TC

Reg.No.

COMMON SECOND MID TERM TEST - 2019

STANDARD - XII

Time : 1.30 hrs

Business Mathematics & Statistics

Marks: 50

Part - I

10 x 1 = 10

Note: 1) Answer all the questions. ii) Each question carrie one mark. iii) Choose the correct

1. The probability function of a random variable is defined as

X = x	-1	-2	0	1	2
P(x)	K	2K	3K	4K	5K

Then K is equal to

a) zero b) 1/4 c) 1/15 d) one

2. Given E(X) = 5 and E(Y) = -2 then E(X-Y) is

a) 3 b) 5 c) 7 d) -2

3. Forty percent of the passengers who fly on a certain route do not check in any lugage. The planes on this route seat 15 passengers. For a full flight, what is the mean of the number of passengers who do not check in any luggage?

a) 6.00 b) 6.45 c) 7.20 d) 7.50

4. Errors in sampling are of

a) two types b) three types c) four types d) five types

5. The stanard error of sample mean is

a) $\frac{\sigma}{\sqrt{2n}}$ b) $\frac{\sigma}{n}$ c) $\frac{\sigma}{\sqrt{n}}$ d) $\frac{\sigma^2}{\sqrt{n}}$

6. A manufacture produces switches and experiences that 2 percent switches are defective. The probability that in a box of 50 switches, there are, atmost two defective is

a) $2.5\ e^{-1}$ b) e^{-1} c) $2e^{-1}$ d) none of the above

7. If X~N (9, 81) the standard normal variate Z will be

a) $Z = \frac{X-81}{9}$ b) $Z = \frac{X-9}{81}$ c) $Z = \frac{X-9}{9}$ d) $Z = \frac{9-X}{9}$

8. If $P(X) = \frac{1}{10}$, x = 10 then E(X) is

a) zero b) 6/8 c) 1 d) -1

9. The distribution function F(x) is equal to

a) P(X = x) b) $P(X \leq x)$ c) $P(x \geq x)$ d) all of these

10. If we have f(x) = 2x, $0 \leq x \leq 1$ then f(x) is a

a) probability distribution b) probability density function
c) distribution function d) continuous random variable

Part - II

Note: i) Answer any four questions. ii) Each question carries 2 marks. iii) Question No. 16 is compulsory.

4 x 2 = 8

11. Constant cumulative distribution function for the given probability distribution

X	0	1	2	3
P(X=x)	0.3	0.2	0.4	0.1

12. State any two properties of distribution function.
13. The following information is the probability distribution of successes.

No. of successes	0	1	2
Probablity	6/11	9/22	1/22

Determine the expected number of sucess.
14. A fair coin is tossed 6 times. Find the probability that exactly 2 heads occurs
15. The mortality rate for a certain disease is 7 in 1000. What is the prbability for just 2 deaths on account of this disease in a group of 400? (Given $e^{(-2.8)} = 0.06$)
16. What is standard error?

Part - III

Note: Answer any four questions. ii) Each question carries 3 marks. iii) Question No.22 is compulsory **4 x 3 = 12**

17. A continuous random variable X has the following p.d.f. $f(x) = ax, 0 \le x \le 1$. Determine the constant' a and also find $P[x \le 1/2]$
18. The time to failure in thousands of hours of an important piece of electronic equipment used in a manufactured DVD player has the density function.

$$f(x) = \begin{cases} 3e^{-3x} & , \ x > 0 \\ 0 & , \ \text{otherwise} \end{cases}$$

Find the expected life of the piece of equipment.
19. A person tosses a coin and is to receive ` 4 for a head and is to pay ` 2 for a tail. Find the expectation and variance of his gains.
20. Consider a random variable X with probability density function $f(x) = \begin{cases} 4x^3 & , \ \text{if } 0 < x < 1 \\ 0 & , \ \text{otherwise} \end{cases}$

Find E(X) and V(X)
21. The mean of binomial distribution is 20 and standard deviation is 4. Find the parameterss of the distribution.
22. A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Find the standard error of the population for an unbiased die.

