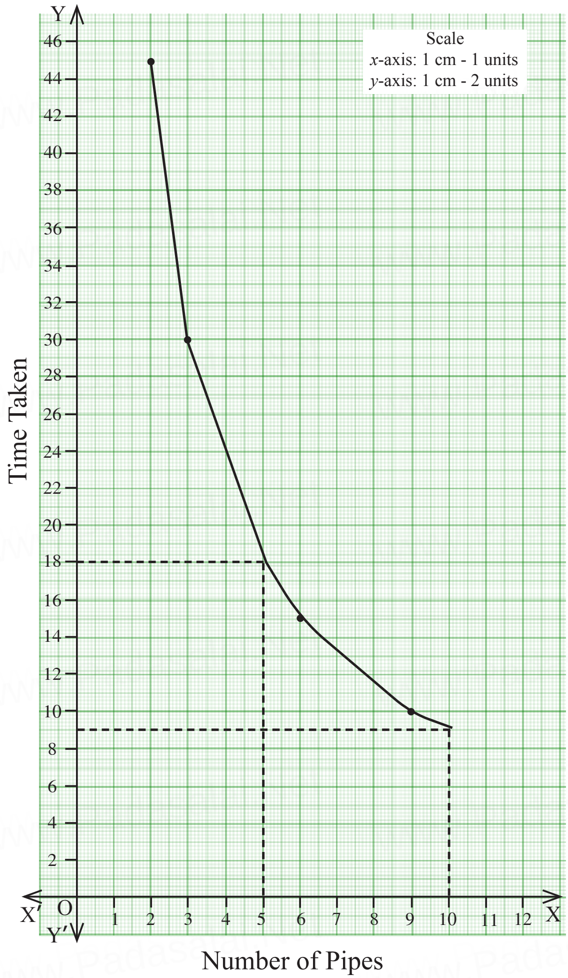
From the table we see that as x increases y decreases.
So it is an indirect variation.

We have $xy = k$; $xy = 90$; i.e $k = 90$

Points to be plotted = {(2, 45), (3, 30), (6, 15), (9, 10)}

(i) When $x = 5$ we've $y = 18$
i.e Time taken to fill the tank using 5 pipes = 18 minutes

(ii) When $y = 9$ we've $x = 10$
i.e To fill the tank in 9 minutes we need 10 pipes.



19. A school announces that for a certain competitions, the cash price will be distributed for all the participants equally as show below.

No. of participants (x)	2	4	6	8	10
Amount for each participant in ₹ (y)	180	90	60	45	36

(i) Find the constant of variation.

(ii) Graph the above data and hence, find how much will each participant get if the number of participants are 12.

Solution:

From the table we see that as x increases, y decreases. So it is an indirect variation.

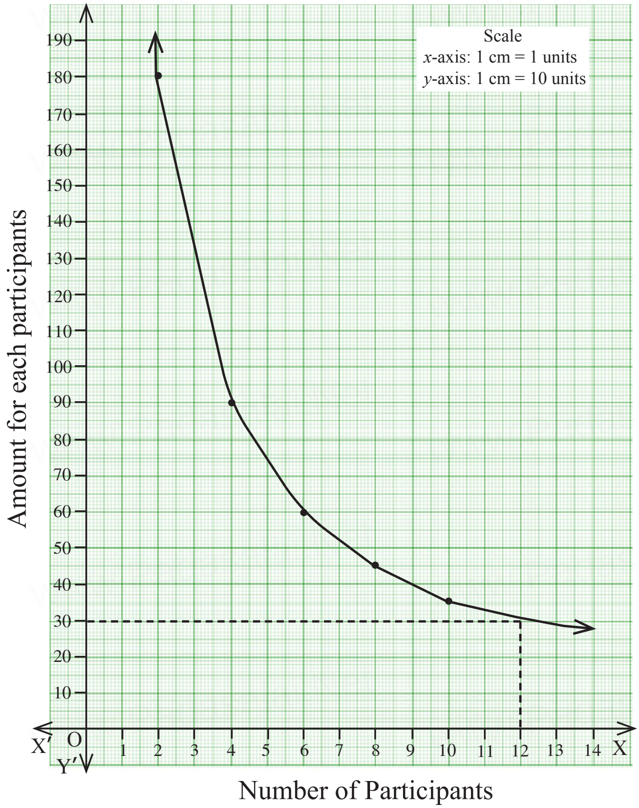
(i) We have $xy = k \Rightarrow xy = 360$

i.e $k = 360$

Points to be plotted: $\{(2, 180), (4, 90), (6, 60), (8, 45), (10, 36)\}$

(ii) When $x = 12$ we've $y = 30$

i.e Amount per each participant = ₹ 30



20. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	**4**	**8**	**12**	**24**
Amount ₹ (y)	**60**	**120**	**180**	**360**

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i) find the amount to be paid when parking time is 6 hr; (ii) find the parking duration when the amount paid is ₹150.

Solution:

From the table we see that as x increases y also increases. So it is a direct variation.

We have $y = kx$; i.e $y = 15x$ $\therefore$ $k = 15$

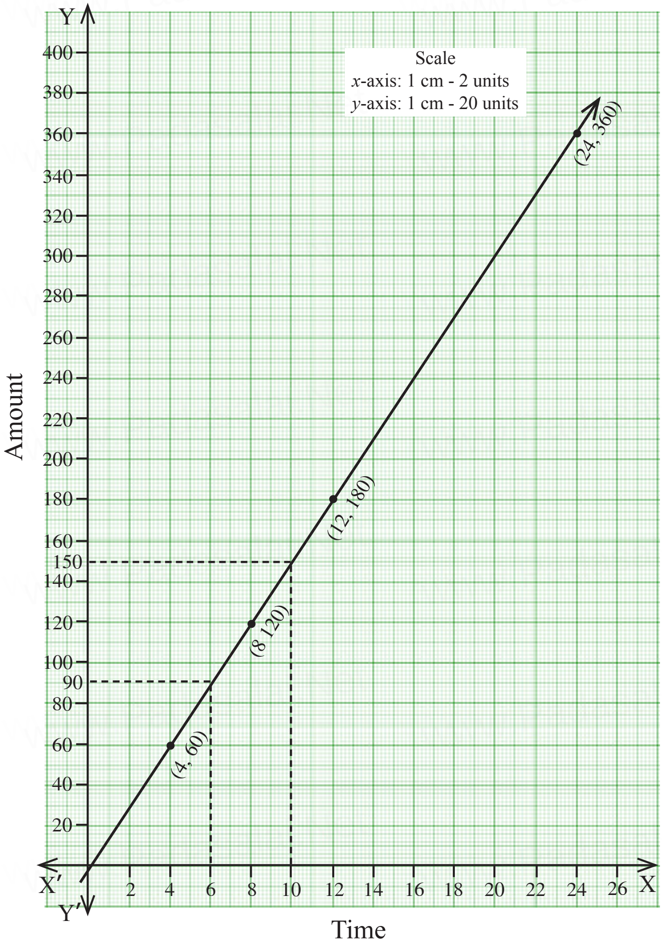
Points to be plotted: {(4, 60), (8, 120), (12, 180), (24, 360)}

(i) When $x = 6$ we've $y = 90$

Amount to be paid = ₹90

(ii) When $y =$ ₹150 we've $x = 10$

The parking duration will be 10 hours.



For Practice **5 marks**

1. Draw the graph for the quadratic equation $x^2 + x - 12 = 0$ and state their nature of solutions.
2. Draw the graph for the quadratic equation $x^2 - 8x + 16 = 0$ and state their nature of solutions.

3. Draw the graph for the quadratic equation $x^2 + 2x + 5 = 0$ and state their nature of solutions.
4. Draw the graph for the quadratic equation $x^2 - 9 = 0$ and state their nature of solutions.
5. Draw the graph for the quadratic equation $x^2 - 6x + 9 = 0$ and state their nature of solutions.
6. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$.
7. Draw the graph of $y = x^2+4x+3$ and hence solve $x^2+x+1 = 0$.
8. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.
9. Draw the graph of $y = x^2+3x+2$ and hence solve $x^2+2x+1 = 0$.
10. Draw the graph of $y = x^2+3x-4$ and hence solve $x^2+3x-4 = 0$.
11. Read the following table carefully.

Weight of Apples in kg	1	2	3	4	5
Cost of Apples in Rupees	200	400	600	800	1000

and answer the questions given below.
i) Find the proportionality constant.
ii) Draw the graph and use it to find the weight of 7 kg of apples.
iii) How many kg of apples can we buy for ₹500?

12.

No. of workers ‘x’	3	4	6	8	9	16
No. of days ‘y’	96	72	48	36	32	18

Draw the graph for the data given in the table. Hence Find the No. of days taken by 12 workers to complete the work.

13. The cost of the milk per is ₹45. Draw the graph for the relation between the quantity and cost. Hence Find
i) The proportionality constant ii) The cost of 3 litres.
14. Draw the graph of $xy = 16$, x, $y > 0$ and hence find the square root of 16.
15. The distance travelled by a motorist is given as below

Time in minutes	4	8	12	16
Distance in km	8	16	24	32

i) Find the constant of proportionality.
ii) Draw the graph and hence find the distance travelled by him in 10 minutes.
iii) In how many minutes can he cover a distance of 40 km.

☙★★★❧

UNIT 4

GEOMETRY

Objective Type Questions — 1 mark

1. If in triangles ABC and EDF, $\dfrac{AB}{DE} = \dfrac{BC}{FD}$ then they will be similar, when

1) $\angle B = \angle E$ 2) $\angle A = \angle D$
3) $\angle B = \angle D$ 4) $\angle A = \angle F$ **Ans:** 3)

2. In ΔLMN, $\angle L = 60°$, $\angle M = 50°$, If $\Delta LMN \sim \Delta PQR$ then the value of, $\angle R$ is

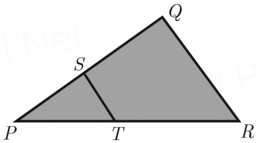
1) 40° 2) 70°
3) 30° 4) 110° **Ans:** 2)

3. If ΔABC is an isosceles triangle with $\angle C = 90°$ and AC = 5 cm, then AB is

1) 2.5 cm 2) 5 cm
3) 10 cm 4) $5\sqrt{2}$ cm **Ans:** 4)

4. In a given figure ST || QR, PS = 2cm and SQ = 3cm. Then the ratio of the area of ΔPQR to the area of ΔPST is

1) 25 : 4
2) 25 : 7
3) 25 : 11
4) 25 : 13



Ans: 1)

5. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ = 10cm, then length of AB is

1) $6\frac{2}{3}$ cm 2) $\dfrac{10\sqrt{6}}{3}$ cm
3) $66\frac{2}{3}$ cm 4) 15 cm **Ans:** 4)

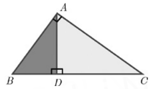
6. If in ΔABC, DE || BC, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is

1) 1.4 cm 2) 1.8 cm
3) 1.2 cm 4) 1.05 cm **Ans:** 1)

7. In a ΔABC, AD is the bisector of, $\angle BAC$. If AB = 8 cm BD = 6 cm and DC = 3 cm The length of the side AC is

1) 6 cm   2) 4 cm
3) 3 cm   4) 8 cm   **Ans:** 2)

8. In the adjacent figure $\angle BAC = 90°$ and AD $\perp$ BC then,

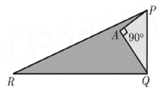


1) $BD.CD = BC^2$ 2) $AB.AC = BC^2$
3) $BD.CD = AD^2$ 4) $AB.AC = AD^2$ **Ans:** 3)\

9. Two poles of heights 6m and 11m stand vertically on a plane ground. If the distance between their feet is 12m. What is the distance between their tops?

1) 13 m   2) 14 m
3) 15 m   4) 12.8 m   **Ans:** 1)

10. In the given figure, PR = 26 cm, QR = 24 cm, $\angle PAQ = 90°$, PA = 6 cm and QA = 8 cm. Find $\angle PQR$



1) 80°   2) 85°
3) 75°   4) 90°   **Ans:** 4)

11. A tangent is perpendicular to the radius at the

1) centre   2) point of contact
3) infinity   4) chord   **Ans:** 2)

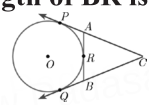
12. How many tangents can be drawn to the circle from an exterior point?

1) one   2) two
3) infinite   4) zero   **Ans:** 2)

13. The two tangents from an external points P to a circle with centre at O arc PA and PB. If $\angle APB = 70°$ then the value of $\angle AOB$ is

1) 100°   2) 110°
3) 120°   4) 130°   **Ans:** 2)

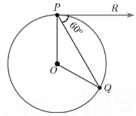
14. If figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7cm, then the length of BR is



1) 6 cm   2) 5 cm
3) 8 cm   4) 4 cm   **Ans:** 4)

15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is

1) 120° 2) 100°
3) 110° 4) 90°



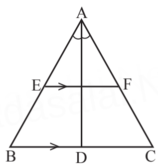
Ans: 1)

Created Questions — 1 mark

1. Which one of the following is not true?

1) If the areas of two similar triangles are in the ratio 64 : 81, then their sides, are in the ratio 8 : 9.
2) If the point of concurrence divides each of the deviances in the ratio 2 : 1 in a triangle, then the point of concurrence is called an orthocentre.
3) If two circles touch each other internally then distance between their centre is equal to difference of the radii.
4) Area of the square drawn on the side of an hypotenuse in a right angled triangle is equal to sum of the areas of squares drawn on other sides of the right angled triangle. **Ans:** 2)

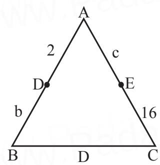
2. From the figure the appropriate condition is



1) $BC \times AC = \dfrac{EF}{BF}$
2) $BD \times AF = AE \times DC$
3) $BE \times EF = AD \times FA$
4) $BD = DC$

Ans: 2)

3. From the figure 2, b, c, 16 are in G.P and their summation is 30 in the figure



1) DE is parallel to BC
2) DE is not parallel to BC
3) It does not obey BPT
4) Data is insufficient

Ans: 1)

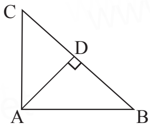
4. If D, E, F are the mid-points of sides BC, CA and AB respectively of ΔABC then the ratio of the area of triangles DEF and ABC is

1) 1 : 4 2) 1 : 2
3) 2 : 3 4) 4 : 5

Ans: 1)

5. In the figure, ABC is a triangle right angled at A and $AD \perp BC$ if AC = 3cm, AB = 4cm and BD = 3.2cm then AD is

1) 2.4
2) 4.2
3) 5.6
4) 3



Ans: 1)

6. **The distance between the centres of two circles is 13 cm and the radii are 8 cm and 3 cm respectively. The length of their direct common tangent is**
   1) 8 cm 2) 5 cm
   3) 13 cm 4) 12 cm **Ans:** 4)

7. **Which of the following is/are true**
   1) All circles are similar.
   2) All Squares are similar.
   3) Two polygons of the same number of sides are similar if their corresponding angels are equal and their corresponding sides are proportional.
   4) All equilateral triangle are similar.
   1) 1 and 2 2) 1, 2 and 4
   3) 1, 2 and 3 4) All the above **Ans:** 4)

8. **A vertical stick 20m long casts a shadow 10m long on the ground at the same time, a tower casts a shadow 50m long on the ground the height of the tower is**
   1) 100 m 2) 120 m
   3) 25 m 4) 200 m **Ans:** 1)

9. **In the given ABC, DE || BC, so that AD = (7x–4) cm, AE = (5x–2) cm, DB = (3x+4) cm, EC = 3x cm then the value of x is**
   1) 3 2) 5
   3) 4 4) 25 **Ans:** 3)

10. **In triangle ABC, $\frac{AB}{AC} = \frac{BD}{DC}$ $\angle$B = 70° and $\angle$C = 50° then $\angle$BAD =**
    1) 30° 2) 40°
    3) 50° 4) 60° **Ans:** 1)

Two Marks Questions | 2 marks

1. **State Pythagoras Theorem.**
In a right angle triange, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

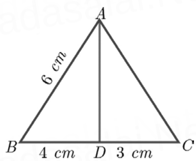
2. **State Converse of Phytagoras Theorem.**
If the square of the longest side of a triangle is equal to the sum of squares of other two sides, then the triangle is a right angle triangle.

3. **State Menelaus Theorem.**
A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1$ where all segments in the formula are directed segments.

4. **State Ceva's Theorem.**
Let ABC be a triangle and let D, E, F be points on the lines BC, CA, AB respectively. Then the cevains AD, BE, CF are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed.

5. **From the figure, AD is the bisector of ∠A. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC.**

Solution:



In ΔABC, AD is the bisector of ∠A.
Therefore by Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

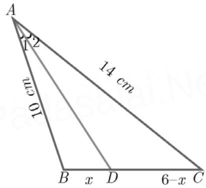
$\frac{4}{3} = \frac{6}{AC}$ gives 4AC = 18. Hence AC = $\frac{9}{2}$ = 4.5 cm

6. **In the Figure, AD is the bisector of ∠BAC, if AB = 10 cm, AC = 14 cm and BC = 6cm. Find BD and DC.**

Solution:

AD is the bisector of ∠BAC
AB=10 cm, AC = 14 cm, BC = 6 cm

By Angle Bisector Theorem



$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{x}{6-x} = \frac{10}{14} \qquad \Rightarrow \frac{x}{6-x} = \frac{5}{7}$$

$$7x = 30 - 5x$$

$$12x = 30 \qquad x = \frac{30}{12}$$

$$x = 2.5 \text{ cm}$$

$$BD = 2.5 \text{ cm}$$

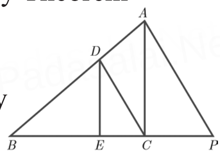
$$DC = 3.5 \text{ cm}$$

7. In the figure, DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.

Solution:

In Δ BPA , DC || AP, By Basic Proportionality Theorem

we get , $\frac{BC}{CP} = \frac{BD}{DA}$ .... (1)



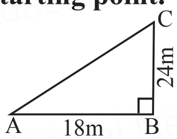
In Δ BCA, DE || AC, By Basic Proportionality Theorem

we get , $\frac{BE}{EC} = \frac{BD}{DA}$ .... (2)

From (1) and (2) $\frac{BE}{EC} = \frac{BC}{CP}$. Hence Proved

8. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point.

Solution:



$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (18)^2 + (24)^2 = 324 + 576$$

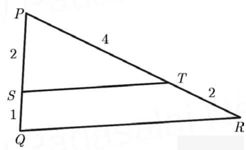
$$AC^2 = 900 \qquad AC = \sqrt{900} \qquad \Rightarrow AC = 30 \text{ m}$$

∴ The distance from the starting point is 30 m.

9. Show that ΔPST ~ Δ PQR.

Solution:

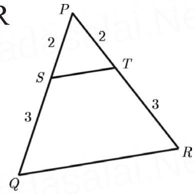
i) In ΔPST and Δ PQR



$$\frac{PS}{PQ} = \frac{2}{2+1} = \frac{2}{3}, \quad \frac{PT}{PR} = \frac{4}{4+2} = \frac{2}{3}$$

Thus, $\dfrac{PS}{PQ} = \dfrac{PT}{PR}$ and $\angle$P is common.

Therefore, by SAS similarity ΔPST~Δ PQR



ii) In ΔPST and Δ PQR

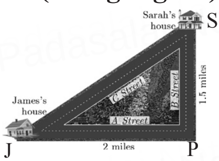
$$\frac{PS}{PQ} = \frac{2}{2+3} = \frac{2}{5}, \quad \frac{PT}{PR} = \frac{2}{2+3} = \frac{2}{5}$$

Thus, $\dfrac{PS}{PQ} = \dfrac{PT}{PR}$ and $\angle$P is common.

Therefore, by SAS similarity $\Delta PST \sim \Delta PQR$.

10. There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take B street and then A street. How much shorter is the direct path along C street (Using Figure)

Solution:



$$SJ = \sqrt{(1.5)^2 + (2)^2}$$
$$= \sqrt{2.25+4} = \sqrt{6.25} = 2.5 \text{ miles}$$

If one chooses A street and B street he has to go

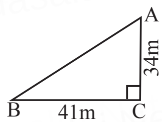
SP + PJ = 1.5 + 2 = 3.5 miles

If one chooses C street the distance from James house to Sarah's house is 2.5 miles

$\therefore$ choosing "C" street, 1 km will be shorter.

11. To get from point A to point B you must avoid walking through a pond. You must walk 34m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?

Solution:



To make a Straight way through the pond

$$AB^2 = AC^2 + BC^2$$
$$= (34)^2 + (41)^2 = 1156 + 1681$$
$$AB^2 = 2837 \Rightarrow AB = \sqrt{2837} = 53.26 \text{ m}$$

Through C one must walk AB = AC + BC = 34 + 41 = 75 m

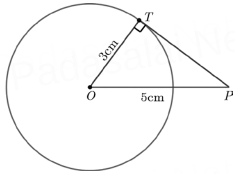
walking through the pond one must come only 53.2 m.

The difference is (75 – 53.26) m = 21.74 m.

To the nearest, one can save 21.74 m.

12. Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5cm and radius of the circle is 3 cm.

Solution:



Given OP = 5 cm, radius r = 3 cm

To find the length of tangent PT.

In right angled Triangle OTP

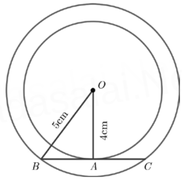
$OP^2 = OT^2 + PT^2$ (By Phythagorous Theorem)

$5^2 = 3^2 + PT^2 \Rightarrow PT^2 = 25 - 9 = 16$

Length of the tangent PT = 4 cm.

13. If radii of two concentric circles are 4cm and 5cm then find the length of the chord of one circle which is a tangent to the other circle.

Solution:



OA = 4 cm, OB = 5 cm, also OA ⊥ BC

$OB^2 = OA^2 + AB^2$

$5^2 = 4^2 + AB^2$

$AB^2 = 25 - 16 = 9$

Therefore AB = 3cm, BC = 2AB. hence, BC = 2 × 3 = 6cm

14. If ΔABC is similar to ΔDEF such that BC = 3cm, EF = 4cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Solution:

Since the ratio of area of two similar triangles is equal to the ratio of the squares of any two corresponding sides, we have

$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2} \text{ gives } \frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$$

$$\text{Area } (\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

15. D and E are respectively the points on the sides of AB and AC of a ΔABC, such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm Show that DE || BC.

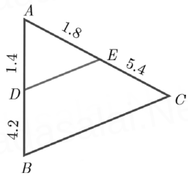
Solution:

AB = 5.6 cm , AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

BD = AB – AD = 5.6 – 1.4 = 4.2 cm

and EC = AC – AE = 7.2 – 1.8 = 5.4 cm

$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$



$$\frac{AD}{DE} = \frac{AE}{EC}.$$

Therefore, by converse of Basic Proportionality Theorem, we infer that DE is parallel to BC. Hence Proved.

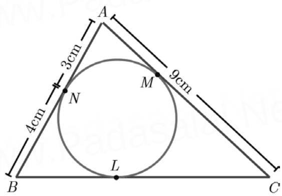
16. In the figure ΔABC is circumscribing a circle. Find the length of BC.

Solution:

AN = AM = 3 cm (Tangents drawn from same external point are equal)

BN = BL = 4 cm

$$\begin{aligned} CL = CM &= AC - AM \\ &= 9 - 3 = 6 \text{ cm} \\ BC &= BL + CL \\ &= 4 + 6 \\ &= 10 \text{ cm} \end{aligned}$$



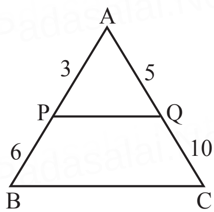
Created Questions with Solution — 2 marks

1. P and Q are points on sides AB and AC respectively of ΔABC. If AP = 3cm, PB = 6cm, AQ = 5cm and QC = 10cm, show that BC = 3 PQ.

Solution:

$$\frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ} \Rightarrow \frac{9}{3} = \frac{15}{5} = \frac{BC}{PQ}$$

$$\frac{BC}{PQ} = 3 \Rightarrow BC = 3PQ$$



2. The altitude of a triangle is 4cm greater than the base of the triangle. If the area of the triangle is 48 cm^2. Find its base and altitude.

Solution:

Let Base = x

Altitude = $x + 4$

$$\text{Area of the Triangle} = 48\ \text{cm}^2$$

$$\text{Area of the Triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$48 = \frac{1}{2} \times x(x+4)$$

$$96 = x^2 + 4x$$

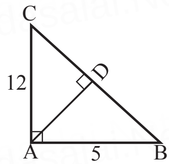
$$x^2 + 4x - 96 = 0$$

$$(x+12)(x-8) = 0$$

$$x = -12 \text{ (Not possible) (or)} \quad x = 8$$

$\therefore$ Base = 8 cm, Altitude = 12 cm

3. In triangle ABC $\angle A = 90°$, AB = 5 cm and AC = 12 cm if AD $\perp$ BC then, AD is equal to?



Solution:

Area of triangle ABC $= \frac{1}{2} \times 5 \times 12 = 30\ \text{cm}^2$

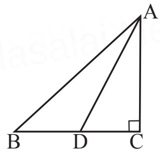
Again area of triangle ABC $= \frac{1}{2} \times$ BC . AD

$$30 = \frac{1}{2} \times 13 \times AD \qquad [BC = \sqrt{12^2 + 5^2} = 13]$$

$$60 = 13\ AD$$

$$AD = \frac{60}{13}\ \text{cm}$$

4. In a right triangle ABC, D is the mid point of BC and $\angle C = 90°$ then prove that $AB^2 = (4AD^2 - 3AC^2)$



Solution:

In triangle ABC $\angle C = 90°$

$\therefore \quad AB^2 = AC^2 + BC^2$

$\qquad = AC^2 + (2CD)^2$

[Since D is the mid point of BC. $\therefore BC = 2CD$]

$\qquad = AC^2 + 4CD^2$

$\qquad = AC^2 + 4[AD^2 - AC^2]$

$\qquad = AC^2 + 4AD^2 - 4AC^2$

Hence $AB^2 = 4AD^2 - 3AC^2$

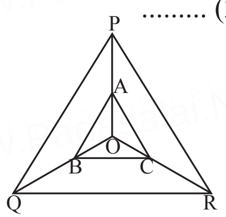
5. In the figure A, B and C are the points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

Solution:

In triangle OPQ AB || PQ by BPT $\frac{OA}{AP}=\frac{OB}{BQ}$ .... (1)

Again In triangle OPR AC || PR

by BPT $\frac{OA}{AP}=\frac{OC}{CR}$ .... (2)



$$\frac{OA}{AP}=\frac{OB}{BQ}=\frac{OC}{CR}$$

$$\frac{OB}{BQ}=\frac{OC}{CR}$$

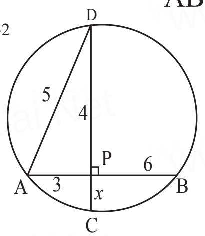
Now in triangle BC || QR.

6. **AB and CD are two perpendicular chords of the circle. AD = 5cm, AP = 3cm, PB = 6cm. Find CP where P is the point of intersection of AB and CD.**

Solution:

In triangle APD

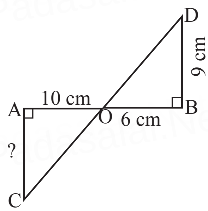
$PD^2 = AD^2 - AP^2$
$= 5^2 - 3^2$
$= 25 - 9$
$= 16$
$PD = 4$ cm



AB and CD are two chords

$AP \cdot PB = CP \cdot PD$
$3\times6 = x\times4$
$4x = 18$
$x = \frac{18}{4} = 4.5$
$CP = 4.5$ cm

7. **In figure CA and DB are perpendicular to AB. If AO = 10 cm, BO = 6 cm and DB = 9 cm. Find AC.**



Solution:

In triangle OAC and OBD

$\angle OAC = \angle OBD = 90$

$\angle AOC = \angle BOD$ (vertically opposite angles)

By AA criterion of similarity

$\Delta AOC \,|||\, \Delta BOD$

$$\frac{AO}{BO}=\frac{OC}{OD}=\frac{AC}{BD} \Rightarrow \frac{AO}{BO}=\frac{AC}{BD} \Rightarrow \frac{10}{6}=\frac{AC}{9}$$

Hence AC = 15 cm

For Practice — 2 marks

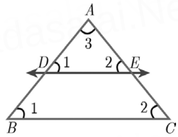
1. What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4ft from the wall? Round off your answer to the next tenth place?
2. Check whether AD is bisector of $\angle A$ of ΔABC in each of the following
   (i) AB = 5cm, AC = 10cm, BD = 1.5cm and CD = 3.5cm
   (ii) AB = 4cm, AC = 6cm, BD = 1.6cm and CD = 2.4cm
3. The length of the tangent to a circle from a point P, which is 25cm away from the centre is 24cm. What is the radius of the circle?
4. In two concentric circles, a chord of length 16cm of larger circle becomes a tangent to the smaller circle whose radius is 6cm. Find the radius of the larger circle.
5. In ΔABC if DE || BC, AD = x, DB = $x - 2$, AE = $x + 2$ and EC = $x - 1$ then, find the lengths of the sides AB and AC.

