COMMON QUARTERLY EXAMINATION – 2022

Thoothukudi District

Std – XI

Physics

Time : 3.00 Hours **Marks : 70**

Part – I

Note : i) Answer all the questions. Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer. **15 X 1 = 15**

1) d) $[LT^{-3}]$
2) b) torque and energy
3) c) 10^{-8}
4) a) increase
5) b) +z direction
6) c) greater than 1
7) b) need not to be zero
8) c) $\sqrt{5gR}$
9) b) 0.2%
10) c) becomes two times
11) a) pure rotation
12) d) $\frac{L}{\sqrt{2}}$
13) c) m
14) b) g/2
15) b) less than potential energy

Part – II

Note : Answer any six Questions. Question No. 24 is Compulsory. **6 x 2 = 12**

16. Why is the cylinder used in defining kilogram made of platinum –iridium alloy?

The cylinder used in defining kilogram made up of platinum iridium alloy because platinum iridium alloy is least affected by environment and time.

17. Define Radian.

One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

18. An iron ball and a feather are both falling from a height of 10 m . What are the time taken by both to reach the ground?

- Since kinematic equations are independent of mass of the object and hence the time taken by both iron ball and feather to reach the ground are the same.
- This is given by

$$T = \sqrt{\frac{2\,h}{g}} = \sqrt{\frac{2 X 10}{10}} = \sqrt{2} \cong \mathbf{1.414\ s}$$

- Both iron and feather reach the earth with same speed and it is given by

$$v = \sqrt{2\,g\,h} = \sqrt{2 X 10 X 10} = \sqrt{200} = 10\sqrt{2} \cong \mathbf{14.14\ m\ s^{-1}}$$

19. State Newton's second law of motion.

The force acting on an object is equal to the rate of change of its momentum

F= dp/dt

F= m a.

20. Write down the various types of potential energy.

(i) The energy possessed by the body due to gravitational force gives rise to **gravitational potential energy**.

ii)The energy due to spring force and other similar forces give rise to **elastic potential energy**.

(iii) The energy due to electrostatic force on charges gives rise to **electrostatic potential energy.**

21. Define Power

Power is a measure of how fast or slow a work is done.

Power is defined as the rate of work done or energy delivered.

P=W/t

Power is a scalar quantity. Its dimension is $[ML^2\,T^{-3}]$.

The SI unit of power is watt (W)

22. State law of conservation of angular momentum.

The absence of external torque, the angular momentum of the rigid body or system of particles is conserved. If $\tau = 0$ then, dL/ dt = 0; L= constant This is known as law of conservation of angular momentum.

23. Why is the energy of a satellite negative?

Implies that the satellite is bound to the Earth and it cannot escape from the Earth. As h approaches ∞, the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distance.

24. A cyclist while negotiating a circular path with speed $20ms^{-1}$ in found to bend an angle by 30^0 with vertical. What is the radius of the circular path? (Given g= $10ms^{-2}$).

Solution

Speed of the cyclist, $v = 20\ m\ s^{-1}$

Angle of bending with vertical, $\theta = 30°$

Equation for angle of bending, $\tan\theta = \frac{v^2}{rg}$

Rewriting the above equation for radius

$r = \frac{v^2}{\tan\theta\ g}$

Substituting,

$$r = \frac{(20)^2}{(\tan 30°)\times 10} = \frac{20\times 20}{(\tan 30°)\times 10}$$

$$= \frac{400}{\left(\frac{1}{\sqrt{3}}\right)\times 10}$$

$$r = \left(\sqrt{3}\right)\times 40 = 1.732\times 40$$

$$r = 69.28\,m$$

Part – III

Note : Answer any six questions. Question No. 33 is compulsory. 6 x 3 = 18

25. Write the rules for determing significant figures

	Rule	Example
i)	All non-zero digits are significant	1342 has **four** significant figures
ii)	All zeros between two non zero digits are significant	2008 has **four** significant figures
iii)	All zeros to the right of a non-zero digit but to the left of a decimal point are significant.	30700. has **five** significant figures
iv)	a) The number without a decimal point, the terminal or trailing zero(s) are not significant. b) All zeros are significant if they come from a measurement	a) 30700 has **three** significant figures b) 30700 m has **five** significant figures
v)	If the number is less than 1, the zero (s) on the right of the decimal point but to left of the first non zero digit are not significant.	0.00345 has **three** significant figures
vi)	All zeros to the right of a decimal point and to the right of non-zero digit are significant.	40.00 has **four** significant figures and 0.030400 has **five** significant figures
vii)	The number of significant figures does not depend on the system of units used	1.53 cm, 0.0153 m, 0.0000153 km, all have **three** significant figures

26. Discuss the properties of vector product

The vector product of two vectors will have maximum magnitude when sin θ = 1, i.e., θ = 90° i.e., when the vectors G A and G B are orthogonal to each other.

