Sri Ramakrishna Garden M.Hr.S. School

Time: 3 hrs. XI - Mathematics Total Marks: 90

Part - A

$20 \times 1 = 20$

I Answer All the Questions:

1. If $a_{ij} = \frac{1}{2}(3i - 2j)$ and $A = [a_{ij}]_{2\times 2}$ is

a) $\begin{bmatrix} \frac{1}{2} & 2 \\ -\frac{1}{2} & 1 \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ d) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1 & 2 \end{bmatrix}$

2. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of λ, $A^2 = 0$?

a) 0 b) ± 1 c) -1 d) 1

3. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is

a) $(2, -1)$ b) $(-2, 1)$ c) $(2, 1)$ d) $(-2, -1)$.

4. If the points $(x, -2)$, $(5, 2)$, $(8, 8)$ are collinear, then x is equal to

a) -3 b) $\frac{1}{3}$ c) 1 d) 3.

5. The value of x, for which the matrix $A = \begin{bmatrix} e^{x-2} & e^{7+x} \\ e^{2+x} & e^{2x+3} \end{bmatrix}$ is singular.

a) 9 b) 8 c) 7 d) 6.

6. If the square of the matrix $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is the unit matrix of order 2, then α, β, γ should satisfy the relation.

a) $1 + \alpha^2 + \beta\gamma = 0$ b) $1 - \alpha^2 - \beta\gamma = 0$ c) $1 - \alpha^2 + \beta\gamma = 0$ d) $1 + \alpha^2 - \beta\gamma = 0$

7. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is

a) 6 b) 3 c) 0 d) -6.

8. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by

a) $B = 4A$ b) $B = -4A$ c) $B = -A$ d) $B = 6A$

9. The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is

a) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

10. If $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then $(A+I)(A-I)$ is equal to

a) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -5 & 4 \\ -8 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 4 \\ 8 & 9 \end{bmatrix}$ d) $\begin{bmatrix} -5 & -4 \\ -8 & -9 \end{bmatrix}$

11. The value of $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD}$ is

a) $\vec{AD}$ b) $\vec{CA}$ c) $\vec{0}$ d) $-\vec{AD}$.

12. If $\vec{a} + 2\vec{b}$ and $3\vec{a} + m\vec{b}$ are parallel, then the value of m is

a) 3 b) $\frac{1}{3}$ c) 6 d) $\frac{1}{6}$.

13. If $\vec{BA} = 3\vec{i}+2\vec{j}+\vec{k}$ and the position vector of B is $\vec{i}+3\vec{j}-\vec{k}$, then the position vector A is

a) $4\vec{i}+2\vec{j}+\vec{k}$ b) $4\vec{i}+5\vec{j}$ c) $4\vec{i}$ d) $-4\vec{i}$

14. If ABCD is a parallelogram, then $\vec{AB}+\vec{AD}+\vec{CB}+\vec{CD}$ is equal to

a) $2(\vec{AB}+\vec{AD})$ b) $4\vec{AC}$ c) $4\vec{BD}$ d) $\vec{0}$.

15. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of three collinear points, then which of the following is true?

a) $\vec{a}=\vec{b}+\vec{c}$ b) $2\vec{a}=\vec{b}+\vec{c}$ c) $\vec{b}=\vec{c}+\vec{a}$ d) $4\vec{a}+\vec{b}+\vec{c}=\vec{0}$.

16. If $\lambda\vec{i}+2\lambda\vec{j}+2\lambda\vec{k}$ is a unit vector, then the value of λ is

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{2}$.

17. If $|\vec{a}+\vec{b}|=60$, $|\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$, then $|\vec{a}|$ is

a) 42 b) 12 c) 22 d) 32.

18. If the projection of $5\vec{i}-\vec{j}-3\vec{k}$ on the vector $\vec{i}+3\vec{j}+\lambda\vec{k}$ is same as the projection of $\vec{i}+3\vec{j}+\lambda\vec{k}$ on $5\vec{i}-\vec{j}-3\vec{k}$, then λ is

a) ± 4 b) ± 3 c) ± 5 d) ± 1.

19. If the points whose position vectors $10\vec{i}+3\vec{j}$, $12\vec{i}-5\vec{j}$ and $a\vec{i}+11\vec{j}$ are collinear then a is equal to

a) 6 b) 3 c) 5 d) 8

20. If $\vec{a}=\vec{i}+\vec{j}+\vec{k}$, $\vec{b}=2\vec{i}+x\vec{j}+\vec{k}$, $\vec{c}=\vec{i}-\vec{j}+4\vec{k}$ and $\vec{a}\cdot(\vec{b}\times\vec{c})=70$, then x is equal to

a) 5 b) 7 c) 26 d) 10.

