Date : 15-11-22 Thanjavur District.

SECOND MID TERM TEST-NOVEMBER 2022

11 TH STANDARD

REG NO. Pattukkottai

TIME:1.30Hrs **MATHEMATICS** **Marks:45**

PART-I

Choose the correct answer 10X1=10

1. The value of x, for which the matrix $A=\begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular is

(1) 9 (2) 8 (3) 7 (4) 6

2. If A is a square matrix, then which of the following is not symmetric?

(1) $A+A^T$ (2) AA^T (3) A^TA (4) $A-A^T$

3. If the square of the matrix $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is the unit matrix of order 2, then α, β and γ should satisfy the relation.

(1) $1+\alpha^2+\beta\gamma=0$ (2) $1-\alpha^2-\beta\gamma=0$ (3) $1-\alpha^2+\beta\gamma=0$ (4) $1+\alpha^2-\beta\gamma=0$

4. If A and B are square matrices of order 2,than $(A+B)^2$ =

(1) $A^2+2AB+B^2$ (2) $A^2+AB+BA+B^2$ (3) $A^2+2BA+B^2$ (4) None of these

5. If AB = C, then matrices A, B, C are

(1) $A_{2X3}, B_{3X2}, C_{2X3}$ (2) $A_{3X2}, B_{2X3}, C_{3X2}$ (3) $A_{3X3}, B_{2X3}, C_{3X3}$ (4) $A_{3X2}, B_{2X3}, C_{3X3}$

6. If the points whose position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear then a is equal to

(1) 6 (2) 3 (3) 5 (4) 8

7. If $|\vec{a}+\vec{b}|=60, |\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$, then $|\vec{a}|$ is

(1) 42 (2) 12 (3) 22 (4) 32

8. If $ABCD$ is a parallelogram, then $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}$ is equal to

(1) $2(\overrightarrow{AB}+\overrightarrow{AD})$ (2) $4\overrightarrow{AC}$ (3) $4\overrightarrow{BD}$ (4) $\vec{0}$

9. Two vertices of a triangle have position vectors $3\hat{i}+4\hat{j}-4\hat{k}$ and $2\hat{i}+3\hat{j}+4\hat{k}$. If the position vector of the centroid is $\hat{i}+2\hat{j}+3\hat{k}$, then the position vector of the third vertex is

(1) $-2\hat{i}-\hat{j}+9\hat{k}$ (2) $-2\hat{i}-\hat{j}-6\hat{k}$ (3) $2\hat{i}-\hat{j}+6\hat{k}$ (4) $-2\hat{i}+\hat{j}+6\hat{k}$

10 Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta=120°$. If $|\vec{a}|=1, |b|=2$, then $[(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})]^2$ is equal to

(1) 225 (2) 275 (3) 325 (4) 300

PART – II

Answer any 3 questions (Question No. 15 is Compulsory) 3×2=6

11. Determine the value of $x+y$ if $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix}=\begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$. $x+y=5$

12. Find the value of the product, $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$. $=6$

13. Find the value or values of m for which $m(\hat{i}+\hat{j}+\hat{k})$ is a unit vector. $m=\pm 1/\sqrt{3}$

14. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=10, |\vec{b}|=15$ and $\vec{a}\cdot\vec{b}=75\sqrt{2}$, find the angle between $\vec{a}$ and $\vec{b}$. $\theta=\pi/4$

15. For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k}$. Example - 8.15 [Book Page No: 12]

PART – III

Answer any 3 questions (Question No. 20 is Compulsory) 3×3=9

16. Construct an $m \times n$ matrix $A = [a_{ij}]$, where a_{ij} is given by $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$ $\frac{1}{2}\begin{bmatrix} 1 & 9 & 25 \\ 0 & 1 & 16 \end{bmatrix}$

17. Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ if $x, y, z \neq 1$. Answer = 0

18. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3, |\vec{c}| = 4$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a}\cdot\vec{b} + 3\vec{b}\cdot\vec{c} + 3\vec{c}\cdot\vec{a}$. = -42

19. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$. [EXERCISE - 8.3 – 9th sum] Book Page No: 80

20. Find the matrix A which satisfies the matrix relation $A\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$. $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

PART – IV

Answer all the questions 4×5=20

21. a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. EXERCISE - 7.2 (4th sum) Book Page No: 29

(or)

b) Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr\,(p+q+r)^3$. Example - 7.25 [Book Page: 32]

22. a) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$. Example - 7.29 [Book Page No: 35]

(or)

b) If $ABCD$ is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$. [EXERCISE - 8.1 – 14th sum] Book Page No: 60

23. a) If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$. EXERCISE - 7.2 – 14th sum Book Page: 30

(or)

b) Prove that The medians of a triangle are concurrent. Theorem - 8.3 [Book Page No: 58]

24. a) Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar. Example - 8.10 [Book Page No: 67]

(or)

b) For any vector $\vec{a}$ prove that $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = 2|\vec{a}|^2$. EXERCISE - 8.4 – 8th sum [Book Page No: 80]

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