The vector product of two non–zero vectors will be minimum when sinθ = 0, ie or . θ =0°or 180 °i.e., the vector product of two non–zero vectors vanishes, if the vectors are either parallel or anti parallel.

The self–cross product, i.e., product of a vector with itself is the null vector . In physics the null vector is simply denoted as zero4

The self–vector products of unit vectors are thus zero.

27. **Consider a circular road of radius 20 meter banked at an angle of 15 degree. With what speed a car has to move on the turn so that it will have safe turn?**

$V = \sqrt{(rg \tan\theta)}$
$V = \sqrt{20 \times 9.8 \times tan 15^0}$
$V = \sqrt{20 \times 9.8 \times 0.26}$
$V = 7.1\ ms^{-1}$

The safe speed of the car on this road is $7.1\ ms^{-1}$

28.State Lami's theorem

Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

$$|\vec{F}_1| \propto \sin\alpha$$

$$|\vec{F}_2| \propto \sin\beta$$

$$|\vec{F}_3| \propto \sin\gamma$$

Therefore, $$\frac{|\vec{F}_1|}{\sin\alpha} = \frac{|\vec{F}_2|}{\sin\beta} = \frac{|\vec{F}_3|}{\sin\gamma}$$

29.What are conservative forces? Give any two example.

A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.

Conservative force is equal to the negative gradient of the potential energy.

Examples for conservative forces are elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

30. Calculate the energy consumed in electrical units when a 75 watt fan is used for 8 hours daily for one month.

Electrical energy = power × time of usage

= $P \times t$

= 75 watt x24 hour

= 18000 watt hour

= 18 kilowatt hour or 18kWh

31. Deduce the relation between torque and angular momentum

An external torque on a rigid body fixed to an axis produces rate of change of angular momentum in the body about that axis.

$\tau = dL/dt$.

32. State Kepler's laws of planetary motion.

Kepler's first law : All the planets in the solar system orbit the Sun in elliptical orbits with the Sun at one of the foci.

Kepler's 2nd law : The radial vector line joining the Sun to a planet sweeps equal areas in equal intervals of time.

Kepler's 3rd law : The ratio of the square of the time period of planet to the cubic power of semi major axis is constant for all the planets in the solar system.

33. Define gravitational potential

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r. It is a scalar quantity.

$V = -Gm/r$.

Part – IV

Note : Answer all the questions. **5 x 5 = 25**

34. a) Explain the propagation of Errors in addition and multiplication.

A number of measured quantities may be involved in the final calculation of an experiment. Different types of instruments might have been used for taking readings. Then we may have to look at the errors in measuring various quantities, collectively

Error in the sum of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively.

Then, Measured value of $A = A \pm \Delta A$

Measured value of $B = B \pm \Delta B$

Consider the sum, $Z = A + B$

The error ΔZ in Z is then given by

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$= (A + B) \pm (\Delta A + \Delta B)$

$= Z \pm (\Delta A + \Delta B)$ (or)

$\Delta Z = \Delta A + \Delta B$

The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

Error in the product of two quantities

Let ΔA and ΔB be the absolute errors in the two quantities A, and B, respectively. Consider the product $Z = AB$

The error ΔZ in Z is given by

$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= (AB) \pm (A\,\Delta B) \pm (B\,\Delta A) \pm (\Delta A \,.\, \Delta B)$

Dividing L.H.S by Z and R.H.S by AB,

we get, $1 \pm \Delta Z / Z = 1 \pm \Delta B / B \pm \Delta A / A \pm \Delta A / A . \Delta B / B$

As ΔA /A, ΔB / B are both small quantities, their product term ΔA / A. ΔB /B can be neglected. The maximum fractional error in Z is

$\Delta Z / Z = \pm (\Delta A / A + \Delta B / B)$

The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities

34. b) If the value of universal gravitational constant in SI is 6.6×10^{-11} $Nm^2 kg^{-2}$, then find its value in CGS System?