Part - IV

Note: i) Answer any 4 questions. ii) Each question carrries 5 marks. **4 x 5 = 20**

23. A discrete random variable X has the following probablity function.

Value of X = x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) find K ii) Evaluate $P(X < 6)$, $P(X \ge 6)$ and $P(0<x<5)$
iii) If $P(X \le x) > 1/2$, then find the minimum value of x. (OR)
24. If the prbability that an individual suffers a bad reaction from injection of a given serum is 0.001, determines the prbability that out of 2000 individuals (a) exactly 3 and (2) more than 2 individuals will suffer a bad reaction. $[e^{-2} = 0.1353]$
25. The amount of bread (in hundreads of pounds) x that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function f(x) is given by

$$f(x) = \begin{cases} Ax & , \quad 0 \le x < 10 \\ A(20-x), & 10 \le x < 20 \\ 0 & , \quad otherwise \end{cases}$$

a) Find the value of A

b) What is the probability that the number of pounds of bread that will be sold tomorrow is

i) More than 10 pounds ii) Less than 10 pounds and iii) Between 5 and 15 pound

(OR)

26. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

27. A sample of 125 dry battery cells tested to find the length of life produced the following results with mean 12 and SD 3 hours. Assuming that the data to be normally distributed, what percentage of battery cells are expected to have life.

i) more than 13 hours

ii) less than 5 hours

iii) between 9 and 14 hours

Z	0.333	2.333	1	0.667
Area	0.1293	0.4901	0.3423	0.2486

(OR)

28. In a business venture a man can make a profit of ` 2000 with a probability of 0.4 or have a loss of ` 1000 with a probability of 0.6 what is his expected variance and standard deviation of the profit.

29. A and B play a game in which their chance of winning are in the ratio 3 : 2. Find A's chance of winning atleast three games out of five games played. **(OR)**

30. An insurance company has discovered that only about 0.1 percent of the population is involved in a certain type of accident each year. Is its 10000 policy holders were randomly selected from the population what is the probability that not more than 5 of its clients are involved in such an accident next year. (e^{-10} = .000045)

31. An ambulance service claims that it taken on the average 8.9 minutes to reach its destinction in emergency calls. To check on this claim, the agency which lincenses ambulance services has than timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.6 minutes. What can they conclude at the level of significance.**(OR)**

32. The marsk obtained in a certain exam follow normal distribution with mean 45 and SD 10. If 1300 students appeared at the examination. Calculate the number of students scoring i) less than 35 marks ii) more than 65 marks.

Z	1	2
Area	0.3413	0.4772

XII – Second Mid Term Test - 2019
Business Maths & Statistics Key

Part - I one marks

1. (c) $1/15$
2. (c) 7
3. (a) 6.00
4. (a) two types
5. (c) $\sigma/\sqrt{n}$
6. (a) $2.5e^{-1}$
7. (c) $Z = \frac{x-9}{9}$
8. (c) 1
9. (b) $P(X \le x)$
10. (b) Probability density function

Part - II (Two marks)

(11) $F(x) = P(X \le x)$

$F(0) = P(X \le 0) = P(X=0) = 0.3$

$F(1) = P(X \le 1) = 0.3 + 0.2 = 0.5$

$F(2) = P(X \le 2) = 0.3 + 0.2 + 0.4 = 0.9$

$F(3) = P(X \le 3) = 0.3 + 0.2 + 0.4 + 0.1 = 1.0$

The cumulative distribution function is

X :	0	1	2	3
F(x) :	0.3	0.5	0.9	1

(12) (i) $0 \le F(x) \le 1, -\infty < x < \infty$

(ii) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$

$F(\infty) = \lim_{x \to \infty} F(x) = 1$

(13) $E(x) = \Sigma x\, p(x)$

$= 0 \times \frac{6}{11} + 1 \times \frac{9}{22} + 2 \times \frac{1}{22}$

$= \frac{9}{22} + \frac{2}{22} = \frac{11}{22}$

$E(x) = 0.5$

(14) Let x be r.v. with

$p = \frac{1}{2}$; $q = \frac{1}{2}$; $n = 6$

$P(X=x) = nC_x\, p^x q^{n-x}$

$P(X=2) = 6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2}$

$= \frac{6 \times 5}{2 \times 1} \times \frac{1}{4} \times \frac{1}{16}$

$P(X=2) = \frac{15}{64}$

(15) $n = 400$ $\therefore \lambda = 400 \times \frac{7}{1000}$

$p = \frac{7}{1000}$ $\boxed{\lambda = 2.8}$

$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$P(X=2) = \frac{e^{-2.8} (2.8)^2}{2!}$

$= (0.06)(3.92)$

$P(X=2) = 0.2352$

(16) The standard deviation of the sampling distribution of a statistic is known as its standard error.