Created Questions — 2 marks

1. Prove that the line joining the mid points of two sides of a triangle is parallel to the third side.
2. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.
3. Check whether AD is the bisector of A of ΔABC. AB = 4cm, AC = 6cm, BD = 1.6cm and CD = 2.4cm
4. The line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side prove.
5. Two circles intersect each other A and B. The common chord AB is produced to meet the common tangent PQ to the Circle at C Prove that CP = CQ.
6. D, E, F are the mid points of the sides BC, CA and AB respectively of triangle ABC, Determine the ration of the areas of triangles ABC and DEF.

Five Marks Questions — 5 marks

1. **State and Prove Basic Proportionality Theorem (BPT) or Thales Theorem.**



Statement:

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof:

Given: In ΔABC, D is a point on AB and E is a point on AC.

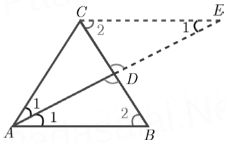
To Prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$. Construction: Draw a line DE || BC.

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because DE || BC.
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because DE || BC.
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle.
4.	$\Delta ABC \sim \Delta ADE$ $\dfrac{AB}{AD} = \dfrac{AC}{AE}$ $\dfrac{AD+DB}{AD} = \dfrac{AE+EC}{AE}$ $1+\dfrac{DB}{AD} = 1+\dfrac{EC}{AE}$ $\dfrac{DB}{AD} = \dfrac{EC}{AE}$ $\dfrac{AD}{DB} = \dfrac{AE}{EC}$	By AAA similarity. Corresponding sides are proportional. Split AB and AC using the points D and E. On Simplification. Cancelling 1 on both sides. Taking reciprocals.
	Hence Proved.	

2. **State and Prove Angle Bisector Theorem.**

Statement:

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.



Proof:

Given: In ΔABC,

AD is the internal bisector.

To Prove: $\dfrac{AB}{AC} = \dfrac{BD}{CD}$.

Construction: Draw a line through C parallel to AB.
Extend AD to meet line through C at E.

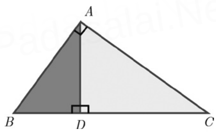
No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles $AC = CE$ …. (1)	In ΔACE, $\angle CAE = \angle CEA$.
3.	$\Delta ABD \sim \Delta ECD$ $\dfrac{AB}{CE} = \dfrac{BD}{CD}$	By AA Similarity.
4.	$\dfrac{AB}{AC} = \dfrac{BD}{CD}$	From (1) $AC = CE$. Hence Proved.

3. **State and Prove Pythagoras Theorem.**

Statement:

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof:



Given: In ΔABC, $\angle A = 90°$

To Prove: $AB^2 + AC^2 = BC^2$

Construction: Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔABD. $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \sim \Delta ABD$ $\dfrac{AB}{BD} = \dfrac{BC}{AB}$ $AB^2 = BC \times BD$ .... (1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity.

2.	Compare ΔABC and ΔADC. $\angle C$ is common. $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \sim \Delta ADC$ $\dfrac{BC}{AC} = \dfrac{AC}{DC}$ $AC^2 = BC \times DC$ .... (2)	Given $\angle BAC = 90°$ and by construction $\angle CDA = 90°$ By AA similarity.

Adding (1) and (2) we get,

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$
$$= BC(BD + DC)$$
$$AB^2 + AC^2 = BC \times BC = BC^2.$$

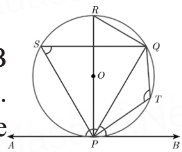
Hence the theorem is proved.

4. State and Prove Alternate Segment Theorem.

Statement:

If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

Proof:



Given: A circle with centre at O, tangent AB touches the circle at P and PQ is a chord. S and T are two points on the circle in the opposite sides of chord PQ.

To Prove: (i) $\angle QPB = \angle PSQ$ and (ii) $\angle QPA = \angle PTQ$

Construction: Draw the diameter POR. Draw QR, QS and PS.

No.	Statement	Reason
1.	$\angle RPB = 90°$ $\angle RPQ + \angle QPB = 90°$ ...... (1)	Diameter RP is perpendicular to tangent AB.
2.	In ΔRPQ, $\angle PQR = 90°$ ... (2)	Angle in a semicircle is 90°.
3.	$\angle QRP + \angle RPQ = 90°$....... (3)	In a right angled triangle, sum of the two acute angles is 90°.
4.	$\angle RPQ + \angle QPB = \angle QRP + \angle RPQ$ $\Rightarrow \angle QPB = \angle QRP$ .... (4)	From (1) and (3)

5.	$\angle QRP = \angle PSQ$ .... (5)	Angles in the same segment are equal.
6.	$\angle QPB = \angle PSQ$ .... (6)	From (4) and (5); Hence (i) is proved.
7.	$\angle QPB + \angle QPA = 180°$ .... (7)	Linear pair of angles.
8.	$\angle PSQ + \angle PTQ = 180°$..... (8)	Sum of opposite angles of a cyclic quadrilateral is 180°.
9.	$\angle QPB + \angle QPA = \angle PSQ + \angle PTQ$	From (7) and (8)
10.	$\angle QPB + \angle QPA = \angle QPB + \angle PTQ$	$\angle QPB = \angle PSQ$ from (6)
11.	$\angle QPA = \angle PTQ$	Hence (ii) is proved. This completes the proof.

5. Show that in a triangle, the medians are concurrent.

Solution:

Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus medians are the cevains where D, E, F are midpoints of BC, CA and AB respectively.

Since D is midpoint of BC, BD = DC. So $\frac{BD}{DC} = 1$ ...... (1)

Since E is midpoint of CA, CE = EA. So $\frac{CE}{EA} = 1$ ...... (2)

Since F is midpoint of AB, AF = FB. So $\frac{AF}{FB} = 1$ ...... (3)

Thus, multiplying (1), (2), (3)

we get $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \times 1 \times 1 = 1$

And so, Ceva's theorem is satisfied.

Hence the Medians are concurrent.

6. The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Solution:

Let x be the shortest side

z be the hypotenuse

y be the 3[rd] side

$$z = 2x + 6$$
$$y = z - 2 = 2x + 6 - 2 = 2x + 4$$

In ΔABC,

By Pythagoras Theorem

$$x^2 + y^2 = z^2$$
$$x^2 + (2x+4)^2 = (2x+6)^2$$
$$x^2 + 4x^2 + 16x + 16 = 4x^2 + 24x + 36$$
$$x^2 - 8x - 20 = 0$$
$$(x - 10)(x + 2) = 0$$

$\therefore x = 10$m

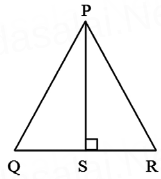
$\therefore y = 2(10) + 4 = 24$m

$\therefore z = 2(10) + 6 = 26$m

$\therefore$ The length of 3 sides are 10m, 24m, 26m.

7. The perpendicular PS on the base QR of a ΔPQR intersects QR at S, such that QS = 3 SR. Prove that $2PQ^2 = 2PR^2 + QR^2$

Solution:



Given: QS = 3SR

To prove: $2PQ^2 = 2PR^2 + QR^2$

$$\therefore QR = QS + SR \Rightarrow 3SR + SR$$
$$QR = 4SR$$
$$SR = \frac{1}{4} QR$$

In Δ PQS, $PQ^2 = PS^2 + QS^2$ .... (1)

In Δ PRS, $PR^2 = PS^2 + SR^2$ .... (2)

$$(1) - (2) \Rightarrow PQ^2 - PR^2 = QS^2 - SR^2$$
$$= (3.SR)^2 - SR^2$$
$$= 9SR^2 - SR^2 = 8SR^2$$
$$= 8(\frac{1}{4}QR)^2 \Rightarrow 8(\frac{1}{16}QR^2)$$
$$PQ^2 - PR^2 = \frac{QR^2}{2}$$
$$2PQ^2 - 2PR^2 = QR^2$$
$$2PQ^2 = 2PR^2 + QR^2$$

Hence Proved.

Created Questions with Solution — 5 marks

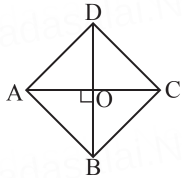
1. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of it's diagonals.

Solution:

Let ABCD be a rhombus

diagonal of a rhombus bisect each other at right angles.

OA = OC and OB = OD



In right triangle AOB

$AB^2 = OA^2 + OB^2$

$BC^2 = OB^2 + OC^2$

$CD^2 = OC^2 + OD^2$

$DA^2 = OA^2 + OB^2$

$$AB^2+BC^2+CD^2+DA^2 = (OA^2+OB^2) + (OB^2+OC^2) + (OC^2+OD^2) + (OA^2+OD^2)$$

$$= 2[OA^2+OB^2+OC^2+OD^2]$$

$$= 2[OA^2+OB^2+OA^2+OB^2]$$

$$= 2[2OA^2+2OB^2]$$

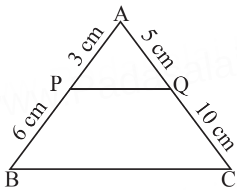
$$= 2\left[2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2\right]$$

$$= 2\left[\frac{1}{2}AC^2 + \frac{1}{2}BD^2\right] = 2\frac{1}{2}\left[AC^2 + BD^2\right]$$

$$= AC^2 + BD^2$$

Thus, Sum of the squares of the sides of a rhombus is equal to the sum of the squares of the diagonals.

2. P and Q are points on sides AB and AC respectively of ΔABC. If AP = 3cm, PB = 6cm, AQ = 5cm and QC = 10cm. Show that BC = 3PQ.



Solution:

$$\frac{AP}{AB} = \frac{3}{9} = \frac{1}{3} \qquad \frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$$

$$\therefore \frac{AP}{AB} = \frac{AQ}{AC}$$

Thus in triangle APQ and ABC, we have

$\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

by SAS criterion of similarity we have,

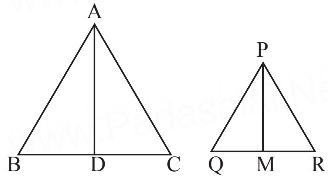
$\Delta APQ \sim \Delta ABC$

$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} \quad \Rightarrow \frac{AQ}{AC} = \frac{PQ}{BC} \quad \Rightarrow \frac{1}{3} = \frac{PQ}{BC}$$

BC = 3PQ. Hence solved.

3. If AD and PM are medians of triangle ABC and PQR respectively where, $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Solution:



We have $\Delta ABC \sim \Delta PQR$ such that AD and PM are the medians corresponding to the sides BC and QR respectively.

$\Delta ABC \sim \Delta PQR$ and the corresponding sides are proportional

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \quad \text{.......... (1)}$$

Corresponding angles are equal in two similar triangles.

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ .... (2)

Since AD and PM are medians

$\therefore$ BC = 2BD and QR = 2QM

From (1) $\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$ .... (3)

and $\angle B = \angle Q \Rightarrow \angle ABD = \angle PQM \rightarrow (4)$

From (3) and (4) we have

$\Delta ABD \sim \Delta PQM$

$\therefore$ their corresponding sides are proportional

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

For Practice — 5 marks

1. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC.

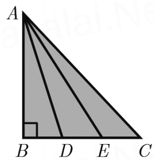
   (i) $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, find AE.

   (ii) If AD = $8x - 7$, DB = $5x - 3$, AE = $4x - 3$ and EC = $3x - 1$, find the value of x.

2. ABCD is a quadrilateral in which AB = AD, the bisector of $\angle BAC$ and $\angle CAD$ intersect the sides BC and CD at the points E and F respectively. Prove that EF || BD.

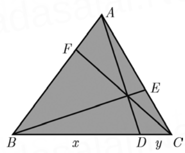
3. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively, such that PQ || DC if PD = 18cm, BQ = 35cm and QC = 15cm, find AD.

4. In the adjacent figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.

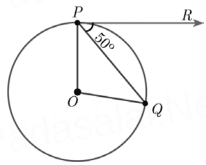


5. Suppose AB, AC and BC have lengths 13, 14 and 15 respectively.

   If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{CE}{EA} = \frac{5}{8}$. Find BD and DC.

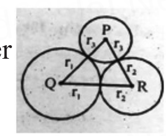


6. In figure O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.



Created Questions — 5 marks

1. With the vertices of a triangle P, Q, R as centres, three circles are described each touching the other two externally. If the sides of the triangle are 4cm, 6cm and 8cm, find the radii of the circles.



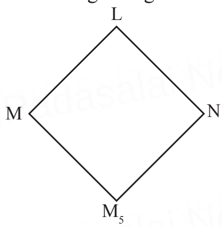
2. The bisector of the angle B and C of a triangle ABC, meet the opposite sides in D and E respectively, if DE || BC, prove that the triangle is isosceles.
3. A lotus is 20cm above the water surface in a pond and its stem is partly below the water surface. As the wind blew, the stem is pushed aside so that the lotus touched the water 40cm away from the original position of the stem. How much of the stem was below the water surface originally?

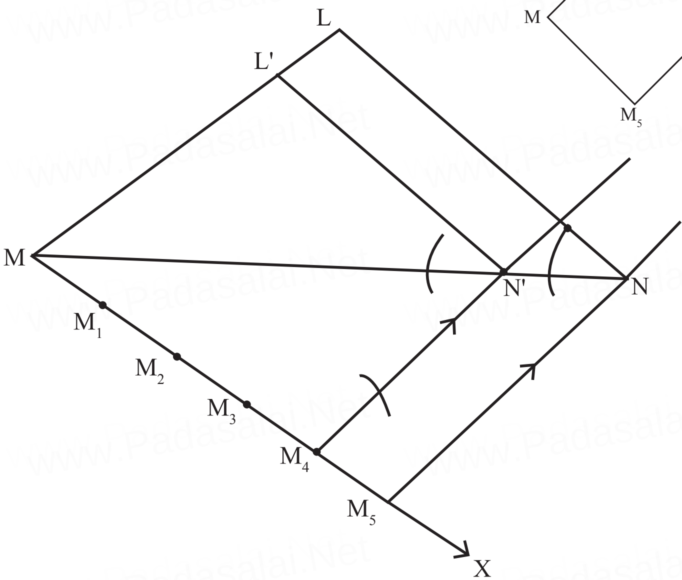
★★★

Practical Geometry — 5 marks

1. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5} < 1$).

Rough Diagram



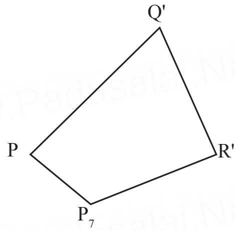


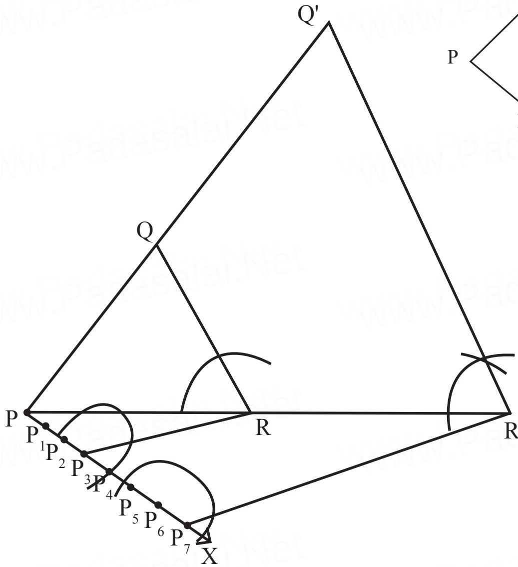
CONSTRUCTION:

1. Construct a ΔLMN with any measurement.
2. Draw a ray MX making an acute angle with MN on the side opposite to vertex L. Locate 5 points (greater of 4 and 5 in $\frac{4}{5}$) points. M_1, M_2, M_3, M_4, and M_5 so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$
3. Join M_5 to N and draw a line through M_4 parallel to M_5N to intersect MN at N'.
4. Draw line through N' parallel to the line LN intersecting line segment ML to L'.
5. Then $\Delta L'MN'$ is the required Δ.

2. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (Scale Factor $\frac{7}{3}>1$)**

Rough Diagram



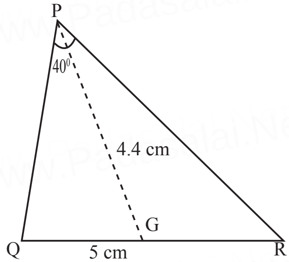


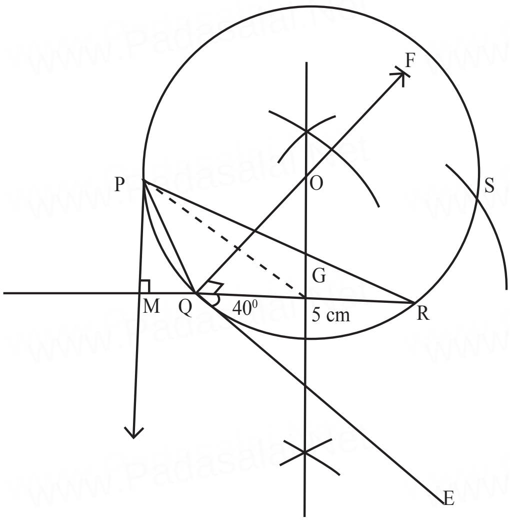
CONSTRUCTION

1. Construct a ΔPQR With any measurement.
2. Draw a ray PX making an acute angle with PR on the side opposite to vertex Q. Locate 7 points (Greater of 3 and 7 in $\frac{7}{3}$) points. P_1, P_2, P_3, P_4, P_5, P_6, P_7 on PX so that $PP_1 = P_1P_2 = P_2P_3 \ldots\ldots = P_6P_7$.
3. Join P_3R and draw a line through P_7 parallel to P_3R to intersecting the extended line segment PR at R'. Draw line through R' parallel to QR intersect the extended line segment PQ to Q'.
4. Then $\Delta\, PQ'R'$ is the required Δ.

3. Construct a ΔPQR in which $QR = 5$ cm $\angle P = 40°$ and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

Rough Diagram



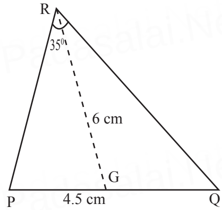


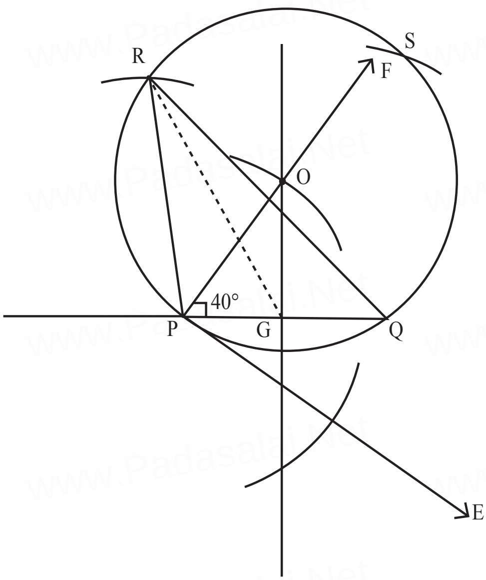
CONSTRUCTION:

1. Draw a line segment QR = 5cm. At Q, draw QE such that $\angle RQE = 40°$. At Q, dra QF such that $\angle EQF = 90°$.
2. Draw the perpendicular bisector to QR, meets QF at O and QR at G.
3. With O as centre and OQ as radius draw a circle.
4. From G mark arcs on the circle with radius 4.4cm.
5. Join PR, PQ. Then ΔPQR is the required Δ.
6. Length of altitude is PM = 3 cm.

4. Construct a ΔPQR which the base $PQ = 4.5$ cm, $\angle R = 35°$ and the median from R to PQ is 6 cm.

Rough Diagram



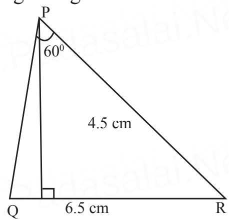


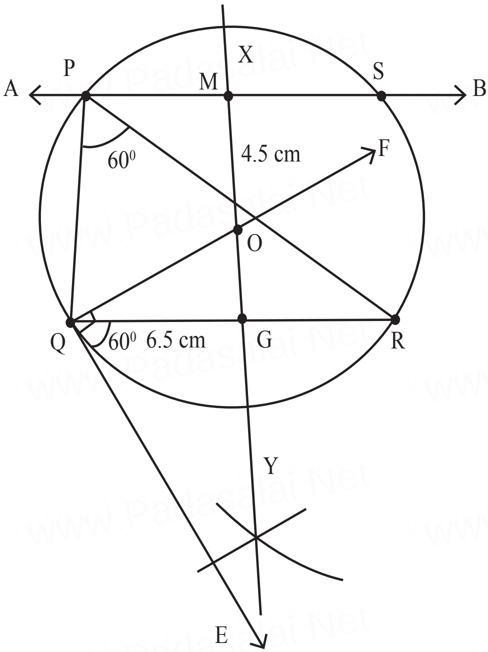
CONSTRUCTION:

1. Draw a line segment $PQ = 4.5$cm. At P, draw PE such that $\angle QPE = 35°$. At P, draw PF such that $\angle EPF = 90°$.
2. Draw the perpendicular bisector to PQ, meets PF at O and PQ at G.
3. With O as centre and OP as radius draw a circle.
4. From G mark arcs of 6 cm on the circle at R and S.
5. Join PR, RQ. Then ΔPQR is the required Δ.
6. Join RG, which is the median.

5. Construct a ΔPQR such that $QR = 6.5$ cm, $\angle P = 60°$ and the altitude from P to QR is of length 4.5 cm.

Rough Diagram

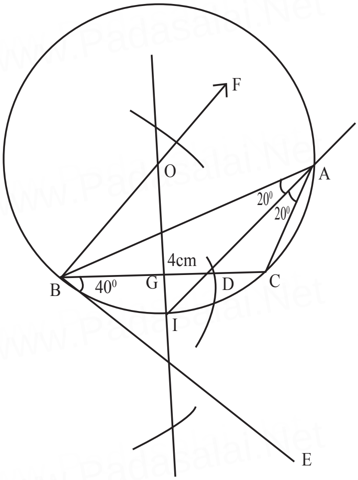




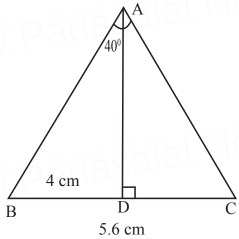
CONSTRUCTION:

1. Draw a line segment $QR = 6.5$cm. At Q, draw QE such that $\angle RQE = 60°$. At Q, draw QF such that $\angle EQF = 90°$.
2. Draw the perpendicular bisector to XY to QR intersects QF at O & QR at G.
3. With O as centre and OQ as radius draw a circle.
4. XY intersects QR at G. On XY, from G, mark arc M such that $GM = 4.5$cm.
5. Draw AB, through M which is parallel to QR. AB meets the circle at P and S.
6. Join QP, RP. Then ΔPQR is the required Δ.

6. Draw a triangle ABC of base BC = 5.6cm, ∠A=40° and the bisector of ∠A meets BC at D such that CD = 4cm.



Rough Diagram

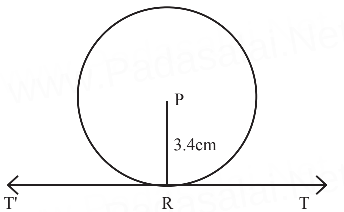


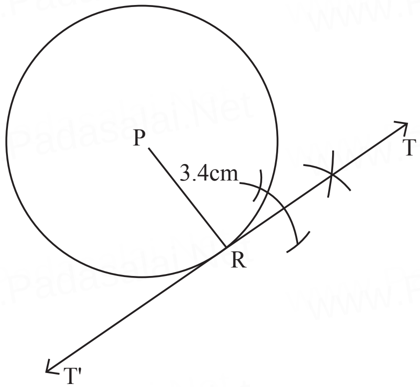
CONSTRUCTION:

1. Draw a line segment BC = 5.6cm. At B, draw BE such that ∠CBE = 40°. At B, draw BF such that ∠EBF = 90°.
2. Draw the perpendicular bisector to BC meets BF at O & BC at G.
3. With O as centre and OB as radius draw a circle.
4. From B, mark an arc of 4cm on BC at D. The perpendicular meets the circle at I & Join ID.
5. ID producted meets the circle at A. Join AB & AC.
6. Then ΔABC is the required Δ.

7. Draw a tangent at any point R on the circle of radius 3.4cm and centre at P.

Rough Diagram

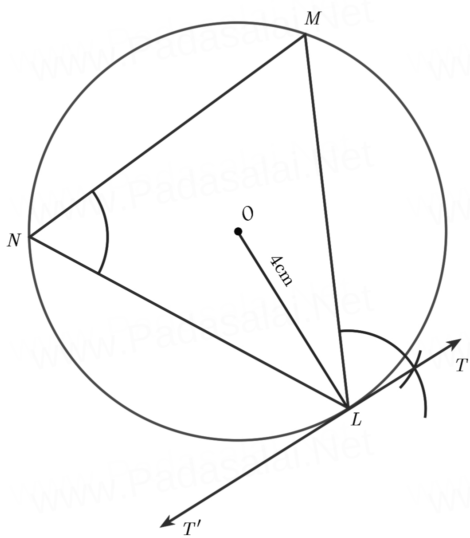


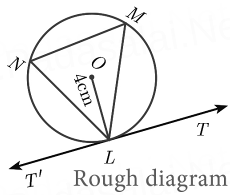


CONSTRUCTION:

1. Draw a circle with centre at P of radius 3.4 cm.
2. Take a point R on the circle and Join PR.
3. Draw perpendicular line TT' to PR which passes through R.
4. TT' is the required tangent.

8. Draw a circle of radius 4cm. At a point L on it draw a tangent to the circle using the alternate segment.





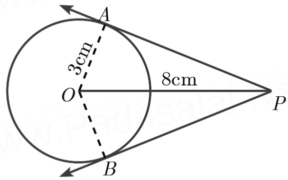
Rough diagram

CONSTRUCTION:

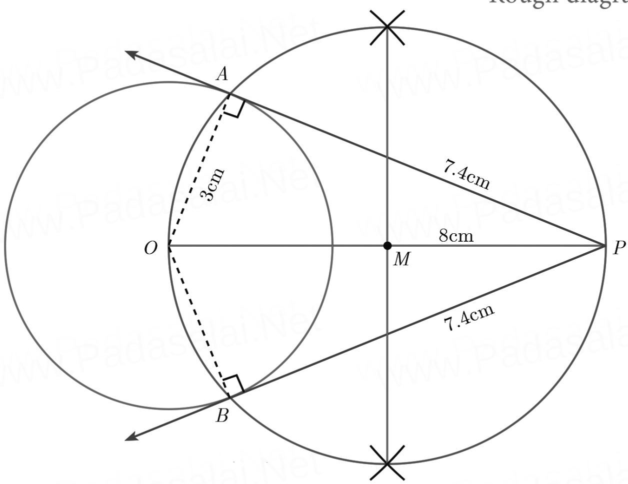
1. With O as the centre, draw a circle of radius 4 cm.

2. Take a point L on the circle. Through L draw any chord LM.
3. Take a point N distinct from L and M on the circle, so that L, M and N are in anticlockwise direction. Join LN and NM.
4. Through L draw a tangent TT' such that ∠TLM = ∠MNL.
5. TT' is the required tangent.

9. Draw a circle of diameter 6cm from a point P, which is 8cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.



Rough diagram

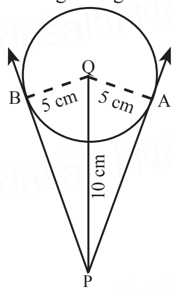


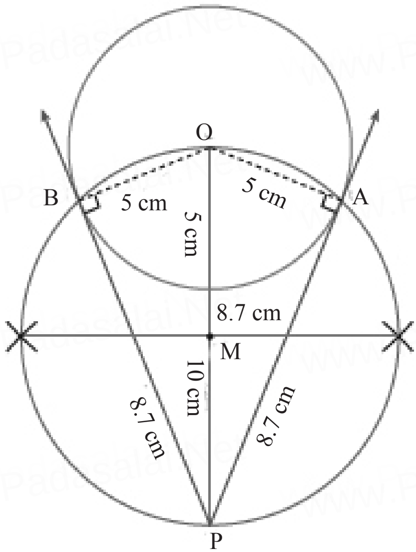
CONSTRUCTION:

1. With centre at O, draw a circle of radius 3cm.
2. Draw a line OP of length 8cm.
3. Draw a perpendicular bisector of OP which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP, AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4cm.

10. **Draw the two tangents from a point which is 10cm away from the centre of a circle of radius 5cm. Also, measure the lengths of the tangents.**

Rough Diagram





CONSTRUCTION:

1. With centre at O, draw a circle of radius 5 cm.
2. Draw a line OP = 10 cm.
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 8.7 cm.

Proof:

In ΔOPA

$$PA^2 = OP^2 - OA^2 = 10^2 - 5^2$$
$$= 100 - 25 = 75$$

$PA = \sqrt{75} = 8.6$ cm. (approx)

For Practice | **5 marks**

1. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR. (scale factor $\frac{3}{5} < 1$).

2. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR. (scale factor $\frac{2}{3} < 1$).

3. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR. (scale factor $\frac{7}{4} > 1$).

4. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$).

5. Construct a Δ PQR such that QR=5 cm, $\angle P = 30°$ and the altitude from P to QR is of length 4.2 cm.

6. Construct a ΔABC such that AB = 5.5 cm, $\angle C = 25°$ and the altitude from C to AB is 4 cm.

7. Construct a Δ PQR in which PQ = 8 cm, $\angle R = 60°$,and the median RG from R to PQ is 5.8cm. Find the length of the altitude from R to PQ.

8. Draw a ΔABC of base BC=8cm, $\angle A = 60°$ and the bisector of $\angle A$ meets BC at D such that BD = 6cm.

9. Draw the two tangents from a point which is 5cm away from the centre of a circle of diameter 6cm. Also measure the length of the tangents.

10. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that

point.

11. Construct a Δ PQR in which QR = 5 cm, $\angle P = 40°$, and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

12. Draw ΔPQR such that PQ = 6.8cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.

★★★

UNIT 5 COORDINATE GEOMETRY

Objective Type Questions — 1 mark

1. **The area of triangle formed by the points (–5, 0), (0, –5) and (5, 0) is**
   1) 0 sq.units 2) 25 sq .units
   3) 5 sq.units 4) none of these **Ans:** 2)

2. **A man walks near a wall, such that the distance between him and the wall is 10 units. consider the wall to be the Y axis. The path travelled by the man is**
   1) $x = 10$ 2) $y = 10$
   3) $x = 0$ 4) y = 0 **Ans:** 1)

3. **The straight line given by the equation $x = 11$ is**
   1) Parallel to X-axis
   2) Parallel to Y-axis
   3) Passing through the origin
   4) Passing through the point (0, 11) **Ans:** 2)

4. **If (5, 7), (3, p) and (6, 6) are collinear, then the value of p is**
   1) 3 2) 6
   3) 9 4) 12 **Ans:** 3)

5. **The point of intersection of $3x - y = 4$ and $x + y = 8$ is**
   1) (5, 3) 2) (2, 4)
   3) (3, 5) 4) (4, 4) **Ans:** 3)

6. **The slope of the line joining (12, 3), (4, a) is 1/8. The value of 'a' is**
   1) 1 2) 4
   3) –5 4) 2 **Ans:** 4)

7. **The slope of the line which is perpendicular to a line joining the points (0, 0) and (– 8, 8) is**
   1) –1 2) 1
   3) $\frac{1}{3}$ 4) –8 **Ans:** 2)

8. **If slope of the line PQ is $\frac{1}{\sqrt{3}}$, then slope of the perpendicular bisector of PQ is**

1) $\sqrt{3}$ 2) $-\sqrt{3}$

3) $\frac{1}{\sqrt{3}}$ 4) 0 **Ans:** 2)

9. **If A is a point on the Y-axis whose ordinate is 8 and B is a point on the X-axis whose abscissae is 5 then the equation of the line, AB is**

1) $8x + 5y = 40$ 2) $8x - 5y = 40$

3) $x = 8$ 4) $y = 5$ **Ans:** 1)

10. **The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is**

1) $7x - 3y + 4 = 0$ 2) $3x - 7y + 4 = 0$

3) $3x + 7y = 0$ 4) $7x - 3y = 0$ **Ans:** 3)

11. **Consider four straight lines (i) $l_1 : 3y = 4x + 5$ (ii) $l_2 : 4y = 3x - 1$ (iii) $l_3 : 4y + 3x = 7$ (iv) $l_4 : 4x + 3y = 2$. Which of the following statement is true?**

1) l_1 and l_2 are perpendicular 2) l_1 and l_4 are parallel

3) l_2 and l_4 are perpendicular 4) l_2 and l_3 are parallel **Ans:** 3)

12. **A straight line has equation $8y = 4x + 21$. Which of the following is true?**

1) The slope is 0.5 and the y intercept is 2.6

2) The slop is 5 and the y intercept is1.6

3) The slope is 0.5 and the y intercept is 1.6

4) The slop is 5 and y intercept is 2.6 **Ans:** 1)

13. **When proving that a quadrilateral is a trapezium, it is necessary to show?**

1) Two sides are parallel.

2) Two parallel and two non-parallel sides.

3) Opposite sides are parallel.

4) All sides are of equal length. **Ans:** 2)

14. **When proving that a quadrilateral is a parallelogram by using slopes you must find?**

1) The slopes of two sides.

2) The slopes of two pair of opposite sides.

3) The lengths of all sides.
4) Both the lengths and slopes of two sides. **Ans:** 1)

15. (2, 1) is the point of intersection of two lines.
1) x–y–3=0; 3x–y–7=0 2) x+y=3; 3x+y=7
3) 3x+y=3; x+y=7 4) x+3y–3=0; x–y–7=0
Ans: 2)

Created Questions 1 mark

1. If the points A (6, 1), B (8, 2), C (9, 4) and D (p, 3) are the vertices of a parallelogram, taken in order then the value of p is
1) –7 2) 7
3) 6 4) –6 **Ans:** 2)

2. If the reciprocal of the gradient of a straight line is $\sqrt{3}$. Then the angle of inclination is
1) 60° 2) 30°
3) 45° 4) 90° **Ans:** 2)

3. Find the angle between the lines x = y and $\sqrt{3}\,x - y = 0$
1) 15° 2) 30°
3) 60° 4) 90° **Ans:** 1)

4. A straight road AB (A is in IV quadrant) is such that it bends at B (1, 0) by an angle of 30° towards the right. Considering the line perpendicular to AB through B to be X-axis equation of the two parts of the road are
1) $x = 1,\ \sqrt{3}\,x - y - \sqrt{3} = 0$ 2) $y = 1,\ x - \sqrt{3}\,y + 1 = 0$
3) $x = 0,\ y = \sqrt{3}$ 4) $y = 1,\ x = 1$ **Ans:** 1)

5. The slope of the line which is perpendicular to the line joining the points (0, 0) and (–1, 1) is
1) 1 2) –1
3) $\frac{1}{2}$ 4) –2 **Ans:** 1)

6. If A is point on the Y-axis whose ordinate is 4 and B is a point on the X-axis whose abscissa is 3 then the equation of the line AB is
1) 3x+4y = 12 2) 4x+3y = 12
3) 3x–4y = 0 4) 3x–y+12 = 0 **Ans:** 2)

7. If lines ax – 5y = 5 and 2x + y = 1 are perpendicular then the value of *a* is

1) 2   2) $\frac{5}{2}$

3) $\frac{2}{5}$ 4) $\frac{1}{2}$ **Ans:** 2)

8. c is the midpoint of PQ if P is (4, x), c is (y, –1) and Q is (–2, 4) then x and y respectively are

1) –6 and 1   2) –6 and 2

3) 6 and –1   4) 6 and –2   **Ans:** 1)

9. The points (a, b) (a_1, b_1) and ($a-a_1$, $b-b_1$) are collinear if

1) $ab = a_1b_1$ 2) $ab_1 = a_1b$

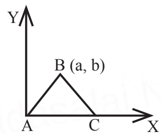
3) a = b   4) $a_1 = b_1$ **Ans:** 2)

10. If the centroid of the triangle is (1, 4) and two of it's vertices are (4, –3) and (–9, 7) then the area of the triangle is

1) 183 sq. units   2) $\frac{183}{2}$ sq. units

3) 366 sq. units   4) $\frac{183}{4}$ sq. units   **Ans:** 2)

11. If the area of the triangle given below is 20 then what are the coordinates of point c?



1) $\left(0, \frac{40}{a}\right)$ 2) (a^2+b^2, 0)

3) $\left(\frac{20}{b}, 0\right)$ 4) $\left(\frac{40}{b}, 0\right)$ **Ans:** 4)

Two Marks Questions — 2 marks

1. Find the area of the triangle whose vertices are (–3, 5), (5, 6) and (5, –2).

Solution:

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} -3 & 5 \\ 5 & 6 \\ 5 & -2 \\ -3 & 5 \end{vmatrix}$$

$$= \frac{1}{2}[(-18-10+25)-(25+30+6)]$$

$$= \frac{1}{2}[-3-61] = \left|\frac{-64}{2}\right|$$

$= 32$ sq.units.

2. **Find the area of the triangle formed by the points (1, –1), (–4, 6) and (–3, –5).**

Solution:

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$$= \frac{1}{2}[(6+20+3)-(4-18-5)]$$

$$= \frac{1}{2}[29+19] = \frac{1}{2}[48]$$

$= 24$ sq.units.

3. **If the area of the triangle formed by the vertices A (–1, 2), B(k, –2) and C (7, 4) (taken in order) is 22 sq.units, find the value of k.**

Solution:

The vertices are A (–1, 2), B (k, –2) and C (7, 4)

Area of ΔABC is 22 sq.units.

$$\frac{1}{2}\begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 22 \Rightarrow \begin{vmatrix} -1 & 2 \\ k & -2 \\ 7 & 4 \\ -1 & 2 \end{vmatrix} = 44$$

$$\{(2+4k+14)-(2k-14-4)\} = 44$$

$$4k+16-2k+18 = 44$$

$$2k+34 = 44$$

$2k = 10.$

Therefore $k = 5$.

4. **Show that the points P(–1.5, 3), Q(6, –2) and R(–3, 4) are collinear.**

Solution:

To Prove Area of Δ PQR = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -1.5 & 3 \\ 6 & -2 \\ -3 & 4 \\ -1.5 & 3 \end{vmatrix} = 0$$

$$= \frac{1}{2}\,[(3+24-9) - (18+6-6)\,] = \frac{1}{2}\,[18 - 18] = 0.$$

∴ Therefore, the given points are collinear.

5. **Find the value of 'a' for which the given points are collinear. (2, 3), (4, a) (6, –3)**

Solution:

A (2, 3), B (4, a) C (6, –3) are collinear.

Area of ΔABC = 0

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} 2 & 3 \\ 4 & a \\ 6 & -3 \\ 2 & 3 \end{vmatrix} = 0$$

$$\frac{1}{2}\,[(2a - 12 + 18) - (12 + 6a - 6)] = 0$$

$$(2a+6) - (6+6a) = 0\times\frac{2}{1} \quad \Rightarrow 2a + 6 - 6 - 6a = 0$$

$$-4a = 0 \Rightarrow a = 0$$

6. **Determine whether the sets of points are collinear (a, b+c), (b, c+a) and (c, a+b)**

Solution:

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & b+c \\ b & c+a \\ c & a+b \\ a & b+c \end{vmatrix}$$

$$= \frac{1}{2}\left[(ac + a^2 + ab + b^2 + bc + c^2) - (b^2 + bc + c^2 + ca + a^2 + ab\right]$$

$$= \frac{1}{2}\left[ac + a^2 + ab + b^2 + bc + c^2 - b^2 - bc - c^2 - ca - a^2 - ab\right]$$

$$= \frac{1}{2}[0] = 0 \text{ sq.units}$$

$\therefore$ The given points are collinear.

Alter:

(a, b+c), (b, c+a), (c, a+b)

x_1, y_1 x_2, y_2 x_3, y_3

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ b+c-c-a & b+c-a-b \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a-b & a-c \\ -(a-b) & -(a-c) \end{vmatrix}$$

$$= \frac{1}{2}[-(a-b)(a-c) + (a-b)(a-c)] \Rightarrow \frac{1}{2}[0] = 0$$

7. Find the value of 'a' for which the given points are collinear (a, 2–2a), (–a+1, 2a) and (–4–a, 6–2a).

Solution:

$\Delta = 0$

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & 2-2a \\ -a+1 & 2a \\ -4-a & 6-2a \\ a & 2-2a \end{vmatrix} = 0$$

$$(2a^2 - 6a + 2a^2 + 6 - 2a - 8 + 8a - 2a + 2a^2)$$
$$- (-2a + 2a^2 + 2 - 2a - 8a - 2a^2 + 6a - 2a^2) = 0$$

$$\Rightarrow (6a^2 - 2a - 2) - (-2a^2 - 6a + 2) = 0$$

$\Rightarrow 8a^2 + 4a - 4 = 0 \quad \div 4$

$\Rightarrow 2a^2 + a - 1 = 0$

$\Rightarrow (2a-1)(a+1) = 0$

$\Rightarrow a = \frac{1}{2}$ or -1

Alter:

Area of $\Delta = 0$

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} a+a-1 & a+4+a \\ 2-2a-2a & 2-2a-6+2a \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2a-1 & 2a+4 \\ 2-4a & -4 \end{vmatrix} = 0$$

$$\begin{aligned} -4(2a-1)-(2-4a)(2a+4) &= 0 \\ -8a+4-[4a+8-8a^2-16a] &= 0 \\ -8a+4-4a-8+8a^2+16a &= 0 \\ 8a^2+4a-4 &= 0 \\ 2a^2+a-1 &= 0 \\ (a+1)(2a-1) &= 0 \\ a &= -1 \text{ (or) } a = \frac{1}{2} \end{aligned}$$

8. Find the equation of a straight line whose slope is 5 and y-intercept is –9.

Solution:

Slope, $m = 5$, y-intercept, $c = -9$

$$\begin{aligned} \text{Equation of a straight line, } y &= mx + c \\ y &= 5x - 9 \\ 0 &= 5x - y - 9 \end{aligned}$$

Equation is $5x - y - 9 = 0$

9. Find the equation of a straight line whose inclination is 45° and y intercept is 11.

Solution:

Inclination, $\theta = 45°$

Slope, $m = \tan\theta$

$$m = \tan 45°$$
$$\text{Slope, } m = 1$$
$$\text{y-Intercept, } C = 11$$
$$\text{Equation of the straight line, } y = mx + C$$
$$y = 1x + 11$$
$$0 = x + 11 - y$$
$$\therefore \text{ Equation is } x - y + 11 = 0$$

10. What is the inclination of a line whose slope is 1?

Solution:

$$\text{Slope, } m = 1$$
$$\text{Slope, } m = \tan\theta$$
$$\tan\theta = 1$$
$$\tan\theta = \tan 45°$$
$$\theta = 45°$$
$$\therefore \text{ Inclination of the line} = 45°$$

11. Find the slope of the straight line $6x + 8y + 7 = 0$

Solution:

$$\text{Slope, } m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{6}{8} = -\frac{3}{4}$$

Slope of the straight line is $-\frac{3}{4}$

12. Find the slope of a line joining the given points (-6, 1) and (-3, 2).

Solution:

A (–6, 1), B (–3, 2)

$$\text{Slope of AB, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-3-(-6)} = \frac{2-1}{-3+6}$$

$$\therefore \text{ Slope } m = \frac{1}{3}$$

13. Calculate the slope and y-intercept of the straight line $8x - 7y + 6 = 0$.

Solution:

$$8x - 7y + 6 = 0$$
$$8x + 6 = 7y \quad \div \text{ by } 7$$

$$\frac{8}{7}x + \frac{6}{7} = \frac{7}{7}y \qquad \Rightarrow \frac{8}{7}x + \frac{6}{7} = y$$

Compare with $mx+c=y$

Slope, $m = \frac{8}{7}$; Y-Intercept, $c = \frac{6}{7}$

14. Find the equation of a line whose intercepts on the x and y axes are given below. (4, –6).

Solution:

x intercept $a = 4$, y-intercept $b = -6$

Equation of the line in intercept form,

$$\frac{x}{a}+\frac{y}{b} = 1 \qquad \frac{x}{4}-\frac{y}{6} = 1$$

$$\frac{x}{4}+\frac{y}{-6} = 1 \qquad \frac{3x-2y}{12} = 1$$

$$3x-2y = 12$$

$$3x-2y-12 = 0$$

15. Find the equation of a line passing through the point (3,–4) and having slope $\frac{-5}{7}$.

Solution:

$(x_1, y_1) = (3, -4)$

Slope, $m = -\frac{5}{7}$

Equation of the straight line

$$y - y_1 = m(x - x_1)$$

$$y-(-4) = -\frac{5}{7}(x-3)$$

$$7(y+4) = -5(x-3)$$

$$7y+28 = -5x+15$$

$$5x+7y+28-15 = 0$$

$$5x+7y+13 = 0$$

16. Find the equation of a straight line passing through (5, –3) and (7, –4).

Solution:

The equation of a straight line passing through the two points

$\quad x_1, y_1 \qquad x_2, y_2$

(5, –3) and (7, –4)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+3}{-4-(-3)} = \frac{x-5}{7-5} \Rightarrow \frac{y+3}{-4+3} = \frac{x-5}{2}$$

$$2(y+3) = -1(x-5)$$

$$2y+6 = -x+5$$

$$x+2y+6-5 = 0$$

Therefore $x + 2y + 1 = 0$

17. The line through the points (–2, a) and (9, 3) has slope $-\frac{1}{2}$. Find the value of a.

Solution:

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-a}{9+2} = \frac{3-a}{11}$$

$$\text{Given Slope} = -\frac{1}{2} \qquad \therefore \frac{3-a}{11} = -\frac{1}{2}$$

$$6-2a = -11$$

$$2a = 17$$

$$a = \frac{17}{2}$$

18. Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Solution:

Slope of the straight line $2x + 3y - 8 = 0$ is

$$m_1 = \frac{-coefficient\ of\ x}{coefficient\ of\ y} = -\frac{2}{3}.$$

Slope of the straight line is $4x + 6y + 18 = 0$

$$m_2 = \frac{-coefficient\ of\ x}{coefficient\ of\ y} = -\frac{4}{6} = -\frac{2}{3}.$$

Here $m_1 = m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

19. Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Solution:

Slope of the straight line $x - 2y + 3 = 0$

$$m_1 = -\left(\frac{a}{b}\right) = -\left(\frac{1}{-2}\right) = \frac{1}{2}$$

Slope of the straight line $6x + 3y + 8 = 0$

$$m_2 = -\left(\frac{a}{b}\right) = -\left(\frac{6}{3}\right) = -2$$

$$m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$$

Hence, the two straight lines are perpendicular.

20. If the straight lines $12y = -(p + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p'.

Solution:

$12y = -(p + 3)x + 12$

The slope of $(p + 3)x + 12y + 12 = 0$

$$m_1 = \frac{-coefficient\ of\ x}{coefficient\ of\ y} = -\frac{(p+3)}{12}.$$

The Slope of $12x - 7y = 16$

$$m_2 = -\frac{12}{-7} = \frac{12}{7}$$

Since both are perpendicular to each other $m_1 \times m_2 = -1$

$$-\frac{(P+3)}{12} \times \frac{12}{7} = -1$$

$$\frac{(P+3)}{7} = 1 \Rightarrow P + 3 = 7 \qquad P = 7 - 3 \qquad P = 4$$

21. The hill in the form of a right triangle has its foot at (19,3). The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.

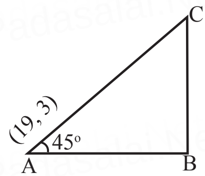
Solution:

Slope m = $\tan\theta = \tan 45° = 1$

$\therefore m = 1$

Equation of AC whose slope 1 and passing through (19, 3) is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= 1(x - 19) \\ x - 19 - y + 3 &= 0 \\ x - y - 16 &= 0 \end{aligned}$$



$\therefore$ Inclination of a line = 45°

22. The line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (*x*, 24). Find the value of *x*.

Solution:

Slope of line joining (–2, 6), (4, 8)

$$m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$$

Slope of line joining (8, 12) (*x*, 24)

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since two lines are perpendicular $m_1 \times m_2 = -1$

$$\frac{1}{3} \times \frac{12}{x-8} = -1 \quad \Rightarrow \quad \frac{4}{x-8} = -1$$

$$\begin{aligned} x - 8 &= -4 \\ x &= 4 \end{aligned}$$

Created Questions with Solution — 2 marks

1. Find the equation of a line passing through the point (–4, 3) and having slope $-\frac{7}{5}$.

Solution:

Slope, $m = -\frac{7}{5}$

The Point $(x_1\ y_1) = (-4, 3)$

$\therefore$ Equation of the straight line is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{7}{5}(x + 4)$$

$$5y - 15 = -7x - 28$$

$$7x + 5y + 13 = 0$$

2. Show that the straight lines $3x - 5y + 7 = 0$ and $15x + 9y + 4 = 0$ are perpendicular.

Solution:

$3x - 5y + 7 = 0$ $\qquad$ $15x + 9y + 4 = 0$

$m_1 = -\left(\frac{a}{b}\right) \Rightarrow -\left(\frac{3}{-5}\right)$ $\qquad$ $m_2 = -\left(\frac{a}{b}\right) \Rightarrow -\left(\frac{15}{9}\right)$

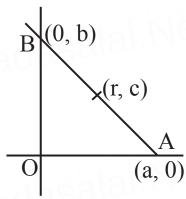
$m_1 = \frac{3}{5}$ $\qquad$ $m_2 = -\frac{5}{3}$

$$m_1 \times m_2 = \frac{3}{5} \times \left(-\frac{5}{3}\right)$$

$$m_1 \times m_2 = -1$$

$\therefore$ $3x - 5y + 7 = 0$ and $15x + 9y + 4 = 0$ are perpendicular to each other.

3. If P(r, c) is the midpoint of the line segment between the axes then show that $\frac{x}{r} + \frac{y}{c} = 2$



Solution:

Let the cuts 'a' intercept on the x-axis
'b' intercept on the y-axis.

Equation of the line intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\text{mid point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(r, c) = \left(\frac{a+0}{2}, \frac{b+0}{2}\right) \Rightarrow \left(\frac{a}{2}, \frac{b}{2}\right) = (r, c)$$

a = 2r and 6 = 2c

The equation of the line is $\frac{x}{2r}+\frac{y}{2c}=1$

Hence $\frac{x}{r}+\frac{y}{c}=2$

4. Find the equation of the straight line passing through (–1, 1) and cutting off equal intercepts, but opposite in signs with the two coordinate axes.

Solution:

Equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

Given a = –b .... (1)

$\frac{x}{-b}+\frac{y}{b}=1$

The line passes through (–1, 1)

$\frac{-1}{-b}+\frac{1}{b} = 1 \Rightarrow \frac{2}{b}=1 \Rightarrow b=2$

(1) ⇒ a = –2

Required equation of straight line is $\frac{x}{-2}+\frac{y}{2}=1$

$2x-2y+4=0$

Hence $x-y+2=0$

5. Prove that the points A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) to be collinear is $x_1y_2+x_2y_3+x_3y_1=x_2y_1+x_3y_2+x_1y_3$

Solution:

Let A (x_1, y_1) B (x_2, y_2), C (x_3, y_3)

Given it's collinear

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0$$

$x_1y_2+x_2y_3+x_3y_1-(x_2y_1+x_3y_2+x_1y_3)=0$

$x_1y_2+x_2y_3+x_3y_1=x_2y_1+x_3y_2+x_1y_3=0$

6. Write the equation of a straight line

i) Parallel to ax+by+c = 0

ii) Perpendicular to ax+by+c = 0

Solution:

i) Equation of the straight line parallel to $ax+by+c = 0$ is $ax+by+k = 0$.

ii) Equation of the straight line perpendicular to $ax+by+c = 0$ is $bx-ay+k = 0$.

7. The foot of the perpendicular from the point (1, 1) to a line is the point (–3, 4). Find the equation of the line.

Solution:

Slope of AB = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-1}{-3-1} = \dfrac{3}{-4}$

Slope of CD = $\dfrac{4}{3}$

Equation of CD is

$$\begin{aligned} y - y_1 &= m(x-x_1) \\ y - 4 &= \frac{4}{3}(x+3) \\ 3y-12 &= 4x+12 \\ 4x - 3y + 24 &= 0 \end{aligned}$$

For Practice — 2 marks

1. Find the area of the triangle formed by the points (–10, – 4), (–8, –1) and (–3, –5).
2. Determine whether the sets of points are collinear. $\left(-\frac{1}{2}, 3\right)$, (–5, 6) and (–8, 8).
3. Find the equation of a line through the given pair of points. (2, 3) and (–7, –1).
4. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1, 2).
5. Find the slope of a line joining the given points (14, 10) and (14, –6).
6. Find the intercepts made by the following lines on the coordinate axes. (i) $3x - 2y - 6 = 0$ (ii) $4x + 3y + 12 = 0$
7. Find the slope of a line joining the points.
   (i) $(5, \sqrt{5})$ with the origin (ii) $(\sin\theta, -\cos\theta)$ and $(-\sin\theta, \cos\theta)$
8. The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and intercept on Y-axis.

9. Find the equation of the line through the given pair of points $\left(2, \frac{2}{3}\right)$ and $\left(\frac{-1}{2}, -2\right)$.

10. Find the slope of a line joining the given points $\left(-\frac{1}{3}, \frac{1}{2}\right)$ and $\left(\frac{2}{7}, \frac{3}{7}\right)$.

11. If the points A (2, 2), B (–2, –3), C (1, –3) and D (x, y) form a parallelogram then find the value of x and y.

12. Find the slope and y intercept of $\sqrt{3}\ x + (1 - \sqrt{3})\ y = 3$.

13. Find the equation of a line whose inclination is 30° and making an intercept –3 on the Y-axis.

Created Questions — 2 marks

1. Find the area of the triangle formed by (a, b+c), (b, c+a), (c, a+b)
2. Find a relation between x and y if the points (x, y), (1, 2), (7, 0) are collinear.
3. Find the equation of the line having an inclination 30 with the positive direction of X axis and cut off an intercept of 4 on the positive side of Y axis.
4. The foot of perpendicular from the point (1, 1) to a line is the point (–3, 4). Find the equation of the line.

Five Marks Questions — 5 marks

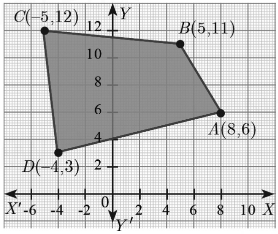
1. Find the Area of the quadrilateral formed by the points (8, 6), (5, 11), (–5, 12) and (–4, 3).

Solution:

Before determining the area of the quadrilateral, plot the vertices in a graph A (8, 6), B (5, 11), C (–5, 12) and D (–4, 3).

Therefore, area of the quadrilateral ABCD.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 8 & 6 \\ 5 & 11 \\ -5 & 12 \\ -4 & 3 \\ 8 & 6 \end{vmatrix}$$



$$= \frac{1}{2}[(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)]$$

$$= \frac{1}{2}[(88 + 60 - 15 - 24 - 30 + 55 + 48 - 24)]$$

$$= \frac{1}{2}[88 + 60 - 15 - 24 - 30 + 55 + 48 - 24]$$

$$= \frac{1}{2}[88 + 60 + 55 + 48 - 15 - 24 - 30 - 24]$$

$$= \frac{1}{2}[251 - 93] \Rightarrow \frac{1}{2}[158] = 79 \text{ sq.units.}$$

Alter:

Area of the quadrilateral ABCD

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$$

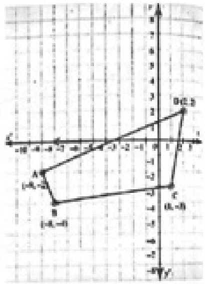
$$= \frac{1}{2}\begin{vmatrix} 8-(-5) & 5-(-4) \\ 6-12 & 11-3 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} 8+5 & 5+4 \\ -6 & 8 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 13 & 9 \\ -6 & 8 \end{vmatrix} \Rightarrow \frac{1}{2}[104 - (-54)]$$

$$= - [104+54] = \frac{1}{2}[158]$$

= 79 sq. units.

2. Find the area of the quadrilateral whose vertices are at (–9, – 2), (–8, –4), (2, 2) and (1, –3).



Solution:

A(–9, –2), B(–8, –4), C(1, –3), D(2, 2)

Area of quadrilateral

$$= \frac{1}{2}\begin{pmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{pmatrix}$$

$$= \frac{1}{2}[(36+24+2–4) – (16–4–6–18)]$$

$$= \frac{1}{2}[58 – (–12)] = \frac{1}{2}[70]$$

= 35 sq.units

Alter:

A(–9, –2), B(–8, –4), C(1, –3), D(2, 2)

x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

$$\text{Area of quadrilateral} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$$

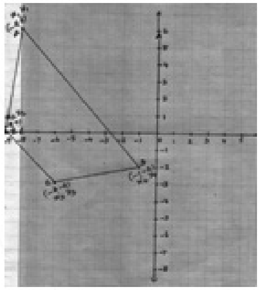
$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} -9-1 & -8-2 \\ -2-(-3) & -4-2 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -10 & -10 \\ -2+3 & -6 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} -10 & -10 \\ 1 & -6 \end{vmatrix}$$

$$= \frac{1}{2}[60–(–10)] = \frac{1}{2}[70]$$

= 35 sq. units

3. Find the area of the quadrilateral whose vertices are at (–9, 0), (–8, 6), (–1, –2) and (–6, –3).



Solution:

A (–8, 6), B (–9, 0),
C (–6, –3), D (–1, –2)

Area of quadrilateral

$$= \frac{1}{2}\begin{pmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{pmatrix}$$

$$= \frac{1}{2}\,[(0+27+12-6)-(-54+0+3+16)]$$

$$= \frac{1}{2}\,[33-(-35)] = \frac{1}{2}\,[68] = 34 \text{ sq.units}$$

Alter:

A(–8, 6), B(–9, 0), C(–6, –3), D(–1, –2)

x_1, y_1 $\quad x_2, y_2$ $\quad x_3, y_3$ $\quad x_4, y_4$

$$\text{Area of quadrilateral} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} -8-(-6) & -9-(-1) \\ 6-(-3) & 0-(-2) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -8+6 & -9+1 \\ 6+3 & 0+2 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} -2 & -8 \\ 9 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\,[-4+72] = \frac{1}{2}[68] = 34 \text{ sq. units}$$

4. Find the value of k, if the area of quadrilateral is 28 sq.units, whose vertices are (–4, –2), (–3, k), (3, –2) and(2, 3).