Solution

Let G_{SI} be the gravitational constant in the SI system and G_{cgs} in the cgs system. Then

$$G_{SI} = 6.6 \times 10^{-11} Nm^2 kg^{-2}$$

$$G_{cgs} = ?$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$G_{cgs} = G_{SI} \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$M_1 = 1$ kg $\quad L_1 = 1$ m $\quad T_1 = 1$ s

$M_2 = 1$ g $\quad L_2 = 1$ cm $\quad T_2 = 1$ s

The dimensional formula for G is $M^{-1}L^3T^{-2}$

a = -1 b = 3 and c = -2

$$G_{cgs} = 6.6 \times 10^{-11} \left[\frac{1kg}{1g}\right]^{-1} \left[\frac{1m}{1cm}\right]^3 \left[\frac{1s}{1s}\right]^{-2}$$

$$= 6.6 \times 10^{-11} \left[\frac{1kg}{10^{-3}kg}\right]^{-1} \left[\frac{1m}{10^{-2}m}\right]^3 \left[\frac{1s}{1s}\right]^{-2}$$

$$= 6.6 \times 10^{-11} \times 10^{-3} \text{x } 10^6 \times 1$$

$$G_{cgs} = 6.6 \times 10^{-8} dyne\ cm^2 g^{-2}$$

35. a) kinematic equations of motion for constant acceleration

Equations of motion :

- Consider a particle moves in a straight line.
- Its initial velocity $= u$
  At time "t" its final velocity $= v$
  Acceleration $= a$

1) **Velocity – time relation :**

- Rate of change of velocity is called acceleration. (i.e.)

$$a = \frac{dv}{dt}$$

$$(or) \qquad dv = a\, dt$$

- Integrate on both sides, we get

$$\int_u^v dv = a \int_0^t dt$$

$$[v]_u^v = a\,[t]_0^t$$

$$v - u = a\,(t - 0)$$

$$v - u = at$$

$$\boldsymbol{v = u + at} \quad ------ \; \textbf{(1)}$$

2) **Displacement – time relation** :

- Rate of change of displacement is called velocity. (i.e,)

$$v = \frac{ds}{dt}$$

$$(or) \qquad ds = v\, dt = (\boldsymbol{u + at})\, dt$$

- Integrate on both sides, we get

$$\int_0^s ds = \int_0^t (u + at)\, dt = \int_0^t u\, dt + \int_0^t a\, t\, dt$$

$$[s]_0^s = u\,[t]_0^t + a \left[\frac{t^2}{2}\right]_0^t$$

$$(s - 0) = u\,(t - 0) + \frac{1}{2}\, a\,(t^2 - 0)$$

$$\boldsymbol{s = u\,t + \frac{1}{2}\, a\, t^2} \quad ------- \; \textbf{(2)}$$

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3) **Velocity – displacement relation** :

- We know, acceleration is first derivative of velocity and velocity is first derivative of displacement. Hence,

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}\; v$$

$$a = \frac{1}{2}\frac{d}{ds}(v^2) \qquad \left[\because \frac{d}{ds}(v^2) = 2\,v\,\frac{dv}{ds}\right]$$

$$(or) \qquad ds = \frac{1}{2\,a}\; d\,(v^2)$$

- Integrate on both sides, we get

$$\int_0^s ds = \int_u^v \frac{1}{2\,a}\; d\,(v^2)$$

$$[s]_0^s = \frac{1}{2\,a}\;[v^2]_u^v$$

$$s = \frac{1}{2\,a}\,(v^2 - u^2)$$

$$2\,a\,s = v^2 - u^2$$

$$\boldsymbol{v^2 = u^2 + 2\,a\,s} \quad ------ \; \textbf{(3)}$$

4) **Displacement – velocity relation :**

- From equantion (1) , $v = u + a\,t \quad (or) \quad a\,t = v - u$
- Put this in equation (2), we get

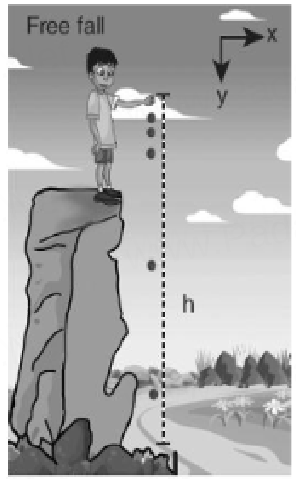
$$s = u\,t + \frac{1}{2}\,a\,t^2 = u\,t + \frac{1}{2}\,(v - u)\,t$$

$$s = u\,t + \frac{v\,t}{2} - \frac{u\,t}{2} = \frac{u\,t}{2} + \frac{v\,t}{2}$$

$$\boldsymbol{s = \frac{(u + v)t}{2}} \quad ------- \; \textbf{(4)}$$

35. b) Derive the equations of motion for a particle (a) falling Vertically (b) projected Vertically.

