

SECOND MIDTERM EXAM - 2022

CLASS : 11
TIME : 1.30 HRS

MATHEMATICS

MARKS : 50

I. Answer all the questions.

10 x 1 = 10

1. If the points $(x, -2)$, $(5, 2)$, $(8, 8)$ are collinear, then x is equal to (a) -3 (b) $1/3$ (c) 1 (d) 3
2. If $A = \begin{pmatrix} \lambda & 1 \\ -1 & \lambda \end{pmatrix}$ then for which value of λ , $A^2 = 0$? (a) 0 (b) ± 1 (c) -1 (d) 1
3. The value of the determinant of $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is
(a) $-2abc$ (b) abc (c) 0 (d) $a^2 + b^2 + c^2$
4. A vector makes equal angle with the positive direction of the coordinate axes. Then each angle is equal to
(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(\frac{2}{3}\right)$ (c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$
5. If $|\vec{a}| = 13$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60^\circ$ then $|\vec{a} * \vec{b}|$ is (a) 15 (b) 35 (c) 45 (d) 25
6. If $\lambda \hat{i} + 2\lambda \hat{j} + 2\lambda \hat{k}$ is a unit Vector, then the value of λ is (a) $1/3$ (b) $1/4$ (c) $1/9$ (d) $1/2$
7. $\lim_{x \rightarrow \infty} \frac{a^x - b^x}{x}$ (a) $\log ab$ (b) $\log(a/b)$ (c) $\log(b/a)$ (d) a/b
8. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$ (a) 1 (b) 0 (c) ∞ (d) $-\infty$
9. $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x \sin 2x}$ (a) 1 (b) $1/2$ (c) 0 (d) 2
10. The value of x , for which the matrix $A = \begin{bmatrix} 4 & 3 \\ -2 & x \end{bmatrix}$ is singular (a) $3/2$ (b) $-3/2$ (c) 3 (d) -2

II. Answer any 5 questions. (Q.No.17 is compulsory)

5 x 2 = 10

11. Find the value of x if $\begin{bmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{bmatrix} = 0$
12. Determine the value of $x+y$ if $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$
13. Find the value of λ for which the vectors \vec{a} and \vec{b} are perpendicular, where $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$
14. Find the magnitude of $\vec{a} * \vec{b}$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

15. Calculate $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 5}{x^3 - 8x + 7}$

16. Find the positive integer n so that $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 27$

17. Show that $\begin{bmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{bmatrix} = 0$

III. Answer any 5 questions. (Q.No.24 is compulsory)

5 x 3 = 15

18. Without expanding, evaluate the following determinants $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{bmatrix}$

19. Show that $\begin{bmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{bmatrix} = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}^2$

20. For any Vector \vec{r} prove that $\vec{r} = (r \cdot \hat{i})\hat{i} + (r \cdot \hat{j})\hat{j} + (r \cdot \hat{k})\hat{k}$

21. If G is the centroid of a triangle ABC , prove that $\vec{GA} + \vec{GB} + \vec{GC} = \vec{0}$

22. Find $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$

23. Prove that $f(x) = 2x^2 + 3x - 5$ is continuous at all points in \mathbb{R}

24. Find the angle between the Vectors $5\hat{i} + 3\hat{j} + 4\hat{k}$ and $6\hat{i} - 8\hat{j} - \hat{k}$

IV. Answer all the questions.

3 x 5 = 15

25. a. Show that $|A| = \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$ by using Factors Theorem. (OR)

b. Prove that the medians of a triangle are concurrent.

26. a. Prove that the points whose position Vectors $2\hat{i} - 4\hat{j} - 3\hat{k}, 4\hat{i} + \hat{j} + 9\hat{k}, 10\hat{i} - \hat{j} + 6\hat{k}$ form a right angled triangle. (OR)

b. Show that $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + \dots + (3n)^2}{(1+2+\dots+5n)(2n+3)} = \frac{9}{25}$

27. a. Find the value of the product $\begin{bmatrix} \log_3^{64} & \log_4^3 \\ \log_3^8 & \log_4^9 \end{bmatrix} * \begin{bmatrix} \log_2^3 & \log_8^3 \\ \log_3^4 & \log_3^4 \end{bmatrix}$ (OR)

b. Show that $\lim_{x \rightarrow 0} x \left[\frac{1}{x} + \frac{2}{x} + \dots + \frac{15}{x} \right] = 120$