CENTUM ACHIEVERS' ACADEMY		
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XII STD(MATHS)	**REVISION EXAMINATION NO.2**	**TIME : 2 ½ Hrs** **MARKS : 100**

PART-I

Choose the correct answer from the given four alternatives : **(20× 1 = 20)**

1. The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is

   (1) (4,11) (2) (4, −11) (3) (−4,11) (4) (−4, −11)

2. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

   (1) $y = 0$ (2) $y = \pm\sqrt{3}$ (3) $y = \frac{1}{2}$ (4) $y = \pm 3$

3. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

   (1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

4. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

   (1) 2 (2) 2.5 (3) 3 (4) 3.5

5. The minimum value of the function $|3 - x| + 9$ is

   (1) 0 (2) 3 (3) 6 (4) 9

6. The maximum value of the function $x^2 e^{-2x}, x > 0$ is

   (1) $\frac{1}{e}$ (2) $\frac{1}{2e}$ (3) $\frac{1}{e^2}$ (4) $\frac{4}{e^4}$

7. The curve $y = ax^4 + bx^2$ with $ab > 0$

   (1) has no horizontal tangent (2) is concave up

   (3) is concave down (4) has no points of inflection

8. The point of inflection of the curve $y = (x - 1)^3$ is

   (1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

9. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

   (1) $-4\sqrt{3}$ (2) −4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

10. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

    (1) 0 (2) 1 (3) 2 (4) ∞

11. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

    (1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%

12. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31 ?

    (1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31

13. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

14. If $u(x,y) = x^2 + 3xy + y - 2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(1) -4 (2) -3 (3) -7 (4) 13

15. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(1) $z - x$ (2) $y - z$ (3) $x - z$ (4) $y - x$

16. If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(1) xye^{xy} (2) $(1+xy)e^{xy}$ (3) $(1+y)e^{xy}$ (4) $(1+x)e^{xy}$

17. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(1) 0.4 cu.cm (2) 0.45 cu.cm (3) 2 cu.cm (4) 4.8 cu.cm

18. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

(1) $12x_0 + dx$ (2) $12x_0dx$ (3) $6x_0dx$ (4) $6x_0 + dx$

19. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdx\ m^3$ (2) $0.03x$ m^3 (3) $0.03x^2m^3$ (4) $0.03x^3$ m^3

20. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

(1) $x + \frac{\pi}{2}$ (2) $-x + \frac{\pi}{2}$ (3) $x - \frac{\pi}{2}$ (4) $-x - \frac{\pi}{2}$

PART-II

(i) Answer any EIGHT questions. (8× 2 = 16)

(ii) Qn.No.30 is compulsory

21. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t)^2, 0 \le t \le 10$. What is the rate at which the person forgets the words 2 days after learning?

22. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

23. A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from $-10°$C to $100°$C. Show that the rate of change of temperature at some time t is $5°$C per second.

24. Evaluate : $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

25. Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2 - 6x + 7}{x + 5}$.

26. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

27. Let $V(x,y,z) = xy + yz + zx,\ x,y,z \in \mathbb{R}$. Find the differential dV.

28. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the Absolute error in calculating the area of the circular plate.

29. Let $g(x,y) = \frac{x^2y}{x^4+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

    Show that $\lim_{(x,y)\to(0,0)} g(x,y) = \frac{k}{1+k^2}$ along every parabola $y = kx^2, k \in \mathbb{R} \setminus \{0\}$.

30. Find Δf for the function f for the indicated values of $x, \Delta x$

    $f(x) = x^2 + 2x + 3\ ;\ \ x = -0.5, \Delta x = dx = 0.1$

PART-III

(i) Answer any EIGHT questions. $(8\times 3 = 24)$

(ii) Qn.No.40 is compulsory

31. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?

32. Find the angle between $y = x^2$ and $y = (x-3)^2$.

33. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval, $f(x) = x^3 - 3x + 2, x \in [-2,2]$

34. Find the intervals of monotonicity and hence find the local extrema for the function $f(x) = x^{\frac{2}{3}}$.

35. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

36. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

37. If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$

38. The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6 cm

    (i) Approximately, how much did the tree's diameter grow?

    (ii) What is the percentage increase in area of the tree's cross-section?

39. Use the linear approximation to find approximate values of $(123)^{\frac{2}{3}}$

40. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3,2]$

PART-IV

Answer the following questions. **$(8\times 5=40)$**

41. a) If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second, at what rate the radius of the baloon changes when the radius is 7 cm ? Also compute the rate at which the surface area changes.(OR)

    b) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall,

    (i) how fast is the top of the ladder moving down the wall?

    (ii) at what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

42. a) Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally.(OR)

    b) Write the Maclaurin series expansion for $\tan^{-1}(x); -1\le x\le 1$

43. a) For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.(OR)

    b) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

44. a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.(OR)

    b) Evaluate : . $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$

45. a) Let $f(x,y)=\sin(xy^2)+e^{x^3+5y}$ for all $(x,y)\in\mathbb{R}^2$. Prove that $\frac{\partial f}{\partial x}=\frac{\partial^2 f}{\partial x\,\partial y}$.(OR)

    b) If $w(x,y,z)=x^2+y^2+z^2, x=e^t, y=e^t\sin t$ and $z=e^t\cos t$, find $\frac{dw}{dt}$.

46. a) Show that $f_{xy}=f_{yx}$ where $f(x,y)=\tan^{-1}\left(\frac{x}{y}\right)$ (OR)

    b) Prove that the curves $x=y^2$ and $xy=k$ cut at right angles if $8k^2=1$.

47. a) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? (OR)

    b) If $u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$.

48. a) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$. (OR)

    b) If $v(x,y)=\log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$