COMMON SECOND MID-TERM TEST - 2019

Reg. No. : ☐☐☐☐☐☐

KA

Standard XII

Marks: 45

Time: 1.30 hours.

MATHEMATICS

Section - A

$10 \times 1 = 10$

I. Choose the correct answer:

1. The volume of a sphere is increasing in volume at the rate of 3π cm³/sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm.

   a) 3 cm / s   b) 2 cm / s   c) 1 cm/s   d) $\frac{1}{2}$ cm/s

2. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0,3]$ is

   a) 1   b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

3. If the curves $y = 2e^x$ and $y = a\,e^{-x}$ intersect orthogonally, then a =

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2   d) $2e^2$

4. Which of the following is in the dterminate form?

   a) $\frac{\infty}{\infty}$ b) $\frac{0}{0}$ c) ∞^0 d) ∞

5. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

   a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5   d) 31

6. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

   a) 0.3x dx m³   b) 0.03 x m³   c) 0.03 x^2 m³   d) 0.03 x^3 m³

7. If $u = y^x$ then $\frac{\partial u}{\partial y}$ at x = y = 1

   a) x y^{x-1} b) y x^{r-1} d) 0   d) 1

8. How many rows are needed for the statement $(p \wedge q) \vee (\sim r \vee \sim s) \wedge (t \wedge \sim x)$

   a) 64   b) 6   c) 2   d) 0

9. The operation * defined by a * b = $\frac{ab}{7}$ is not a binary operation on

   a) Q^* b) Z   c) R   d) C

10. Which one of the following statement has the truth value T ?

    a) sin x is an even fucntion   b) every square matrix is non-singular

    c) the product of complex number and its conjugate is purely imaginary

    d) $\sqrt{5}$ is an irrational number

Section - B

II. Answer any 3 questions: (Ques.No.15 is compulsory) $3 \times 2 = 6$

11. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$

12. A thermometer was taken from a freezer and placed in a boiling water. It took 22 sec for the thermometer to raise from $-10°C$ to $100°C$. Show that the rate of change of temperature at some time t is $5°C$ per second.

13. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.50 m. Find the absolute error in calculating the area of the circular plate.

(2)

14. If $w(x,y,z) = x^2y + y^2z + z^2x$, $x,y,z \in R$, find the differential dw.

15. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find

     $A \vee B$ and $A \wedge B$

Section - C

III. Answer any 3 questions: (Ques.No.20 is compulsory) $3 \times 3 = 9$

16. Find the asymptotes of the curve $f(x) = \dfrac{x^2}{x^2 - 1}$

17. Using L'Hôpital rule, prove that $\underset{x \to 0^+}{\text{Lt}} (1+x)^{\frac{1}{x}} = e$

18. Find the linear approximation for the function $f(x) = x^3 - 5x + 12$ at $x_0 = 2$

19. Show that $f(x,y) = \dfrac{x^2 - y^2}{y^2 + 1}$ is continuous at every $(x,y) \in R^2$

20. State and prove commutatives laws of conjunction and disjunction by uisng Truth table.

Sectoin - D

     $4 \times 5 = 20$

IV. Answer all the questions:

21. a) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

(or)

     b) Find the Maclaurine series expansion of $\tan^{-1}x$; $-1 \leq x \leq 1$

22. a) We have a thin square material of side 12 units and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is which cut produces the box of maximum volume?

(or)

     b) Discuss the curve $f(x) = x^4 - 4x^3$ with respect to concavity and points of inflection.

23. a) If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x} + \sqrt{y}}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\tan u$

(or)

     b) If $w(x,y) = xy + \sin(xy)$ then prove that $\dfrac{\partial^2 w}{\partial y \partial x} + \dfrac{\partial^2 w}{\partial x \partial y}$

24. a) Verify (i) Closure property (ii) Commutative property (iii) Associative proeprty (iv) Existence of identity (v) Existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.

(or)

     b) Prove that $p \to (-q \vee r) = -p \vee (-q \vee r)$ using truth table.

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