Part - III (Three marks)

(17) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^1 ax\,dx = 1$

$a\int_0^1 x\,dx = 1$

$a\left(\frac{x^2}{2}\right)_0^1 = 1$

$\frac{a}{2}[1-0] = 1$

$\boxed{a = 2}$

$\therefore P(X \le \frac{1}{2}) = \int_0^{1/2} 2x\,dx$

$= 2\left(\frac{x^2}{2}\right)_0^{1/2}$

$P(X \le \frac{1}{2}) = \frac{1}{4}$

(18) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$E(x) = 3\int_0^{\infty} x e^{-3x}\,dx$. | $u = x$ $dv = e^{-3x}dx$; $du = dx$ $v = \frac{e^{-3x}}{-3}$

$= 3\left[uv - \int v\,du\right]$

$= 3\left\{\left(x\frac{e^{-3x}}{-3}\right)_0^{\infty} - \int_0^{\infty}\left(\frac{e^{-3x}}{-3}\right)dx\right\}$

$= \int_0^{\infty} e^{-3x}\,dx = \left(\frac{e^{-3x}}{-3}\right)_0^{\infty}$

$E(x) = -\frac{1}{3}\left[e^{-3x}\right]_0^{\infty} = -\frac{1}{3}\left[e^{-\infty} - e^{0}\right]$

$E(x) = \frac{1}{3}$

(19) $x = 4, -2$

$p(\text{head}) = p(x=4) = \frac{1}{2}$

$p(\text{tail}) = p(x=-2) = \frac{1}{2}$

The p.m.f is

x :	4	-2
$p(x)$:	$\frac{1}{2}$	$\frac{1}{2}$

$E(x) = \sum x\, p(x)$

$= 4\times\frac{1}{2} - 2\times\frac{1}{2}$

$= \frac{1}{2}[4-2]$

$= \frac{1}{2}(2)$

Mean $E(x) = 1$

$E(x^2) = \sum x^2 p(x)$

$= 16\times\frac{1}{2} + 4\times\frac{1}{2}$

$= 8+2$

$E(x^2) = 10$

$var(x) = E(x^2) - [E(x)]^2$

$V(x) = 10 - (1)^2$

$= 10-1$

$V(x) = 9$

(20) $f(x) = 4x^3,\ 0<x<1$

$E(x) = \int_0^1 x f(x)\,dx$

$= \int_0^1 4x^4\,dx$

$= 4\left(\frac{x^5}{5}\right)_0^1$

$E(x) = \frac{4}{5}$

$E(x^2) = \int_0^1 4x^5\,dx$

$= 4\left(\frac{x^6}{6}\right)_0^1$

$= \frac{4}{6} = \frac{2}{3}$

$V(x) = E(x^2) - (E(x))^2$

$= \frac{2}{3} - \frac{16}{25} = \frac{2}{75}$

(21) Mean $= np = 20$

$S.D = \sqrt{npq} = 4$

$npq = 16$

$\frac{npq}{np} = \frac{16}{20} \Rightarrow q = \frac{4}{5}$

$p = \frac{1}{5}$

$np = 20$

$\Rightarrow n\left(\frac{1}{5}\right) = 20$

$n = 100$

$\therefore$ parameter of Binomial distribution is $(n, p) = (100, \frac{1}{5})$

(22) $n = 9000$ and $P = \frac{2}{6} = 0.3333$

$p = \frac{3240}{9000} = 0.36 \quad \therefore Q = 1-0.36 =$

$Q = 0.6667$

$SE = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.3333\times 0.6667}{9000}}$

$SE = 0.00496$

Part-IV (five marks)