Case (1): A body falling from a height h



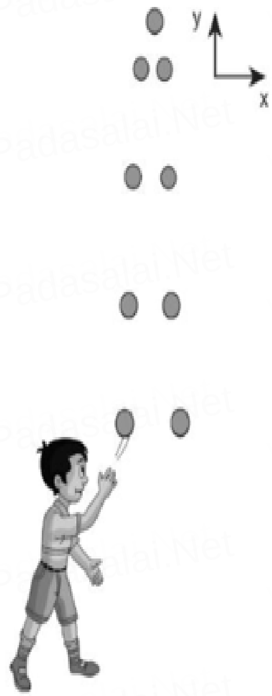
1. Consider an object of mass m falling from a height h.
   Assume there is no air resistance. For convenience, let us choose the downward direction as positive y-axis as shown in the Figure.
2. The object experiences acceleration 'g' due to gravity which is constant near the surface of the Earth. We can use kinematic equations to explain its motion. We have The acceleration $\vec{a} = g\hat{j}$
   By comparing the components, we get
   $A_x = 0$, $a_y = g$, $a_z = 0$ Let us take for simplicity,
   $a_y = a = g$
3. If the particle is thrown with initial velocity 'u' downward which is in negative y axis, then velocity and position at of the particle any time t is given by
   $v = u + gt$ ---------------- (1)
   $y = ut + \frac{1}{2} gt^2$ ------------ (2)
4. The square of the speed of the particle when it is at a distance y from the hill-top, is $v^2 = u^2 + 2gy$ -----------(3)
   Suppose the particle starts from rest. Then $u = 0$
5. Then the velocity v, the position of the particle and v2 at any time t are given by (for a point y from the hill-top)

   $v = gt$ --------- (4)
   $y = \frac{1}{2} gt^2$ ---------(5)
   $v^2 = 2gy$ ----------(6)
6. The time (t =T) taken by the particle to reach the ground (for which y =h), is given by using equation (5),
   $h = y = \frac{1}{2} gT^2$ ---------(7)
   $T = \sqrt{\frac{2h}{g}}$ ---------(8)
   The equation (8) implies that greater the height(*h*), particle takes more time(*T*) to reach the ground. For lesser height(*h*), it takes lesser time to reach the ground.
   The speed of the particle when it reaches the ground (y = h) can be found using equation (6), we get $v_{ground} = \sqrt{2gh}$ ----------- (9)

7. The above equation implies that the body falling from greater height (h) will have higher velocity when it reaches the ground.

The motion of a body falling towards the Earth from a small altitude h << R), purely under the force of gravity is called free fall. (Here R is radius of the Earth)

Case (ii): A body thrown vertically upwards



1. Consider an object of mass m thrown vertically upwards with an initial velocity u. Let us neglect the air friction.
2. In this case we choose the vertical direction as positive y axis as shown in the Figure then the acceleration a = − g (neglect air friction) and g points towards the negative y axis.
3. The kinematic equations for this motion are,

$v = u - gt$ ------------------ (10)

$y = ut - \frac{1}{2} gt^2$ ------------- (11)

The velocity and position of the object at any time t are,

$v^2 = u^2 - 2gy$ ------------(12)

36. a) Explain the motion of blocks connected by string in horizontal motion

i) In this case, mass m_2 is kept on a horizontal table and mass m_1 is hanging through a small pulley. Assume that there is no friction on the surface.

ii) As both the blocks are connected to the un-stretchable string, if m_1 moves with an acceleration *a* downward then m_2 also moves with the same acceleration *a* horizontally.

The forces acting on mass m_2 are

(i) Downward gravitational force (m_2g)

(ii) Upward normal force (N) exerted by the surface

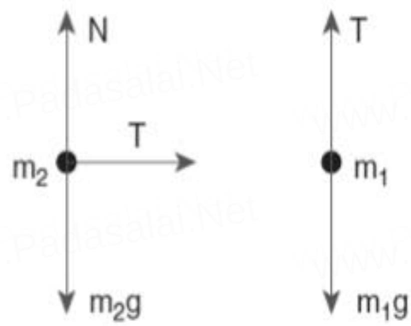
(iii) Horizontal tension (T) exerted by the string

The forces acting on mass m_1 are

(i) Downward gravitational force (m_1g)

(ii) Tension (T) acting upwards

The free body diagrams for both the masses



Applying Newton's second law for m_1

$T\hat{j} - m_1g\hat{j} = m_1a\hat{j}$

By comparing the components on both sides of the above equation,

$\mathbf{T - m_1g = -m_1a}$ ---------- (1)

Applying Newton's second law for m_2

$T\hat{i} - m_2a\hat{i}$, By comparing the components on both sides of above equation,

$\mathbf{T = m_2a}$ ------------(2)

There is no acceleration along y direction for m_2.