Solution:

$$\text{Area of quadrilateral, } \frac{1}{2}\begin{pmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{pmatrix} = 28$$

$$(-4k + 6 + 9 - 4) - (6 + 3k - 4 - 12) = 56$$

$$(11-4k)-(3k-10) = 56$$
$$21-7k = 56$$
$$7k = -35$$
$$k = -5$$

Alter:

(–4, –2), (–3, k), (3, –2), (2, 3)

x_1, y_1 x_2, y_2, x_3, y_3 x_4, y_4

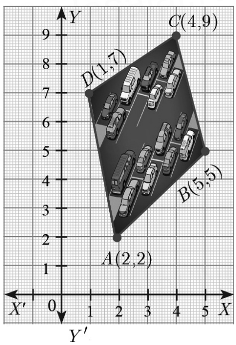
Area of quadrilateral = 28

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = 28 \quad \Rightarrow \quad \frac{1}{2}\begin{vmatrix} x_1-x_3 & x_2-x_4 \\ y_1-y_3 & y_2-y_4 \end{vmatrix} = 28$$

$$\frac{1}{2}\begin{vmatrix} -4-3 & -3-2 \\ -2-(-2) & k-3 \end{vmatrix} = 28 \quad \Rightarrow \quad \begin{vmatrix} -7 & -5 \\ 0 & k-3 \end{vmatrix} = 56$$

$$-7(k-3)+0 = 56$$
$$-7k+21 = 56$$
$$-7k = 56-21$$
$$-7k = 35$$
$$k = \frac{35}{-7} = -5$$

5. **The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?**



Solution:

The parking lot is a quadrilateral whose vertices's are

A (2, 2), B (5, 5), C (4, 9) and D (1, 7).

Therefore, Area of parking lot is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 2 \\ 5 & 5 \\ 4 & 9 \\ 1 & 7 \\ 2 & 2 \end{vmatrix}$$

$$= \frac{1}{2}[(10 + 45 + 28 + 2) - (10 + 20 + 9 + 14)]$$

$$= \frac{1}{2}[85 - 53] \Rightarrow \frac{1}{2}[32]$$

$= 16$ sq.units.

So, Area of parking lot = 16 sq.feet.

Construction rate per square fee = ₹ 1300

Therefore, total cost for constructing the parking lot

$= 16 \times 1300 =$ ₹ 20800

Alter:

A(2, 2), B(5, 5), C(4, 9) D(1, 7)

x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

Area of parking lot is $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} 2-4 & 5-1 \\ 2-9 & 5-7 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -2 & 4 \\ -7 & -2 \end{vmatrix} \Rightarrow \frac{1}{2}[4-(-28)]$$

$$= \frac{1}{2}[4+28] = \frac{1}{2}[32] = 16 \text{ sq. units}$$

So, Area of parking lot = 16 sq.feet.

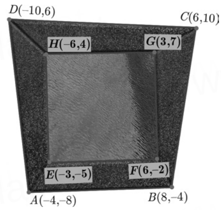
Construction rate per square fee = ₹ 1300

Therefore, total cost for constructing the parking lot.

$= 16 \times 1300 =$ ₹ 20800

6. In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.

Solution:



Area of the patio = Area of ABCD – Area of EFGH

$$= \frac{1}{2}\begin{pmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{pmatrix}$$

$$= \frac{1}{2}[(16 + 80 + 36 + 80) - (-64 - 24 - 100 - 24)]$$
$$- \frac{1}{2}[(6 + 42 + 12 + 30) - (-30 - 6 - 42 - 12)]$$

$$= \frac{1}{2}[212 - (-212)] - \frac{1}{2}[90 - (-90)]$$

$$= \frac{1}{2}[424] - \frac{1}{2}[180]$$

= 212 – 90 = 122 square units.

Alter:

A(–4, –8), B(8, –4) C(6, 10), D(–10, 6)
x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

E(–3, –5), F(6, –2) G(3, 7), H(–6, 4)
x_1, y_1 x_2, y_2 x_3, y_3 x_4, y_4

Area of the patio = Area of ABCD – Area of EFGH

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -4-6 & 8+10 \\ -8-10 & -4-6 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} -3-3 & 6+6 \\ -5-7 & -2-4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -10 & 18 \\ -18 & -10 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} -6 & 12 \\ -12 & -6 \end{vmatrix}$$

$$= \frac{1}{2}[100+324] - \frac{1}{2}[36+144]$$

$$= \frac{1}{2}[424] - \frac{1}{2}[180]$$

$$= 212 - 90 = 122 \text{ sq. units.}$$

7. Show that the given points form a parallelogram A (2.5, 3.5), B (10, –4), C (2.5, –2.5) and D (–5, 5).

Solution:

A (2.5, 3.5), B (10, –4)

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-3.5}{10-2.5} = -\frac{7.5}{7.5} = -1$$

C (2.5, – 2.5), D (–5, 5)

$$\text{Slope of CD} = \frac{5-(-2.5)}{-5-2.5} = \frac{5+2.5}{-7.5} = \frac{7.5}{-7.5} = -1$$

$\therefore$ Slope of AB = Slope of CD. So AB || CD

B (10, –4), C (2.5, –2.5)

$$\text{Slope of BC} = \frac{-2.5-(-4)}{2.5-10} = \frac{-2.5+4}{-7.5}$$

$$= \frac{1.5}{-7.5} \times \frac{10}{10} = \frac{15}{-75} = -\frac{1}{5}$$

A (2.5, 3.5), D (–5, 5),

$$\text{Slope of AD} = \frac{5-(3.5)}{-5-2.5} = \frac{1.5}{-7.5}$$

$$= \frac{1.5}{-7.5} \times \frac{10}{10} = \frac{15}{-75} = -\frac{1}{5}$$

$\therefore$ Slope of BC = Slope of AD. So BC || AD.

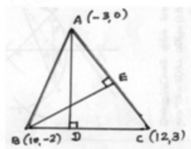
$\therefore$ The given points form a parallelogram.

8. A (–3, 0), B (10, –2) and C (12, 3) are the vertices of ΔABC. Find the equation of the altitude through A and B.

Solution:

Given A (–3, 0), B (10, –2) and C (12, 3).

B (10, –2), C (12, 3).



$$\text{Slope of BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+2}{12-10} = \frac{5}{2}$$

BC $\perp$ AD

$\therefore$ Slope of AD $= \frac{-2}{5}$ A(–3, 0)

$\therefore$ Equation of AD is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-2}{5}(x + 3)$$

$$2x + 5y + 6 = 0$$

A (–3, 0), C (12, 3) Slope of AC $= \frac{3-0}{12+3} = \frac{3}{15} = \frac{1}{5}$

B (10, –2) Slope of BE = –5

$\therefore$ Equation of BE is

$$y - y_1 = m(x - x_1)$$

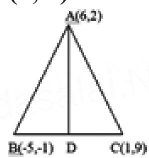
$$y + 2 = -5(x - 10)$$

$$y + 2 = -5x + 50$$

$$5x + y - 48 = 0$$

9. Find the equation of the median and altitude of ΔABC through A where the vertices are A(6, 2), B(–5, –1) and C(1, 9).

Solution:



Equation of the median through A.

$$\text{Midpoint of BC} = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$$

$$= D(-2, 4)$$

Equation of AD is A (6, 2), D (–2, 4)

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8} \Rightarrow \frac{y-2}{1} = \frac{x-6}{-4}$$

$$x - 6 = -4y + 8$$

$$x + 4y - 14 = 0$$

Equation of altitude through 'A'

Slope of BC $= \frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$

Since AD $\perp$ BC, Slope of AD $= \frac{-3}{5}$ and A is (6, 2)

Equation of altitude AD is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-3}{5}(x - 6)$$
$$5y - 10 = -3x + 18$$
$$3x + 5y - 28 = 0$$

10. If the area of the triangle formed by the vertices. (p, p), (5, 6) and (5, –2) [Given in order] is 32, Find the value of "p".

Solution:

Area of $\Delta = 32$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 32 \Rightarrow \frac{1}{2}\begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 32 \Rightarrow \begin{vmatrix} p & p \\ 5 & 6 \\ 5 & -2 \\ p & p \end{vmatrix} = 64$$

$$(6p - 10 + 5p) - (5p + 30 - 2p) = 64$$
$$11p - 10 - (3p + 30) = 64$$
$$11p - 10 - 3p - 30 = 64$$
$$8p - 40 = 64$$
$$8p = 64 + 40$$
$$8p = 104$$
$$p = \frac{104}{8} \Rightarrow p = 13$$

Alter:

(p, p), (5, 6), (5, –2)

x_1, y_1 x_2, y_2 x_3, y_3

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 32$$

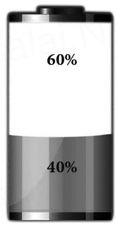
$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = 32$$

$$\frac{1}{2}\begin{vmatrix} p-5 & p-5 \\ p-6 & p+2 \end{vmatrix} = 32 \Rightarrow \begin{vmatrix} p-5 & p-5 \\ p-6 & p+2 \end{vmatrix} = 32\times 2$$

$$(p-5))(p+2) - (p-5)(p-6) = 64$$

$$\begin{aligned} (p-5)[p+2-p+6] &= 64 \\ (p-5)\,[8] &= 64 \\ 8p - 40 &= 64 \\ 8p &= 104 \\ p &= \frac{104}{8} \\ p &= 13 \end{aligned}$$

11. A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as y = – 0.25x + 1.



(i) Find the number of hours elapsed if the battery power is 40%.

(ii) How much time does it take so that the battery has no power?

Solution:

i) To find the time when the battery power is 40%, we have to take, y = 0.40

$$\begin{aligned} y &= -0.25x + 1 \\ 0.40 &= -0.25x + 1 \\ 0.40 + 0.25x &= 1 \\ 0.25x &= 1 - 0.40 \\ 0.25x &= 0.60 \\ x &= \frac{0.60}{0.25} \times \frac{100}{100} = \frac{60}{25} = 2.4 \text{ hours} \end{aligned}$$

ii) If the battery power is 0 then

$$\begin{aligned} y &= 0 \\ 0 &= -0.25x + 1 \\ 0.25x &= 1 \Rightarrow \quad x = \frac{1}{0.25} \\ x &= \frac{1}{0.25} \times \frac{100}{100} = \frac{100}{25} \quad \Rightarrow x = 4 \end{aligned}$$

12. Find the equation of a straight line through the intersection of lines 7x + 3y = 10, 5x – 4y = 1 and parallel to the line 13x + 5y + 12 = 0.

Solution:

$13x + 5y + 12 = 0$

slope, $m = \dfrac{-coefficient\ of\ x}{coefficient\ of\ y}$

$m = -\dfrac{13}{5}$ $\therefore$ Parallel to the slope $= -\dfrac{13}{5}$

we have $7x + 3y = 10$ .... (1)

$5x - 4y = 1$ .... (2)

$$\begin{aligned}
(1) \times 5 \Rightarrow\quad 35x + 15y &= 50 \\
(2) \times 7 \Rightarrow\quad 35x - 28y &= 7 \\
(-)\quad (+)\quad & \quad (-) \\
\hline
43y &= 43 \\
y &= 1
\end{aligned}$$

Substitute y in (1) $7x + 3(1) = 10$

$7x = 10 - 3 \Rightarrow 7x = 7$

$x = 1$

Therefore the intersecting point is (1, 1).

The equation of straight line is $y - y_1 = m(x - x_1)$

$y - 1 = -\dfrac{13}{5}(x - 1) \Rightarrow 5y - 5 = -13x + 13$

$13x + 5y - 18 = 0$

$\therefore$ Required equation is $13x + 5y - 18 = 0$

13. Find the equation of a straight line through the intersection of lines $5x - 6y = 2$, $3x + 2y = 10$ and perpendicular to the line $4x - 7y + 13 = 0$.

Solution:

$5x - 6y = 2$ .... (1)

$3x + 2y = 10$ .... (2)

$$\begin{aligned}
(1) \times 1 \Rightarrow\quad 5x - 6y &= 2 \\
(2) \times 3 \Rightarrow\quad 9x + 6y &= 30 \\
\hline
14x &= 32 \\
x &= \frac{32}{14}
\end{aligned}$$

$$x = \frac{16}{7}$$

Substitute x in (1)

$$5\left(\frac{16}{7}\right) - 6y = 2 \quad \Rightarrow \frac{80}{7} - 6y = 2 \Rightarrow - 6y = 2 - \frac{80}{7}$$

$$\Rightarrow -6y = \frac{14-80}{7} \quad \Rightarrow \; - 6y = -\frac{66}{7} \quad \Rightarrow \; y = \frac{-66}{7\times -6} \Rightarrow \; y = \frac{11}{7}$$

$\therefore$ Intersecting Point is $\left(\frac{16}{7}, \frac{11}{7}\right)$.

$4x - 7y + 13 = 0$

Slope, m = $\frac{4}{7}$

Perpendicular Slope, m = $-\frac{7}{4}$, Point $\left(\frac{16}{7}, \frac{11}{7}\right)$.

Equation of straight line, $y - y_1 = m(x - x_1)$

$$\therefore \text{ Required Equation is } y - \frac{11}{7} = -\frac{7}{4}\left(x - \frac{16}{7}\right)$$

$$4y - 4\left(\frac{11}{7}\right) = -7\left(\frac{7x-16}{7}\right)$$

$$4y - \frac{44}{7} = -7x + 16$$

$$28y - 44 = -49x + 112$$

$49x + 28y - 156 = 0$ is the required equation of the line.

14. The area of a triangle is 5 sq. units. Two of its vertices are (2, 1) and (3, –2). The third vertex is (x, y) where $y = x + 3$. Find the coordinates of the third vertex.

Solution:

The third vertex of the triangle be (x, y)

$$\text{Area of the triangle} = \frac{1}{2}\begin{vmatrix} 2 & 1 \\ 3 & -2 \\ x & y \\ 2 & 1 \end{vmatrix}$$

$$5 = \frac{1}{2}[(-4 + 3y + x) - (3 - 2x + 2y)]$$
$$10 = [x + 3y - 4 - 3 + 2x - 2y)]$$
$$10 = 3x + y - 7$$
$$3x + y = 17 \quad \text{.......... (1)}$$
$$x - y = -3 \quad \text{.......... (2)}$$
$$(1) + (2) \Rightarrow \quad 4x = 14$$
$$x = \frac{7}{2}$$

Substitute x in eqn (2)

$$\frac{7}{2} - y = -3$$
$$-y = -3 - \frac{7}{2} \quad \Rightarrow -y = -\frac{13}{2} \quad \Rightarrow y = \frac{13}{2}$$

$\therefore$ Third vertex is $(x, y) = \left(\frac{7}{2}, \frac{13}{2}\right)$

Aliter:

$(2, 1)$, $(3, -2)$, (x, y)

x_1, y_1 x_2, y_2 x_3, y_3

The third vertex of the triangle be (x, y)

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

$$5 = \frac{1}{2}\begin{vmatrix} 2-3 & 2-x \\ 1+2 & 1-y \end{vmatrix} \quad \Rightarrow 10 = \begin{vmatrix} -1 & 2-x \\ 3 & 1-y \end{vmatrix}$$

$$-1(1-y)-3(2-x) = 10$$
$$-1+y-6+3x = 10$$

$$(1) \Rightarrow \quad 3x + y = 17$$
$$(2) \Rightarrow \quad x - y = -3$$
$$4x = 14$$
$$x = \frac{14}{4}$$

$$x = \frac{7}{2}$$

Substitute x in eqn (2)

$$\frac{7}{2} - y = -3 \quad \Rightarrow \quad \frac{7}{2} + 3 = y$$

$$\frac{7+6}{2} = y \quad \Rightarrow \quad y = \frac{13}{2}$$

the third vertex is $(x, y) = \left(\frac{7}{2}, \frac{13}{2}\right)$

15. If the points A (–3, 9), B (a, b) and C (4, –5) are collinear and if a + b = 1, then find a and b.

Solution:

Given A (–3, 9), B (a, b), C (4, –5) are collinear and a+b=1

Area of the triangle formed by 3 points = 0

(i.e.) $\frac{1}{2}\begin{pmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{pmatrix} = 0$

$$(-3b - 5a + 36) - (9a + 4b + 15) = 0$$

$$-5a - 3b + 36 - 9a - 4b - 15 = 0$$

$$-14a - 7b + 21 = 0 \ (\div 7)$$

$$2a + b = 3 \qquad \text{.......... (1)}$$

$$a + b = 1 \qquad \text{.......... (2)}$$

$(1) - (2) \Rightarrow a = 2 \quad b = -1$

Aliter:

A(–3, 9), B(a, b), C(4, –5)

$x_1, y_1 \quad x_2, y_2 \quad x_3, y_3$

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} -3-a & -3-4 \\ 9-b & 9+5 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} -3-a & -7 \\ 9-b & 14 \end{vmatrix} = 0$$

$$14[-3-a] + 7[9-6] = 0$$

$$-42-14a+63-76 = 0$$
$$-14a-76+21 = 0$$
$$\div 7 \qquad 14a+7b = 21$$
$$2a+b = 3 \qquad \text{..........} (1)$$
$$a+b = 1 \qquad \text{..........} (2)$$

$(1)-(2) \Rightarrow a=2$

$(2) \Rightarrow b=-1$

16. Without using Pythagoras theorem, show that the points (1, –4), (2, –3) and (4, –7) form a right angled triangle.

Solution:

Let the points be A (1, –4) , B (2, –3) and C (4, –7).

The Slope of AB $= \dfrac{-3+4}{2-1} = \dfrac{1}{1} = 1$

The Slope of BC $= \dfrac{-7+3}{4-2} = \dfrac{-4}{2} = -2$

The Slope of AC $= \dfrac{-7+4}{4-1} = \dfrac{-3}{3} = -1$

Slope of AB × Slope of AC = (1) (–1) = –1

AB is perpendicular to AC. $\angle A = 90°$

Therefore, ΔABC is a right angled triangle.

Created Questions with Solution — 5 marks

1. The Area of the triangle formed by the points [P, 2–2P], [1–P, 2P] and [–4–P, 6–2P] is 56 sq. units. Find the number of possible integer values of "P".

Solution:

Let A (P, 2–2P), B (1–P, 2P) and C (–4–P, 6–2P)

Area of triangle ABC = 70 sq. units

$$\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \text{Area of the triangle ABC.}$$

$$\begin{vmatrix} P & 2-2P \\ 1-P & 2P \\ -4-P & 6-2P \\ P & 2-2P \end{vmatrix} = 56$$

$$\begin{aligned} [2P^2 + (1-P)(6-2P) - (4+P)(2-2P)] \\ - [(1-P)(2-2P) - (4+P)(2P) + P(6-2P)] &= 56 \\ [2P^2 + 6-8P+2P^2 - (8-6P-2P^2)] \\ - [2-4P+2P^2 - 8P-2P^2+6P-2P^2] &= 56 \\ 4P^2 - 8P + 6 + 2P^2 + 6P-8 - [-2P^2 - 6P + 2] &= 56 \\ 6P^2 - 2P - 2 + 2P^2 + 6P - 2 &= 56 \end{aligned}$$

$\div$ by 4

$$\begin{aligned} 8P^2+4P-60 &= 0 \\ 2P^2+P-15 &= 0 \\ (P+3)(2P-5) &= 0 \\ (P+3) = 0 \text{ (or) } (2P-5) &= 0 \end{aligned}$$

$\therefore$ P = –3 (or) P = $\frac{5}{2}$

$\therefore$ The number of possible Integer values of P is 1.

2. Is the line through (–2, 3) and (4, 1) perpendicular to the line 3x–y–1 = 0? Does the line 3x–y–1 = 0 bisect the join of (–2, 3) and (4, 1).

Solution:

Let A (–2, 3) and B (4, 1) be the given points

Slope of AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{1-3}{4+2} = \frac{-2}{6} = \frac{-1}{3} = m_1$ (say)

Slope of the given line = $\frac{-3}{-1} = 3 = m_2$

$m_1 \times m_2 = -1$ therefore the lines are perpendicular.

Midpoint of AB = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+4}{2}, \frac{3+1}{2}\right) = (1, 2)$

Substituting (1, 2) in the equation $3x-y-1 = 0$

$3(1) - 2 - 1 = 0$

$3 - 3 = 0$

i.e the equation is satisfied.

$\therefore$ the given line bisects the join of two points.

3. Find the circumcentre of the triangle whose vertices are A (4, 2), B (3, 1) and C (3, 3).

Solution:

Let D and E be the midpoint of the sides AB and AC respectively.

$$D\,(x, y) = \left(\frac{4+3}{2}, \frac{2+1}{2}\right) = \left(\frac{7}{2}, \frac{3}{2}\right)$$

The coordinates of E are $\left[\frac{4+3}{2}, \frac{2+3}{2}\right] = \left(\frac{7}{2}, \frac{5}{2}\right)$

Slope of AB = $\dfrac{2-1}{4-3}$

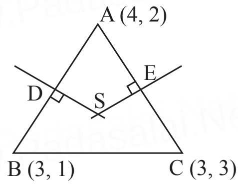
Slope of perpendicular of AB is –1.

Equation of perpendicular bisector of AB.

$$\begin{aligned} y - \frac{3}{2} &= 1\left(x - \frac{7}{2}\right) \\ 2y-3 &= 2x-7 \\ x+y-5 &= 0 \\ x+y &= 5 \qquad \text{..........} \ (1) \end{aligned}$$

Slope of the perpendicular bisector of side AC is 1

Equation of perpendicular of AC is



$$\begin{aligned} y - \frac{5}{2} &= -1\left(x - \frac{7}{2}\right) \\ 2y - 5 &= 2x - 7 \\ 2x - 2y - 2 &= 0 \\ x - y - 1 &= 0 \\ x - y &= 1 \qquad \text{..........} \ (2) \end{aligned}$$

Solving (1) and (2) we get,

$$\begin{aligned} x + \cancel{y} &= 5 \\ x - \cancel{y} &= 1 \\ \hline 2x &= 6 \\ x &= 3 \end{aligned}$$

Substitute $x = 3$ in equation (1)

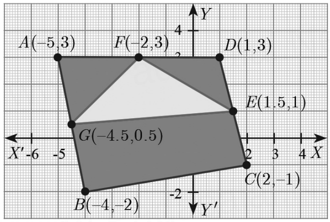
$$\begin{aligned} 3+y &= 5 \\ y &= 2 \end{aligned}$$

The circumcentre of the triangle is (3, 2).

For Practice	5 marks

1. A triangular shaped glass with vertices at A (–5, –4), B (1, 6) and C (7, –4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

2. In the figure, find the area of
   (i) triangle AGF
   (ii) triangle FED
   (iii) Quadrilateral BCEG.



3. Show that the given points are collinear: (–3, –4), (7, 2) and (12, 5).
4. If the three points (3, –1), (a, 3) and (1, –3) are collinear, find the value of a.
5. If the points A (2, 2), B(–2, –3) , C(1, –3) and D(x, y) form a parallelogram then find the value of x and y.
6. The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (3, –2), (–1, –1) and (1, 2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.
7. If (0, 0), (P, 8) and (6, 2) are the Vertices of a triangle taken in order. Then find the value of 'p'. If the area of the triangle is 20 sq. units.
8. Show that the points (–2, 5), (6, –1) and (2, 2) are collinear.
9. Find the intercepts made by the line $4x - 9y + 36 = 0$ on the coordinate axes.
10. A quadrilateral has vertices at A (–4, –2), B (5, –1), C (6, 5) and D (–7, 6). Show that the mid-points of the sides form a parallelogram.
11. You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$
    (i) Find the total MB of the song.

(ii) after how many seconds will 75% of the songs get downloaded?

(iii) after how many seconds the song will be downloaded completely?

12. Find the equation of a straight line joining the point of intersection of $3x + y + 2 = 0$ and $x - 2y - 4 = 0$ to the point of intersection of $7x - 3y = -12$ and $2y = x + 3$.

13. Find the equation of a straight line joining the point of intersection of lines $8x + 3y = 18$, $4x + 5y = 9$ and bisecting the line segment joining the points (5, –4) and (–7, 6).

14. Let P (11, 7), Q (13.5, 4) and R (9.5, 4) be the mid points of the sides AB, BC and AC respectively of ΔABC. Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR.

Created Questions — 5 marks

1. Find the equation of the lines through the intersection of $2x+3y-1 = 0$ and $3x-4y-6 = 0$ and (i) parallel to the line $5x-y-2 = 0$ and (ii) Perpendicular to the line $x+2y-1 = 0$

2. Find the equation of the line through the point of intersection of the line $2x+y-5 = 0$ and $x+y-3 = 0$ and bisecting the line segment joining the points (3, –2) and (–5, 6).

3. Name the type of quadrilateral formed if the points and given reason for your answer. (–1, –2) (1, 0) (–1, 2) (–3, 0).

★★★

UNIT 6 TRIGONOMETRY

Objective Type Questions — 1 mark

1. **The value of $\sin^2\theta + \dfrac{1}{1+\tan^2\theta}$ is equal to**
   1) $\tan^2\theta$ 2) 1
   3) $\cot^2\theta$ 4) 0 **Ans:** 2)

2. **The value of $\tan\theta \operatorname{cosec}^2\theta - \tan\theta$**
   1) $\sec\theta$ 2) $\cot^2\theta$
   3) $\sin\theta$ 4) $\cot\theta$ **Ans:** 4)

3. **If $(\sin\alpha + \operatorname{cosec}\alpha)^2 + (\cos\alpha + \sec\alpha)^2 = k + \tan^2\alpha + \cot^2\alpha$ then the value of k is equal to**
   1) 9 2) 7
   3) 5 4) 3 **Ans:** 2)

4. **If $\sin\theta + \cos\theta = a$ and $\sec\theta + \operatorname{cosec}\theta = b$, then the value of $b(a^2 - 1)$ is equal to**
   1) 2a 2) 3a
   3) 0 4) 2ab **Ans:** 1)

5. **If $5x = \sec\theta$ and $\dfrac{5}{x} = \tan\theta$ then, $x^2 - \dfrac{1}{x^2}$ is equal to**
   1) 25 2) $\dfrac{1}{25}$
   3) 5 4) 1 **Ans:** 2)

6. **If $\sin\theta = \cos\theta$ and $2\tan^2\theta + \sin^2\theta - 1$ is equal to**
   1) $\dfrac{-3}{2}$ 2) $\dfrac{3}{2}$
   3) $\dfrac{2}{3}$ 4) $\dfrac{-2}{3}$ **Ans:** 2)

7. **$x = a\tan\theta$ and $y = b\sec\theta$ then**
   1) $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$ 2) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
   3) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ 4) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$ **Ans:** 1)

8. **(1+tanθ+secθ) (1+cotθ–cosecθ) is equal to**
   1) 0 2) 1
   3) 2 4) –1 **Ans:** 3)

9. **acot θ + bcosec θ = p and bcot θ + acosec θ = q then, $p^2 - q^2$ is equal to**
   1) $a^2 - b^2$ 2) $b^2 - a^2$
   3) $a^2 + b^2$ 4) b – a **Ans:** 2)

10. **If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}$: 1, then the angle of elevation of the sun has measure**
   1) 45° 2) 30°
   3) 90° 4) 60° **Ans:** 4)

11. **The electric pole subtends an angle of 30°at point on the same level as its foot. at a second point "b" metres above the first, the depression of the foot of the tower is 60°. The height of the tower (in metres) is equal to**
   1) $\sqrt{3}$ b 2) $\frac{b}{a}$
   3) $\frac{b}{2}$ 4) $\frac{b}{3}$ **Ans:** 4)

12. **A tower is 60°m height. its shadow is *x* metres shorter when the sun altitude is 45° then when it has been 30°, then *x* is equal to**
   1) 41.92 m 2) 43.92 m
   3) 43 m 4) 45.6 m **Ans:** 2)

13. **The angle of depression of the top and bottom of 20 m tall building from the top of a multi storied building are 30° and 60° respectively. The height of the multi storied building and the distance between two building(in metres)**
   1) 20, $10\sqrt{3}$ 2) 30, $5\sqrt{3}$
   3) 20, 10 4) 30, $10\sqrt{3}$ **Ans:** 4)

14. **Two persons are standing x" metres apart form each other and the height of the first person is double that of other. if from the middle point of the line joining their feet an observer find the angular elevation of their tops to be**

complementary, then the height of the shorter person(in metres) is

1) $\sqrt{2x}$ 2) $\dfrac{x}{2\sqrt{2}}$

3) $\sqrt{\ }$ 4) $2x$ **Ans:** 2)

15. **The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is**

1) $\dfrac{h(1+\tan\beta)}{1-\tan\beta}$ 2) $\dfrac{h(1-\tan\beta)}{1+\tan\beta}$

3) h tan(45° – β) 4) none of these **Ans:** 1)

Created Questions — 1 mark

1. **The angle of elevation and depression are usually measured by a device called**

1) Theodolite 2) Kaleidoscope

3) Periscope 4) Telescope **Ans:** 1)

2. $\dfrac{\sin(90-\theta)}{\tan\theta}+\dfrac{\cos(90-\theta)}{\cot\theta}=$

1) tanθ 2) 1

3) –1 4) sin θ **Ans:** 2)

3. **If cos (40°+x) = sin 30°, then x is equal to**

1) 20° 2) 30°

3) 60° 4) 0° **Ans:** 1)

4. **If cosecθ = $x+\dfrac{1}{4x}$, then the value of cosecθ + cotθ is**

1) $2x$ 2) $-2x$

3) $\dfrac{2}{x}$ 4) $-\dfrac{1}{2x}$ **Ans:** 1)

5. **secA (1–sinA) (secA+tanA) =**

1) 0 2) 1

3) cosθ 4) sinθ **Ans:** 2)

6. **The angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to it's height is**

1) 30° 2) 45°
3) 60° 4) 90° **Ans:** 2)

7. The angle of elevation of the top of a tower from a distance of 100 m from it's foot is 30° the height of the tower is

1) $100\sqrt{3}$ m 2) $\frac{100}{\sqrt{3}}$ m
3) $50\sqrt{3}$ m 4) $\frac{200}{\sqrt{3}}$ m **Ans:** 2)

8. $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} =$

1) 1 2) $1 - \sin\theta\cos\theta$
3) $\sin\theta + \cos\theta$ 4) $\tan\theta$ **Ans:** 2)

9. The length of the string a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$ how height is the kite?

1) 75 m 2) 78.05 m
3) 226 m 4) none of these **Ans:** 1)

10. The ratio of the length of a rod and it's shadow is 1: $\sqrt{3}$ the altitude of the sum is

1) 30° 2) 45°
3) 60° 4) 90° **Ans:** 1)

11. The value of tanθ from the equation $3(\sec^2\theta - 1) + 16\tan\theta + 5 = 0$ are

1) $\left[\frac{1}{3}, 5\right]$ 2) $\left[\frac{1}{5}, 3\right]$
3) $\left[\frac{-1}{3}, \frac{-1}{5}\right]$ 4) $\left[\frac{-1}{3}, -5\right]$ **Ans:** 4)

Two Marks Questions — 2 marks

1. Prove that secθ – cosθ = tanθ sinθ.

Solution:

$$\text{LHS} = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta} . \sin\theta = \tan\theta \sin\theta$$

$\therefore$ LHS = RHS

2. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ = cosecθ + cotθ.