(23) (i) $\sum p(x) = 1$

$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$10K^2 + 9K = 1$

$10K^2 + 9K - 1 = 0$

-10; $-\frac{1}{10}$, $\frac{10}{10} = 1$; 9

$(10K-1)(K+1) = 0$

$10K-1=0 \mid K+1=0$

$10K = 1 \mid K = -1$

$\boxed{K = \frac{1}{10}}$

(ii) (a) $P(X<6) = 0+K+2K+2K+3K+K^2$

$= K^2 + 8K$

$= \frac{1}{100} + \frac{8}{10} = 0.01 + 0.8$

$P(X<6) = 0.81$

(b) $P(X\geq 6) = P(6) + P(7) = 2K^2 + 7K^2 + K$

$= 9K^2 + K = \frac{9}{100} + \frac{1}{10}$

$= 0.09 + 0.1 = 0.19$

(c) $P(0<x<5) = P(1)+P(2)+P(3)+P(4)$

$= K+2K+2K+3K$

$= 8K$

$= \frac{8}{10}$

$= 0.8$

(iii) $P(X\le x) > \frac{1}{2}$

Put $x=0$, $P(X\le 0) = P(0) = 0 < \frac{1}{2}$

Put $x=1$, $P(X\le 1) = 0+K = K = \frac{1}{10} < \frac{1}{2}$

Put $x=2$, $P(X\le 2) = 0+K+2K = 3K$

$= \frac{3}{10} < \frac{1}{2}$

Put $x=3$, $P(X\le 3) = 0+K+2K+2K$

$= 5K = \frac{5}{10} = \frac{1}{2}$

Put $x=4$, $P(X\le 4) = 0+K+2K+2K+3K$

$= 8K = \frac{8}{10} = \frac{2}{5} < \frac{1}{2}$

$\therefore$ when $x=4$, $P(X\le x) > \frac{1}{2}$.

(OR)

(24) $p = 0.001$

$n = 2000$

$\lambda = np = 2000\times 0.001$

$\lambda = 2$

$\therefore P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

(a) P(exactly 3)

$= P(X=3)$

$= \frac{e^{-2}(2)^3}{3!}$

$= 0.1804$

(b) P(more than 2)

$= P(X>2)$

$= 1-P(X\le 2)$

$= 1-[P(0)+P(1)+P(2)]$

$= 1-\left[e^{-2}\frac{2^0}{0!}+e^{-2}\frac{2^1}{1!}+e^{-2}\frac{2^2}{2!}\right]$

$= 1-e^{-2}[1+2+2]$

$= 1-e^{-2}(5)$

$= 1-0.1353(5)$

$= 1-0.6765$

$= 0.3235$

(25) (a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^{10} Ax\,dx + \int_{10}^{20} A(20-x)\,dx = 1$

$A\left\{\left(\frac{x^2}{2}\right)_0^{10} + \left(20x-\frac{x^2}{2}\right)_{10}^{20}\right\} = 1$

$A[(50-0)+(400-200)-(200-50)] = 1$

$100A = 1$

$A = \frac{1}{100}$

(b)(i) $P(10<X<20) = \int_{10}^{20}\frac{1}{100}(20-x)\,dx$

$= \frac{1}{100}\left[20x-\frac{x^2}{2}\right]_{10}^{20}$

$= \frac{1}{100}[(400-200)-(200-50)]$

$= 0.5$

(ii) $P(0<X<10) = \int_0^{10}\frac{1}{100}x\,dx$

$= \frac{1}{100}\left(\frac{x^2}{2}\right)_0^{10} = \frac{1}{100}(50-0)$

$= 0.5$

(iii) $P(5<X<15) = \int_5^{15} f(x)\,dx$

$= \int_5^{10}\frac{1}{100}x\,dx + \int_{10}^{15}\frac{1}{100}(20-x)\,dx$

$= \frac{1}{100}\left(\frac{x^2}{2}\right)_5^{10} + \frac{1}{100}\left(20x-\frac{x^2}{2}\right)_{10}^{15}$

$= 0.75$

(OR)

26) $n(S) = 36$

Doublets: (1,1), (2,2), (3,3), (4,4) (5,5) (6,6)

$\therefore p = \frac{6}{36} = \frac{1}{6}$; $q = \frac{5}{6}$

$n = 4$

$\therefore P(X=x) = nC_x \, p^x q^{n-x}$

$P(X=2) = 4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2}$

$= 6 \times \frac{1}{36} \times \frac{25}{36}$

$= \frac{25}{216}$

27) $\mu = 12$; $\sigma = 3$

(i) P(more than 13 hours)

$= P(X>13)$

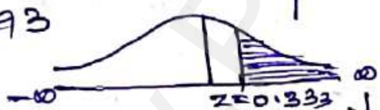
when $x=13$, $Z = \frac{x-\mu}{\sigma}$

$Z = \frac{13-12}{3} = 0.333$

$P(X>13) = P(Z>0.333)$

$= 0.5 - 0.1293$

$= 0.3707$



The expected battery cells life to have more than 13 hrs is $125 \times 0.3707 = 46.34\%$