Part-B.

$\boxed{7\times 2=14}$

II Answer Any 7 Questions:- (Q.No: 30 is compulsory).

21. If $A=\begin{bmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{bmatrix}$, compute A^2.

22. Find $|A|$ if $A=\begin{bmatrix} 0 & \sin\alpha & \cos\alpha \\ \sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$.

23. Prove that $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix} = 0$.

24. Find the area of the triangle whose vertices are $(-2,-3)$, $(3,2)$ and $(-1,-8)$.

25. If $A=\begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$ ~~are~~ singular, find the value of 'b'.

26. If G is the centroid of a triangle ABC, Prove that $\vec{GA}+\vec{GB}+\vec{GC}=\vec{0}$.

27. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ Prove that $\vec{a}$ and $\vec{b}$ are Perpendicular

28. Show that the Points $(2,-1,3)$, $(4,3,1)$ and $(3,1,2)$ are collinear.

29. Find the Projection of the vector $\vec{i}+3\vec{j}+7\vec{k}$ on the $2\vec{i}-3\vec{j}+6\vec{k}$.

30. Show that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})=\vec{0}$

Part-C

$\boxed{7\times3=21}$

III Answer Any 7 Questions:- (Q.No: 40 is compulsory)

31. Determine the matrices A and B if they satisfy
$$2A-B+\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}=0 \quad\text{and}\quad A-2B=\begin{bmatrix}3 & 2 & 8\\ -2 & 1 & -7\end{bmatrix}.$$

32. Show that $\begin{vmatrix}b+c & bc & b^2c^2\\ c+a & ca & c^2a^2\\ a+b & ab & a^2b^2\end{vmatrix}=0$.

33. Solve $\begin{vmatrix}x+a & b & c\\ a & x+b & c\\ a & b & x+c\end{vmatrix}=0$.

34. Show that the Points $(a,b+c)$, $(b,c+a)$ and $(c,a+b)$ are collinear

35. Find the value of $\begin{vmatrix}\log_3 64 & \log_4 3\\ \log_3 8 & \log_4 9\end{vmatrix}\times\begin{vmatrix}\log_2 3 & \log_8 3\\ \log_3 4 & \log_3 4\end{vmatrix}$.

36. If D is the midpoint of the side BC of a triangle ABC, Prove that $\vec{AB}+\vec{AC}=2\vec{AD}$.

37. Find a Point whose Position vector has magnitude 5 and Parallel to the vector $4\vec{i}-3\vec{j}+10\vec{k}$.

38. Find the angle between the vectors $2\vec{i}+3\vec{j}-6\vec{k}$ and $6\vec{i}-3\vec{j}+2\vec{k}$.

39. Find the area of a triangle having the Points A(1,0,0), B(0,1,0) and C(0,0,1) as its vertices.

40. For any vector $\vec{a}$ Prove that $|\vec{a}\times\vec{i}|^2+|\vec{a}\times\vec{j}|^2+|\vec{a}\times\vec{k}|^2=2|\vec{a}|^2$.

Part - D.

$7 \times 5 = 35$

IV. Answer Any 7 Questions:-

41. Express the matrix $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.

42. Without expanding the determinants, show that $|B| = 2|A|$. where $B = \begin{bmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{bmatrix}$ and $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$.

43. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.

44. Using Factor Theorem, Prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = (x-1)^2(x+9)$.

45. If $\cos 2\theta = 0$, determine $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$.

46. If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then Prove that $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$.

47. Prove that the Points whose Position vectors $2\vec{i}+4\vec{j}+3\vec{k}$, $4\vec{i}+\vec{j}+9\vec{k}$ and $10\vec{i}-\vec{j}+6\vec{k}$ form a right angled triangle.

48. Show that the vectors $\vec{i}-2\vec{j}+3\vec{k}$, $-2\vec{i}+3\vec{j}-4\vec{k}$, $-\vec{j}+2\vec{k}$ are coplanar.

49. Show that the Points $A(1,1,1)$, $B(1,2,3)$ and $C(2,-1,1)$ are vertices of an isosceles triangle.

50. Find the vectors of magnitude $10\sqrt{3}$ that are Perpendicular to the plane which contains $\vec{i}+2\vec{j}+\vec{k}$ and $\vec{i}+3\vec{j}+4\vec{k}$.

... —— All the Best —— ...