Solution:

$$\text{LHS} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$= \sqrt{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$$

LHS = cosecθ + cotθ

$\therefore$ LHS = RHS

3. Prove that cotθ + tanθ = secθ cosecθ.

Solution:

$$\text{LHS} = \cot\theta+\tan\theta = \frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \sec\theta\,\mathrm{cosec}\,\theta$$

$\therefore$ LHS = RHS. Hence Proved.

4. Prove that $\frac{\cos\theta}{1+\sin\theta}$ = secθ - tanθ

Solution:

$$\text{LHS} = \frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$$

$\therefore$ LHS = RHS. Hence Proved.

5. Prove that $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Solution:

$$\text{LHS} = \frac{\cot A \quad \cos A}{\cot A \quad \cos A} = \frac{\dfrac{\cos A}{\sin A} - \cos A}{\dfrac{\cos A}{\sin A} + \cos A}$$

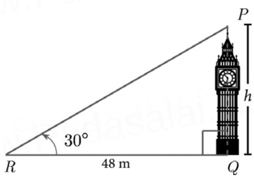
$$= \frac{\cos A\left[\dfrac{1}{\sin A} - 1\right]}{\cos A\left[\dfrac{1}{\sin A} + 1\right]} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$= \text{RHS}$

$\therefore$ LHS = RHS. Hence Proved.

6. A tower stands vertically on the ground. From a point on the ground, which is 48m away from the foot of the tower, the angle of elevation of the top of the tower 30°. Find the height of the tower.

Solution:



$$\text{In } \Delta PQR \quad \tan\theta = \frac{PQ}{QR}$$

$$\tan 30° = \frac{h}{48}$$

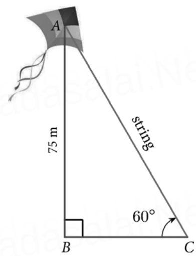
$$\frac{1}{\sqrt{3}} = \frac{h}{48}$$

$$h = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{48\sqrt{3}}{3}$$

Therefore the height of the tower is, $h = 16\sqrt{3}$ m

7. **A kite is flying at a height of 75m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.**

Solution:



$$\text{In } \Delta ABC \ \sin\theta = \frac{AB}{AC}$$

$$\sin 60° = \frac{75}{AC} \quad \Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{AC}$$

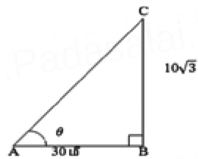
$$AC = \frac{75 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3}$$

$$AC = 50\sqrt{3}$$

$\therefore$ Hence, the length of the string is $50\sqrt{3}$ m.

8. **Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}$ m.**

Solution:



$$\text{In } \Delta ABC \ \tan\theta = \frac{\textit{oppostie side}}{\textit{Adjacent side}}$$

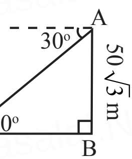
$$\tan\theta = \frac{10\sqrt{3}}{30} \Rightarrow \frac{\sqrt{3}}{3}$$

$$\tan\theta = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

9. **From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.**

Solution:



$$\text{In } \Delta ABC \ \tan\theta = \frac{\textit{oppostie side}}{\textit{Adjacent side}}$$

$$\tan 30° = \frac{50\sqrt{3}}{BC}$$

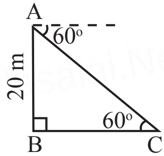
$$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{BC}$$

$$BC = 50\sqrt{3} \times \sqrt{3}$$

$$= 50\,(3) = 150 \text{ m}$$

Created Questions with Solution — 2 marks

1. A player sitting on the top of a tower of height 20m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3}$ = 1.732)



Solution:

Height of the tower = 20 m

$$\text{In } \Delta ABC, \tan\theta = \frac{AB}{BC}$$

$$\tan 60° = \frac{20}{BC} \Rightarrow \sqrt{3} = \frac{20}{BC}$$

$$BC = \frac{20}{\sqrt{3}} \Rightarrow BC = \frac{20\sqrt{3}}{3}$$

$$= 11.55 \text{ cm}$$

2. Prove that $\sqrt{\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}} = \dfrac{1-\sin\theta}{\cos\theta}$

Solution:

$$\text{LHS} = \sqrt{\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}} = \sqrt{\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}}$$

$$= \sqrt{\frac{(\sec\theta-\tan\theta)^2}{\sec^2\theta-\tan^2\theta}} \Rightarrow \sqrt{\frac{(\sec\theta-\tan\theta)^2}{1}}$$

$$= \sec\theta - \tan\theta$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{1-\sin\theta}{\cos\theta} = \text{RHS}$$

3. If $x = a\sec\theta\cos\varphi$, $y = b\sec\theta\sin\varphi$ then show that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1+\tan^2\theta$$

Solution:

$x = a\sec\theta \cos\varphi \Rightarrow x^2 = a^2\sec^2\theta \cos^2\varphi$

$y = b\sec\theta \sin\varphi \Rightarrow y^2 = b^2\sec^2\theta \sin^2\varphi$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = \frac{a^2\sec^2\theta\cos^2\phi}{a^2}+\frac{b^2\sec^2\theta\sin^2\phi}{b^2}$$
$$= \sec^2\theta\cos^2\varphi + \sec^2\theta\sin^2\varphi$$
$$= \sec^2\theta[\cos^2\varphi + \sin^2\varphi]$$
$$= \sec^2\theta$$
$$= 1+\tan^2\theta$$

4. **Prove that $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1-2\sin^2\theta\cos^2\theta)$**

Solution:

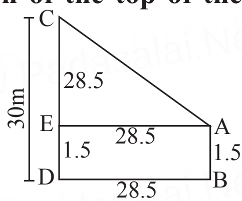
$$\text{LHS} = \sin^8\theta - \cos^8\theta$$
$$= (\sin^4\theta)^2 - (\cos^4\theta)^2$$
$$= (\sin^4\theta - \cos^4\theta)(\sin^4\theta + \cos^4\theta)$$
$$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta)$$
$$= (1)(\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2)$$
$$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$$
$$= (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$$

5. **If $x = r\sin A\cos B$, $y = r\sin A\sin B$ and $z = r\cos A$. Prove that $r^2 = x^2+y^2+z^2$**

Solution:

$$x^2+y^2+z^2 = r^2\sin^2A\cos^2B + r^2\sin^2A\sin^2B + r^2\cos^2A$$
$$= r^2\sin^2A[\sin^2B+\cos^2B] + r^2\cos^2A$$
$$= r^2\sin^2A + r^2\cos^2A \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= r^2[\sin^2A + \cos^2A]$$
$$= r^2$$

6. **Rahul observe 1.5m tall is 28.5m away from the tower 30m height. Determine the angle of elevation of the top of the tower from his eye.**



Solution:

$$CE = CD - DE$$
$$= 30 - 1.5 = 28.5$$

In right triangle CEA

$\tan\theta = \frac{opp}{Adj} = \frac{28.5}{28.5}$

$\tan\theta = 1 \quad \therefore \theta = 45°$

Angle of elevation from the top of the tower = 45°.

For Practice — 2 marks

1. Prove the following identity $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$.
2. Prove the following identity $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$.
3. Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$.
4. A player sitting on the top of a tower of height 20m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. ($\sqrt{3} = 1.732$)

Created Questions — 2 marks

1. Prove that the identity $\frac{\sin\theta}{\operatorname{cosec}\theta} + \frac{\cos\theta}{\sec\theta} = 1$.
2. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$.
   Then prove that $x^2 - y^2 = a^2 - b^2$
3. A ramp for unloading a moving truck , has an angle of elevation of 30°, if the top of the ramp is 0.9m above the ground level, then find the length of the ramp.
4. A ladder leaning against a vertical wall, makes an angle of 60° with the ground. The foot of the ladder is 3.5m away from the wall. Find the length of the ladder.
5. If $\sqrt{3}\tan\theta = 3\sin\theta$ Find the value of $\sin^2\theta - \cos^2\theta$.
6. Prove that: $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$
7. The kite is flying at a height of 60m above the ground the string attacked to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is not slack in the string.

Five Marks Questions — 5 marks

1. If cosecθ + cotθ = p, then prove that cosθ = $\dfrac{p^2-1}{p^2+1}$.

Solution:

Given: $\operatorname{cosec}\theta + \cot\theta = p$ .... (1)

$\Rightarrow \quad \operatorname{cosec}\theta - \cot\theta = \dfrac{1}{p}$ .... (2)

$(1)+(2) \Rightarrow \quad 2\operatorname{cosec}\theta = p + \dfrac{1}{p}$

$2\operatorname{cosec}\theta = \dfrac{p^2+1}{p}$ .... (3)

$(1)-(2) \Rightarrow \quad 2\cot\theta = p - \dfrac{1}{p}$

$2\cot\theta = \dfrac{p^2-1}{p}$ .... (4)

$$(4)/(3) \Rightarrow \frac{2\cot\theta}{2\operatorname{cosec}\theta} = \frac{\frac{p^2-1}{p}}{\frac{p^2+1}{p}} \Rightarrow \frac{\cot\theta}{\operatorname{cosec}\theta} = \frac{p^2-1}{p^2+1}$$

$$\frac{\cos\theta}{\sin\theta}\times\sin\theta = \frac{p^2-1}{p^2+1}$$

$$\cos\theta = \frac{p^2-1}{p^2+1}.$$

Hence Proved.

2. Prove that the following identity

$$\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\frac{\sin^3 A-\cos^3 A}{\sin A-\cos A}=2$$

Solution:

$$\text{LHS} = \frac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\frac{\sin^3 A-\cos^3 A}{\sin A-\cos A}$$

$$= \frac{(\sin A+\cos A)(\sin^2 A+\cos^2 A-\sin A\cos A)}{\sin A+\cos A}$$

$$+\frac{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}{\sin A-\cos A}$$

$= \sin^2A + \cos^2A - \sin A\cos A + \sin^2A + \cos^2A + \sin A\cos A$

$= \sin^2A + \cos^2A + \sin^2A + \cos^2A$

$= 1+1 = 2 =$ R.H.S.

3. If $\frac{\cos\alpha}{\cos\beta}$ =m and $\frac{\cos\alpha}{\sin\beta}$ =n, then prove that $(m^2+n^2)\cos^2\beta = n^2$

Solution:

$$m = \frac{\cos\alpha}{\cos\beta},\ n = \frac{\cos\alpha}{\sin\beta} \Rightarrow m^2 = \frac{\cos^2\alpha}{\cos^2\beta},\ n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$m^2 + n^2 = \frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta}+\frac{1}{\sin^2\beta}\right)$$

$$= \cos^2\alpha\left(\frac{\sin^2\beta+\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)$$

$$(m^2+n^2)\cos^2\beta = \cos^2\alpha\left(\frac{1}{\cancel{\cos^2\beta}\sin^2\beta}\right)\cancel{\cos^2}\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta}$$

$(m^2 + n^2)\cos^2\beta = n^2$

Hence Proved.

4. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \mathrm{cosec}\theta = q$, then prove that $q(p^2-1) = 2p$.

Solution:

$p = \sin\theta+\cos\theta$

$p^2 = (\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$p^2 - 1 = 1 + 2\sin\theta\cos\theta - 1$

$$= 2\sin\theta\cos\theta$$

$$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)(2\sin\theta\cos\theta)$$

$$= \left[\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right](2\sin\theta\cos\theta)$$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}(2\sin\theta\cos\theta)$$

$$= 2(\sin\theta + \cos\theta) = 2p$$

5. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1}{a}$**, then prove that** $\dfrac{a^2-1}{a^2+1} = \sin\theta$**.**

Solution:

$$\frac{1}{a} = \frac{\cos\theta}{1+\sin\theta} \Rightarrow a = \frac{1+\sin\theta}{\cos\theta}$$

$$a = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta \quad \text{.......... (1)}$$

$$|||^{ly}\ \frac{1}{a} = \sec\theta - \tan\theta \quad \text{.......... (2)}$$

$$(1) - (2) \Rightarrow \quad a - \frac{1}{a} = \sec\theta + \tan\theta - (\sec\theta - \tan\theta)$$

$$\frac{a^2-1}{a} = \sec\theta + \tan\theta - \sec\theta + \tan\theta$$

$$\frac{a^2-1}{a} = 2\tan\theta \quad \text{.......... (3)}$$

Similarly we've $$\frac{a^2+1}{a} = 2\sec\theta \quad \text{.......... (4)}$$

$$(3)/(4) \Rightarrow \quad \frac{a^2-1}{a^2+1} = \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta}\times\cos\theta$$

$$\therefore \frac{a^2-1}{a^2+1} = \sin\theta$$

6. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ **then,**
prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$**.**

Solution:

Now $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring on both sides,

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$
$$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$
$$2\cos^2\theta - \cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$$
$$\cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$$
$$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\sin\theta\cos\theta$$
$$\cos\theta - \sin\theta = \frac{2\sin\theta\cos\theta}{\cos\theta + \sin\theta}$$
$$= \frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta} = \sqrt{2}\sin\theta$$

Therefore,

$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$. (since $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$)

7. If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Solution:

LHS

$\sqrt{3}\sin\theta - \cos\theta = 0 \Rightarrow \sqrt{3}\sin\theta = \cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\sqrt{3}} \qquad \Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} \qquad \Rightarrow \theta = 30°$

$$= \tan 3\theta$$
$$= \tan 3(30°)$$
$$= \tan 90°$$
$$= \infty$$

RHS

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \frac{3\tan 30^o - \tan^3 30^o}{1 - 3\tan^2 30^o}$$
$$= \frac{3.\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}}{1 - 3.\frac{1}{3}} \Rightarrow \frac{3.\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}}{1-1}$$
$$= \frac{\frac{9-1}{3\sqrt{3}}}{0} \Rightarrow \frac{\frac{8}{3\sqrt{3}}}{0} = \infty$$

8. Prove that $\left(\dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$

Solution:

$$\text{LHS} = \left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right)$$

$$= \left(\frac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \cos A \sin A)}{\cos A - \sin A}\right) - \left(\frac{(\cos A + \sin A)(\cos^2 A + \sin^2 A - \cos A \sin A)}{\cos A + \sin A}\right)$$

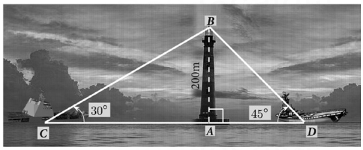
$= (1 + \cos A\sin A) - (1 - \cos A\sin A)$

[Since $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$]

$= 1 + \cos A \sin A - 1 + \cos A\sin A$

$= 2\cos A\sin A$

9. Two ships are sailing in the sea on either sides of a light house. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m high, find the distance between the two ships. ($\sqrt{3}$ = 1.732)



Solution:

Let AB be the light house.

Let C and D be the positions of the two ships. Then, AB = 200 m

$\angle ACB = 30°$, $\angle ADB = 45°$

In right triangle BAC, $\tan 30° = \dfrac{AB}{AC}$

$\dfrac{1}{\sqrt{3}} = \dfrac{200}{AC}$ gives, $AC = 200\sqrt{3}$

In right triangle BAD, $\tan 45° = \dfrac{AB}{AD}$

$1 = \dfrac{200}{AD}$ gives $AD = 200$

Now, $CD = AC + AD = 200\sqrt{3} + 200$ [by (1) and (2)]

$CD = 200(\sqrt{3} + 1) = 200 \times 2.732 = 546.4$

Distance between two ships is 546.4 m

10. From the top of a tower 50m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. ($\sqrt{3}$ = 1.732)

Solution:

Height of the tower AB = H = 50m

Let the height of the tree = CD = h

$\angle ACM = \alpha = 30°$ $\qquad$ $\angle ADB = \beta = 45°$

$$\text{Height of the tree} = h = \frac{H[\tan\beta - \tan\alpha]}{\tan\beta}$$

$$= \frac{50[\tan 45^o - \tan 30^o]}{\tan 45^o} \Rightarrow \frac{50\left[1-\frac{1}{\sqrt{3}}\right]}{1}$$

$$= 50\left[1-\frac{1}{\sqrt{3}}\right] \Rightarrow 50 - \frac{50}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= 50 - \frac{50\sqrt{3}}{3} \Rightarrow 50 - \frac{50\times 1.732}{3}$$

$$= 50 - 28.87$$

Height of the tree (h) = 21.13 m

11. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30m high building are 45° and 60° respectively. Find the height of the tower ($\sqrt{3}$ = 1.732)

Solution:

In ΔAPB $\tan\theta = \frac{\textit{opposite side}}{\textit{Adjacent side}} = \frac{AB}{BP}$

$\tan 45° = \frac{30}{BP} \Rightarrow 1 = \frac{30}{BP}$

BP = 30 m

In ΔBPC $\tan 60° = \frac{BC}{BP} = \frac{AB+AC}{BP}$

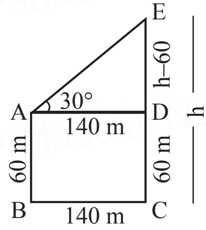
$$\sqrt{3} = \frac{h+30}{30} \Rightarrow 30\sqrt{3} = h + 30$$

$$h = 30\sqrt{3} - 30$$

$$= 30(1.732 - 1) = 30(0.732)$$

$$= 21.960$$

12. The horizontal distance between two buildings is 140m. The angle of depression of the top of the first building when seen from the top of the second building is 30°. If the height of the first building is 60m, find the height of the second building. ($\sqrt{3}$ = 1.732)



Solution:

$$\text{In } \Delta AEC \tan\theta = \frac{\textit{opposite side}}{\textit{Adjacent side}}$$

$$\tan 30° = \frac{h-60}{140}$$

$$\frac{1}{\sqrt{3}} = \frac{h-60}{140}$$

$$h - 60 = \frac{140}{\sqrt{3}} \Rightarrow \frac{140\sqrt{3}}{3}$$

$$h - 60 = \frac{140 \times 1.732}{3}$$

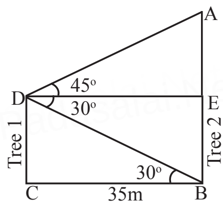
$$h - 60 = 80.83$$

$$h = 80.23 + 60$$

$$h = 140.83 \text{ m.}$$

Therefore the height of the second building is 140.78 m.

13. From the top of a tree of height 13m the angle of elevation and depression of the top and bottom of another tree are 45° and 30° respectively. Find the height of the second tree. ($\sqrt{3}$ = 1.732)



Solution:

Height of Tree 1 CD = h = 13 m

Elevation $\angle ADE = \alpha = 45°$; Depression $\angle EDB = \beta = 30°$

$$\text{Height of Tree 2 AB} = H = h[1+\tan\alpha \cot\beta]$$

$$H = h[1+\tan 45° \cot 30°]$$

$$= 13[1+1\times\sqrt{3}]$$

$$= 13[1+\sqrt{3}]$$

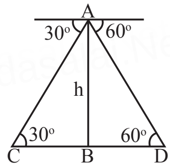
$$= 13[1+1.732]$$

$$= 13[1+2.732]$$

Height of Tree 2, H = 35.52 m

14. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Solution:



C, D – Positions of the two ships

Height of the Light House AB = h m

$$\text{In } \Delta\, ABC \tan\theta = \frac{\textit{opposite side}}{\textit{Adjacent side}}$$

$$\tan 30° = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3} \qquad \text{Where } BC = x$$

In Δ ABD

$$\tan 60° = \frac{h}{y} \qquad \text{Where } BD = y$$

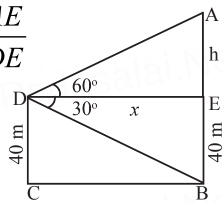
$$\sqrt{3} = \frac{h}{y} \Rightarrow y = \frac{h}{\sqrt{3}}$$

$$\text{Distance between two ships} = (x + y) = h\sqrt{3} + \frac{h}{\sqrt{3}}$$

$$d = \frac{4h}{\sqrt{3}} \text{ m}$$

15. A man is standing on the deck of a ship, which is 40m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill. ($\sqrt{3}$ = 1.732)

Solution:



$$\text{In } \Delta AED \ \tan\theta = \frac{\textit{opposite side}}{\textit{Adjacent side}} = \frac{AE}{DE}$$

$$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \qquad .... (1)$$

$$\text{In } \Delta BDE \tan\theta = \frac{opposite\ side}{Adjacent\ side}$$

$$\tan 30° = \frac{BE}{DE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$x = 40\sqrt{3} \quad \text{....... (2)}$$

From (1) & (2)

$$\frac{h}{\sqrt{3}} = 40\sqrt{3}$$

$$h = 40\sqrt{3}\,(\sqrt{3}) = 40(3)$$

$$h = 120 \text{ m}$$

substitute h in equation (1)

$$x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{120\sqrt{3}}{3}$$

$$x = 40\sqrt{3} \text{ m.}$$

∴ The distance of the hill from the ship is AC = $x = 40\sqrt{3}$

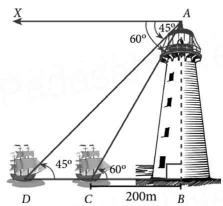
$x = 40 \times 1.732$

$x = 69.28$ m

∴ The height of the hill AB = 120 + 40 = 160 m.

16. A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat(in km/hr), assuming that it is sailing in still water? ($\sqrt{3}$ = 1.732)

Solution:



Let AB be the tower.

Let C and D be the positions of the boat

∠XAC = 60° = ∠ACB and

∠XAD = 45° = ∠ADB, BC = 200m

$$\text{In right triangle, ABC, } \tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{200}$$

$$AB = 200\sqrt{3} \quad \text{.......... (1)}$$

In right triangle, ABD $\tan 45° = \dfrac{AB}{BD}$

$$1 = \frac{200\sqrt{3}}{BD} \quad \text{[by (1)]}$$

We get, $BD = 200\sqrt{3}$

Now, $CD = 200\sqrt{3} - 200$

$= 200(\sqrt{3} - 1) = 146.4$

It is given that the distance CD is covered in 10 seconds.

That is, the distance of 146.4m is covered in10 seconds.

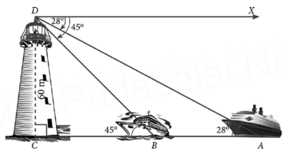
Therefore , speed of the boat $= \dfrac{distance}{time} = \dfrac{146.4}{10}$

$= 14.64$ m/s

$= 14.64 \times \dfrac{3600}{1000}$ km/hr

$= 52.704$ km/hr

17. As observed from the top of a 60m high light house from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (tan28° = 0.5317).



Solution:

Height of the light house = CD = 60m

Position of the observer = D

From the diagram $\angle XDA = 28° = \angle DAC$ and

$\angle XDB = 45° = \angle DBC$

From the triangle DCB, we have

$$\tan 45° = \frac{DC}{BC} \Rightarrow 1 = \frac{60}{BC}$$

$BC = 60$ m

From the triangle DCA, we have

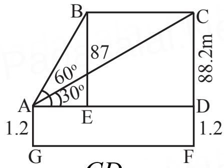
$$\tan 28° = \frac{DC}{AC} \Rightarrow 0.5317 = \frac{60}{AC} \Rightarrow AC = \frac{60}{0.5317}$$

$AC = 112.85$ m

Distance between two ships AB = AC – BC = 52.85 m

Created Questions with Solution — 5 marks

1. A 1.2m tall girl Jasmine spots a balloon moving with the wind in a horizontal line at a height of 88.2m from the ground. The angle of elevation of the balloon from the eyes of the girl at an instant is 60°. After some time the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Solution:

AG = Height of the Jasmine = 1.2m

Position of the balloon from the ground=88.2m

In ΔABC $\tan\theta = \dfrac{BE}{AE}$

$\tan 60° = \dfrac{87}{AE}$

$\sqrt{} = \dfrac{87}{AE}$

$AE = \dfrac{87}{\sqrt{3}} = 29\sqrt{3}$

In ΔACD $\tan\theta = \dfrac{CD}{AD}$

$\tan 30° = \dfrac{CD}{AE + ED}$

$\dfrac{1}{\sqrt{3}} = \dfrac{87}{29\sqrt{3} + ED}$

$29\sqrt{3} + ED = 87\sqrt{3}$

$ED = 87\sqrt{3} - 29\sqrt{3}$

$ED = 58\sqrt{3}$

$ED = 58 \times 1.732$

$= 100.46$ m

Here ED = BC Distance travelled by the balloon = 100.46 m

2. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$. Prove that $x^2+y^2 = 1$.

Solution:

We have

$$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta) + y\cos\theta(\cos^2\theta) = \sin\theta\cos\theta$$
$$x\sin\theta(\sin^2\theta) + x\sin\theta(\cos^2\theta) = \sin\theta\cos\theta$$

[since $x\sin\theta = y\cos\theta$]

$$x\sin\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$
$$x\sin\theta = \sin\theta\cos\theta$$
$$x = \cos\theta$$

Now
$$x\sin\theta = y\cos\theta$$
$$\cos\theta\sin\theta = y\cos\theta$$
$$y = \sin\theta$$

Hence $x^2+y^2 = \cos^2\theta + \sin^2\theta = 1$

3. Prove the identity $\left(\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\dfrac{1-\cos\theta}{1+\cos\theta}$

Solution:

$$\text{L.H.S} = \left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2$$

$$=\frac{(1)^2+(\sin\theta)^2+(-\cos\theta)^2+2(1)\sin\theta+2\sin\theta(-\cos\theta)+2(-\cos\theta)(1)}{(1)^2+(\sin\theta)^2+(\cos\theta)^2+2(1)\sin\theta+2\sin\theta\cos\theta+2\cos\theta(1)}$$

$$=\frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+\sin^2\theta+\cos^2\theta+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$=\frac{1+1+2\sin\theta-2\cos\theta(1+\sin\theta)}{1+1+2\sin\theta+2\cos\theta(1+\sin\theta)}$$

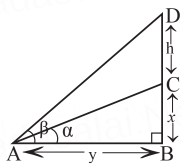
$$=\frac{2+2\sin\theta-2\cos\theta(1+\sin\theta)}{2+2\sin\theta+2\cos\theta(1+\sin\theta)}$$

$$=\frac{2(1+\sin\theta)-2\cos\theta(1+\sin\theta)}{2(1+\sin\theta)+2\cos\theta(1+\sin\theta)}$$

$$=\frac{2(1+\sin\theta)(1-\cos\theta)}{2(1+\sin\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

4. A vertical tower stands and horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β. Prove that the height of the tower is $\dfrac{h\tan\alpha}{\tan\beta-\tan\alpha}$

Solution:



Let height of the tower BC = x and CD be the flagstaff.