$N\hat{j} - m_2g\hat{j} = 0$, By comparing the components on both sides of the above equation

$\mathbf{N - m_2g = 0 ; N = m_2g}$ ---------- (3)

By substituting equation (2) in equation (1), we can find the tension T

$m_2a - m_1g = - m_1a$; $m_2a + m_1a = m_1g$; $\mathbf{a = \frac{m_1}{m_1+m_2}g}$ ---------(4)

Tension in the string can be obtained by substituting equation (4) in equation (2)

$\mathbf{T = \frac{m_1m_2}{m_1+m_2}g}$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.

36. b) Prove the law of conservation of linear momentum.

i) The force on each particle (Newton's second law) can be written as
$\vec{F}_{12} = \frac{d\vec{p}_1}{dt}$ and $\vec{F}_{21} = \frac{d\vec{p}_2}{dt}$

ii) Here $\vec{p}_1$ is the momentum of particle 1which changes due to the force $\vec{F}_{12}$ exerted by particle 2. Further $\vec{p}_2$ is the momentum of particle 2. These changes due to $\vec{F}_{21}$ exerted by particle 1.
$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}$; $\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$; $\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$

iii) It implies that $\vec{p}_1 + \vec{p}_2$ = constant vector (always).

iv) $\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system.

v) If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector.

37.a) State and explain work-energy principle. Mention three example for it

Work - Kinetic energy theorem :

- Work done by the force on the body changes the kinetic energy of the body. This is called work - kinetic energy theorem.

Explanation :

- Let a body of mass "m" rest on a frictionless horizontal surface.
- The work done by the constant force "F" for a displacement "s" is

$$W = F\,s = m\,a\,s \quad --------- \quad (1)$$

- From equation of motion,

$$v^2 = u^2 + 2\,a\,s$$
$$2\,a\,s = v^2 - u^2$$

$$a = \frac{v^2 - u^2}{2\,s} \quad --------- \quad (2)$$

- Put equation (2) in (1),

$$W = m\left[\frac{v^2 - u^2}{2\,s}\right]s$$
$$= \frac{m}{2}\,[v^2 - u^2]$$
$$W = \frac{1}{2}\,m\,v^2 - \frac{1}{2}\,m\,u^2 = (KE)_{final} - (KE)_{initial}$$
$$\boldsymbol{W = \Delta\,KE}$$

- Thus work done by the force, change the kinetic energy of the body. This is known as work - kinetic energy theorem.

Conclusion :

- If work done by the force on the body is positive, then kinetic energy increases.
- If work done by the force on the body is negative, then kinetic energy decreases.
- If work done by the force on the body is zero (no work), there is no change in kinetic energy, which means the body moves with constant speed.

37. b) Derive the expression for Escape speed

1) Consider an object of mass M on the surface of the Earth. **When it is thrown up with an initial speed v_i, the initial total energy of the object is $E_i = \frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E}$** ———— 1

Where M_E, is the mass of the Earth and R_E - the radius of the Earth.

The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

2) When the object reaches a height far away from Earth and hence treated as approaching infinity, **the gravitational potential energy becomes zero [U (∞) = 0] and the kinetic energy becomes zero as well.** Therefore, **the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape.** Otherwise Kinetic energy can be non-zero.

$E_f = 0$, According to the law of energy conservation, $E_i = E_f$ ——— 2

Substituting (1) in (2) we get,

$\frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E} = 0$

$\frac{1}{2} MV_i^2 = \frac{GMM_E}{R_E}$ ———— 3

3) The escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace, V_i with V_e. i.e,

$\frac{1}{2} MV_e^2 = \frac{GMM_E}{R_E}$

$V_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M}$; $V_e^2 = \frac{2GM_E}{R_E}$ ———— 4

Using $g = \frac{GM_E}{R_e}$ ———————————— 5

$V_e^2 = 2gR_E$; $V_e = \sqrt{2gRE}$ ———— 6

From equation (6) the escape speed depends on two factors: **acceleration due to gravity and radius of the Earth**. It is completely independent of the mass of the object.