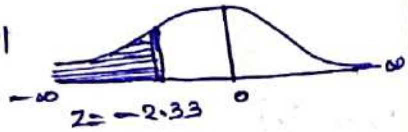
(ii) P(Less than 5 hrs) $= P(X<5)$

when $x=5$, $Z = \frac{5-12}{3} = -2.333$

$P(Z<-2.333)$

$= 0.5 - 0.4901$

$= 0.0099$



The expected battery life to have than 5 hrs is 125×0.0099

$= 1.23\%$

(iii) P(between 9 and 14 hrs)

$= P(9<X<14)$

when $x=9$, $Z = \frac{9-12}{3} = -1$

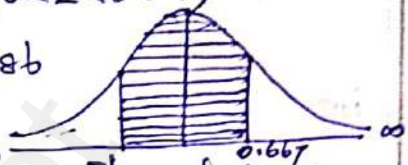
when $x=14$, $Z = \frac{14-12}{3} = 0.667$

$P(9<X<14) = P(-1<Z<0.667)$

$= P(0<Z<1) + P(0<Z<0.667)$

$= 0.3413 + 0.2486$

$= 0.5899$



The expected battery cells life to have between 9 and 14 hrs is

$= 125 \times 0.5899 = 73.73\%$

(OR)

28) Let X be the r.v of getting the amount.

$X = 2000, -1000$

$P(X=2000) = 0.4$

$P(X=-1000) = 0.6$

Mean $= E(X) = \sum x\,p(x)$

$= 2000 \times 0.4 + (-1000) \times 0.6$

$= 800 - 600$

Mean $= E(X) = 200$

$E(X^2) = (2000)^2(0.4) + (-1000)^2(0.6)$

$= 1600000 + 600000$

$E(X^2) = 2200000$

$V(X) = E(X^2) - [E(X)]^2$

$= 2200000 - 40000$

$V(X) = ₹\ 2160000$

$S.D = \sqrt{V(X)} = \sqrt{2160000}$

$\sigma = ₹\ 1469.69$

(29) Let p be the Probability that 'A' wins the game.

$\therefore p = \frac{3}{5}$; $q = \frac{2}{5}$; $n = 5$

$P(X=x) = nC_x \, p^x q^{n-x}$

$P(X=x) = 5C_x \left(\frac{3}{5}\right)^x \left(\frac{2}{5}\right)^{5-x}$

$P(X \geq 3) = P(3) + P(4) + P(5)$

$= 5C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^2 + 5C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^1 + 5C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^0$

$= 0.6826.$

(OR)

(30) $P = \frac{0.1}{100} = \frac{1}{1000}$

$n = 10000$

$\lambda = 10000 \times \frac{1}{1000} = 10$

$P($ not more than $5)$

$= P(X \leq 5)$

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= e^{-10} \left[1 + \frac{10}{1!} + \frac{10^2}{2!} + \frac{10^3}{3!} + \frac{10^4}{4!} + \frac{10^5}{5!} \right]$

$= 0.06651$

(31) $n = 50$; $\bar{x} = 9.3$; $s = 1.6$; $\mu = 8.9$

Null Hypothesis $H_0: \mu = 8.9$

Alternate Hypothesis $H_1: \mu \neq 8.9$

Test Statistic, $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$Z = \frac{9.3 - 8.9}{1.6/\sqrt{50}} = \frac{0.4}{0.2263} = 1.7676$

At 5% level $Z_{\alpha/2} = 1.96$

$1.7676 < 1.96$

$|Z| < Z_{\alpha/2}$

$\therefore H_0$ is accepted.

$\therefore$ we conclude that an ambulance service claims on the average 8.9 min to reach its destination in emergency calls.

(OR)

(32) Let x be the normal variate

Here $\mu = 45$; $\sigma = 10$

(i) $P($less than $35) = P(X < 35)$

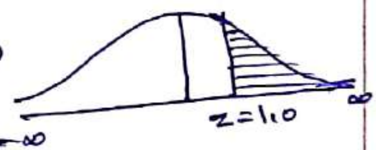
when $x = 35$, $Z = \frac{35-45}{10} = -1$

$P(X < 35) = P(Z < -1) = P(Z > 1)$

$= 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$



Expected no. of students are scoring less than 35 marks

0.1587×1300

$= 206$

(ii) $P($more than $65) = P(X > 65)$

when $x = 65$, $Z = \frac{65-45}{10} = 2$

$P(X > 65) = P(Z > 2)$

$= 0.5 - P(0 < Z < 2.0)$

$= 0.5 - 0.4772 = 0.0228$

$\therefore$ Expected no. of students are scoring more than 65 marks

$0.0228 \times 1300 = 30.$

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