Let CD = h; distance AB = y

In right triangle ABC tan α = $\dfrac{BC}{AB}$

In right triangle ABD tan α $=\dfrac{x}{y}$ $\Rightarrow$ y = $\dfrac{x}{\tan\alpha}$ ..... (1)

tan β $=\dfrac{x+h}{y}$ $\Rightarrow$ y = $\dfrac{x+h}{\tan\beta}$ ..... (2)

From (1) and (2) we get

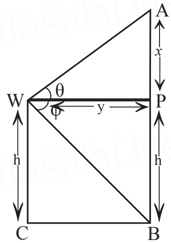
$$\frac{x+h}{\tan\beta} = \frac{x}{\tan\alpha} \quad \Rightarrow \frac{x}{\tan\beta} + \frac{h}{\tan\beta} = \frac{x}{\tan\alpha}$$

$$\frac{h}{\tan\beta} = \frac{x}{\tan\alpha} - \frac{x}{\tan\beta}$$

$$\frac{h}{\cancel{\tan\beta}} = x\left[\frac{\tan\beta - \tan\alpha}{\tan\alpha\,\cancel{\tan\beta}}\right] \Rightarrow x = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence height of the tower = $\dfrac{h\tan\alpha}{\tan\beta - \tan\alpha}$

5. From the window (h metres height above the ground) of a house in a street, the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and φ respectively. Show that the height of the opposite house is h (1+tanθ cotφ)



Solution:

Let W be the window and AB be the house on the opposite side. the WP be the width of the street

Let BP = h m, AP = x m and WP = y m.

In triangle BPW $\tan\varphi = \dfrac{BP}{WP} \Rightarrow \dfrac{h}{y}$

$$y = \frac{h}{\tan\varphi} \Rightarrow y = h\cot\varphi$$

Now in triangle APW we have

$$\tan\theta = \frac{AP}{WP} \quad \Rightarrow \tan\theta = \frac{x}{y}$$

$$x = y\tan\theta$$

$$x = h\cot\varphi\tan\theta$$

Height of the opposite house = AP + BP

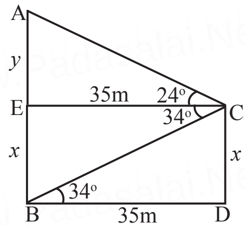
$\Rightarrow x + h = h\cot\varphi\tan\theta + h$

$= h[1 + \tan\theta\cot\varphi]$

6. A building and a statue are in opposite side of a street from each other 35m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle

of depression of base of the statue is 34°. Find the height of the statue? (tan24° = 0.4452, tan 34° = 0.6745)

Solution:



CD = x m = height of the building = EB

AB = x + y m = height of the statue

$$\Delta CBD,\ \tan 34° = \frac{CD}{BD} = \frac{x}{35}$$

$$x = 35\tan 34° \ ...... (1)$$

$$\Delta ACE,\ \tan 24° = \frac{AE}{EC} = \frac{y}{35}$$

$$y = 35\tan 24° \ ...... (2)$$

$$\begin{aligned} \text{Height of the Statue} &= x + y \\ &= 35(\tan 34° + \tan 24°) \\ &= 35(0.6745 + 0.4452) \\ &= 35(1.1197) \\ &= 39.1895 \\ &\approx 39.19 \text{ m} \end{aligned}$$

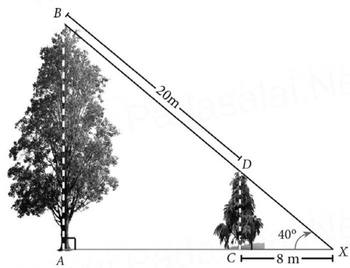
For Practice | **5 marks**

1. If $\sin\theta(1 + \sin^2\theta) = \cos^2\theta$ then, prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$
2. If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$ then, prove that $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$.
3. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66m high apartment are 60° and 30° respectively. Find (i) The height of the lamp post. (ii) The difference between height of the lamp post and the apartment. ($\sqrt{3}$ = 1.732)
4. As observed from the top of a 60m high light house from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (tan28° = 0.5317).

5. If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. Prove that the height that the cloud is located from the ground is $\dfrac{h(\tan\theta_1 + \tan\theta_2)}{\tan\theta_2 + \tan\theta_1}$

6. An aeroplane sets off from G on a bearing of 24° towards H, a point 250km away. At H it changes course and heads towards J on a bearing of 55 and a distance of 180km away.
   (i) How far is H to the North of G?
   (ii) How far is H to the East of G?
   (iii) How far is J to the North of H?
   (iv) How far is J to the East of H?

7. Two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point X on the ground is 40°. If the horizontal distance between X and the smaller tree is 8m and the distance of the top of the two trees is 20m, calculate



   (i) the distance between the point X and top of the smaller tree.
   (ii) the horizontal distance between the two trees.
   (cos 40° = 0.7660)

8. From the top of a 12m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

9. A pole 5m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression to the point 'A' from the top of the tower is 45°. Find the height of the tower. ($\sqrt{3}$ = 1.732)

10. $36x^4 - 60x^3 + 61x^2 - mx + n$. Find the value of m and n.

Created Questions — 5 marks

1. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$ and $m \neq n$, then show that $m^2 - n^2 = 4\sqrt{mn}$.

2. If A, B, C are the interior angles of a triangle, prove that $\tan\left(\frac{B+C}{2}\right) = \cot\left(\frac{A}{2}\right)$

3. The angle of elevation of the top of a hill from the foot and top of a building 28m high are 60° and 30° respectively. Find the height of the hill.

4. A vertical tree was broken by a wind. The top of the tree touches the ground and makes an angle 30° with it. If the top of the tree touches the ground 30m away from the foot, then find the actual height of the tree.

5. Prove the identities:
$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

6. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$. Prove that $x^2+y^2 = 1$.

7. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \operatorname{cosec}\theta\sec\theta$

8. Two ships are sailing in the sea on the either side of the light house, the angle of depression of two ships as observed from the top of the light house are 60° and 45° respectively. If the distance between the ship is $100\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ m then find the height of the light house.

☙★★★❧

UNIT 7 MENSURATION

Objective Type Questions — 1 mark

1. **The curved surface area of a right circular cone of height 15cm and base diameter 16 cm**
   1) 60π cm^2 2) 68π cm^2
   3) 120π cm^2 4) 136π cm^2 **Ans:** 4)

2. **If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is**
   1) $4\pi r^2$ sq.units 2) $6\pi r^2$ sq.units
   3) $3\pi r^2$ sq.units 4) $8\pi r^2$ sq.units **Ans:** 1)

3. **The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be**
   (1) 12 cm 2) 10 cm
   3) 13 cm 4) 5 cm **Ans:** 1)

4. **If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is**
   1) 1 : 2 2) 1 : 4
   3) 1 : 6 4) 1 : 8 **Ans:** 2)

5. **The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is**
   1) $\frac{9\pi h^2}{8}$ sq.units 2) $24\pi h^2$ sq. units
   3) $\frac{8\pi h^2}{9}$ sq. units 4) $\frac{56\pi h^2}{9}$ sq.units **Ans:** 3)

6. **In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is**
   1) 5600π cm^3 2) 11200π cm^3
   3) 56π cm^3 4) 3600π cm^3 **Ans:** 2)

7. If the radius of the base of a cone is tripled and the height is double then the volume is?

1) Made 6 times
2) made 18 times
3) made 12 times
4) unchanged

Ans: 2)

8. The total surface area of a hemi-sphere is how much times the square of its radius..

1) π
2) 4π
3) 3π
4) 2π

Ans: 3)

9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is

1) $3x$ cm
2) x cm
3) $4x$ cm
4) $2x$ cm

Ans: 3)

10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. Then the volume of the frustum is

1) 3328π cm^3
2) 3228π cm^3
3) 3240π cm^3
4) 3340π cm^3

Ans: 1)

11. A shuttle cock used for playing badminton has the shape of the combination of

1) a cylinder and a sphere
2) a hemisphere and a cone
3) a sphere and a cone
4) frustum of a cone and a hemisphere

Ans: 4)

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

1) 2 : 1
2) 1 : 2
3) 4 : 1
4) 1 : 4

Ans: 1)

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

1) $\frac{4}{3}\pi$
2) $\frac{10}{3}\pi$
3) 5π
4) $\frac{20}{3}\pi$

Ans: 1)

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is

1) 1 : 3 2) 1 : 2
3) 2 : 1 4) 3 : 1 **Ans:** 2)

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is

1) 1 : 2 : 3 2) 2 : 1 : 3
3) 1 : 3 : 2 4) 3 : 1 : 2 **Ans:** 4)

Created Questions — 1 mark

1. If the volume of sphere is 36π cm^3, then its radius is equal to

1) 3 cm 2) 2 cm
3) 5 cm 4) 10 cm **Ans:** 1)

2. C.S.A of solid sphere is equal to

1) T.S.A of solid sphere 2) T.S.A of hemisphere
3) C.S.A of hemisphere 4) None of these **Ans:** 1)

3. The volume of the cylinder is 180 cm^3. Then the volume of the cone having the same radius and height as that of the cylinder is

1) 180 cm^3 2) 540 cm^3
3) 60 cm^3 4) 360 cm^3 **Ans:** 3)

4. The height of a cylinder is 14 cm and it's curved surface area is 264 cm^2 the volume of the cylinder is

1) 296 cm^3 2) 396 cm^3
3) 369 cm^3 4) 503 cm^3 **Ans:** 2)

5. The ratio between the radius of the base and the height of the cylinder is 2:3 if it's volume is 1617 cm^3, the total surface area of the cylinder is

1) 208 cm^2 2) 77 cm^2
3) 707 cm^2 4) 770 cm^2 **Ans:** 4)

6. The ratio of the volumes of two spheres is 8 : 27 the ratio between their surface area is

1) 2 : 3 2) 4 : 27
3) 8 : 9 4) 4 : 9 **Ans:** 4)

7. **The number of spherical balls each of radius 1 cm, that can be made from a solid sphere of lead of radius 8 cm is**
1) 60 2) 512
3) 4096 4) 8 **Ans:** 2)

8. **The radius of the slant height of the cone are in the ratio 4:7 and it's curved surface area is 792 cm^2, then the radius is**
1) 10 cm 2) 8 cm
3) 12 cm 4) 9 cm **Ans:** 3)

9. **A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 1 cm and the diameter of the base is 2 cm. If a right circular cylinder circumscribes the solid the volume of unused space inside the cylinder is**
1) 2π cc 2) $\frac{\pi}{2}$ cc
3) π cc 4) π^2 cc **Ans:** 3)

10. **If two cylinders have their radii in the ratio 4 : 5 and heights in the ratio 5 : 6 then the ratio of their volume is**
1) 8 : 15 2) 15 : 8
3) 6 : 5 4) 4 : 5 **Ans:** 1)

11. **A child reshape a cone made up of china clay of height 24 cm and radius 6 cm into a sphere the radius of the sphere is**
1) 24 cm 2) 12 cm
3) 6 cm 4) 48 cm **Ans:** 3)

12. **From a solid cylinder of height 10 cm and radius of the base 6cm, a cone of same height and same base removed. The volume of the remaining solid is**
1) 240π cm^3 2) 5280π cm^3
3) 620π cm^3 4) 360π cm^3 **Ans:** 1)

Two Marks Questions — 2 marks

1. **A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.**

Solution:

Given that, height of the cylinder h = 20 cm; radius r = 14 cm.

T.S.A of the cylinder = $2\pi r(h + r)$ sq.units

$$= 2 \times \frac{22}{7} \times 14\,(20+14) = 88 \times 34$$

$$= 2992 \text{ cm}^2$$

C.S.A of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 20 = 1760 \text{ cm}^2$$

Therefore, C.S.A = 1760 cm^2 and T.S.A = 2992 cm^2

2. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

Solution:

Given that C.S.A of the cylinder = $2\pi rh$ sq.units

$$2\pi rh = 88$$

$$2 \times \frac{22}{7} \times r \times 14 = 88 \text{ (Given h = 14 cm)}$$

$$2r = \frac{88}{14} \times \frac{7}{22} \Rightarrow 2r = 2$$

Therefore, diameter = 2 cm.

3. A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions.

Solution:

diameter d = 2.8 m; height = 3 m; radius r = 1.4 m

Area covered in one revolution = curved surface area of the cylinder.

$= 2\pi rh$ sq.units.

$$= 2 \times \frac{22}{7} \times 1.4 \times 3$$

Area covered in 1 revolution = 26.4 m^2

Area covered in 8 revolutions = $8 \times 26.4 = 211.2$

Therefore, area covered is 211.2 m^2

4. If the total surface area of a cone of radius 7 cm is 704 cm^2, then find its slant height.

Solution:

Total surface area = 704 cm^2

$$\pi r(l + r) = 704$$

$$\frac{22}{7} \times 7\,(l+7) = 704$$

$$l + 7 = \frac{704}{22} = \frac{64}{2} = 32$$

$l + 7 = 32$, $l = 32 - 7 = 25$ cm.

Therefore, slant height of the cone is 25 cm.

5. 4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm² of the floor area, then find the height of the tent.

Solution:

$$\text{Base area of the cone} = 4 \times 22 = 88 \text{ cm}^2$$

$$\pi r^2 = 88$$

$$\frac{22}{7} \times r^2 = 88$$

$$r^2 = 88 \times \frac{7}{22} = 28 \text{ cm}^2$$

$$l = 19 \text{ cm} \qquad l^2 = 361$$

$$h = \sqrt{l^2 - r^2} = \sqrt{361 - 28} = \sqrt{333}$$

Height of the tent = 18.25 cm.

6. Find the diameter of a sphere whose surface area is 154 m²

Solution:

$$\text{Surface area of sphere} = 154 \text{ m}^2$$

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$\text{gives} \quad r^2 = \frac{154}{4} \times \frac{7}{22} \Rightarrow r^2 = \frac{49}{4} \Rightarrow r = \frac{7}{2}$$

Radius of sphere $r = \frac{7}{2}$ m; Diameter of sphere d = 7 m

7. The radius of a spherical balloon increase from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Solution:

Let r_1 and r_2 be the radii of the balloons.

Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Ratio of C.S.A.of balloons = $\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Therefore, ratio of C.S.A of balloons is 9 : 16

8. If the base area of a hemispherical solid is 1386 sq.metres, then find its total surface area.

Solution:

Base area of a hemispherical solid, πr^2 =1386 sq.m

Total Surface area of hemispherical = $3\pi r^2$

= 3 × 1386 = 4158

Therefore, T.S.A of the hemispherical solid is 4158 m^2

9. The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution:

l = 5 cm, R = 4 cm, r = 1 cm

$$\text{C.S.A of the frustum} = \pi (R + r)\, l \text{ sq.units}$$

$$= \frac{22}{7}(4+1) \times 5$$

$$= \frac{22 \times 5 \times 5}{7} = \frac{550}{7} = 78.57 \text{ m}^2$$

10. The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

Solution:

Let r and h be the radius and height of the cone respectively.

Given: h = 24 cm, volume = 11088 cm^3

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = 441$$

∴ Radius of the cone r = 21 cm

11. The ratio of the radii of two right circular cones of same height is 1 : 3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Solution:

$r_1 : r_2 = 1 : 3$

$h_1 = 3r_1$ $\qquad$ $h_2 = 3r_2$

$r_1 = x$ $\qquad$ $r_2 = 3x$

$h_1 = 3x$ $\qquad$ $h_2 = 3x$

$$l_1 = \sqrt{h_1^2 + r_1^2} = \sqrt{9x^2 + x^2} = \sqrt{10x^2} = \sqrt{10}x$$

$$l_2 = \sqrt{h_2^2 + r_2^2} = \sqrt{9x^2 + 9x^2} = 3\sqrt{2}x$$

$$\therefore \text{Ratio of their CSA} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{x^2\sqrt{10}}{3x(3\sqrt{2}x)}$$

$$= \frac{\sqrt{10}x^2}{9\sqrt{2}x^2} = \frac{\sqrt{10}}{9\sqrt{2}} = \frac{\sqrt{5}}{9}$$

$\therefore$ Ratio of their CSA $= \sqrt{5} : 9$

12. A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Solution:

Height of the cone = 24 cm, Radius are same.

$$\frac{1}{3}\pi r^2 h_1 = \pi r^2 h_2$$

$$\frac{1}{3} \times 24 = h_2$$

$$h_2 = 8 \text{ cm}$$

13. The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights.

Solution:

$$\text{Ratio of the volumes of two cones} = \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2$$

$$= h_1 : h_2$$

$$= 3600 : 5040$$

$$= 360 : 504$$

$$= 40 : 56$$

$$= 5 : 7$$

14. **A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$: 4.**

Solution:

Given

Total surface area of a solid sphere = Total surface Area of a solid hemisphere

$$4\pi R^2 = 3\pi r^2$$

$$\frac{R^2}{r^2} = \frac{3}{4} \Rightarrow \frac{R}{r} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{ Ratio of their volumes } = \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3} \Rightarrow \frac{2R^3}{r^3} \Rightarrow 2\left[\frac{R}{r}\right]^3$$

$$= 2\left[\frac{\sqrt{3}}{2}\right]^3 \Rightarrow 2\times\frac{3\sqrt{3}}{8} \Rightarrow \frac{3\sqrt{3}}{4}$$

$\therefore$ Ratio of their volumes = $3\sqrt{3}$: 4

Created Questions with Solution — 2 marks

1. **Find the number of spherical lead shots, each of diameter 6cm that can be made from a solid cuboids of lead having dimensions 24 cm × 22 cm × 12 cm.**

Solution:

Volume of the cuboid, $l \times b \times h = 24 \times 22 \times 12$

Diameter of the spherical lead shot = 6 cm

Radius of the spherical lead shot, r = 3 cm

$$\text{Numberofsphericalleadshots} = \frac{\textit{volume of the cuboid}}{\textit{volume of the sperical lead shot}}$$

$$= \frac{l\times b\times h}{\frac{4}{3}\pi r^3} \Rightarrow \frac{24\times22\times12}{\frac{4}{3}\times\frac{22}{7}\times3\times3\times3}$$

$$= \frac{24\times22\times12\times3\times7}{4\times22\times3\times3\times3} = 56$$

$\therefore$ 56 spherical lead shots can be made.

2. **The slant height of the frustum of a cone is 4 cm and the perimeters (circumference) for it's circular ends are 18 cm and 6cm. Find the curved surface area of the frustum.**

Solution:

We have slant height (l) = 4 cm

$2\pi r_1 = 18; \qquad 2\pi r_2 = 6$

$\pi r_1 = 9 \qquad \pi r_2 = 3$

$$\begin{aligned}\text{Curved surface are of the frustum} &= \pi r_1 l + \pi r_2 l \\ &= l\,[\pi r_1 + \pi r_2] \\ &= 4\,[9+3] \\ &= 4[12] \\ &= 48 \text{ cm}^2\end{aligned}$$

3. **A solid is hemispherical at the bottom and the conical from the top. If the surface areas of the two parts are equal then find the ratio of it's radius and height of it's conical part.**

Solution:

Given:

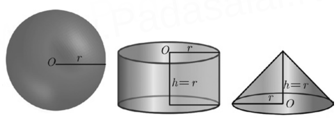
Surface area of the cone = Surface area of hemisphere

$$\begin{aligned}\pi r l &= 2\pi r^2 \\ l &= 2r \\ l &= \sqrt{h^2+r^2} \\ 2r &= \sqrt{h^2+r^2} \\ 4r^2 &= h^2+r^2\end{aligned}$$

$$\begin{aligned}3r^2 &= h^2 \\ \sqrt{3}\,r &= h \\ \frac{r}{h} &= \frac{1}{\sqrt{3}} \\ r : h &= 1 : \sqrt{3}\end{aligned}$$

For Practice | **2 marks**

1. A sphere, a cylinder and a cone (Figure) are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.



2. If the ratio of radii of two spheres is 4 : 7. Find the ration of their volumes.
3. A aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8cm. Find the height of the cylinder.

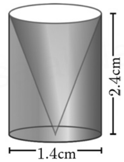
4. The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.
5. Find the volume of a cylinder whose height is 2 m and whose base area is 250 m^2.
6. A metallic sphere of radius 16 cm is melted and recase into small spheres each of radius 2 cm. How many small spheres can be obtained.
7. The radius and height of a cylinder are in the ratio 5 : 7 and its curved surface area is 5500 sq.cm. Find its radius and height.
8. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of embankment.

Created Questions — 2 marks

1. Find the slant height of the cone whose base has the diameter 10 cm and whose height is 12 cm.
2. Two cones have their height in the ratio 5 : 3 and radii of their bases in the ratio 2 : 1. Find the ratio of their volumes.
3. The volume of the sphere is numerically equal to its surface area. Find its diameter.
4. How many litres of water will a hemispherical tank hold, whose diameter is 4.2 m?
5. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.
6. The volume of the cube is 2744 cm^3. Find it's surface area.
7. Find the ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height is 20 cm.
8. A right triangle with sides 3 cm, 4 cm and 5 cm is revolved about the side 3 cm. Find the volume of the cone so formed.
9. The area of the base right circular cone is 78.5 sq.m. If it's height is 12 m find it's volume.

Five Marks Questions — 5 marks

1. **From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out in figure. Find the total surface area of the remaining solid.**



Solution:

h = 2.4 cm, d = 1.4 cm,

Total surface area of the remaining solid

= C.S.A of the cylinder + C.S.A. of the cone + Area of the bottom

= $(2\pi rh + \pi rl + \pi r^2)$ sq.units

= $\pi r(2h + l + r)$ sq.units

$$l = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2}$$

$$= \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5 \text{ cm}$$

Area of the remaining solid $= \pi r(2h + l + r)$ sq.units

$$= \frac{22}{7} \times 0.7[2(2.4)+2.5+0.7)]$$

$$= 22 \times 0.1\,(4.8+2.5+0.7)$$

$$= 22 \times 0.1 \times 8.0$$

$$= 2.2 \times 8 = 17.6 \text{ cm}^2$$

Therefore, total surface area of the remaining solid is 17.6 cm^2

2. **An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10m and 4 m and whose height is 4m. Find the curved and total surface area of the bucket.**

Solution:

Diameter of the top = 10 m; Radius of the top, R = 5 m

Diameter of the bottom = 4 m; Radius of the bottom, r = 2 m, height h = 4 m

$$\text{Now, } l = \sqrt{h^2 + (R-r)^2} = \sqrt{4^2 + (5-2)^2} = \sqrt{16+9}$$

$$= \sqrt{25} = 5\text{m}$$

$$\Rightarrow \quad l = 5\text{m}$$

C.S.A $= \pi(R+r)l$ sq.units

$= \frac{22}{7}(5+2)\times 5 \quad \Rightarrow \frac{22}{7}\times 7\times 5 = 110\ m^2$

T.S.A $= [\pi(R+r)\,l + \pi R^2 + \pi r^2]$ sq. units

$= \pi[(R+r)\,l + R^2 + r^2]$

$= \frac{22}{7}[(5+2)5+5^2+2^2] \Rightarrow \frac{22}{7}(35+25+4)$

$= \frac{1408}{7} = 201.14\ m^2$

Therefore, C.S.A = $110\ m^2$ and T.S.A = $201.14\ m^2$

3. **If the radii of the circular ends of a frustum which is 45 cm hight are 28 cm and 7 cm, find the volume of the frustum.**

Solution:

height of the frustum, h = 45 cm, bottom radii, R = 28 cm, top radii, r = 7 cm

Volume of the frustum $= \frac{1}{3}\pi h[R^2 + Rr + r^2]$cu.units

$= \frac{1}{3}\times\frac{22}{7}\times 45[28^2+28\times 7+7^2]$

$= \frac{1}{3}\times\frac{22}{7}\times 45[784+196+49]$

$= \frac{1}{3}\times\frac{22}{7}\times 45\times 1029$

$= 22\times 15\times 147 = 48510\ cm^3$

4. **A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill a container at the rate of ₹ 40 per litre.**

Solution:

h = 16 cm, r = 8 cm, R = 20 cm

Volume of the frustum $= \frac{1}{3}\pi h[R^2 + Rr + r^2]$ cu. units

$$= \frac{1}{3} \times \frac{22}{7} \times 16[20^2+20(8)+8^2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 16[400+160+64]$$

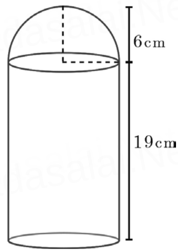
$$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 624$$

$$= 10459 \text{ cm}^3 = 10.459 \text{ litre}$$

The cost of milk is ₹ 40 per litre

The cost of 10.459 litres milk = 10.459 × 40 = ₹ 418.36

5. A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.



Solution:

diameter, d = 12 cm, radius, r = 6 cm

height of the cylindrical portion, h = 25–6=19 cm

T.S.A of the toy = C.S.A of the cylinder + C.S.A of the hemisphere + Base Area of the cyliner

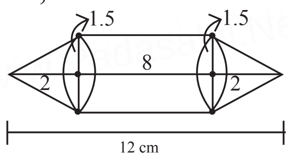
$= 2\pi rh + 2\pi r^2 + \pi r^2$ sq.units

$= 2\pi rh + 3\pi r^2 = \pi r\,(2h + 3r)$

$= \frac{22}{7} \times 6 \times (38+18) \quad \Rightarrow \frac{22}{7} \times 6 \times 56$

$= 1056 \text{ cm}^2$

6. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12cm. If each cone has a height of 2cm, find the volume of the model that Nathan Made.



Solution:

Cylinder:

diameter, d = 3cm, radius, r = 1.5cm

height h_1=12–(2+2) = 8cm

Cone:

radius, $r = \frac{3}{2}$ cm, height $h_1 = 2$cm

Volume of the model = Volume of the cylinder + Volume of 2 cones

$$= \pi r^2 h_1 + 2\frac{1}{3}\pi r^2 h_2$$

$$= \pi r^2[h_1+2\frac{1}{3}h_2]$$

$$= \frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}[8+\frac{2}{3}\times 2]$$

$$= \frac{22}{7}\times\frac{9}{4}[8+\frac{4}{3}]$$

$$= \frac{99}{14}\left[\frac{28}{3}\right]$$

$= 66$ cm

7. **A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm. Find the internal diameter.**

Solution:

diameter of the cone = 14 cm, radius of the cone = 7 cm, height of the cone, h = 8 cm

$$\frac{4}{3}\pi(R^3-r^3) = \frac{1}{3}\pi r^2 h \Rightarrow \frac{4}{3}\pi(5^3-r^3) = \frac{1}{3}\pi\times7\times7\times8$$

$$125 - r^3 = \frac{7\times7\times8}{4}$$

$$125 - r^3 = 98 \Rightarrow r^3 = 27 \Rightarrow r^3 = 3^3$$

$$r = 3$$

Internal Diameter of the sphere = 2(r) = 2(3) = 6 cm

8. **The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, then find the height of the cyliner.**

Solution:

Hollow Hemisphere

External Diameter = 10 cm

Radius, R = 5 cm

Internal Diameter = 6 cm

Radius, r = 3 cm

Cylinder

Diameter = 14 cm

Radius, r = 7 cm

Height, h = ?

Volume of the cylinder = Volume of the Hollow hemisphere

$$\pi r^2 h = \frac{2}{3}\pi(R^3 - r^3) \qquad 7\times7\times h = \frac{2}{3}(125-27)$$

$$\pi\times7\times7\times h = \frac{2}{3}\pi(5^3 - 3^3) \qquad h = \frac{2}{3}\times\frac{98}{7\times7}$$

Height of Cylinder, $h = \frac{4}{3} = 1.33$ cm

9. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.

Solution:

Sphere: Radius, r = 6 cm

Hollow Cylinder: External Radius, R=5cm, height, h=32, r=?

Volume of hollow cylinder = Volume of sphere

$$\pi(R^2 - r^2)h = \frac{4}{3}\pi r^3$$

$$\pi(5^2 - r^2)32 = \frac{4}{3}\pi\times6\times6\times6$$

$$(25 - r^2) = \frac{4\times6\times6\times6}{3\times32}$$

$$25 - r^2 = 9$$

$$25 - 9 = r^2$$

$$r^2 = 16 \Rightarrow r = 4$$

Thickness of cylinder $= R - r = 5 - 4 = 1$ cm

10. Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60cm and height 105 cm. Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sump which has been full, initially.