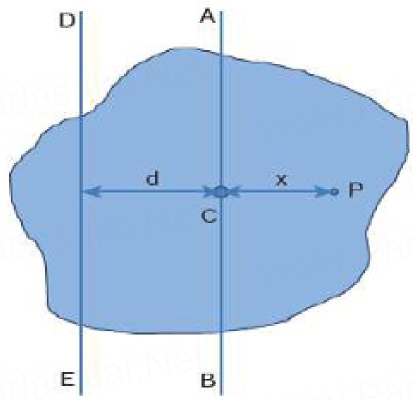
38. a) State and prove parallel axis theorem

Parallel axis theorem :

- Parallel axis theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

Proof :

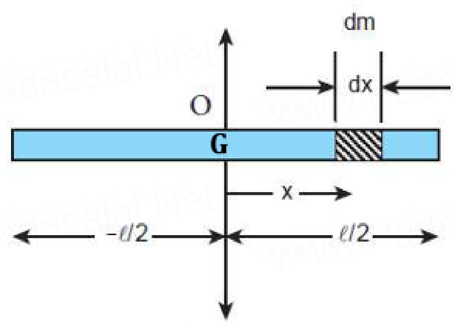
- Let us consider a rigid body whose moment of inertia about an axis AB passing through the centre of mass is I_C.
- DE is another axis parallel to AB at a perpendicular distance d from AB.
- The moment of inertia of the bodv about DE is I.



- Let us consider a point mass m on the body at position x from its centre of mass.
- Here, moment of inertia of the body about the centre of mass (ACB) is
  $I_C = \sum m\ x^2 \qquad ------ \quad (1)$
- The moment of inertia I of the whole body about the parallel axis DE is ,.
  $I = \sum m\ (x + d)^2 \qquad ------ \quad (2)$
- This equation could further be written as,
  $I = \sum m\ (x^2 + d^2 + 2\,x\,d\,)$
  $I = \sum m\,x^2 + \sum m\,d^2 + 2\,d\,\sum m\,x$
- where,
  $\sum m\,x^2 = I_C$ $\quad -- \rightarrow$ moment of inertia of the body about the centre of mass
  $\sum m\,x = 0$ $\quad -- \rightarrow$ because, x can take positive and negative values with respect to the axis AB.
  $\sum m = M$ $\quad -- \rightarrow$ Total mass of the object
- Hence equation (2) becomes,
  $$I = I_C + M\,d^2 + 2\,d\,(0)$$
  $$\boldsymbol{I = I_C + M\,d^2} \qquad --------- \quad (3)$$
- Hence, the parallel axis theorem is proved.

38. b) Derive the expression for moment of inertia of a rod about an axis passing through its centre and perpendicular to the rod.

Moment of Inertia of a Uniform Rod :



- Let us consider a uniform rod. Let us find an expression for moment of inertia of this rod about an axis that passes through the centre of mass and perpendicular to the rod.
- Mass of the rod $= M$
  Length of the rod $= l$
  Position of centre of mass $= G$
  Distance of infinitesimally small mass dm from centre $= x$
  Length of infinitesimally small mass dm $= dx$
  The mass per unit length of the rod $= \lambda$
- The Moment of inertia of the small mass (dm) about the perpendicular axis

$$dI_G = (dm)\, x^2 \qquad --------- (1)$$

- As the mass is uniformly distributed

$$M = \lambda\, l \quad (or) \quad \lambda = \frac{M}{l}$$

- Hence the mass of the infinitesimally small length as,

$$dm = \lambda\, dx = \frac{M}{l}\, dx$$

- Put this in equation (1)

$$dI_G = \left(\frac{M}{l}\, dx\right) x^2 \qquad -------- (2)$$

- Thus the moment of inertia (I_G) of the entire rod about the perpendicular axis passes through its centre can be found by integrating equation (2),

$$I_G = \int_{-l/2}^{l/2} \left(\frac{M}{l}\, dx\right) x^2 = \frac{M}{l} \int_{-l/2}^{l/2} x^2\, dx$$

$$I_G = \frac{M}{l} \left[\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{M}{3\,l}\left[\frac{l^3}{8} - \left(-\frac{l^3}{8}\right)\right] = \frac{M}{24\,l}\,(l^3 + l^3) = \frac{M}{24\,l}\,(2\,l^3)$$

$$\boldsymbol{I_G = \frac{1}{12}\, M\, l^2}$$

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