2SM **SECOND MID TERM TEST - 2019**

12 - STD **MATHEMATICS**

Time : 1.30 Marks : 50

I Answer all the questions 10 X 1 = 10

1. The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangent is - 0.25?
   a) -8 b) - 4 c) -2 d) 0
2. The minimum value of the function $|3 - x| + 9$ is ....
   a) 0 b) 3 c) 6 d) 9
3. The curve $y = ax^4 + bx^2$, with $ab > 0$
   a) has no horizontal tangent b) is concave up
   c) is concave down d) has no point of inflection
4. The value of the $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right)$, $m \in N$ is ....
   a) 0 b) 1 c) ∞ d) does not exist
5. If $V(x, y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}=$ ....
   a) $e^x + e^y$ b) $\frac{1}{e^x+e^y}$ c) 2 d) 1
6. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is ....
   a) $0.3\,xdx\ m^3$ b) $0.03x\ m^3$ c) $0.03x^2\ m^3$ d) $0.03x^3\ m^3$
7. Linear approximation for $g(x)=\cos x$ at $x=\frac{\pi}{2}$ is ....
   a) $x+\frac{\pi}{2}$ b) $-x+\frac{\pi}{2}$ c) $x-\frac{\pi}{2}$ d) $-x-\frac{\pi}{2}$
8. If $V(x, y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=$ ....
   a) 1 b) 2 c) 3 d) ½
9. The critical number of the function $f(x) = (x - 2)^{2/3}(2x + 1)$ are ....
   a) -1, 2 b) 1 c) 1, - ½ d) 1, 2
10. If $x=r\cos\theta$, $y=r\sin\theta$ then $\frac{\partial r}{\partial x}=$ ....
    a) $\sec\theta$ b) $\sin\theta$ c) $\cos\theta$ d) $\operatorname{cosec}\theta$

II Answer any 5 questions. (Q.No. 17 is compulsory) 5 X 2 = 10

11. Find the angle between the curves $y = x^2$ and $x = y^2$ at the point of intersection (0,0).
12. Compute the value of "C" satisfied by the Rolle's theorem for the function $f(x)=\sqrt{x}-\frac{x}{3}$ $x\in[0, 9]$.

13. Write Maclaurin series expansion of the functions f(x) = sinx.
14. A sphere is made of ice having radius 10cm. Its radius decreases from 10cm to 9.8cm. Find the approximation for change in its surface area.
15. If $u(x, y) = x^2y + 3xy^4$, $x = e^t$ and $y = \sin t$, find $\frac{du}{dt}$ and evaluate it at $t = 0$.
16. Let $u(x, y) = e^{-2y}\cos(2x)$ $\forall (x,y) \in R^2$. Prove that u is a harmonic function in R^2.
17. Prove that the function $f(x) = x^3$ is strictly increasing on $(-\infty,\infty)$.

III Answer any 5 questions. (Q.No. 24 is compulsory) 5 X 3 = 15

18. A store is dropped into a point causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2cm/sec. When the radius is 5cm find the rate of changing of the total area of the disturbed water?
19. Using mean value theorem, prove that $|\sin\alpha - \sin\beta| \le |\alpha - \beta|$, $\alpha,\beta \in R$.
20. Find the intervals of concavity and points of inflextion for the function $f(x) = \frac{1}{2}(e^x - e^{-x})$.
21. Let $g(x,y) = \frac{e^y \sin x}{x}$, $x \ne 0$ and $g(0, 0) = 1$. Show that g is continuous at (0,0).
22. Let $U(x, y, z) = x^2 - xy + 3\sin z$, $x, y, z \in R$. Find the linear approximation for U at (2, -1, 0).
23. If $z(x, y) = x\tan^{-1}(xy)$, $x = t^2$, $y = se^t$, $s,t \in R$ find $\frac{\partial z}{\partial s}$ at $s = t = 1$.
24. Evaluate : $\lim_{x \to 0} |x|^{\sin x}$

IV Answer all the questions. 3 X 5 = 15

25. a) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$, Find the intervals of monotonicity, local extrema, intervals of concavity and point of inflection. *(OR)*
b) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of cone.
26. a) Sketch the curve $y = \frac{x^2 - 3x}{(x-1)}$. *(OR)*
b) Prove that $g(x, y) = x.\log(y/x)$ is homogenous; what is the degree? Verify Euler's theorem for g.
27. a) Let P be a point on the curve $y = x^3$ and suppose that the tangent line at P intersects the curve again at Q. Prove that the slop of Q is four times the slope at P. *(OR)*
b) Verify Clairaut's theorem, for the function $f(x, y) = \log(x^2 + y^2 + 3)$ $\forall (x,y) \in R^2$.