Solution:

Overhead Tank (Cylinder):

radius, r = 60 cm, height, h = 105 cm

Cuboid:

l = 2m = 200cm, b = 1.5m = 150cm, h = 1m = 100cm

Volume of remaining water left in sump

= Volume of water in sump (cuboid) – Volume of water overhead tank (cylinder)

$= l \times b \times h - \pi r^2 h$

$= 200 \times 150 \times 100 - \frac{22}{7} \times 60 \times 60 \times 105$

$= 3000000 - 1188000$

$= 1812000 \text{ cm}^3 = 1812$ litres.

11. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.

Solution:

Cylinder (Pipe)

Diameter, R= 14 cm

Radius, r = 7 cm

$r = \frac{7}{100}$ m

Cuboid Tank

Length = 50 m

Width, b = 44 m

Height, $h = \frac{21}{100}$ m

Speed of the water = 15 km/hour = 15000 m/hour

Volume of water left out from the pipe in time t

= Volume of the rectangular tank

$$\text{base area} \times \text{time} \times \text{speed} = l \times b \times h$$

$$\pi r^2 \times t \times \text{speed} = l \times b \times h$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times t \times 15000 = 50 \times 44 \times \frac{21}{100}$$

$$t = \frac{22 \times 21}{11 \times 7 \times 3}; \ t = 2 \text{ hours}$$

12. A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice creams to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

Solution:

Cylinder	Cone	Hemisphere
r = 6 cm	r = 3 cm	r = 3 cm
h = 15 cm	h = 9 cm	

Volume of Cylinder:

Number of cones needed to fill the ice creams

$$= \frac{volume\ of\ cylinder}{volume\ of\ cone + volume\ of\ hemisphere}$$

$$= \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3} = \frac{\pi r^2 h}{\pi\left(\frac{1}{3}r^2 h + \frac{2}{3}r^3\right)}$$

$$= \frac{6\times 6\times 15}{\frac{1}{3}\times 3\times 3\times 9 + \frac{2}{3}\times 3\times 3\times 3}$$

$$= \frac{6\times 6\times 15}{27+18} = \frac{6\times 6\times 15}{45} = 12$$

$\therefore$ Number of cones needed to fill the ice creams are 12.

13. A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Solution:

Hemispherical Bowl	Cylinder
Radius = r	Radius = r
	$r = h + \frac{1}{2}$ $h = \frac{3}{2}h$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{3}{2}h\right)^3$$

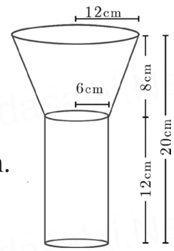
$$= \frac{2}{3}\pi\frac{27}{8}h^3 = \frac{9}{4}\pi h^3$$

$$\text{Volume of cylinder} = \pi r^2 h = \pi\left(\frac{3}{2}h\right)^2 h$$

$$= \pi \times \frac{9}{4} h^2 h = \frac{9}{4} \pi h^3$$

$\therefore$ Both the volumes are equal. 100% of juice that can be transferred from the bowl into the cylindrical vessel.

14. A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.



Solution:

Let h_1, h_2 be the heights of the frustum and cylinder respectively.

Let R, r be the top and bottom radii of the frustum.

Given that,

R=12cm, r=6cm, h_2=12cm, h_1=20–12=8cm

Slant height of the frustum $l = \sqrt{(R-r)^2 + h_1^2}$ units

$= \sqrt{36+64}$

l = 10 cm

Outer Surface Area $= 2\pi r h_2 + \pi(R+r)l$ sq.units

$= \pi[2rh_2 + (R+r)l]$

$= \pi[(2 \times 6 \times 12) + (18) \times (10)]$

$= \pi(144+180)$

$= \frac{22}{7} \times 324 = 1018.28$

Therefore, outer surface area of the funnel is 1018.28 cm^2

15. A metallic sheet in the form of a sector of a circle of radius 21cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

Solution:

Central angle, D = 216°

Radius of the sector = Slant height of cone

(i.e.) l = 21 cm

Length of arc of the sector = Perimeter of the base of cone

$$\frac{D}{360} \times 2\pi R = 2\pi r \qquad \Rightarrow \frac{216}{360} \times 21 = r \qquad \Rightarrow r = 12.6$$

$$h = \sqrt{l^2 - r^2} \qquad \Rightarrow \sqrt{21^2 - (12.6)^2}$$

$$= \sqrt{441 \quad 158.76} \Rightarrow \sqrt{282.24}$$

$$= 16.8 \text{ cm}$$

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{} \times 12.6 \times 12.6 \times 16.8$$

$$= 22 \times 4.2 \times 12.6 \times 2.4$$

$$= 2794.17 \text{ cm}^3$$

16. The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

Solution:

Radius of the conical tent, r = 7 m; height, h = 24 m

$$\text{Hence, } l = \sqrt{h^2 + r^2}$$

$$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625}$$

$$l = 25 \text{ m}$$

$$\text{C.S.A. of the conical tent} = \pi r l$$

$$= \frac{22}{7} \times 7 \times 25$$

$$= 550 \text{ m}^2$$

Width of the rectangular canvas = 4 m

$$\text{Area of the rectangular canvas} = l \times b$$

$$= l \times 4$$

$$\text{Area of the rectangular canvas} = \text{CSA of the conical tent}$$

$$4l = 550$$

$$l = 550/4 \Rightarrow l = 137.5 \text{ m}$$

17. If the slant height of the frustum cone is 1 cm and perimeters of its circular base are 18 cm and 28 cm respectively. What is the curved surface area of a the frustum?

Solution:

Slant height of the frustum cone, $l = 10$ cm

$$\begin{aligned}
\text{Circumference of the top} &= 28 \text{ cm} \\
2\pi R &= 28 \\
2 \times \frac{22}{7} \times R &= 28 \\
R &= 28 \times \frac{1}{2} \times \frac{7}{22} \\
R &= \frac{49}{11}
\end{aligned}$$

Circumference of the bottom = 18 cm

$$\begin{aligned}
2\pi r &= 18 \\
2 \times \frac{22}{7} \times r &= 18 \\
r &= 18 \times \frac{1}{2} \times \frac{7}{22} \\
r &= \frac{63}{22}
\end{aligned}$$

CSA of the frustum

$$\begin{aligned}
= \pi(R+r)l &= \frac{22}{7}\left(\frac{98}{22}+\frac{63}{22}\right) \times 10 \\
&= \frac{22}{7}\left(\frac{161}{22}\right) \times 10 \\
&= 230 \text{ cm}^2
\end{aligned}$$

18. Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular of height 10 cm and diameter 4.5 cm.

Solution:

diameter of the coin = 1.5 cm (smaller cylinder)

$r = 1.5/2 = 0.75$ cm; $h = 2$ mm $= 0.2$ cm

diameter of the bigger cylinder = 4.5 cm

$R = 2.25$ cm; $H = 10$ cm

$$\begin{aligned}
\text{Number of coins} &= \frac{\textit{volume of the bigger cylinder}}{\textit{volume of the smaller cylinder}} \\
&= \frac{\pi R^2 H}{\pi r^2 h} \Rightarrow \frac{\frac{9}{4}\times\frac{9}{4}\times 10}{\frac{3}{4}\times\frac{3}{4}\times\frac{2}{10}} \\
&= 450 \text{ coins.}
\end{aligned}$$

Created Questions with Solution — 5 marks

1. **A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand completely. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.**

Solution:

Volume of the cylindrical bucket $= \pi r^2 h$

$= \pi \times 18 \times 18 \times 32$

Volume of the cone $= \frac{1}{3}\pi r^2 h \Rightarrow \frac{1}{3}\pi r^2 \times 24$

Volume of the cone = Volume of the cylinder

$$\frac{1}{3}\pi r^2 \times 24 = \pi \times 18 \times 18 \times 32$$

$$r^2 = 18 \times 18 \times 2 \times 2$$

$$r = 18 \times 2 = 36 \text{ cm.}$$

$\therefore$ Radius of the cone = 36 cm.

$$l^2 = r^2 + h^2 = 36^2 + 24^2 = 1296 + 576 = 1872$$

$$l = \sqrt{1872}$$

$$l = 2\times2\times3\times\sqrt{13}$$

$$l = 12\sqrt{13}$$

$\therefore$ Slant height of the cone $= 12\sqrt{13}$ cm

2. **A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of the embankment.**

Solution:

Diameter = 3 m

$\therefore$ Radius, $r = \frac{3}{2}$ m; $R - r = 4 \Rightarrow R - \frac{3}{2} = 4 \quad \therefore R = \frac{11}{2}$

Height, h = 14 m

Volume of the sand $= \pi r^2 h$

$= \pi \times \frac{3}{2} \times \frac{3}{2} \times 14$

Volume of the embankment around the well

$= \pi(R^2 - r^2)\, h \qquad = \pi \times \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times h$

$$= \pi \times \left[\frac{121}{4} - \frac{9}{4}\right] \times h$$

$$= \pi \times \frac{112}{4} \times h$$

$$= \pi \times 28 \times h$$

$$\therefore \pi \times 28 \times h = \pi \times \frac{3}{2} \times \frac{3}{2} \times 14$$

$$h = \frac{1}{2} \times \frac{9}{4}$$

$$h = \frac{9}{8}$$

$\therefore$ Required Height $= \frac{9}{8}$ cm

3. A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of the radius 3.5 cm and height is 3 cm. Find the number of cones so formed.

Solution:

The volume of the solid metallic sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5 \quad [r = 10.5 \text{ cm}]$$

Volume of the cone $= \frac{1}{3}\pi r^2 h \quad \Rightarrow \frac{1}{3}\pi \times 3.5 \times 3.5 \times 3$

Number of cones so formed $= \dfrac{\frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5}{\frac{1}{3}\pi \times 3.5 \times 3.5 \times 3}$

$$= 126$$

4. Metallic sphere of radii 6 cm, 8 cm, 10 cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

Large sphere	**Small Sphere**
$R = ?$	$r_1 = 6$ cm
	$r_2 = 8$ cm
	$r_3 = 10$ cm

Volume of large sphere = Total volume of small sphere

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3]$$

$$R^3 = r_1^3 + r_2^3 + r_3^3$$

$$R^3 = (6)^3 + (8)^3 + (10)^3$$

$$R^3 = 216 + 512 + 1000$$
$$R^3 = 1728$$
$$R = 12 \text{ cm}$$

Thus the required radius of the resulting sphere = 12 cm

5. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form to cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Solution:

For a Circular coin:

diameter = 1.75 cm

radius = $\frac{175}{200}$ cm

Thickness(h) = 2mm = $\frac{2}{10}$ cm

For a Cuboid:

length (h) = 10 cm

breadth (b) = 5.5 cm

height (h) = 3.5 cm

$$\text{Number of coins} = \frac{\textit{Volume of cuboid}}{\textit{Volume of one coin}} = \frac{l \times b \times h}{\pi r^2 h}$$
$$= \frac{10 \times 5.5 \times 3.5}{\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}}$$
$$= \frac{10 \times 5.5 \times 3.5 \times 7 \times 200 \times 200 \times 10}{22 \times 175 \times 175 \times 2}$$
$$= 400$$

6. The perimeter of the ends of a frustum are 48 cm and 36 cm. If the height of the frustum is 11 cm find it's volume.

Solution:

Given $2\pi R = 48$ and $2\pi r = 36$ $h = 11$ cm

$R = \frac{24}{\pi}$ $r = \frac{18}{\pi}$

$$\text{Volume fo the frustum} = \frac{1}{3}\pi h\,[R^2+r^2+Rr]$$
$$= \frac{1}{3}\pi(11)\left[\left(\frac{24}{\pi}\right)^2 + \left(\frac{18}{\pi}\right)^2 + \frac{24}{\pi}\left(\frac{18}{\pi}\right)\right]$$
$$= \frac{11}{3}\pi\left[\frac{576+324+432}{\pi^2}\right]$$

$$= \frac{11}{3\pi}[1332] \Rightarrow \frac{\cancel{11} \times 7}{3 \times \cancel{22}_{2}}[1332]$$

$$= \frac{7(1332)}{6}$$

$$= 1554 \text{ cm}^3$$

7. A vessel in the form of an inverted cone it's height is 8 cm and radius of it's bop, which is open, is 5 cm It is filled with water up to the 6min, when leadshots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flours out. Find the number of leadshots dropped in the vessel.

Solution:

h = 8 cm; r = 5 cm

Volume of water in the conical vessel

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8$$

$$= \frac{4400}{21} \text{ cm}^3$$

Now the volume of leadshots

$$= \frac{1}{4} \text{ of [volume of water in the cone]}$$

$$= \frac{1}{4} \times \frac{4400}{21} = \frac{1100}{21} \text{ cm}^3$$

since the radius of the leadshots (r) = 0.5 cm

$$\text{Volume of leadshot} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.5)^3$$

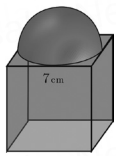
$$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10}$$

$$\text{Number of leadshots} = \frac{\textit{Total volume of leadshots}}{\textit{Volume of 1 leadshot}}$$

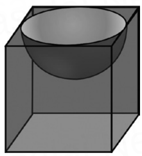
$$= \frac{\frac{1100}{21}}{\frac{4\times22\times5\times5\times5}{3\times7\times100}} = 10$$

For Practice — 5 marks

1. A capsule is in the space of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12mm and diameter of the capsule is 3 mm, how much medicine it can hold?
2. From a solid cylinder whoses height is 2.4cm and the diameter 1.4cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.
3. A solid consisting of a right circular cone of height 12cm and radius 6cm standing on a hemisphere of radius 6cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6cm and height is 18cm.
4. As shown in figure a cubical block of side 7cm is surmounted by a hemisphere. Find the surface area of the solid.



5. A hemispherical section is cut out from on face of a cubical block (fig) such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.



6. Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surrounded by a cone. Each person occupies 4 sq.m of the space on ground and 40cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 cm?

Created Questions — 5 marks

1. The diameter of a road roller of length 120 cm is 84 cm if that takes 500 complete revolutions to level a playground, then find the cost of leveling it at the cost of 75 paise per square meter (Take $\pi = 22/7$).

2. A spherical solid material of radius 18cm is melted and recast into 3 small solid spherical spheres of different sizes. If the radii of two spheres are 2cm and 12cm. Find the radius of third sphere.

3. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19cm and the diameter of the cylinder is 7 cm. Find the total surface area of the solid.

4. A tent is int the shape of a cylinder surmounted by a conical top. If the height of the diameter of the cylindrical part are 2.1m and 4m respectively, and the slant height of the top is 2.8m, find the area of the canvas used for making the tent. Also find the cost of the canvas of the tent at the rate of ₹500/m^2 [Note that the base of the tent will not be covered with canvas]

5. A vessel is in the form of an inverted cone. It's height is 8 cm and the radius of it's top, which is open, is 5cm. It is filled with water up to the brim. When leadshots, each of which is 9 sphere of radius 0.5 cm are dropped into the vessel, one forth of the water flows out. Find the number of lead shots dropped in the vessel.

6. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of icecream. The icecream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with icecream.

7. A hollow spherical shell has an inner radius of 8cm. If the volume of the material is $\frac{1952\pi}{3}$ c.c. Find the thickness of the shell.

8. The radii of the internal and external surface area of hollow spherical shell and 3cm and 5cm respectively. If it's melted and recast into solar cylinder of height $2\frac{2}{3}$ cm. Find the diameter of the cylinder.
9. A medicine capsule is in the shape of cylinder with two hemispheres stuck to each of it's ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find it's surface area.
10. A solid consisting of a right circular cone of height 120cm and radius 60cm standing on a hemisphere of radius 60cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60cm and it's height is 180cm.

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UNIT 8 STATISTICS AND PROBABILITY

Objective Type Questions — 1 mark

1. **Which of the following is not a measure of dispersion?**
   1) Range 2) Standard deviation
   3) Arithmetic mean 4) Variance **Ans:** 3)

2. **The range of the data 8, 8, 8, 8, 8 …. 8 is**
   1) 0 2) 1
   3) 8 4) 3 **Ans:** 1)

3. **The sum of all deviations of the data from its mean is**
   1) always positive 2) always negative
   3) zero 4) non-zero integer **Ans:** 3)

4. **The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is**
   1) 40000 2) 160900
   3) 160000 4) 30000 **Ans:** 2)

5. **Variance of first 20 natural numbers is**
   1) 32.25 2) 44.25
   3) 33.25 4) 30 **Ans:** 3)

6. **The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is**
   1) 3 2) 15
   3) 5 4) 225 **Ans:** 4)

7. **If the standard of x, y, z is p then the standard deviation of, 3x + 5, 3y + 5, 3z + 5 is**
   1) 3p + 5 2) 3p
   3) p + 5 4) 9p + 15 **Ans:** 2)

8. **If the mean and coefficient of variation of a data are 4 and 87.5% than the standard deviation is**
   1) 3.5 2) 3
   3) 4.5 4) 2.5 **Ans:** 1)

9. **Which of the following is incorrect?**
   1) $P(A) > 1$ 2) $0 \le P(A) \le 1$
   3) $P(\varphi) = 0$ 4) $p(A) + P(\overline{A}) = 1$ **Ans:** 1)

10. **The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is**
   1) $\dfrac{q}{P+q+r}$ 2) $\dfrac{P}{P+q+r}$
   3) $\dfrac{P+q}{P+q+r}$ 4) $\dfrac{P+r}{P+q+r}$ **Ans:** 2)

11. **A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is**
   1) $\dfrac{3}{10}$ 2) $\dfrac{7}{10}$
   3) $\dfrac{3}{9}$ 4) $\dfrac{7}{9}$ **Ans:** 2)

12. **The probability of getting job for a person is $\dfrac{x}{3}$. If the probability of not getting the job is $\dfrac{2}{3}$ than value of x is**
   1) 2 2) 1
   3) 3 4) 1.5 **Ans:** 2)

13. **Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\dfrac{1}{9}$, then the number of tickets bought by Kamalam is**
   1) 5 2) 10
   3) 15 4) 20 **Ans:** 3)

14. **If a letter is chosen at random from the English alphabets (a, b, …, z) then the probability that the letter chosen precedes x?**
   1) $\dfrac{12}{13}$ 2) $\dfrac{1}{13}$
   3) $\dfrac{23}{26}$ 4) $\dfrac{3}{26}$ **Ans:** 3)

15. **A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?**

1) $\frac{1}{5}$ 2) $\frac{3}{10}$

3) $\frac{2}{3}$ 4) $\frac{4}{5}$ **Ans:** 4)

Created Questions — 1 mark

1. **The range of first 10 prime numbers is**

1) 9 2) 20

3) 27 4) 5 **Ans:** 3)

2. **The average of first 'n' natural numbers is**

1) $\frac{(n(n+1))}{2}$ 2) $\frac{n}{2}$

3) $\frac{n+1}{2}$ 4) n **Ans:** 3)

3. **Probability of getting 3 heads or 3 tails in tossing a coin 3 times is**

1) $\frac{1}{8}$ 2) $\frac{1}{4}$

3) $\frac{3}{8}$ 4) $\frac{1}{2}$ **Ans:** 2)

4. **A fair die is thrown once. The probability of getting a prime (or) composite number is**

1) 1 2) 0

3) $\frac{5}{6}$ 4) $\frac{1}{6}$ **Ans:** 3)

5. **If the coefficient of range is $\frac{1}{3}$ for the data $x-8, x-5, x-3, x, x+2, x+3, x+5$ and $x+7$ where $x \in Z+$ then the value of "x" is**

1) 32 2) 23

3) 30 4) 1 **Ans:** 2)

6. **Which one of the following is not true?**
   1) The positive square root of the variance is called the standard deviation.
   2) Quartile deviation is one among the measures of dispersion
   3) The probability of an event may be $\frac{\sqrt{5}-1}{2}$
   4) p[A and B] represents P(A∪B) **Ans:** 4)

7. **Which of the following statements is/are true**
   1. Probability of an Event E + Probability of the event not E' is 1.
   2. The Probability of an event that cannot happen is 0. Such an event is called impossible event.
   3. The probability of an event that is certain to happen is 0. such an event is called sure or certain event.
   4. The probability of an event is greater than or equal to 0 and less than or equal to 1.
   1) 1, 2 and 3 2) 1, 2 and 4
   3) 1, 3 and 4 4) all the above **Ans:** 2)

8. **A man and woman appear in an interview for two vacancies in the same post. The probability of man's selection is $\frac{1}{4}$ and that of the women's selection is $\frac{1}{3}$ what is the probability that none of them will be selected**
   1) $\frac{1}{2}$ 2) $\frac{1}{3}$
   3) $\frac{1}{4}$ 4) $\frac{1}{8}$ **Ans:** 1)

9. **One card is drawn from a well shuffled deck of 52 cards, then which of the following is true?**
   1) The probability that the card will be diamond is $\frac{1}{2}$.
   2) The probability of an ace of heart is $\frac{1}{52}$.
   3) The probability not a heart is $\frac{1}{4}$.
   4) The probability of king or queen is $\frac{1}{26}$. **Ans:** 2)

10. Two dice are thrown simultaneously, select the correct option.

1) The probability of not getting doublet is $\frac{5}{6}$.

2) The probability of getting a total of atleast 10 is $\frac{1}{6}$.

3) The probability of not getting a total as a perfect square is $\frac{29}{36}$.

4) All of these. **Ans:** 4)

11. The probability of getting neither an ace nor a king from a pack of 52 cards is

1) $\frac{2}{13}$ 2) $\frac{11}{13}$

3) $\frac{4}{13}$ 4) $\frac{8}{13}$ **Ans:** 2)

Two Marks Questions — 2 marks

1. Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution:

Largest Value, L = 67; Smallest Value, S = 18.

Range, R = L – S = 67 – 18 = 49

$$\text{Coefficient of Range} = \frac{L - S}{L + S}$$

$$\text{Coefficient of Range} = \frac{67 - 18}{67 + 18} = \frac{49}{85} = 0.576$$

2. Find the range of the following distribution.

Age (in Years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution:

Here Largest value, L = 28

Smallest Value, S = 18

Range R = L – S ⇒ R = 28 – 18 = 10 Years.

3. **The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.**

Solution:

Range,	R	= 13.67
Largest Value,	L	= 70.08
Range,	R	= L – S
	13.67	= 70.08 – S
	S	= 70.08 – 13.67 = 56.41

Therefore, the smallest value is 56.41.

4. **Find the range and coefficient of range of following data: 43.5, 13.6, 18.9, 38.4, 61.4, 29.8**

Solution:

43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4, S = 13.6

Range, R = L – S = 61.4 – 13.6= 47.8

Coefficient of Range = $= \frac{L - S}{L + S} = \frac{47.8}{61.4 + 13.6} = \frac{47.8}{75.0} = 0.64$

5. **Find the standard deviation of first 21 natural numbers.**

Solution:

Standard Deviation of first 21 natural numbers.

$$\sigma = \sqrt{\frac{n^2 - 1}{12}} = \sqrt{\frac{(21)^2 - 1}{12}} = \sqrt{\frac{441 - 1}{12}} = \sqrt{\frac{440}{12}} = \sqrt{36.66}$$

$= 6.05$

6. **If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.**

Solution:

Standard deviation of the data, σ = 4.5, each value of the data decreased by 5, the new standard deviation does not change and it is also 4.5.

7. **The standard deviations of 20 observations is $\sqrt{6}$. If each observation is multiplied by 3, find the standard deviation and variance of the resulting observations.**

Solution:

Given: Standard deviations of 20 observations = $\sqrt{6}$

Each observation is multiplied by 3, the new standard deviation = $3\sqrt{6}$

Variance of the resulting observations $= (3\sqrt{6})^2$

$= 9(6) = 54$

8. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution:

The standard deviation of the data is 3.6 and each of the data is divided by 3 then the new standard deviation is also divided by 3.

$\therefore$ The new standard deviation $= \frac{3.6}{3} = 1.2$

The new variance = (Standard Deviation)$^2 = \sigma^2 = (1.2)^2 = 1.44$

9. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the value of coefficient of variation.

Solution:

Co-efficient of variation C.V. $= \frac{\sigma}{\overline{x}} \times 100$

$\sigma = 6.5, \quad \overline{x} = 12.5$

$$CV = \frac{\sigma}{\overline{x}} \times 100 = \frac{6.5}{12.5} \times 100 = \frac{6500}{125} = 52\ \%$$

10. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution:

$\sigma = 1.2, \quad CV = 25.6, \quad \overline{x} = ?$

$$CV = \frac{\sigma}{\overline{x}} \times 100 \Rightarrow \overline{x} = \frac{\sigma}{cv} \times 100 = \frac{1.2}{25.6} \times 100 = \frac{1200}{256}$$

$\overline{x} = 4.7$

11. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution:

$\overline{x} = 15$, C.V. = 48,

$CV = \frac{\sigma}{\overline{x}} \times 100$

$\sigma = \frac{CV \times \overline{x}}{100} = \frac{48 \times 15}{100} = \frac{720}{100} = 7.2$

12. If n = 5, $\overline{x}$ = 6, $\sum x^2$ = 765, then calculate the coefficient of variation.

Solution:

n = 5, $\overline{x}$ = 6, Σx^2 = 765

$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

$= \sqrt{\frac{765}{5} - (6)^2}$

$= \sqrt{153 - 36}$

$= \sqrt{117}$

$= 10.8$

$CV = \frac{\sigma}{\overline{x}} \times 100\ \%$

$= \frac{10.8}{6} \times 100$

$= \frac{1080}{6}$

$= 180\ \%$

13. The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution:

Coefficient of variation $= \frac{\sigma}{\overline{x}} \times 100$

$18.75 = \frac{\sigma}{25.6} \times 100$

$\sigma = \frac{18.75 \times 25.6}{100} = 4.8$

14. If A and B are two mutually exclusive events of a random experiment and P (not A) = 0.45, P(A∪B) = 0.65, then find P(B).

Solution:

Given A and B are mutually exclusive events.

P(not A) = 0.45

P(A∪B) = 0.65

P(A∪B) = P(A) + P(B)

0.65 = 0.55 + P(B)

P(B) = 0.10

15. A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8.

Solution:

$P(A) = 0.5$

$P(A \cap B) = 0.3$

We have $P(A \cup B) \leq 1$

$P(A) + P(B) - P(A \cap B) \leq 1$

$0.5 + P(B) - 0.3 \leq 1$

$P(B) \leq 1 - 0.2$

$P(B) \leq 0.8$

Therefore, probability of B getting selected is atmost 0.8.

16. Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution:

When two coins are tossed together, the sample space is

$S = \{HH, HT, TH, TT\}$; $n(S) = 4$

Let A be the event of getting difference faces on the coins.

$A = \{HT, TH\}$; $n(A) = 2$

Probability of getting different faces on the coins is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

17. What is the probability that a leap year selected at random will contain 53 Saturdays. (Hint $366 = 52 \times 7 + 2$)

Solution:

A leap year has 366 days. So it has 52 full weeks and 2 days.

52 Saturdays must be in 52 full weeks.

S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)}

$n(S) = 7$

Let A be the event of getting 53rd Saturday.

Then A = {Fri-Sat, Sat-Sun} $n(A) = 2$

Probability of getting 53 Saturdays in a leap year is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

18. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution:

S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T}

n(S) = 12

Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H} ; n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

19. A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution:

Total number of possible outcomes n(S) = 5 + 4 = 9

(i) Let A be the event of getting a blue ball.

Number of favourable outcomes for the event A = n(A) = 5

Probability that the ball drawn is blue.

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

(ii) $\overline{A}$ will be the event of not getting a blue ball.

$$\text{So } P(A) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$$

20. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{3}$ then find $P(A \cap B)$.

Solution:

$$P(A) = \frac{2}{3},\ P(B) = \frac{2}{5},\ P(A \cup B) = \frac{1}{3}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{2}{3} + \frac{2}{5} - \frac{1}{3} \Rightarrow \frac{(10+6-5)}{15} \Rightarrow P(A \cap B) = \frac{11}{15}$$

21. From a well- shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Solution:

n(S) = 52

A = Probability of getting a red king cards.

$n(A) = 2, P(A) = \frac{2}{52}$

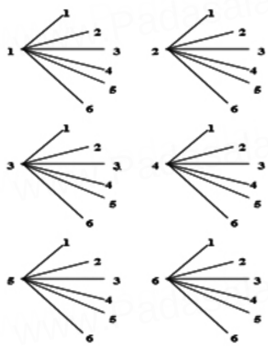
B = Probability of getting a black queen cards.

$n(B) = 2 \quad P(B) = \frac{2}{52}$

$P(AUB) = P(A) + P(B) = \frac{2}{52} + \frac{2}{52} = \frac{4}{52} = \frac{1}{13}$

$\therefore$ The probability of being either a red king or a black queen $= \frac{1}{13}$

22. Write the sample space for selecting two balls from balls from a bag containint 6 balls numbered 1 to 6 using tree diagram (with replacement)



Solution:

Sample Space "S" =

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Created Questions with Solution — 2 marks

1. If "5" coins are tossed, what is the chance that all will show beads?

Solution:

$n(S) = 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

A: Event of getting Head in all the coins = {(H, H, H, H, H)}

n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} \Rightarrow \frac{1}{32}$$

2. A letter is chosen at random from the word "ASSASSINATION". Find the probability that the letter is i) a vowel ii) a consonant

Solution:

Sample space "S" = {A, S, S, A, S, S, I, N, A, T, I, O, N}

n(S) = 13

i) A: Probability of getting a vowel

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{13}$$

ii) B: Probability of getting a consonant

n(B) = 7

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{13}$$

3. The sum of the squares of the deviations from the mean of "6" variables is 54. What is the variance.

Solution:

Given that $\Sigma d^2 = 54$ and n = 6

$$\text{Variance } \sigma^2 = \frac{\sum d^2}{n} = \frac{54}{6} = 9$$

For Practice | **2 marks**

1. Find the range and coefficient of range of the following data. 63, 89, 98, 125, 79, 108, 117, 68.
2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
3. A coin is tossed thrice. What is the probability of getting 2 consecutive tails?

4. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

5. What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards.
6. If P(A) = 0.37, P(B) = 0.42, P(A∩B) = 0.09, then find P(A∪B).
7. A and B are two events such that, P(A) = 0.42, P(B) = 0.48 and P(A∩B) = 0.16. Find (i) P(not A) (ii) P(not B) (iii) P(A or B)
8. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is 3/8 then, find the number of defective bulbs.

Created Questions — 2 marks

1. Cards marked with numbers 1 to 100 are placed in a bag and mixed one card is drawn at random. Find the probability that the number on the card is a prime number less than 30.
2. A card is drawn from a well shuffled pack of 52 cards. Find the probability that the card drawn is neither a red card nor a king card.
3. Three coins are tossed simultaneously. Find the probability of getting (i) atleast 2 tails (ii) Exactly 2 tails.
4. If the mean of numbers 27+x, 31+x, 89+x, 107+x, 156+x is 82 then find the mean of 130+x, 126+x, 68+x, 50+x, 1+x.

Five Marks Questions — 5 marks

1. Find the mean and variance of the first n natural numbers.

Solution:

$$\text{Mean } \bar{x} = \frac{\textit{Sum of all the observations}}{\textit{Number of observations}}$$

$$= \frac{\sum x_i}{n} = \frac{1+2+3+\ldots+n}{n} = \frac{n(n+1)}{2\times n}$$

$$\text{Mean } \bar{x} = \frac{n+1}{2}$$

$$\text{Variance}\,\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \left[\begin{array}{l}\sum x_i^2 = 1^2+2^2+3^2+\ldots+n^2 \\ \left(\sum x_i\right)^2 = (1+2+3+\ldots.+n)^2\end{array}\right]$$

$$= \frac{n(n+1)(2n+1)}{6\times n} - \left[\frac{n(n+1)}{2\times n}\right]^2$$

$$= \frac{(n+1)(2n+1)}{2\times 3} - \left[\frac{(n+1)}{2}\right]^2$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{(n+1)}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right] = \frac{n+1}{2}\left[\frac{n-1}{6}\right]$$

$$\sigma^2 = \frac{n^2-1}{12}$$

$$\text{Variance } \sigma^2 = \frac{n+1}{2}\left[\frac{n-1}{6}\right] = \frac{n^2-1}{12}$$

2. Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13.

Solution:

When we roll two dice, the sample space is given by

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}; n(S) =36

(i) Let A be the event of getting the sum of outcome values equal to 4.

A = {(1,3), (2,2), (3,1)}; n(A) = 3 $P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(ii) Let B be the event of getting the sum of outcome values greater than 10.

Then B = {(5,6), (6,5), (6,6)}; n(B) = 3 $P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$

(iii) Let C be the event of getting the sum of outcomes less that 13. Here all the outcomes have the sum value less than 13. Hence C = S.
n(C) = n(S) = 36. $P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$

3. From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card.

Solution:

n(S) = 52

(i) Let A be the event of getting a red card. n(A) = 26
Probability of getting a red card is $P(A) = \frac{26}{52} = \frac{1}{2}$

(ii) Let B be the event of getting a heart card. n(B) = 13
Probability of getting a heart card is $P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

(iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king. n(C) = 2
Probability of getting a red king card is $P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

(iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen (Q), and King (K). n(D) = 4 × 3 = 12
Probability of getting a face card is $P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

(v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10. n(E) = 4 × 9 = 36
Probability of getting a number card is $P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$

4. If A is an event of a random experiment such that $P(A):P(\overline{A}) = 17:15$ and n(S)=640 then find (i) $P(\overline{A})$ (ii) n(A).

Solution:

$$\frac{P(A)}{P(\overline{A})} = \frac{17}{15}$$

$$\frac{1-P(\overline{A})}{P(\overline{A})} = \frac{17}{15}$$

$15\,[1 - P(A)] = 17\,P(\overline{A}) \Rightarrow 15 - 15\,P(\overline{A}) = 17\,P(\overline{A})$

$15 = 15\,P(\overline{A}) + 17\,P(\overline{A}) \Rightarrow 32\,P(\overline{A}) = 15$

$P(\overline{A}) = \frac{15}{32}$ $\Rightarrow P(A) = 1 - P(\overline{A})$

$= 1 - \frac{15}{32} = \frac{32-15}{32} = \frac{17}{32}$

$P(A) = \frac{n(A)}{n(S)}\ ;\ \frac{17}{32} = \frac{n(A)}{640} \Rightarrow n(A) = \frac{17\times 640}{32}$ $n(A) = 340$

5. Two unbiased dice are rolled once. Find the probability of getting. (i) a doublet (equal numbers on both dice) (ii) the product of the Faces as a prime number (iii) the sum of the Faces as a prime number (iv) the sum as 1.

Solution:

n(S) = 36

(i) A = Probability of getting Doublets (Equal numbers on both dice)

A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

$n(A) = 6\ ;\ P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) B = Probability of getting the product as a prime number

B = {(1,2), (1,3), (1,5), (2,1),(3,1), (5,1)}

$n(B) = 6\ ;\ P(B) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(iii)C = Probability of getting sum as a prime number.

C ={(1,2), (1,4) ,(1,6), (2,3), (2,5), (3,4), (5,6) (1,1) (2,1) (3,2) (4,1), (4,3), (5,2), (6,1), (6,5)}

$n(C) = 15\ ;\ P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

D = Event of getting the sum as 1

$n(D) = 0\ ;\ P(D) = \frac{n(D)}{n(S)} = 0$

6. Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails.

Solution:

Possible Outcomes = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

No. of possible outcomes, n(S) = $2 \times 2 \times 2 = 8$

(i) A = Probability of getting all heads

A = {HHH} n(A) = 1 $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{8}$

(ii) B = Probability of getting atleast one tail.

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

n(B) = 7 $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{7}{8}$

(iii) C = Probability of getting atmost one head.

C = {TTT, TTH, THT, HTT}

n(C) = 4 $\quad P(C) = \dfrac{n(C)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

(iv) D = Probability of getting atmost two tails.

D = {TTH, THT, HTT, HHT, HTH, THH, HHH}

n(D) = 7 $\quad P(D) = \dfrac{n(D)}{n(S)} = \dfrac{7}{8}$

7. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) White (ii) Black or Red (iii) Not white (iv) Neither white nor black

Solution:

S = {5 Red, 6 White, 7 Green, 8 Black} n(S) = 26

i) A – probability of getting White Balls

n(A) = 6 $\quad ; P(A) = \dfrac{6}{26} = \dfrac{3}{13}$

ii) B – Probability of getting Black (or) Red Balls

n(B) = 5 + 8 = 13 $\quad ; P(B) = \dfrac{13}{26} = \dfrac{1}{2}$

iii) C – Probability of not getting White Balls

$n(C) = 20$; $P(C) = \frac{20}{26} = \frac{10}{13}$

iv) D – Probability of getting Neither White nor Black

$n(D) = 12$; $P(D) = \frac{12}{26} = \frac{6}{13}$

8. The king and queen of diamonds, queen and jack of hearts, Jack and king of spades are removed form a deck of 52 playing cards and then well shuffled. Now one cared is drawn at random form the remaining cards. Determine the probability that the card is (i) a claver (ii) a queen of red card (iii) a king of black card.

Solution:

Removed cards: The king and queen of diamonds, queen and jack of hearts, jack and king of spades.

(i.e) remaining number of cards = 52 – 6 = 46

$n(S) = 46$

(i) A is event of getting Claver cards

$n(A) = 13$ $P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$

ii) B is event of getting a Queen of red card.

$n(B) = 0$ $P(B) = \frac{n(B)}{n(S)} = \frac{0}{46} = 0$

iii) C is event of getting King of black card.

$n(C) = 1$ $P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$

9. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution:

$n(S) = 36.$

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then,

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$ $n(A) = 6$ $P(A) = \frac{6}{36}$

B = {(1,3), (2,2), (3,1)} n(B) = 3 P(B) = $\frac{3}{36}$

Therefore, A∩B = {(2,2)}, n(A∩B) = 1

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{36}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

10. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution:

Total number of cards = 52; n(S) = 52.

Let A be the event of getting a king card.

$$n(A) = 4 ;\ P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a heart card

$$n(B) = 13 ;\ P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Let C be the event of getting a red card

$$n(C) = 26 ;\ P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$$

P(A∩B) = P(getting heart king) = $\frac{1}{52}$

P(B∩C) = P(getting red and heart) = $\frac{13}{52}$

P(A∩C) = P(getting red king) = $\frac{2}{52}$

P(A∩B∩C) = P(getting heart, king which is red) = $\frac{1}{52}$

Therefore, required probability is

P (A∪B∪C) = P(A) + P(B) + P(C) – P(A∩B) – P(B∩C) – P(A∩C) + P(A∩B∩C)

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$$

$$= \frac{28}{52} = \frac{7}{13}$$

11. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)} n(S) =36

A = Probability of getting an even number in the first die.

A ={(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

n(A) = 18 ; $P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

B = Probability of getting a total face sum is 8

B = {(2,6), (3,5), (4,4), (5,3), (6,2)}

n(B) = 5; $P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$P(A \cap B) = \frac{}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$$

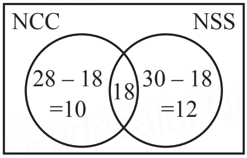
12. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Solution:



Total number of students n(S) = 50.

(i) A : opted only NCC but not NSS

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{50} = \frac{1}{5}$$

(ii) B : opted only NSS but not NCC

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{50} = \frac{6}{25}$$

(iii)C : opted only one

$$P(C) = \frac{n(C)}{n(S)} = \frac{(10+12)}{50} = \frac{22}{50} = \frac{11}{25}$$

13. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution:

n(S) = 8

A = Probability of getting atmost 2 tails

A = {HHH, HHT, HTH, THH, TTH, THT, HTT}

n(A) = 7 $\quad\quad P(A) = \frac{7}{8}$

B = Probability of getting atleast 2 heads

B = {HHT, HTH, THH, HHH}

n(B) = 4 $\quad\quad P(B) = \frac{4}{8}$

$$P(A \cap B) = \frac{4}{8}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$$

14. A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. find (i) Number of black balls (ii) Total number of balls.

Solution:

Number of green balls = n(G) = 6

Number of red balls = n(R) = x

Therefore, number of black balls n(B) = $2x$

Total number of balls n(S) = $6 + x + 2x = 6 + 3x$

It is given that, P(G) = 3 × P(R)

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

$3x = 6$ gives, $x = 2$

(i) Number of black balls = 2 × 2 = 4

(ii)Total Number of balls = 6 + (3 × 2) = 12

15. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution:

S = {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

n(S) = 8

A = Exactly 2 Heads

A = {HHT, HTH, THH}

n(A) = 3 $\qquad$ $P(A) = \frac{3}{8}$

B = Atleaset one tail

B = {HHT, HTH, THH, TTT, TTH, THT, HTT}

n(B) = 7 $\qquad$ $P(B) = \frac{7}{8}$

C = Consecutively 2 heads

C = {HHH, HHT,THH}

n(C) = 3 $\qquad$ $P(C) = \frac{3}{8}$

$P(A\cap B) = \frac{3}{8}$; $\qquad$ $P(B\cap C) = \frac{2}{8}$

$P(A\cap C) = \frac{2}{8}$; $\qquad$ $P(A\cap B\cap C) = \frac{2}{8}$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

$$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$$

$$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8} = \frac{8}{8} = 1$$

16. A wall clock strikes the bell once at 1 o'clock, 2 times 2 o' clock, 3 times at 3 o'clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Solution:

A clock strikes bell at 1 o' clock once

twice at 2 o'clock

thrice at 3 o'clock.....

Number of times it strikes in a particular day

$= 2(1 + 2 + 3 + ... + 12)$

$= 2 \times \left[\frac{n(n+1)}{2}\right] \qquad = 12\,(12 + 1)$

$= 156$ times clock strikes in a particular day.

$\sigma = 2\sqrt{\frac{n^2 - 1}{12}} \qquad = 2 \times \sqrt{\frac{12^2 - 1}{12}}$

$= 2 \times \sqrt{\frac{144 - 1}{12}} \qquad = 2 \times \sqrt{\frac{143}{12}}$

$= 2 \times \sqrt{11.91} \qquad = 2 \times 3.45 = 6.9$

$\therefore \sigma \simeq 6.9$

17. Find the variance and standard deviation of the wages of 9 workers given below:
₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280

Solution:

$$\bar{x} = \frac{\sum x}{n} = \frac{2700}{9} = 300$$

x	$d = x - \bar{x}$	d^2
310	10	100
290	–10	100
320	20	400
280	–20	400
300	0	0
290	–10	100

320	20	400
310	10	100
280	–20	400
Σx=2700	**0**	**2000**

$$\text{Variance} = \frac{\sum d^2}{n} = \frac{2000}{9}$$

$$= 222.22$$

$$\text{Standard Deviation, } \sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{222.22} = 14.91$$

18. The rainfall recorded in various places of five districts in a week are given below.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Solution:

Rain fall xi (mm)	Number of places fi	fi xi	$d = x - \bar{x}$	d_i^2	fid_i^2
45	5	225	-11	121	605
50	13	650	-6	36	468
55	4	220	-1	1	4
60	9	540	4	16	144
65	5	325	9	81	405
70	4	280	14	196	784
	N = 40	**Σ fi xi = 2240**			**Σfid_i^2 = 2410**

$$\text{Mean} = \bar{x} = \frac{\sum \text{fi xi}}{N} = \frac{2240}{40} = 56$$

$$\text{Standard Deviation, } \sigma = \sqrt{\frac{\sum \text{fi}\, d_i^2}{N}} = \sqrt{\frac{2410}{40}}$$

$$= \sqrt{60.25} = 7.76$$

19. In a study about viral fever, the number of people affected in a town were noted as

Age in Years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Solution:

Let the Assumed mean A = 35, C = 10

Age in Years (x)	Number of people affected f_i	Mid Point x_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	d_i^2	$f_i d_i^2$
0 – 10	3	5	–30	–3	–9	9	27
10 – 20	5	15	–20	–2	–10	4	20
20 – 30	16	25	–10	–1	–16	1	16
30 – 40	18	35	0	0	0	0	0
40 – 50	12	45	10	1	12	1	12
50 – 60	7	55	20	2	14	4	28
60 – 70	4	65	30	3	12	9	36
	N = 65				$\Sigma f_i x_i = 3$		$\Sigma f_i d_i^2 = 139$

$$\text{Standard Deviation, } \sigma = C \times \sqrt{\frac{\sum \text{fi}d_i^2}{N} - \left(\frac{\sum \text{fi}d}{N}\right)^2}$$

$$= 10 \times \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2}$$

$$= 10 \times \sqrt{2.138 - (0.046)^2}$$

$$= 10 \times \sqrt{2.138 - 0.002116}$$

$$= 10 \times \sqrt{2.136}$$

$$= 10 \times 1.46$$

$$= 14.6$$

20. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.

Solution:

The pages yet to be completed by them are

60 – 32, 60 – 35, 60 – 37, 60 – 30, 60 – 33, 60 – 36, 60 – 35, 60 – 37

= 28, 25, 23, 30, 27, 24, 25, 23

To find the SD of the data 28, 25, 23, 30, 27, 24, 25, 23.

A = 25

x	$d=x-A$	d^2
23	–2	4
23	–2	4
24	–1	1
25	0	0
25	0	0
27	2	4
28	3	9
30	5	25
	Σd=5	**Σd²= 47**

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{47}{8} - \left(\frac{5}{8}\right)^2} = \sqrt{\frac{47}{8} - \frac{25}{64}}$$

$$= \sqrt{\frac{376-25}{64}} = \sqrt{\frac{351}{64}} = \frac{18.73}{8}$$

$$\therefore \sigma \simeq 2.34$$

21. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Solution:

x	$d = x - \bar{x}$	d^2
24	–6	36
26	–4	16
33	3	9
37	7	49
29	–1	1
31	1	1
180	**Σd = 0**	**112**

Mean $= \bar{x} = \frac{\sum x}{n} = \frac{180}{6} = 30$

Standard Deviation, $\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{112}{6}} = \sqrt{18.66} = 4.32$

Coefficient of Variation $= \frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{4.32}{30} \times 100\% = 14.4\ \%$$

22. The marks scored by the students in a slip test are given below. Find the standard deviation.

x	4	6	8	10	12
f	7	3	5	9	5

Solution:

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
4	7	–4	–28	112
6	3	–2	6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
	N = 29		$\Sigma f_i d_i = 4$	$\Sigma f_i d_i^2 = 240$

Let the Assumed mean, A = 8

$$\text{Standard Deviation, } \sigma = \sqrt{\frac{\sum f_i d_i^2}{n} - \left(\frac{\sum f_i d_i}{n}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$$

$$\sigma = \sqrt{\frac{6944}{29 \times 29}}$$

$$\sigma \simeq 2.87$$

23. The temperature of two cities A and B in a winter season are given below.

Temperature of city A (in degree Celsius)	**18**	**20**	**22**	**24**	**26**
Temperature of city B (in degree Celsius)	**11**	**14**	**15**	**17**	**18**

Find which city is more consistent in temperature changes?

Solution:

Temperature of City ‘A’ 18, 20, 22, 24, 26

$$\bar{x} = \frac{110}{5} = 22$$

x	$d = \frac{x-22}{2}$	d^2
18	–2	4
20	–1	1
22	0	0
24	1	1
26	2	4
	0	**10**

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} \times C$$

$$= \sqrt{\frac{10}{5}} - 0 \times 2 \qquad = 2\sqrt{2}$$

$$\therefore \text{ CV for city A} = \frac{\sigma}{\bar{x}} \times 100 \qquad = \frac{2\sqrt{2}}{22} \times 100$$

$$= \frac{100 \times 1.414}{11} \qquad = 12.85\%$$

Temperature of City 'B' 11, 14, 15, 17, 18

$$\bar{x} = \frac{75}{5} = 15$$

x	$d = x - 15$	d^2
11	–4	16
14	–1	1
15	0	0
17	2	4
18	3	9
	0	**30**

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} \times C$$

$$= \sqrt{\frac{30}{5}} - 0 \qquad = \sqrt{6} \qquad = 2.45$$

$$\therefore \text{CV for city 'B'} = \frac{\sigma}{\bar{x}} \times 100 = \frac{2.45}{15} \times 100 = 16.33$$

∴ CV for city 'A' < ∴ CV for city 'B'

∴ City A is more consistent in temperature changes.

24. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time taken (sec)	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Number of students	6	8	17	10	9

Solution:

C.I	Mid Value (x)	f	d= x–11	d^2	f.d	$f.d^2$
8.5-9.5	9	6	–2	4	–12	24
9.5-10.5	10	8	–1	1	–8	8
10.5-11.5	11	17	0	0	0	0
11.5-12.5	12	10	1	1	10	10
12.5-13.5	13	9	2	4	18	36
		Σf=50			Σfd=8	$\Sigma f d^2$=78

Standard Deviation, σ $= \sqrt{\dfrac{\sum fd^2}{n} - \left(\dfrac{\sum fd}{n}\right)^2}$

$$= \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2} \qquad = \sqrt{\frac{3900-64}{50^2}}$$

$$= \sqrt{\frac{78}{50} - \frac{64}{50^2}} \qquad = \sqrt{\frac{3836}{50^2}} = \frac{61.935}{50}$$

$$= 1.238$$

Standard Deviation, σ $= 1.24$

25. The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	Weight
Mean	155 cm	46.50 cm
Variance	72.25 cm	28.09 cm

Which is more varying than the other?

Solution:

For comparing two data, first we have to find their coefficient of variations

Mean $\overline{x_1}$ = 155 cm, Variance, $\sigma_1^2 = 72.25\ cm^2$

Therefore Standard Deviation, $\sigma_1 = 8.5$

Coefficient of Variation, $C.V_1 = \frac{\sigma_1}{\overline{x_1}} \times 100\%$

$$C.V_1 = \frac{8.5}{155} \times 100\%$$

$$= 5.48\% \text{ (For Heights)}$$

Mean $\overline{x_2}$ = 46.50 kg, Variance, $\sigma_2^2 = 28.09\ kg^2$

Therefore Standard Deviation, $\sigma_2 = 5.3$ kg

Coefficient of Variation, $C.V_2 = \frac{\sigma_2}{\overline{x_2}} \times 100\%$

$$C.V_2 = \frac{5.3}{46.50} \times 100\%$$

$$= 11.40\% \text{ (For Weights)}$$

$C.V_1 = 5.48\%$ and $C.V_2 = 11.40\%$

Since $C.V_2 > C.V_1$, the weight of the students is more varying than the height.

Created Questions with Solution — 5 marks

1. In a class of 50 students, 28 opted for NCC, 28 opted for NSS and 10 opted both NCC and NSS. One of the students is selected at random, Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Solution:

Total Number of students, n(S) = 50

Let A and B be the events of students opted for NCC and NSS respectively.

$n(A) = 28, n(B) = 28, n(A\cap B) = 10$

$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$; $\quad P(B) = \frac{n(B)}{n(S)} = \frac{28}{50}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{10}{50}$

(i) Probability of the students opted for NCC but not NSS

$P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{28}{50} - \frac{10}{50} = \frac{18}{50}$

(ii) Probability of the students opted for NSS but not NCC

$P(\overline{A} \cap B) = P(B) - P(A \cap B) = \frac{28}{50} - \frac{10}{50} = \frac{18}{50}$

(iii)Probability of the students opted for exactly one of them

$= P[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$

$= P(A \cap \overline{B}) + P(\overline{A} \cap B) = \frac{18}{50} + \frac{18}{50} = \frac{36}{50} = \frac{18}{25}$

2. Find the coefficient of variation of 18, 20, 15, 12, 25.

Solution:

x	$d = x - \overline{x}$	d^2
18	0	0
20	2	4
15	–3	9
12	–6	36
25	7	49
$\Sigma x = 90$	0	$\Sigma d^2 = 98$

$\overline{x} = \frac{\sum x}{n} = \frac{90}{5} = 18$

Standard Deviation $\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{98}{5}} = \sqrt{19.6} = 4.42$

Coefficient of Variation $= \frac{\sigma}{\overline{x}} \times 100 \Rightarrow \frac{4.42}{18} \times 100$

$= 24.5\ \%$

3. A card is drawn from a pack of 52 cards. Find the probability of getting a Queen or a diamond or a black card.

Solution:

n(s) = 52

Event A : Selection of a Queen Card

n(A) = 4

$$\therefore \ P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Event B : Selection of a Diamond Card

n(B) = 13

$$\therefore \ P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Event C : Selection of a Black Card

n(C) = 26

$$\therefore \ P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$$

$$P(A \cap B) = \frac{1}{52}, \quad P(B \cap C) = 0$$

$$P(A \cap C) = \frac{2}{52} \quad P(A \cap B \cap C) = 0$$

$$\therefore \ P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{\ }{52} - 0 - \frac{2}{52} + 0$$

$$= \frac{4+13+26-1-2}{52} = \frac{43-3}{52}$$

$$= \frac{40}{52} = \frac{10}{13}$$

4. A pair of dice is thrown once. Find the probability that neither a doublet nor a total of 7 will appear.

Solution:

Sample Space "S" = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) = 36

A: Event of getting a doublet

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

B: Event of getting a total of 7

B = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

n(B) = 6

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Since "A" and "B" are mutually exclusive we've

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{6} + \frac{1}{6} \Rightarrow \frac{2}{6} \Rightarrow \frac{1}{3}$$

$$\text{Required Probability} = 1 - \frac{1}{3} = \frac{2}{3}$$

5. A Jar contains only green, white and yellow marbles. The probabilities of selecting a green marble or a white marble randomly from the jar are $\frac{1}{4}$ and $\frac{1}{3}$ respectively. If the Jar contains 10 yellow marbles, what is the total number of marbles in the jar?

Solution:

Total Number of marbles in the Jar = x (say)

A: Event of selecting a green marble

B: Event of selecting a white marble

C: Event of selecting a yellow marble

$$P(A) = \frac{1}{4}, \; P(B) = \frac{1}{3}, \; P(C) = \frac{10}{x}$$

We've "Total probability" = 1

$$\frac{1}{4} + \frac{1}{3} + \frac{10}{x} = 1$$

$$\frac{3x+4x+120}{12x} = 1$$

$$7x + 120 = 12x$$

$$120 = 12x - 7x$$

$$5x = 120$$

$$x = \frac{120}{5}$$

$$x = 24$$

$\therefore$ Total marbles in the jar = 24

6. If the range and the co-efficient of range of a statistical data are 20 and 0.2 respectively. Then find the greatest and smallest values of the data.

Solution:

Range = L – S = 20

$$\text{Coefficient of Range} = \frac{L-S}{L+S} = 0.2$$

$$\frac{20}{L+S} = 0.2$$

$$0.2(L+S) = 20$$

$$L+S = \frac{20}{0.2} \Rightarrow \frac{20}{0.2} \times \frac{10}{10} \Rightarrow \frac{200}{2}$$

$$L+S = 100$$

$$L + \cancel{S} = 100$$

$$L - \cancel{S} = 20$$

$$2L = 120$$

$$L = \frac{120}{2} \Rightarrow L = 60$$

We've $L+S = 100$

$$60+S = 100$$

$$S = 100 - 60$$

$$S = 40$$

$\therefore$ Largest value = 60

Smallest value = 40

7. **Find the standard deviation of all the composite numbers upto 10.**

Solution:

The composite numbers up to 10 are 4, 6, 8, 9, 10.

x	x^2
4	16
6	36
8	64
9	81
10	100
$\Sigma x = 37$	$\Sigma x^2 = 297$

Standard deviation $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left[\frac{\Sigma x}{n}\right]^2}$

$$= \sqrt{\frac{297}{5} - \left[\frac{37}{5}\right]^2} = \sqrt{\frac{297}{5} - \frac{1369}{25}}$$

$$= \sqrt{\frac{1485 - 1369}{25}} = \sqrt{\frac{116}{25}}$$

$$= \sqrt{4.64} = 2.15$$

	2 . 15
2	4 . 64, 00
	4
41	64
	41
425	2300
	2125
	175

8. **The Number of people affected by corona virus on 16/07/2020 in various cities are as shown below.**

City	Chennai	Chengal pattu	Madurai	Thiru-vallur	Vellore
No. of people affected	113	117	110	125	145

Find the coefficient of variation of the data.

Solution:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{610}{5} = 122$$

x	$d = x - \bar{x}$	d^2
113	–9	81
117	–5	25
110	–12	144
125	3	9
145	23	529
	$\Sigma d = 0$	$\Sigma d^2 = 788$

$$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{788}{5}} = \sqrt{157.6}$$

$$= 12.6$$

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100 = \frac{12.6}{122} \times 100 = \frac{1260}{122} = \frac{630}{61}$$

$$= 10.33\%$$

For Practice **5 marks**

1. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$ $P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$, then find P(A), P(B) and P(C).

2. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1,2,3,…12. What is the probability that it will point to (i) 7 (ii) a Prime number (iii) a composite number.

3. In a game the entry fee is ₹150. The game consists of tossing a coin three times. Dhana bought a ticket for entry if one or two heads show, she gets her entry fee back. If she throws three heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) Gets double entry fee (ii) Just gets her entry fee (iii) Loses the entry fee.

4. A Box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

5. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?

6. For a group of 100 candidates the mean and the standard deviation of their marks where found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 where wrongly entered as 40 and 27. Find the correct mean and standard deviation.

7. The number of televisions sold in each day of week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

8. The mark scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

9. 48 students are asked to write the total number of hours per week. They spent on watching television. Write this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

10. The marks scored by the students in a slip test are given below.

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

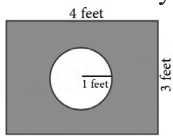
11. Marks of the students in a particular subject of a class are given below. Find the standard deviation.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of Students	8	12	17	14	9	7	4

12. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

13. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) What is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

14. Some boys are playing a game, in which the stone thrown by them, landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



Created Questions — 5 marks

1. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box what is the probability that it will be a black ball? if 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

2. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that is green is 2/3. Find the number of blue marbles in the jar.